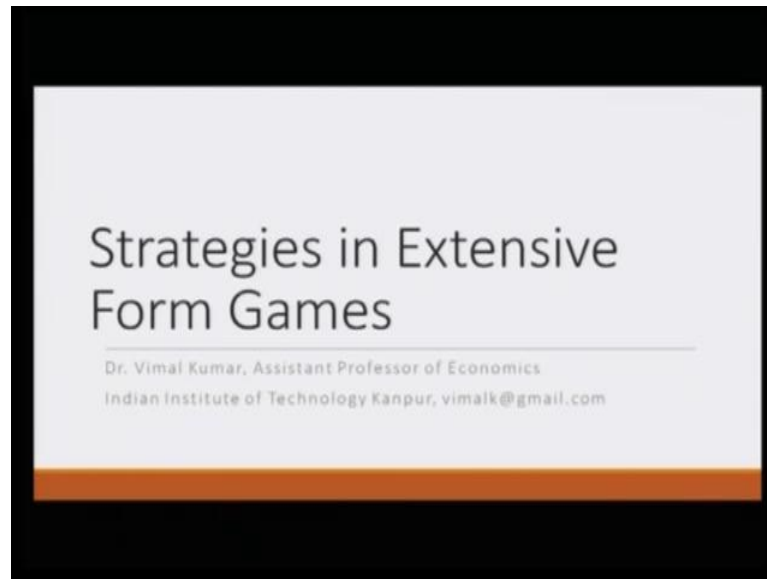**Strategy: An Introduction to Game Theory**
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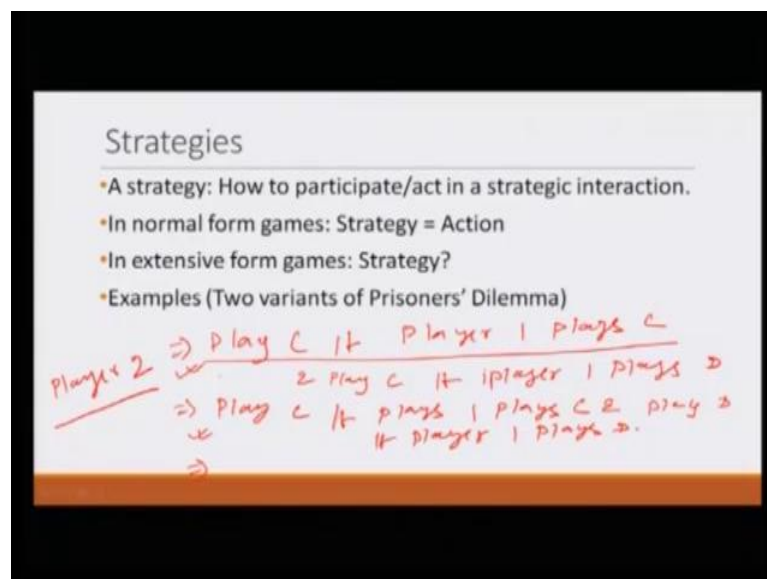
Lecture – 25

Welcome to mooc lectures on Strategy, An Introduction to Game Theory.

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In this module, I am going to talk about to Strategies in Extensive Form Games.

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We have already discussed, what do you mean by a game? Game is basically a module representing a strategic interaction. So, whenever we have a strategic interaction we

think, how to participate, how to act and that is strategic interaction. And so far, what we have seen in normal form game, because players have to move simultaneously and once and for all. So, the way to take, way to move in that game is to have, to take one of the action or one of the strategies available to that player.

But, notice in extensive form game a player can be called upon to move at different instances in that game. So, how should we represent, how should we describe the move made by a player? One way is to simply say, that when game reaches to a particular node these are the action a player can take. But, what would be the nice compact way of representing the all the moves of that player? For example, let us take we have been discussing the prisoners dilemma and again I keep on going back to the same example.

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So, I will again go back to this prisoners dilemma example and here, we have prisoners dilemma from previous module. In this game, it is clear that player 1 has two strategies, P 1 has two strategies C and D, whether we call it is strategy or action, it does not matter in the simultaneous move game. And prisoner 2 again has two strategy, because notice arbitrarily we said that prisoner 1 moves first and then, the prisoner 2 moves, we can say that prisoner 2 moves first, but prisoner 1 is not aware of the actual move.

So, no matter how we do it, both the players will have the same set of strategies C and D, C and D, but in the first game that we discussed. What do we have? Let us look at the actions available to player 1 at node 1, these are C and D two actions, for player 2 at N 2

again C and D and at N 3 for player 2 again C and D. But, what are if we want to represent it in the compact manner, what are the strategies for player 1.

Notice, that whenever we start taking about the strategy, we are trying to describe this, what is going on in the players mind before the strategic interaction has begun. So, we can say player 1 might be thinking, that he would either play C or D, so C or D gives complete plan of his action. So, we can say the C and D are the two strategies for player 1. How about for player 2? Player 2 knows, because games structure is in the common knowledge.
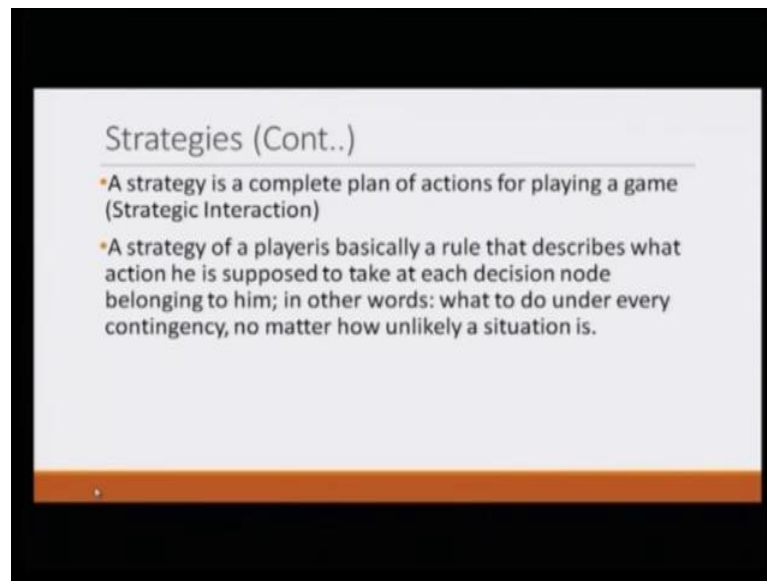
So, player 2 knows that he gets to move only after player 1 has made his move. So, what would be the strategy for player 2? Player 2 has to think about two different eventualities, what if player 1 moves takes actions C or what if player 1 takes action D. So, we can say player 2 can have four different strategies and what are those four different strategies one play C, let us let me go back there ((Refer Time: 03:53)) play C, if player we are writing for player 2, play C if player 1 play C.

Because, notice player 2 knows what player 1 is move, when player 2 has to move player 2 knows what player 1 has done. So, he is planning his action based on what player 1 is going to do. Similarly, we can say and but this is not the complete plan, because he does not know, when we are talking about his strategies, game has not yet begin. So, this is not complete plan in the sense that player 2 may not know, what would happen if player 1 plays D, so we have to describe the complete plan.

So, this is a complete plan, when I add play C if player 1 plays D and I can write it slightly differently, that plan for this strategy for player 2 is to play C irrespective of what player 1 does. But, this is only one of the strategies, what are the other three play C, if player 1 plays C and play D, if player 1 plays D; in other word I can say that player 2 is mimicking the player 1. Here, we have play C irrespective of what player 1 is doing, here mimic the player 1.

The third possibility is I am not going to write it in detail, so play D irrespective of what player 1 has done, that is the third one and the forth one could be to play opposite of what player 1 has done. So, play D if player 1 plays C and plays C if player 1 plays D. So, these are the four different strategies.
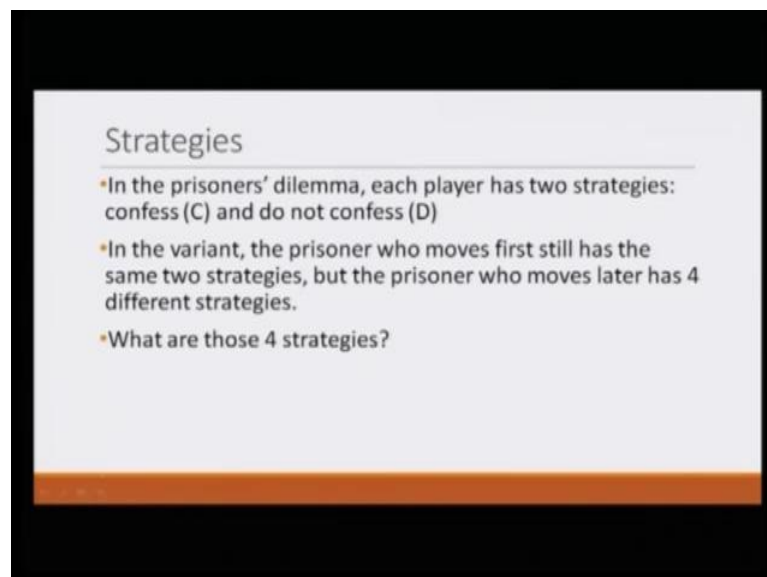
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So, what do we see, that a strategy is a complete plan of action for playing a game. More detail a strategy of a player is, basically a rule that describes, what actions he is supposed to take at each decision node belonging to him. In other words, what to do under every contingence is, no matter how unlikely that situation is.

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So, you can also think of it that let us think yourself for a moment as a player and let us say that you have to participate in a strategic interaction. But, you are busy, see you are sending your friend or your representative to participate in this on your behalf. So, you are giving a complete set of instruction how to behave, as if this is what you would have done and that is what the strategy is.

We have already talked about the prisoners dilemma in simultaneous move game, both the players have two strategies each. But, in the variant player 1 has two strategies and player 2 has four strategies.

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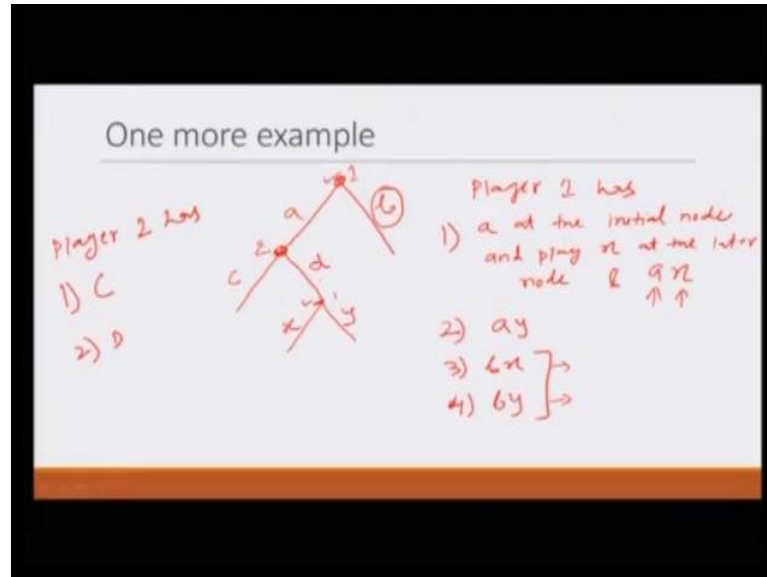
Let us take another example, what we have, player 1 moves and can take one of the action a or b, if it takes action a, then player 2 gets to move. Of course, I am not clearly identifying the node, but I am just being lazy, you should be clear from the context. If player 1 takes action a, then player 2 gets to move and player 2 can either take action c or d, are if player 1 takes action b game ends. Similarly, if player 2 takes action c game ends, but if player 2 takes action d, then player 1 gets to move and he can take one of the action x or y.

In this game, how many strategies do you think player 1 has and how many strategies, do you think player 2 has, pass for a moment and think about it. Player 1 has, let me say player 1 get to move either here and here, at the initial node player 1 has two actions available a and b. And at the other node, where player 1 has an opportunity, may have an opportunity to move again, again player 1 has two different actions available.

So, the different possibilities for player 1 we can say there are four different possibilities for player 1, that play a at the initial node and play x at the later node, if you have an opportunity. In short we can write it as a x, because sequence is clear, that first one represents player 1's actions at the initial node and later one indicates his action at the later node. Similarly other than a x, he can have a y, he can have b x, he can have b y, let

hold down if you worried, I will come back to this that why I am indicating b x and b y as two different strategies.

How about for player 2? Player 2 gets to move only once in this game and when player 2 gets to move, the action available to him are c and d, so the only strategies player 2 has two strategies C or D. Now, let us come back to the point that is bothering you, that why I am saying that player 1 has four strategies. You must be thinking that if player 1 is taking an action b in the beginning, why do we have to think, what would he do at the later node? As he knows that if he takes action b, game would end.

There are different justifications I am going to give you, one justification is very simple ((Refer Time: 10:47)), the definition the way we are giving. The strategy of player, basically is a rule that describes what action he is suppose to take at each decision node belonging to him. So, here this is the node belongs to him and this decision node belongs to him, so strategy has to describe one action at each decision node, but you may say that is very bad, why are you defining in this particular manner.

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So, let us come to this question one other way to think about it is, that what if let me draw the game again here player 1 2 player 1 a b c d x y one may to think about it would be let has sent your friend to participate in this event and you have given instruction to play b. But, b we are here, the way we are describing think it is not for one particulars situation. But, we are coming up with the general rule how to describe again.

So, we do not know what kind of action a and b are it may be possible it is a possibility that a and b are only slightly different. So, by just by mistake or somehow your agent indented to take action b, but he may in some taking action a, then of course, game would proceed in this direction rather than inning without giving an apparently to player b player 2 to play. What would your friend do? What would you represented representative do in the situation.

So, we need to describe, what if had, when if her player 1 has an opportunity to move in this game or when if here player mean we have move in this game, then we need to describe the action that player the you are representative if can take that is one justification for the strategy. So, I given you do two justification, so one is mathematical second is based on the fact. What, if player 1 player 1 makes mistake rather than playing be hints a playing a the third one is also the third one is very important that again, we are doing this for a general case not for the special case.

So, let us write down the payoffs, then it would become clear, so let me write the payoff then things would becomes clearer. Then, these are again random number as I always say just do illustrate a point there is nothing especially now these points. But, again mind you that the rule, that we are come up with, how to describe this strategies for general case, we do not want the that we can describe as strategy if game is if this nature we can describe is strategy of some different way, if give me of other nature.

So, these are the payoffs that will help us illustrate and if we use backwards induction, let me erase this. So, what would happen is we use backward induction here player 1 is taking that action, so if we takes action x we gets two if we takes action y we gets 1. So, of course, 2 is greater than 1, so if game reaches to this point player 1 will take action x. Now, let us come to this node, we are moving in the form the back side that is why it is called backward induction.

Player 2 if player 2 takes action c player 2 gets 5, now we do not have to pay attention to the first number indicates the payoff of the player 1 and the second number indicates the payoff of player 2. So, if player 2 takes action c, then player 2 gets 5, if player 2 takes action d then player 1 gets to move and as using common knowledge we figured out, that player 1 will take action x and player 2 would be aware of that. So, if player 2 takes action d, then he is payoff would be 4, 5 is greater than 4, so player 2 will take action c.

So, coming back to the initial node at initial node player 1 has two actions either take action a or action b. If, at he takes action a then his gets 0, because after that clear to will get action c and if you take action b game hints and he gets 1, so of course, 1 is greater than 0, so game of in this direction. So, what happens player 1 the sides right at the beginning to take action b and game ends player 2 does not get to move.

But, notice how we have add to y player 1 is taking action d because he knows that if we takes action a player 2 will take action c and that will get him 0. What if he believe that player 2 is going to take action d in that case he will take action x and he will get two instead of 1. So, the key point is that player once this is and to play p depends on, what player 2 is going to do, it does not only depend on, what he decides it is remember it is a strategic interaction.

So, his action is b because he believes clear to is going to play c, but mind you let us say go for player 2 y player 2 is playing c, because if player 2 has believe that if he takes action d and player 1 takes action y, then he gets 6, which is greater than 5. But, then y is taking action c, because he is rational and he is basing on the fact that if game moves to the next node if he place d game would move to next node and player 1 would get to play and player 1 would take action x and that would get him 4 not 6.

So, player 2 is deciding to play c, because as player 1 it given a chance player 1 would play x say. So, again if you go back player 1 is playing b, because player 2 is going to play c here and y play 2 is going to play c here, because player 1 is going to play x here. So, overall plan of player 1 maters, if player 1 has decided to play b here and y here, then game would happen best response from player to would happen different.

So, it is very, very important to take the complete plane of action in the account if in for the notes, which will not be reached in the game. Because, in the strategic interaction it is not only important what you do, what you would have done, also place very, very important role. So, in that sense strategy describe gives the complete contingency wherever at clear may have a move in the game in at that point this strategy would describe in action for that place.

Thank you.