Strategy: An Introduction to Game Theory
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Lecture – 21

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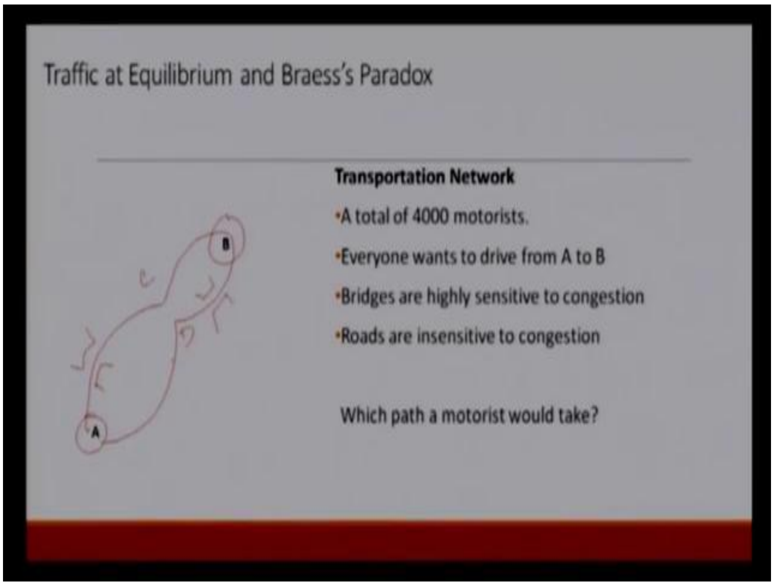

Welcome back to the mooc lectures on Strategy an Introduction to Game theory. In this module, I am going to talk about Braess's paradox. So, we have a simple transportation network. What we have is the total of 4000 motorists. Let us say they work at a place called A and they would like to go to place called B, where there is site. And, that is what I am saying everybody wants to drive from A to B. Now, let us say this is the way to go. From A to B, this is the path. And, let us say we have a bridge here and there is another way to go like this. And in this case, it is the bridge here. What we have that the bridges are sensitive to congestion; while roads are insensitive to congestion. So, we can say this path is made of bridge which is sensitive to congestion. And, so is this part. And, you can give this a name C and D. Now, the question is which path a motorist would take. Will he take? Will he go from A to B via C or will he go to B via D? Let us try to modulate as a game.

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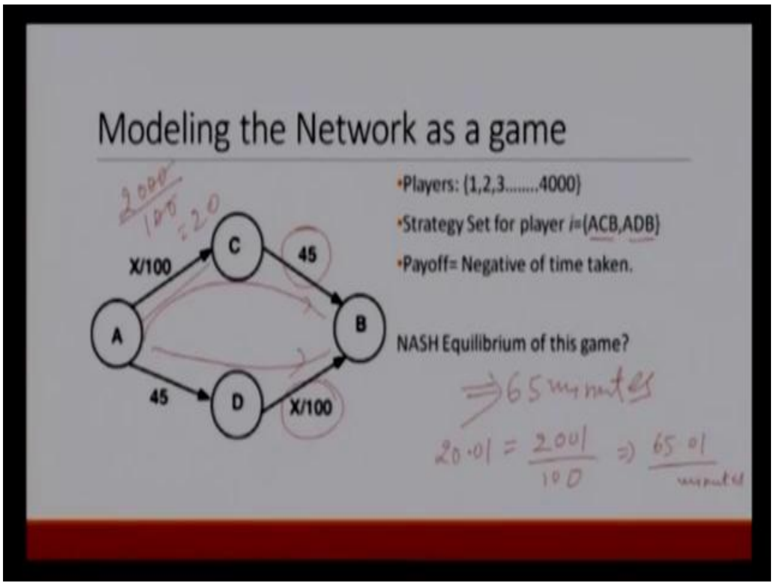

What do we have? As we have 4000 motorist, we can say we have 4000 players. We can name them one, two, three and so on. What are the strategies for player? There are two paths they can take. So, their strategies ACB or ADB. A C B means going to B via C and A D B means going to B via D. And, what is the payoff? Basically, they are interested in decreasing their time to reach B from A. So, one can say negative of time taken is payoff. Higher the number is; worse they are.

Can we get the Nash equilibrium of the game? So, for that we have to give more information. As we have said that bridge is prone to congestion, so we can say the time taken to travel from A to C which has the bridge, depends on the number of cars on that segment. So, let us say that X is the number of cars in that segment. So, if there are 1000 cars, then it takes 1000 divided by 100; 10 minutes. It is a game. These numbers are made up numbers; nothing important. It is just to illustrate a point.

And from C to B, road which is broad enough; so, there is no congestion. So, no matter how many cars are on this part of this road. It always takes 45 minutes from C to B. And, similarly A to D is same as C to B and D to B is same as A to C. So, here again it takes X to 100.

How can we get the Nash equilibrium? This is a finite game as we have finite number of players; 4000 motorists. And, all the players have only two strategies. But the problem is, again it would be difficult to draw table as we have 4000 players. In two players case we can easily draw the table. So, what we can do? How can we model? We can think that we can propose a particular strategy profile as Nash equilibrium and see whether one of the players would be interested in changing his or her path. If there is one player who would be interested in changing, then their strategy profile would not be the Nash equilibrium.

So, if we; let us start with this symmetry argument. Let us say the 2000 players; a 2000 motorists would take this path and 2000 motorists would take this path. So, we have a strategy profile in which without any loss of generality we say that from motorist number one to 2000 is going to take path A C B and from 2001 to 4000 is going to take ADB.

Now, let us calculate the time taken by a motorist. Since there are 2000 motorists on this path, so 2000 is divided by 100. So, it takes 20 minutes from A to C and again 45 minutes from C to B. So, everyone on A C B path takes 65 minutes. How about motorists travelling on A D B? Again, there are 2000 motorists on AD. So, sorry, again 2000 motorists are on D B. So, everyone would take 20 minutes from D to B and A to D it is given as 45 minutes. So, again everyone would take 65 minutes.

Now to say it is a Nash equilibrium, we have to check whether one of the motorist have incentive to deviate or not. Let us say one motorist changes his path from A C B to A D B. What happens then? Now, he would take 45 minutes from A to D. And now, we have 2001 motors on D B; not just 2000. It means D B would take now 20.01. As it is equal to 2001 divided by 100. So, total time would increase from 65 minutes to 65.01 minute. So, the payoff would decrease as the payoff is negative of time taken. So, none of the motorists have been incentive. None of the motorists who are taking A C B had incentive to change their paths to A D B. And, same is true for the other motorists also; who are taking A D B. So, this is a Nash equilibrium.

In fact, this strategy profile is not the only Nash equilibrium. In all the strategy profile in which 2000 motorists are taking A C B and remaining 2000s are taking A D B would be a Nash equilibrium. So, outcome would remain the same. The 2000 motorists would take A C B and 2000 motorists would take A D B. Now, one can say the rational planner may think that this segment C to D is very near, what if we build a road? Will it help the traffic because it is, you know, building a new road; the notion is building a new road would never make the traffic worse off. So, let us see what happens. Let us say C and D, they are very near to each other. So, roads takes negligible or zero amount of time.

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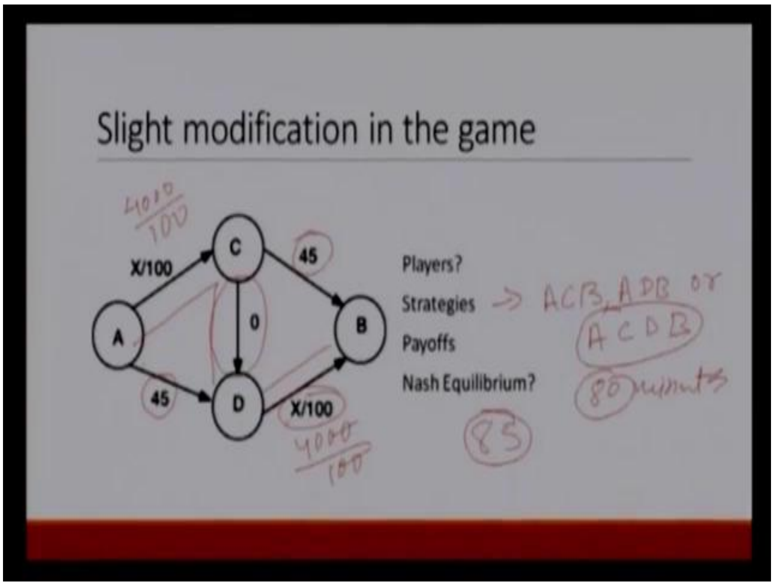

And here, I show schematically. Here, we have C to D. And, just to control traffic it is announced that C to D people can travel only from C to D, not from D to C. It does not matter if you change the direction. It would remain the same. Now, we have slightly different game. And, what is this game? Again, we have the same players. So, players do not change. We have 4000 players. Strategy; for each player now we have a different strategies set. In the earlier case, they could either take path A C B or they could take A D B. But, now they have one more option that is A C D B. So, there will be three strategies; A C B, A D B or A C D B. Payoff would remain the same. It is negative of the time taken. What would be the Nash equilibrium of this game? Can you think of what would be the Nash equilibrium of the game? Now, again we have to propose a solution and we would check whether that is an equilibrium or not. L et us propose that all the motorists take A C D B and none of them take A C B or A D B. what happens now? Time taken would be here, 4000 divided by 100. And, again here 4000 divided by 100. So, they all will take 80 minutes. Now, we have to check whether one of them have any incentive to deviate or not.

Notice, basically C and D are the same point because time taken to travel from C to D is 0. So once someone reaches C, if he takes C to B, he spends 45 minutes. And even if everyone is driving on this road, time taken is only 40 minutes.

So after reaching C, it is not a good idea for any of the motorists to take C B. They will all come to this. So, they would not. None of them would deviate to A C B. But, how about A D B? Again, in the beginning they have two options. Either take A to C or A to D. And C and D, they are negligible. Sorry, not, from D to C is not possible. It is not possible to go. So, they will take 45 minutes. And again even if all of them are taking this path, then 4000 divided by 100; 40 minutes. So, 85 minutes. So, we see none of the players have any incentive to deviate and all of them would drive on A C D B. And, all of them would take 80 minutes to reach from A to B.

You may be thinking that what about any other Nash equilibrium. My friend, in this game there is no other Nash equilibrium. This is the only possibility. So, all of them would take now 80 minutes to reach. In the earlier case when C and D were not connected, they were taking 65 minutes. So, building this road makes everyone worse off and that is why we call it Braess's paradox. Braess was the first one to come up with it. Braess's paradox is not exactly a paradox. It is a counterintuitive result in which like prisoner's dilemma, collective good gets sacrificed because of self's interest.

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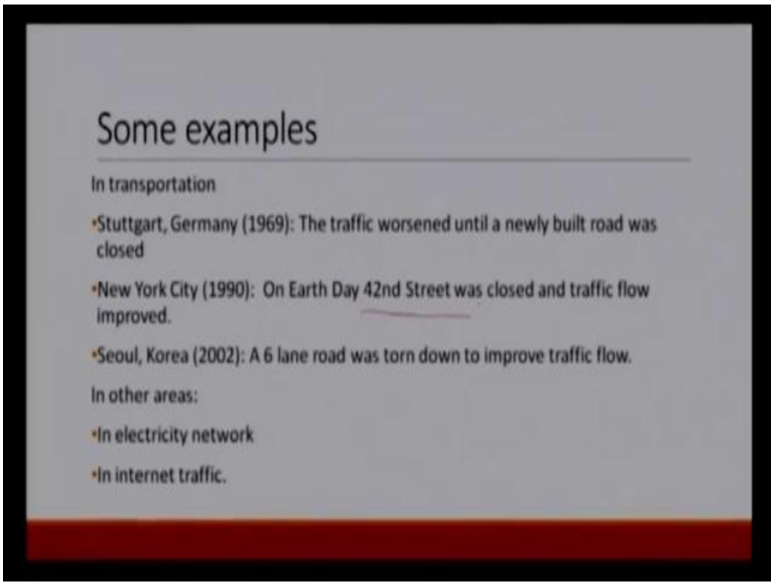

And, this happens in reality also. I have given some; listed some examples. Stuttgart, Germany; in 1969, the traffic worsened until a newly built road was closed. Same happened in New York City in 1990 on Earth Day. To celebrate the Earth Day, the forty second street was closed. And, what was found? That traffic flow improves. By the way in 2009 in New York, New York City Government decided to close down forty second street because of Braess's paradox. And also in Seoul, Korea, a six lane road was torn down to improve traffic flow.

By the way, you may be thinking that, you know building road is a bad idea as this example shows. This is not true. Building road is not always a bad idea. Most of the time it improves the traffic flow. But some of the time, if roads are not built after putting lot of thought or putting design, then it may worsen the situation. And, this is observed not only in transportation sector, but also it is observed in electricity network as well as internet traffic.

Thank you very much.

This was Braess's paradox. We will meet again and talk about some more examples.

Thank you.