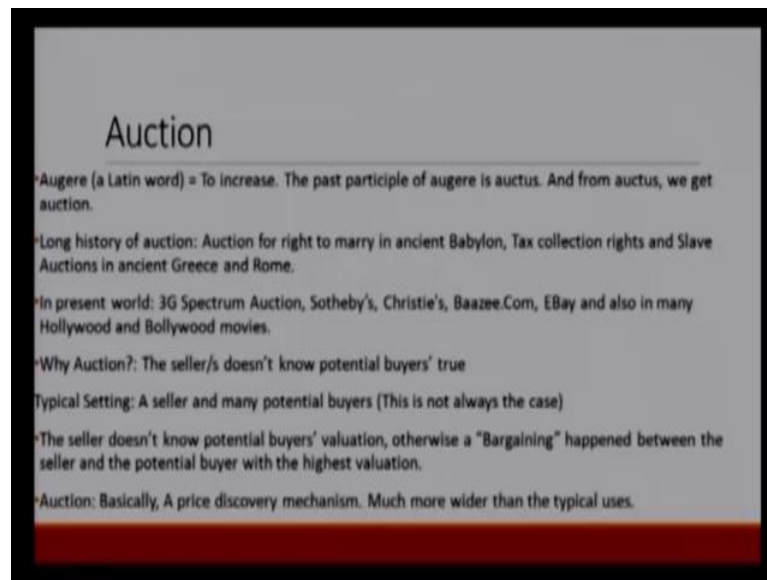


Strategy: An Introduction to Game Theory
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Lecture – 19

Welcome to the Mooc lectures on Strategy, An Introduction to Game Theory. In this module I am going to talk about Auction. In this module, particularly I will introduce the concept of auction and in the next module; we are going to use game theoretic techniques to model and auction. So, first of all, let us understand the word auction, it comes from a Latin word Augere, which means to increase.

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And not to waste our time on English, but it make sense to understand it, the past participle of augere is auctus and from auctus we get auction. So, the key term in auction is to increase and this is the way, we mostly understand the auction. But, over the period we will see that auction means much more than that increasing the bids. We have very long history of action in the ancient Babylon; the right to marry was soul using auction.

Ancient Greek and Rome also sold tax collection right using auction, slaves were also traded using auction not only in ancient Greece and Rome, but also in United States of America in since 17, 18, 100's. And we should not thing that auction was useful only in the historical perspective, even in today's world, auction is very relevant, very useful. The 3 G spectrum that we have heard so much about was sold in several countries using different auction mechanisms.

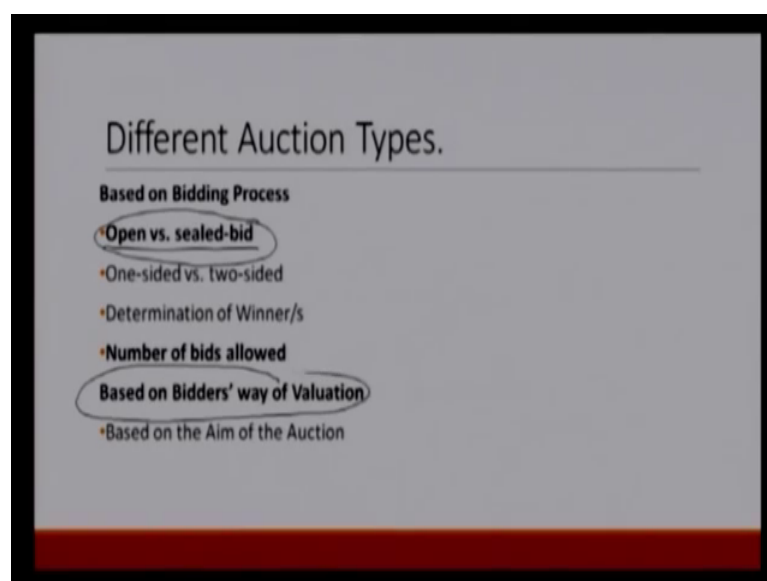
We also seen many Hollywood, Bollywood movies different auctions taking places, mineral exploration right, while drilling rights, these are sold using auction. So, it is very, very useful concept that we should understand. First of all, what do we see that if you want to buy, let us say a pen, we go to market, we ask a shopkeeper, how much does this pen cost and he tells us and if we like, we buy that pen.

Auction is different from this process, typically what happens that seller does not know that how much price he can get for the item that he is willing to sell, he is not aware of the value that potential buyers associate with that object. So, auction is used as a price discovery mechanism, auction we used to figure out the bidder, who is willing to pay the highest amount for that good, so in that sense auction is a price discovery mechanism.

We should understand, it is important that seller does not know the valuation attached by buyers to that particular good. Because, if seller knew, how much money different people are willing to pay, what would he do, he would pick the buyer, potential buyer, who would be willing to pay the maximum amount and he would negotiate or bargain with that particular person. Auction is different from this negotiation of bargaining.

Auction as we will see has very particular mechanisms, but we should also understand that auction not only means a discovery of a price mechanisms, it can be used for scheduling also. And today, we are going to emphasize that this price discovery mechanisms part.

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We should talk about different types of auctions and these different types, depends on

different processes involved in conducting the auction. So, the first one is based on bidding process, first is open versus sealed bid. It depends how much the bidders know about the others bid, whenever they are bidding, they maybe aware of others bid and they may not know the others bid.

When they become aware of others bid, then the process is called open bid auction and when they remain ignorant of others actual bid, then the process is called sealed bid auction. Also, we should also think about slightly different bidding process in which only one side is bidding; that is one sided auction, typically the one we have been talking about, typically what happens, there is a seller and there are many potential buyers.

So, that sort of auction would be called one sided auction, because only potential buyers are submitting the bids. Seller is not submitting any bids, but as oppose to this, this process, there can be several different auctions in which both side bids simultaneously. That typically happens in electricity market and in this case, it is called two sided auction, as sellers also submit their bids and buyers also submit their bid.

We can also distinguish different auction types based on how winners get determined. So, let us say, there can be just one winner or winner can be declared at continuous interval. So, depending on that, we can classify different auction. Also, based on the number of bids allowed or bidders allowed to bid only once or they can revise their bids. So, based on that, also we can distinguish different auction format.

Auction also can be classified based on the fact, how bidders value, how bidders are assigning value to the object; that is being sold through this auction. So, we will come back to this point, this is fairly important point. We are going to emphasize these two in more detail, one, this one and another, this one.

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Sealed Bid Auctions

- One seller and many potential buyers.
- Typically One Sided: Only buyers bid.
- Single round of simultaneous bids
- The buyer who submits the highest bid wins
- Example: Common in Procurement, commodities, and construction
- Two Variants:
 - First Price: The winner pays the amount equal to his bid.
 - Second Price: The winner pays the amount equal to the highest among remaining bids. It is known as Vickrey Auction.
- Why would the seller ever use the Second Price Sealed Bid Auction?

Handwritten examples on the slide:
5, 6, 7, 2, 8
5, 6, 8, 2, 8

So, let us talk about open versus sealed bid auction. What happens in the sealed bid auction? We have typically one seller and there are many potential buyers, this process is typically again one sided as only buyers bid. So, if I am using the term bidder in context of sealed bid auction, then I meet buyers, because they are the only one bidding. What happens in this? In this, there is a single round of simultaneous bids. Typically, the bidders do not get to revise their bids, they submit their bid only once and then, auction ends.

How does this auction end? Once they submit their bids, all the bids get evaluated and the person who submits the highest bid wins. Typically, we observe it in commodity market, construction market and also it is common in procurement. Procurement in one way is not auction, it is reverse of the auction that we are talking about. What happens in the sealed bits auction that we are talking about? There is a seller and many potential buyer.

But, in procurement what happens, there is one buyer and there are many potential sellers and buyer procures the good or the item from the seller, who is willing to sell his good at the lowest price, but the process is very similar. We can talk about two different variants of sealed bid auction, one variant is called first price auction, why because in this case, the winner pays the amount equal to his bid.

So, the first bid one can sell; that is the first price sealed bid auction and the second variant is called second price sealed bid auction. In this case, the winner pays the amount

equal to the highest among remaining bids. Why do we call it second price auction? So, let us take an example in which we have five different bids like 5, 6, 7, 2 and 8. Who wins? The person who submits the highest bid and that is equal to 8.

How much money he will have to pay? He will have to pay amount equal to the highest among remaining bid. So, let us look at the remaining bids, which one is the highest 7. So, he has to pay 7, he submits the bid of 8, but since it is second price sealed bid auction, he has to submit the second one; that is 7. But, we have to be careful. Why did not I say, the winner pays the second highest amount, because this maybe wrong in some cases.

Let us take another example that similar, but slightly different, in this case five people bid 5, 6 and instead of 7, 8 and then, 2 and then, again 8. In this case, there are two people, who submit the highest bid, the third and person 5. Let us say randomly one is decided the winner, let us say randomly this person is decided the winner, third person. So, how much he will have to pay? He will have to pay the amount equal to the highest from the remaining bid.

So, highest from the remaining bid, we have to consider these two and again these two. How much is the highest among remaining bid 8, so he will have to pay 8. So, here it is not second highest, second highest is in fact, 6. But, it is called second price sealed bid auction, because in most of the cases, the winner ends up paying the second highest amount, but you should be careful.

Now, a question that should come to your mind in naturally that why would the seller ever use the second price sealed bid auction, when he can use first price sealed bid auction. Notice in the first price sealed bid auction, the winner has to pay amount equal to his own bid; that would be definitely more than the second highest bid. So, why would a seller ever use the second price sealed bid auction? This question you should keep in mind, either Adithya or I would talk about it again in the future.

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Now, let us talk about open bid auction, earlier we talked about sealed bid auction, now let us talk about open bid auction. Similar to sealed bid auction, typically we will have one seller and many potential buyers, it is also like sealed bid auction, it is one sided; only buyers bid. But, what is the difference between sealed bid auction and open bid auction? In sealed bid auction, all the potential buyers, they submit their bids simultaneously and only once, they do not get to revise their bid.

But, in open bid auction, there are multiple rounds of bids and they get to revise their bid and also in the sealed bid auction, as I said that players or buyers participating in the auction would not know how much other players or bidding. But, since open bid auction will be conducted in open, they would notice how much other bidders are bidding. Again, we will have two variants here, one is called ascending price open bid auction.

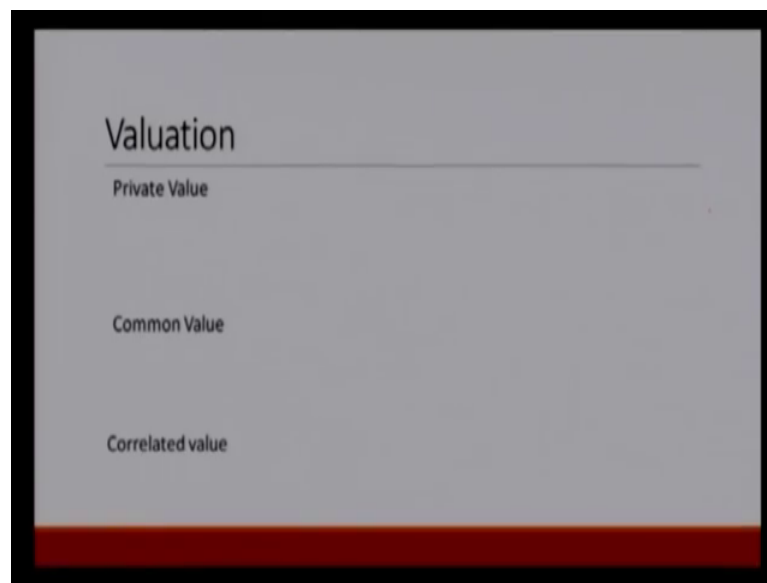
Here, what happens, the bids must be increasing, remember that we have multiple round. So, in the next round bid should increase and the auction closes, when only one bidder is left. It is typically observed in English auctions, you have seen it in movies also, Christian saw tooth they typically conduct auction using this method. Another example is descending price auction, what happens in this, again in the auction progresses, bids must be decreasing.

And in this auction closes as soon as one person bids, because what happens in the descending price, they start the auction at really high price. A price at which no one would be willing to buy the product and gradually, this start decreasing the price, as price

start decreasing or if it reaches to a particular point, one person become willing to buy the object at that particular price.

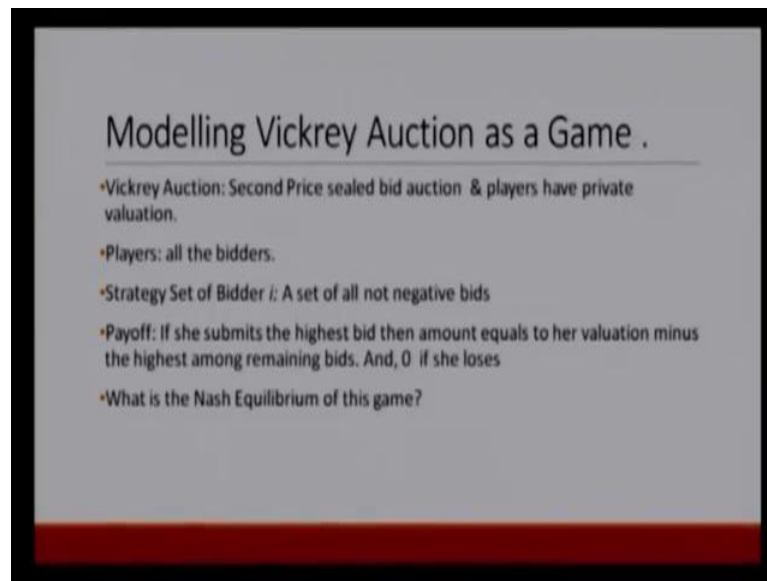
As soon as he indicates that he would be willing to buy the product at this prevailing price auction ends and this is descending price auction. Typically, flowers in Netherlands is sold using this technique, it is very widely used for tulip selling. So, it is also called Dutch auction. Example of open bid auction would be E-Bay auction or Google which sells add, they also use open bid auction and as I already indicated auction by English, auction houses.

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Now, let us talk about valuation, of course, why would someone be willing to buy that good, because he values, the good at higher his valuation for the good is higher at the higher than the selling price. So, that is why, he is interested in buying that product.

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The slide is titled "Modelling Vickrey Auction as a Game" and contains the following text:

- Vickrey Auction: Second Price sealed bid auction & players have private valuation.
- Players: all the bidders.
- Strategy Set of Bidder i : A set of all not negative bids
- Payoff: If she submits the highest bid then amount equals to her valuation minus the highest among remaining bids. And, 0 if she loses
- What is the Nash Equilibrium of this game?

But the key point is, how did he arrive to his valuation, let us talk about, how do these potential bidders value the item that there are interested in buying or they buying the item further personal placer or they are buying the item, so that they can sell it in the market at some later date and make some profit. So, depending on how there are reaching to the valuation of the product, we will have different mechanisms.

So, one is private value, in this, all the bidders independently value the product, because they are mostly interested in consumption. Like, if you are interested in buying Monalisa for your own placer, then you are valuation for Monalisa would be private value or let say if you are interested in buying any art face by Maqbool Fida Husain or any one. If you are not buying the product or the art, for selling it at future rate, you are buying it for your personal placer, then it will be your private value.

Second one is common value ((Refer Time: 15:57)) what happens when you are interested in buying the same Maqbool Fida Husain and painting. So, that you can sell it at later date and make some profit. Then, how would you figure out the valuation of the product, you would think about the money that you would get at future date and that money would give you the valuation. That is why when this is the case we have common value, because there is one true value at which these potential buyers, if they are willing, if they are able to buy the product they we able to sell the product at that particular price.

But, the problem here is typically, these potential bidders would have some idea about the price they would get in future, but they do not know it act exactly. So, that would be

covered in common value and the mix of a private value ((Refer Time: 16:55)) and common value is correlated value. Some of the players are interested in buying the product or personal placer and some of the players are interested in buying. So, that the consulate in the future.

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So, one last thing I want to talk about in this module that we have basically four standard auction format. It is I am not saying that auction is limited to only these for format, these four format are canonical in the sense that, standing them give as basic foundation in auction theory. So, we studied two sealed bid, first price and second price and in open bid, we studied descending price and ascending price.

Can we say that, first price is seem as descending price or first price is same as a ascending price, can we say that second price is same as the descending price or ascending price, let us think about it. How would you participate in sealed bid, of course, here auction is taking place in only one round, all the potential bidders are submitting their bid simultaneously, you do not get to resides.

So, what would you doing the first price, after let say it is private value for example, it is private value, you already you know how much you would be willing to pay for this product, you think about it. And then, you say this is the amount I am going to bid and that you would write on a piece of paper sell it in well up and give it to the seller or auction here. And that would be you are bid, but let us say that the same good is being sold using descending price, what would you do, price is very high, you will again think

about a particular bid at which you would be willing to buy.

And if your bid is the highest as soon as the price reaches to that level, you would raise your hand or shout out say you would be willing to buy. So, basically in both the cases, you would be submitting the same bid. Even, if your bid is not the highest one, what would happen, again in first price sealed bid auction, you will write the bid, submit it and in open bid auction, you will wait for your turn, but it will never reach and you would not when the auction.

So, mathematically speaking and you will have the same game, you will have the same bidding process; that is in your mind. So, we can say first price auction is same as descending price auction in the case of private value. But, how about common value, in common value also, think about common value, even if you have common value in sell bid first price auction, you do not get to observe, what others are bidding.

So, in an open bid descending price auction, because as soon as someone says that, he or she is willing to buy the product at that particular price auction ends, you do not get to observe others. So, there is nothing that would modify your behavior. So, the case is very similar to or the same as private valuation. So, first price is same as descending price mathematically speaking, both in case of private value and in case of common value.

Let us think about now second price an ascending price auction, let us first talk about private value, what happens in the second price auction, again you have to submit a bid, you will think about a number that you would submit. In the ascending price, there would be different rounds and you would get to see, what others are bidding. But, since your valuation is private in nature, your valuation does not get affected by what others how much others are willing to pay, you would not change, you would not modify your behavior.

So, second price auction is same as ascending price auction, but you may say, why not descending price auction, because think about it, when does the ascending price auction ends, whenever a person, let say you have a number in your mind. And the next competitor also has a number in mind, as soon as that number reaches, your competitor with lower bid will leave the auction.

And you would not wait, you would not bid the number that you have been in your mind, you will win the auction at that particular bid. So, basically you are paying the amount

equal to the that is equal to the second last persons valuation that is why ascending price is similar to second price auction in the case of private valuation. But, let us think about common valuation, what happens in that case, in that case, second price are sealed bid auction you do not get to observed how much others are bidding.

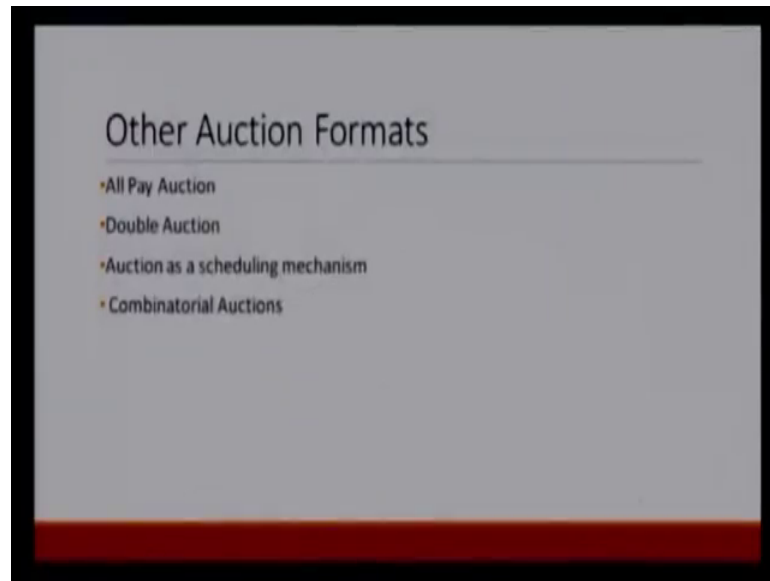
But, in the case of ascending price auction you get to see how much others are bidding, see you may modify your behavior, because you know it is a common valuation, you may think that your estimate is wrong. Because, someone is willing very, very low or your estimate was low because someone was bidding very, very high are you estimate was high is some other people were bidding very, very low.

So, you get to modify your behavior in ascending price that you do not get in the second price sealed bid auction. So, second price sealed bid auction is same as a sending price sealed bid auction, only in the case of private valuation, not in the case of common valuation. Before, we close this module, I would like to talk about some other auction formats, it is important, so that you do not think that the only auction format that we are interested in come from sealed bid auction or open bid auction.

Let us talk about something called aleph auction, but we have been talking about so for is that the only winner pays, but it is also possibility that not only winner pays, but all the bigger paid. But, you may be thinking is it true ever does it ever happen, think about bid in trace, although it does not sound like auction, but it can be model as auction, where all ((Refer Time: 23:39)) is interested in are ending.

Their interested in getting new product and their spending the money, one can say the spending the amount or money as their bid, who wins this, the person whose able to come up with the first product. He gets the patent and he gets to sell that product in the market. So, in this all the RND forms in the spending the money, whether the win penitents race or not this can be modeled as all pay auction.

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Similarly, one can think about double auction. Here, we have been talking about only one sided auction, I have already mentioned that there can be two sided auction which is very prevalent in electricity market. Also, computer engineer and electrical engineer, they also they can use auction as a scheduling mechanisms. Let say, if you have scarce resource such as use of super computer time, how would you schedule different people for the use of that super computer and last one that I want to mention is combinatorial auction, where not only one good is sold, but several related goods are sold in multiple quantities.

And bidders are interested in buying a particular combination of items. So, that would be studied in combinatorial auction, this is very important for 3 G auctions. So, that is enough for today, we will come back to revisit auction once we discuss the deviant games.

Thank you very much.