Strategy: An Introduction to Game Theory
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Lecture – 16

Hello, welcome to another module in this online course Strategy, An introduction to Game Theory. So, we are looking at games with mixed strategy or mixed strategy Nash equilibria. Let us now look at another example of a game with the mixed strategy Nash equilibria, let us look at an investment game or the portfolio management game.

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Let us look at a portfolio management game or an investment game. What is this investment game? We can think and this is an interesting game, it is not readily going to be obvious us to, who the players are.

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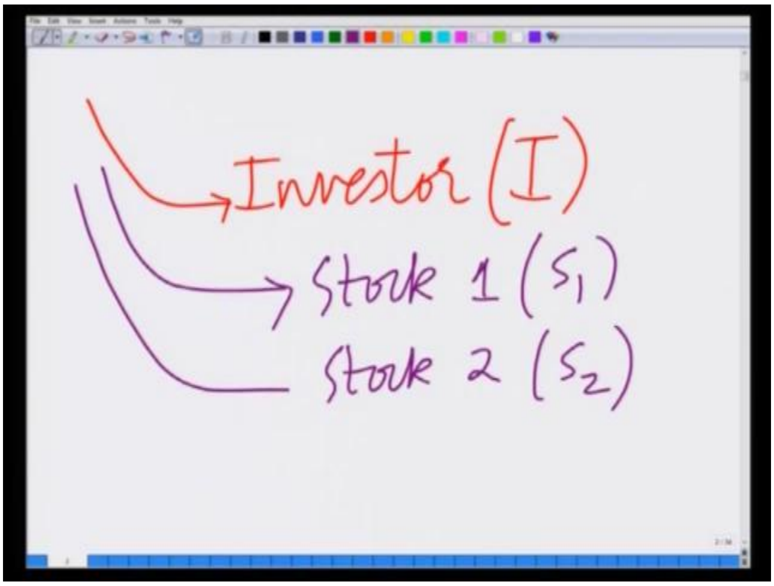

Well, one of the player is of course, going to be an investor, investor I and he has two options, either to invest in stock 1. He has can either invest in stock 1, let us denote this by S 1 or you can invest in stock 2, S 2. So, in investor he can either invest in stock 1 and in stock 2.

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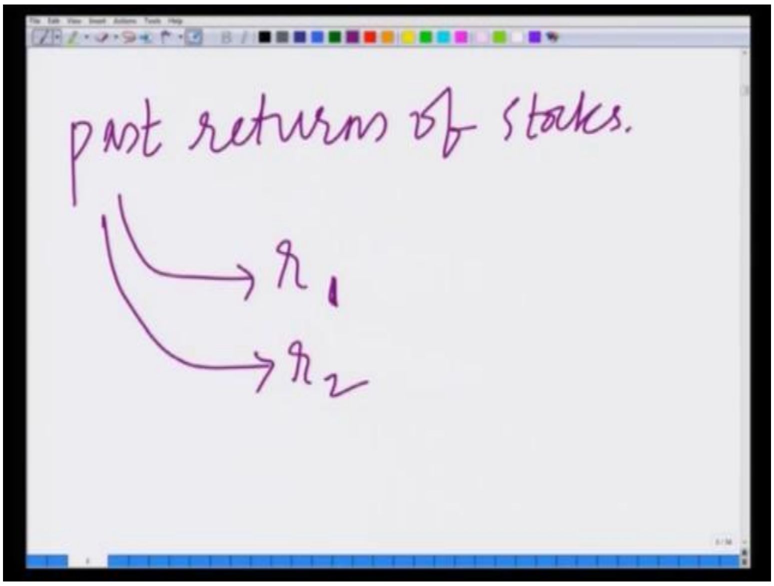

And the information, he has are the returns for these stocks, returns the past returns, past returns of stocks, there is return 1, let say from year 1; that is the rate 1 and rate 2. What are these rates? Well, let say the rate 1 corresponds to the past year and rate 2 corresponds the year before that. He has some history of the returns from that particular stock. So, now, let us represent this game has a game table of course, fully realized and in that at this point we still have only 1 player, we have not indicated who the other player is.

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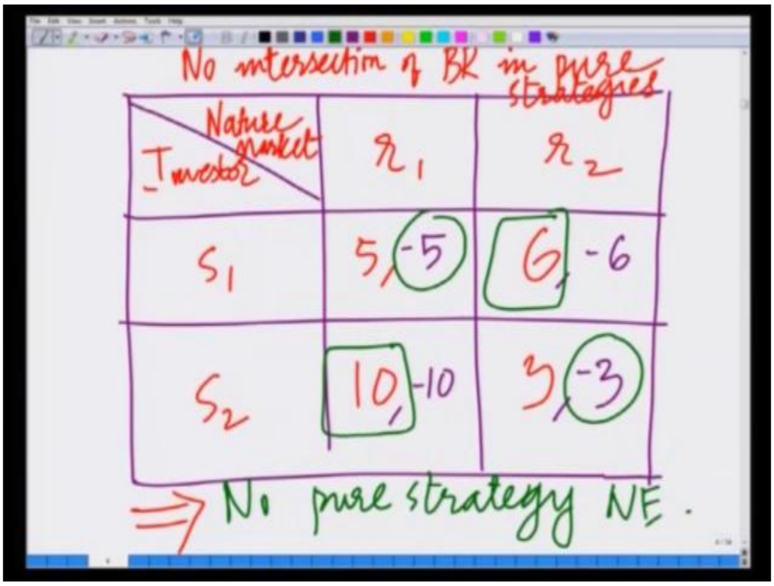

But, as let us try to draw this game table and therefore, the game table for this; it can be drawn as follows. So, there is the investor and he can invest in stock 1 or stock 2 and the rates are r 1 and r 2 and we have not mention yet, who the second player is. And let say stock 1 in the past year; that is corresponding to rate 1 has a return of 5 and corresponding stock 2 has a rate of 10 in the past year.

However, in the year before that in the year that is, stock 1 has a rate of 6 and stock 2 has a rate of 3, which means, let say these are percentages. The stock 1 yields a return on investment of 5 percent, stock 2 yields a, or yielded a return on investment of 10 percent in the past year. And in the year before that, stock 1 yielded and return on investment of 6 percent and stock 2 yielded a return of investment of 3 percent and now, we are trying to figure out for this year, for the current year, how do I invest in stock 1 and stock 2.

Now, how about the second player? Of course, in this game there is no obvious second player; that is the stock broker or the investor is not investing against any other particular payer, but we can think of him as investing against an adversity like the stock market. So, we can think as this adversary or it is opponent being nature or the stock market or the nature or market.

And we can think of as the market being an adversity, we can think as if the market is whatever the stock investor is trying to invest, we can think of the adversary as trying to minimize his return. So, we can think of as the stock market being unpredictable and trying to play an opposite strategy or trying to complete with the investor to minimize his return of the stocks.

And therefore, we can think of the return of or we can think of as the payoff of the stock market being the minus of the payoff of the investor. So, this is an interesting game, this is an interesting investment game in which game, in which we are modeling this as a game between investor and the stock market. The investor is trying to maximize his profit and the stock market is as acting as on adversary and trying to minimize his profit.

And therefore, we can think of this as an investment game in which an investor is trying to matches which again the adversary or trying to maximize his, let us trying to use an optimize strategy to maximize his payoff. And now, again let us try to see, if this game has a pure strategy Nash equilibrium, well if the nature or stock market chooses the return of the previous year r 1, then it is better to invest in this in stock 2.

On the other hand, if the stock market chooses the year before that, then it is better to invest in stock 1. Since, it is yields are return as 6 percent and if the investor chooses to invest in stock 1, then the adversary can choose the past year, because it gets minus 5. While, if the investor chooses to invest he has 2, then the stock market can chooses to choose r 2; that is the year before the past year, so that, it gets a rate of minus 3.

And therefore, we can again see, no intersection, there is no intersection of best response in pure strategies, which implies no pure strategy that is in the stock market investment game, there is no pure strategy Nash equilibria, where, this is a game being model as a game between an investor and the stock market, there is no pure strategy Nash equilibrium. So, naturally we have to look at a mixed strategy Nash equilibrium.

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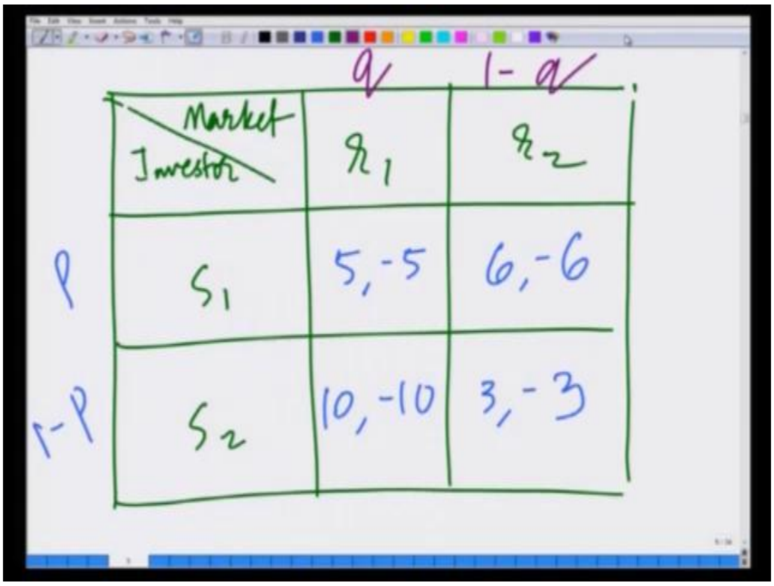

So, let us look at a mixed strategies Nash equilibrium, let me redraw the game table as the investor and the market stock 1, stock 2, rate 1, rate 2 and the payoffs are 5 comma minus 5, 6 comma minus 6, 10 comma minus 10, 3 comma minus 3. And let say the investor is mixing with probabilities P and 1 minus p, if the market always chooses rate 1; that is if it always chooses the previous year, then it is payoff is minus 5 times p plus minus times 10 times 1 minus p.

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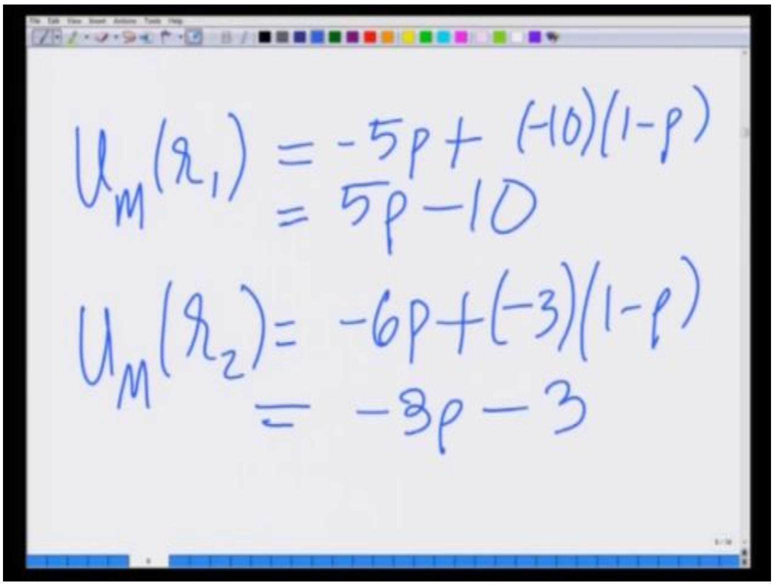

So, if the market m always chooses rate 1, then it is payoff is minus 5 times p plus minus 10 times 1 minus p, which is equal to 5 p minus 10. On the other hand, you can see ((Refer Time: 08:29)), if the market always chooses the year before that; that is chooses rate 2, then it is payoff is minus 6 times p plus minus 3 times 1 minus p; that is the return for the market. If it always chooses rate 2 is minus 6 times p plus minus 3 times 1 minus p, which is equal to well minus 3 p minus 3.

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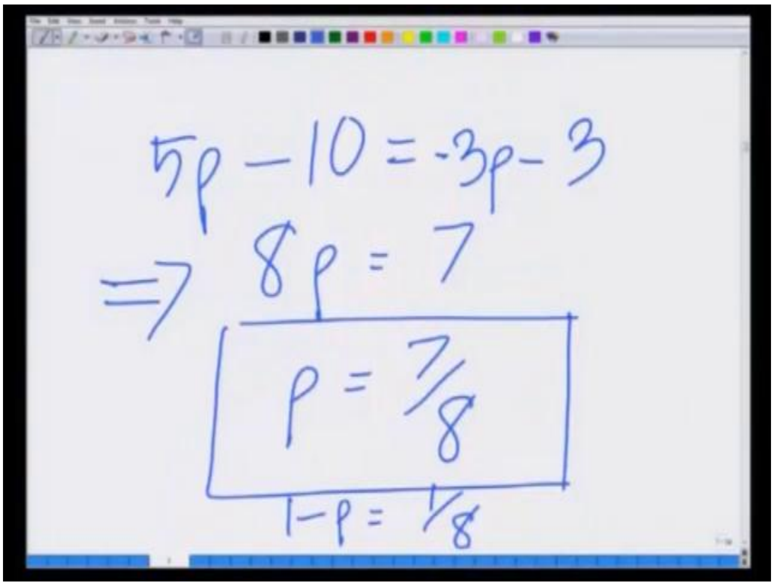

And therefore now, it will use a mixed strategy only when these two payoffs are equal. It will randomly choose, but it is nature or stock market randomly chooses to mix. These two, only when the payoffs are equal, which means 5 p minus 10 is equal to minus 3 p minus 3, which implies 8 p is equal to 7, which implies p is equal to 7 by 8. Therefore, the mixed strategy employed by nature or the stock market is p equal to 7 by 8, 1 minus p is equal to 1 by 8.

So, the mixed strategy employed by the investor, the mixed strategy employed by the investor is p equal to 7 by 8, 1 minus p equal to 1 by 8 and this can be thought as him investing 7 by 8ths of the funds in stock 1 and 1 by 8th of the fund in stock 2.

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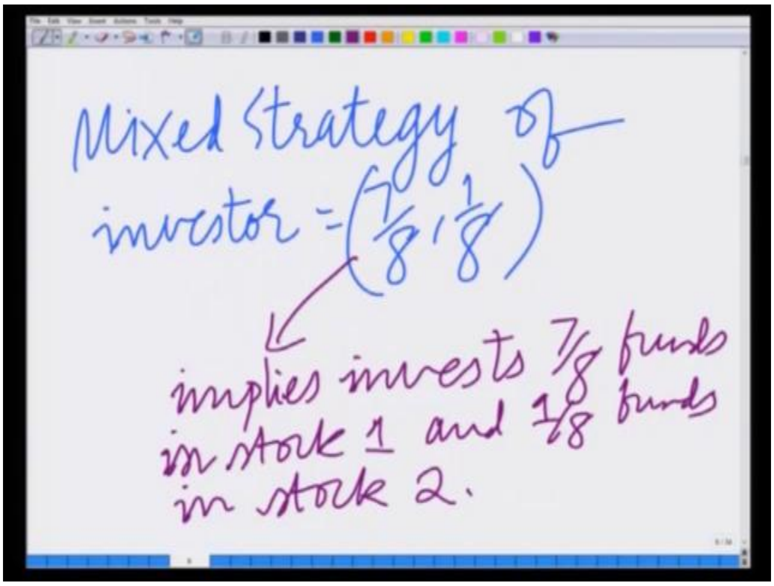

So, mixed strategy implies invests 7 by 8 funds in stock 1 and 1 by 8th funds in stock 2, so he invest 7 by 8 of the funds in stock 1 and 1 by 8th of the funds in stock 2 and similarly and therefore, this the mixed strategy employed by the investor ((Refer Time: 11:04)). Now, we can also find the mixed strategy employed by nature, let say nature is mixing with probability q and 1 minus q. If the investor is always choosing stock 1, he is return is 5 q plus 6 times 1 minus q.

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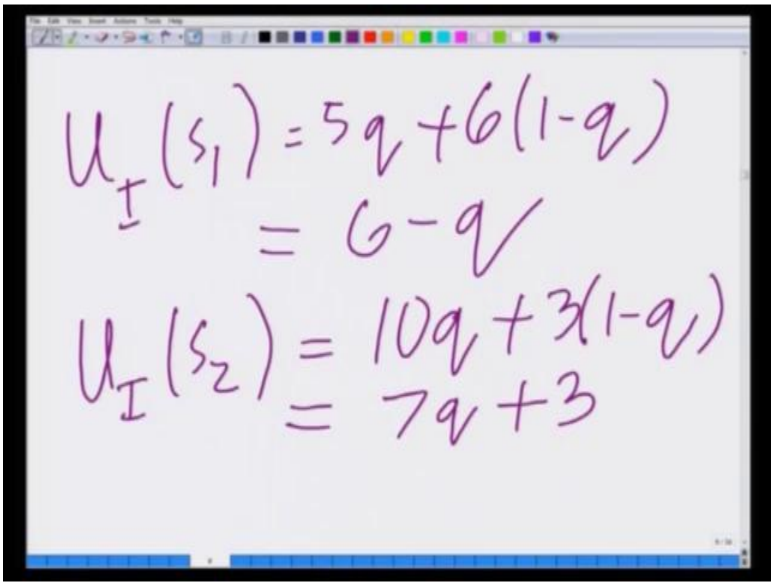

If that is, if the investor is always choosing stock 1 is return is 5 times q plus 6 times 1 minus q, which is equal to well 6 minus q and if the investor is always choosing stock 2 ((Refer Time: 11:41)), his return is 10 times q plus 3 times 1 minus q. That is, if the investor is always choosing stock 2, his return is, well his return is 10 times q plus 3 times 1 minus q, which is equal to 7 q plus 3.

So, if the investor is always choosing to invest in stock 1, he is return is 6 minus q and if he is always choosing to invest in stock 2 is return is 7 q plus 3. And he will choose 2 is a mixed strategy.

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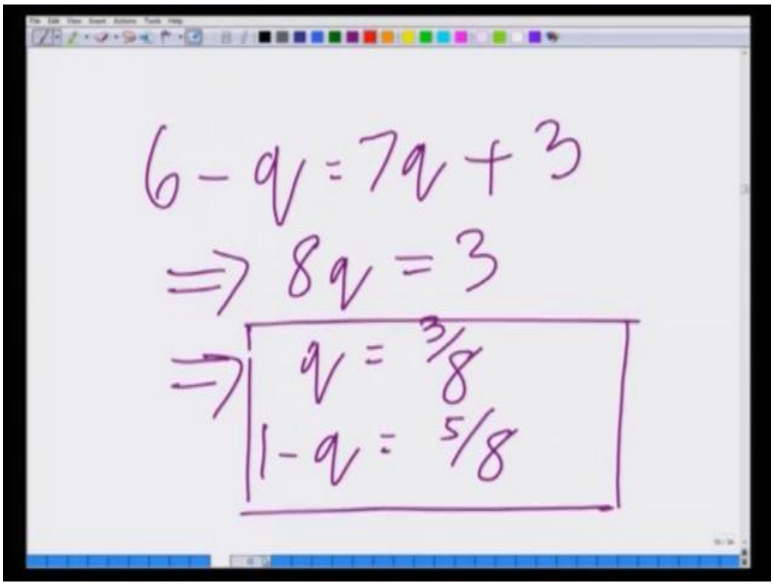

And therefore, he will choose to use a mixed strategy only when 6 minus q equals 7 q plus 3, 6 minus q equals 7 q plus 3 implies 8 q equals 3 implies q equals 3 by 8 and 1 minus q equals 5 by 8. Therefore, this is the mixed strategy employed by his adversary of the stock market.

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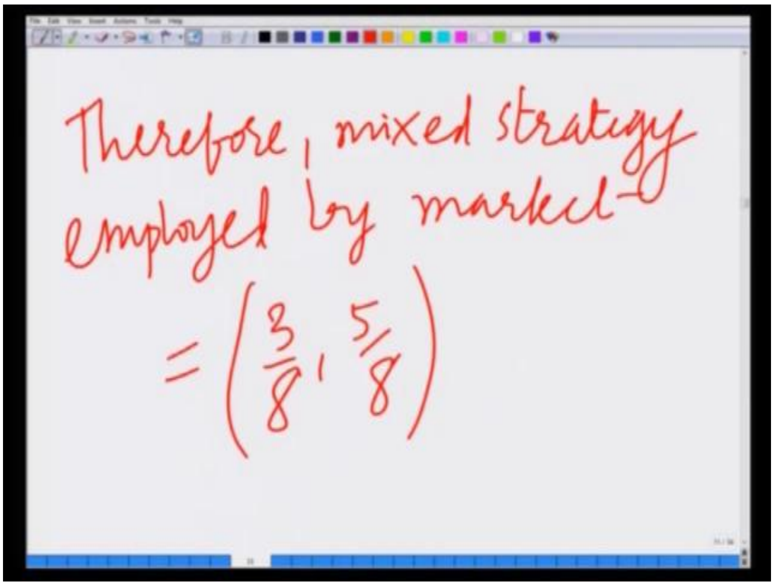

Therefore mixed strategy employed in the market is 3 by 8 comma 5 by 8.

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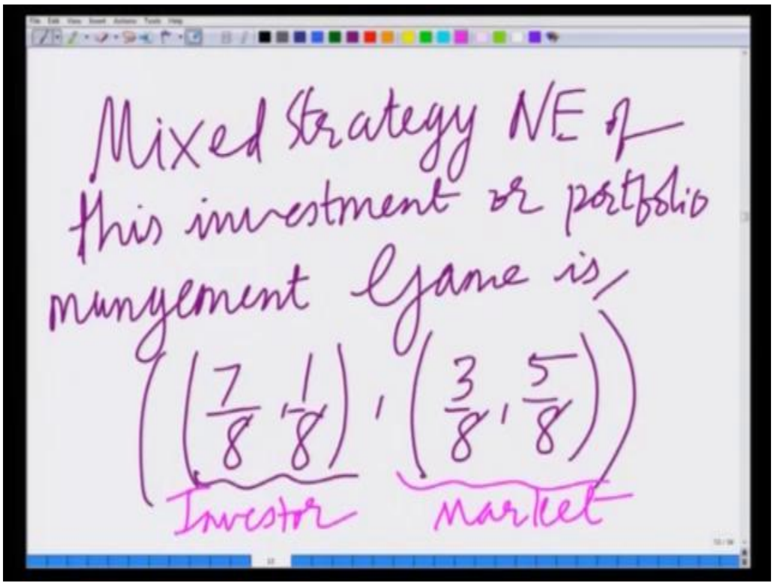

And therefore, the mixed strategy Nash equilibrium of this investment or portfolio management game is, well 7 by 8 comma 1 by 8 for the investor and 3 by 8 comma 5 by 8 for the market. This is the mixed strategy for the investor and this is the mixed strategy for the market, which means the investor is using the mixers 7 by 8, 1 by 8. That is his mix investing 7 by 8th of the funds in stock 1, 1 by 8 of the funds and stock 2 and the market is choosing the return r 1 of the past year with probability 3 by 8 and return r 2 of the year before that with probability 5 by 8.

So, this is the interesting example of a portfolio management or an invest a game and which what they think of investor playing against this adversary, which the stock market and trying to minima maximizes worst case payoff. So, if this, we will conclude our discussion in this module.

Thank you very much.