Strategy: An Introduction to Game Theory
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Lecture - 15

Hello, everyone welcome to another module in this online course Strategy, An Introduction to Game Theory. And we are now looked at several examples of mixed strategy Nash equilibria. Let us continue this and let us look at the couple of other games with mixed strategy Nash equilibria.

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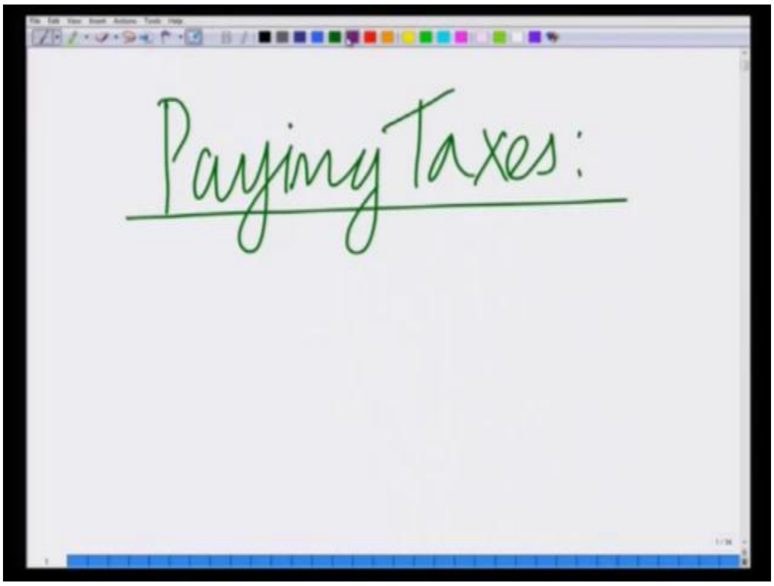

In particular, let us now look at an interesting game of paying taxes, which is applicable in real life scenarios where individuals are paying taxes. And this is the game between the tax payers, the individuals and the auditors of the people that officials, what trying to odd this.

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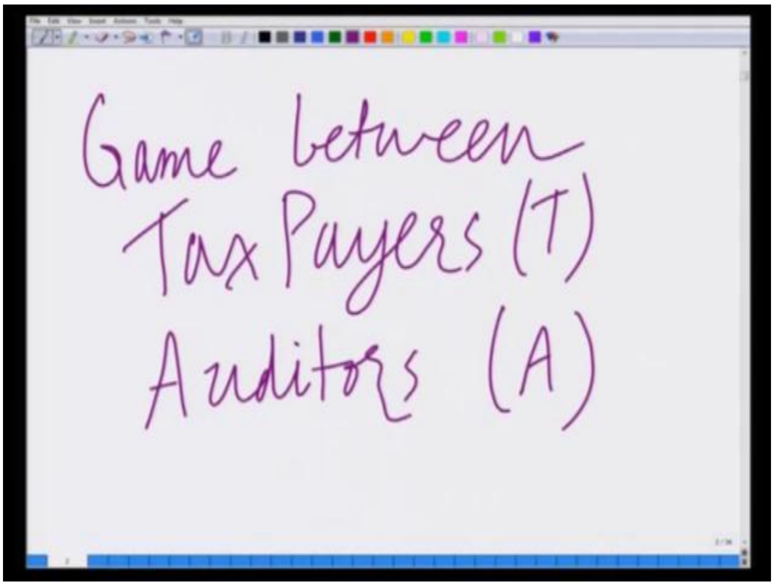

So, this is the game between tax payers and the auditors. So, this is a game between the tax payers and auditors. Let us denote the tax payers by T, let us denote the auditors by A and they have the tax payers and both of them have different actions.

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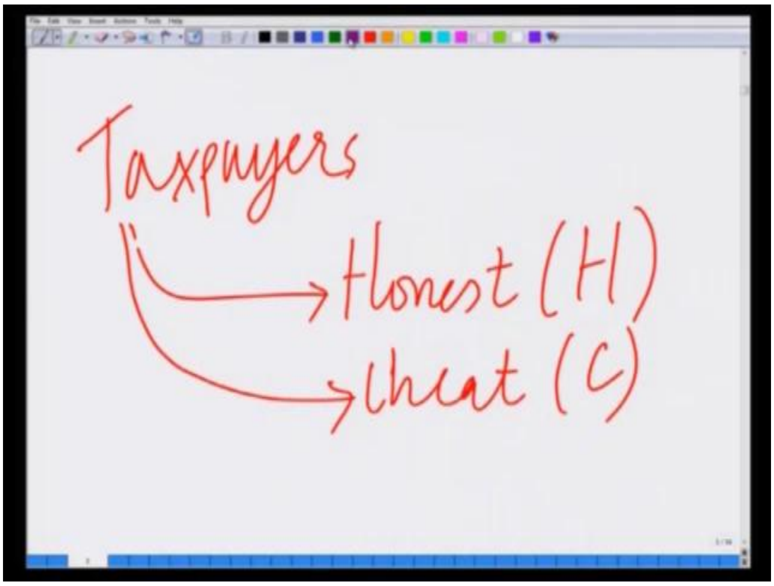

The tax payers can either be honest in reporting, where it act as H or they can either cheat, that is, they can cheat on their taxes, let try to say one, tax money. So, the tax payers can either be honest or cheat.

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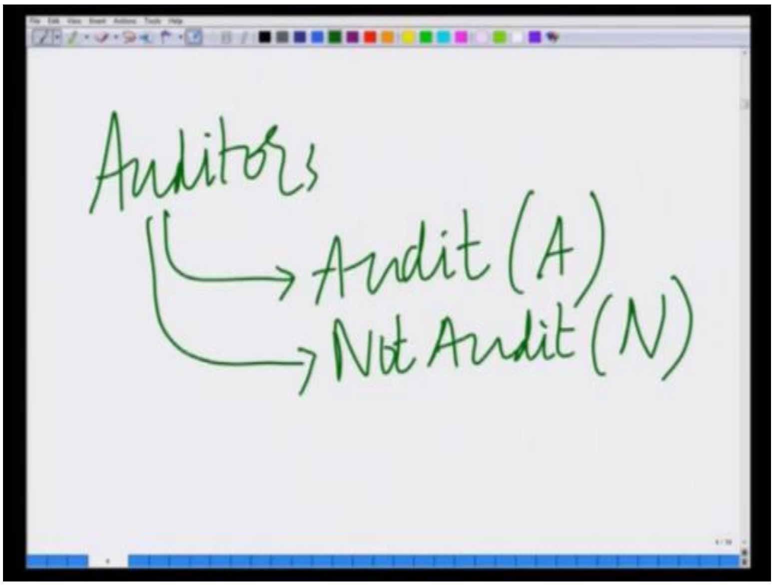

And the auditors, of course, they cannot audit every tax form or every tax return that has been submitted. So, they have an option to randomly either audit; that is A or not audit; that is N. So, we are looking at a game in which both these players, this is in symmetry; that is the actions of both the players are different. There is a tax players can either pay that taxes; that is we honest or cheat and auditors can either audit or not audit.

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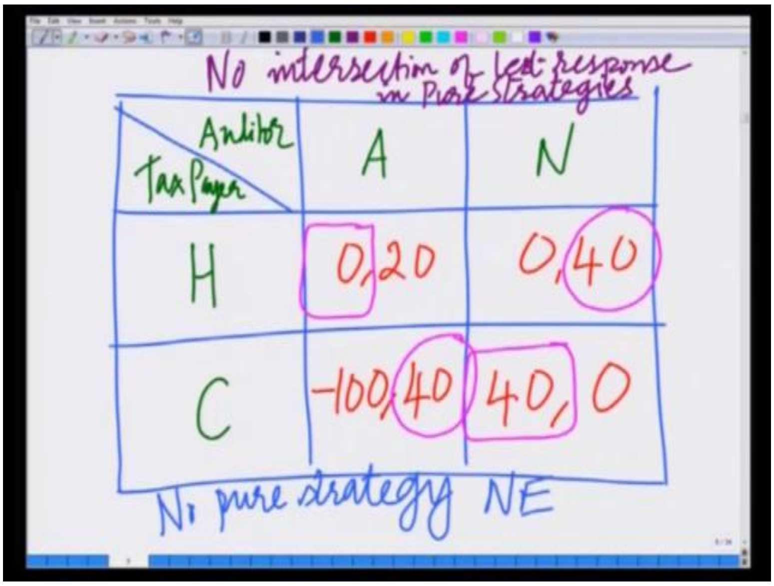

And let us now draw the game table for this game and again, we can draw the game table for this game and I can have the tax payer as the column player and auditor, tax payer as the row player, an auditor as the column player. The tax payer can be honest or he can cheat the auditor can audit or not audit. The tax payer is honest, he get 0, there is no payoff.

While the auditor, if he audits the honest tax payer, he gets a payoff of 20, while if he does not audit an honest tax payer, he gets a payoff of 40, because he is basically save in time and effort by not auditing and honest tax payer. So, if he does not audit, he gets a payoff of 40. On the other hand in the tax payer cheats and it does not get, what is not audited, we get a payoff of 40, the auditor on the other hand gets a payoff of 0, because he missed a cheater.

But, he gets the tax payer cheats and he gets auditor, he gets a payoff minus 100; that is he gets a huge penalty and the auditor gets a payoff of 40. So, this is the representative tax payer game, where the tax payer is honest or cheating and the auditor is auditing or not auditing. The payoffs are the tax payer is honest and auditor audits, the payoffs are 0 comma 20.

If the tax payer is honest and the audit, the auditor does not audit, then the payoffs are 0 or 40. If the tax payer cheats and the auditor audits, then the payoffs are minus 100 and 40 and the tax payer cheats and auditor does not audit, then the payoffs are 40 comma 0. So, this is a simple representation of a tax payer game. Let us now try to see this game has a pure strategy Nash equilibrium. Let us try to see if this game now has a pure strategy Nash equilibrium.

If the auditor is auditing, then the best response of the tax payer is to of course, be honest, because being honest gives him 0, cheating gives him minus 100. If the tax payer is not honest auditing, then the best response of the tax payer is to cheat, because cheating gives him 40, not cheating gives him 0. Similarly, if the tax payer is honest, best response of auditor is to not audit, because he save time and effort that gives him payoff of 40. If the tax payer is cheating best response of auditor is to audit, because that gives him a payoff of 40.

And you can see, there is no intersection in pure strategies, there is no intersection of best response in pure strategies. Therefore, there is no pure strategy Nash equilibrium.

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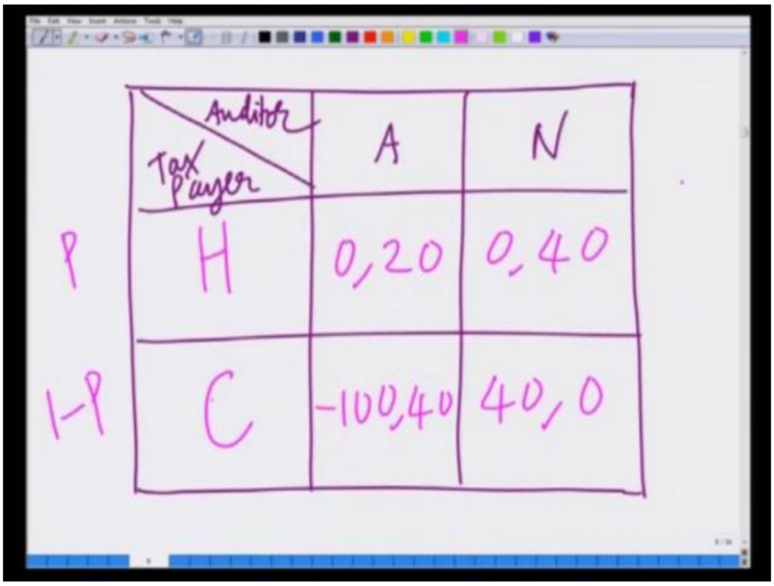

Let us now therefore, try to find a mixed strategy Nash equilibrium, let me redraw this game table. So, that it is clear again, again we have two players, the tax player and the auditor and the tax player can cheat or be honest or the auditor can audit or not audit and the payoffs are 0 comma 20, 0 comma 40 minus 100 comma 40 and 40 comma 0. And if the tax payer is using the mixture p comma 1 by p, let say and let say, the auditor is always choosing to audit.

The payoff of auditor is always choosing to audit is basically, he gets a payoff of 20, because with probability P, he encounters a honest, let me just these options are slightly reversed. Let me just this is the row, first row is the honest tax payer and the second row is the cheater. If the auditor is always auditing, then he gets a payoff of 20 with probability P, when he encounters an honest tax payers and he get a payoff of 40 with probability 1 minus p when encounters, someone who cheats on it is taxes.

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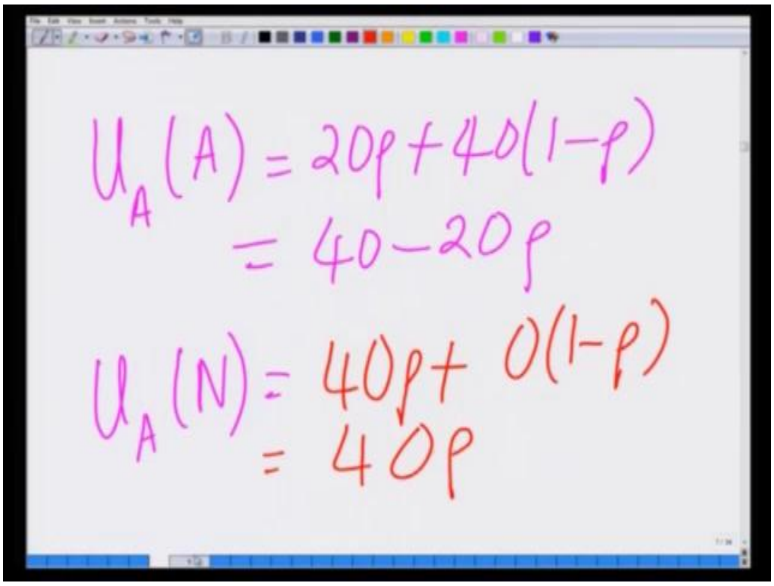

Therefore, his average payoff of auditor who always auditing is well, it is 20 times p plus 40 times 1 minus p equals 40 minus 20 p. On the other hand ((Refer Time: 07:45)), if the auditor always does not audit with probability P, he gets a payoff of 40 with probability 1 minus p, he gets a payoff of 0. So, the payoff of the auditor, who chooses to not audit is basically with probability P, he gets 40, so it is 40 into p plus with probability of 1 minus p, he gets 0, 0 into 1 minus p, which is 40 times p.

And therefore, the auditor, if the payoff from auditing is always greater than the payoff from non audit, not auditing the auditor will always audit. If the payoff from not auditing is always greater than the payoff from auditing, the auditor will not audit. So, if the payoff from auditing is always greater than not auditing the payoff. The auditor will not always audit and if the payoff from not auditing is always greater than the payoff from auditing, he will not auditing. He will randomly choose between auditing and not auditing, only when the payoff from both of these are equal.

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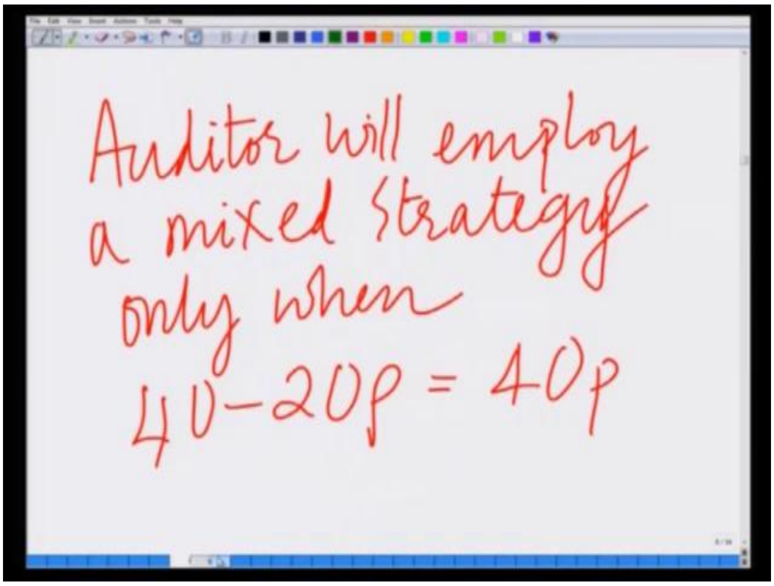

Randomly choose; that is the auditor will employ, he will employ a mixed strategy only when 40 minus 20 p is equal to 40 p.

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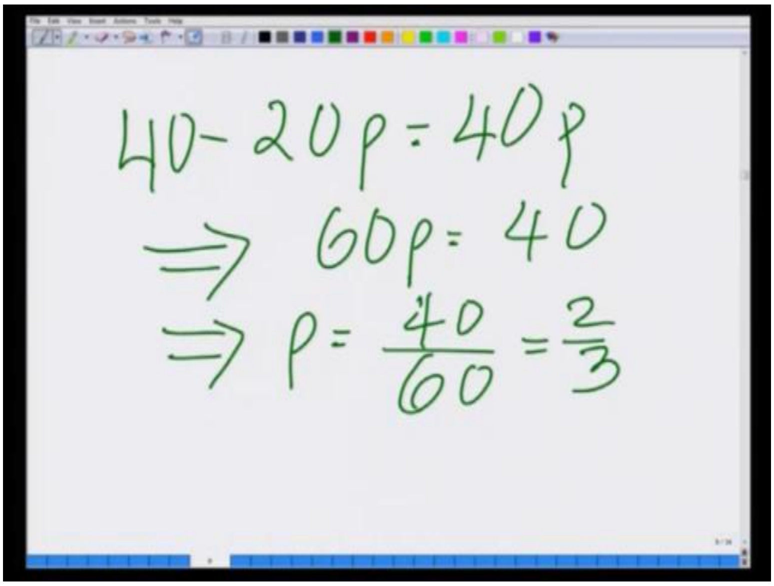

That is 40 minus 20 p is equal to 40 p, implies basically 60 p is equal to 40 implies, p is equal to 40 divided by 60 is equal to 2 by 3. So, the auditor will use the mixed strategy only when p is equal to 2 by 3. Therefore, the mixture of the tax payer is p equal to 2 by 3.

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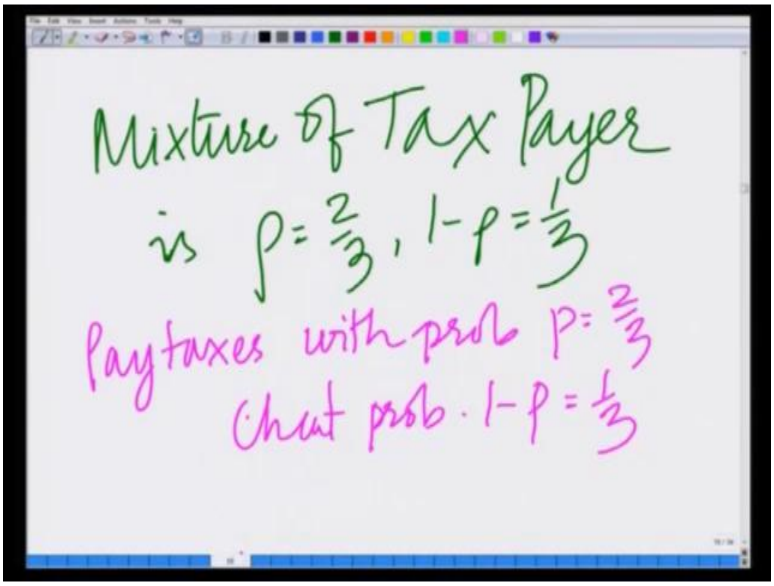

Mixture of tax payer equals 2 by 3, 1 minus p equals 1 by 3; that is pay taxes with probability P equal 2 by 3, cheat probability 1 minus p equals 1 by 3. Now, this can also be thought of as interestingly not one person a different instance of time choosing to pay taxes or not. But, this can be TOT of across a population, different kinds of TOT tax payer, randomly if you pick a person from that population with probability 2 by 3, he is paying honestly paying is taxes with probability 1 by 3 is cheating honest taxes.

So, you need not think of this as again unlike other games, which we thought of as being played repeatedly time. We can think of this game as being played across a population. So, across a population we can say any given individual randomly; that is we will pick an individual from this population randomly with probability to 2 by 3, we will encounter an honest individual, who is paying taxes and with probability 1 minus p equal to 1 by 3, we will encounter and individual ways cheating on it is taxes.

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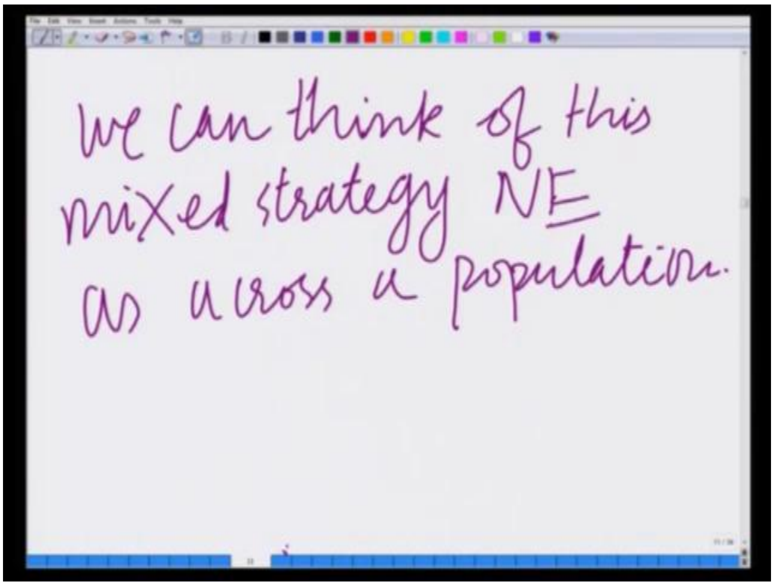

So, we can think of this mixed strategy NE as across a population.

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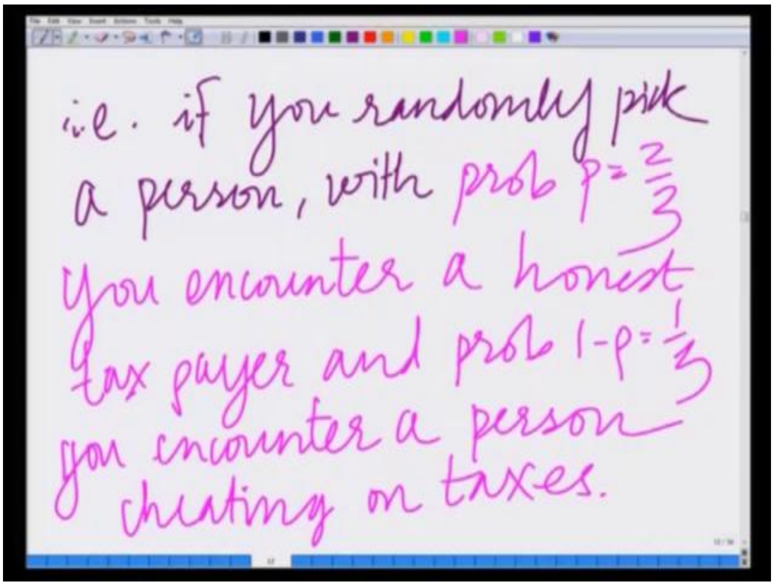

That is, if you randomly pick a person with probability P equal to 2 by 3, you encounter an honest tax payer and probability 1 minus p equal to 1 by 3, you encounter a person cheating on it is taxes. Therefore the mixture of not 1 player, but the mixture across a population can be thought of as being p equal to 2 by 3, 1 minus p equal to 1 by 3.

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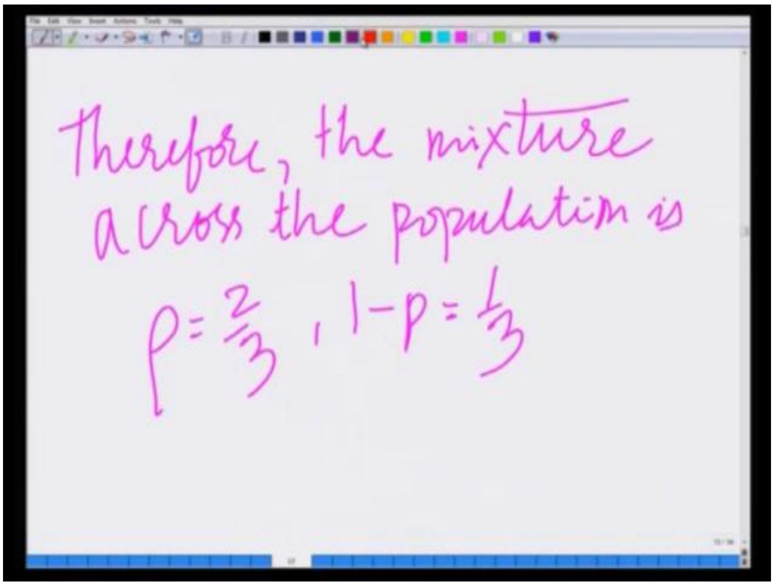

Therefore, the mixture p equal to 2 by 3, 1 minus p equal to 1 by 3, this is the mixture across the population. So, we have found the mixed strategy Nash equilibrium across the population of tax payer. So, we can think as this as the population of the tax payers of the state, of a city or the country and so on. Similarly, now let us try to look at the mixture of the auditor.

((Refer Time: 14:02)) Let say the auditor is randomly auditing with probability q and not auditing with probability 1 minus q. If he is always auditing is payoff is 20, if the tax payer is always honest his payoff is 0 times q plus 0 times 1 minus q.

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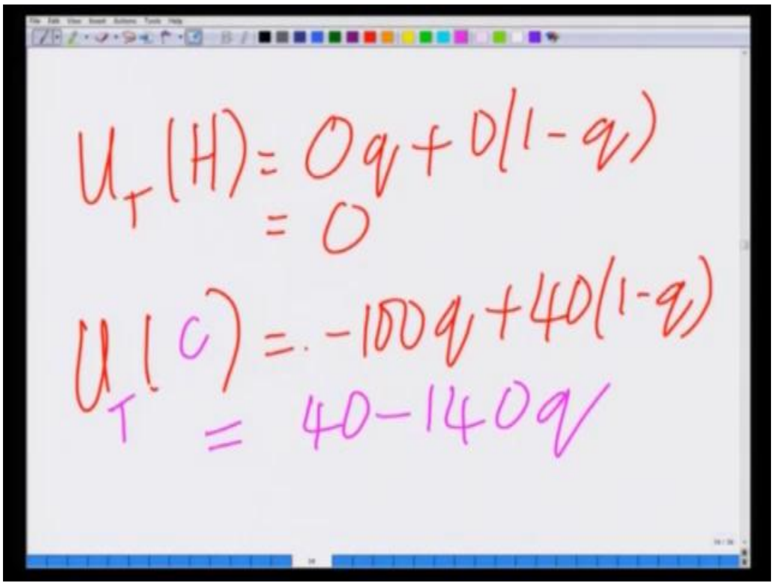

If the tax payer is always honest is payoff is 0 times q plus 0 times 1 minus q, which is equal to 0. On the other hand, if the tax payer is always cheating ((Refer Time: 14:40)), his payoff is minus 100 times minus 100 times q plus 40 times 1 minus q, if the tax payer is always cheating his payoff is minus 100 times q plus 40 times 1 minus q, which is equal to 40 minus 1 40 q. So, if the tax payer is honest his payoff is 0, if the tax payer is cheating u T of C his payoff is 40 times, 40 minus 140 q.

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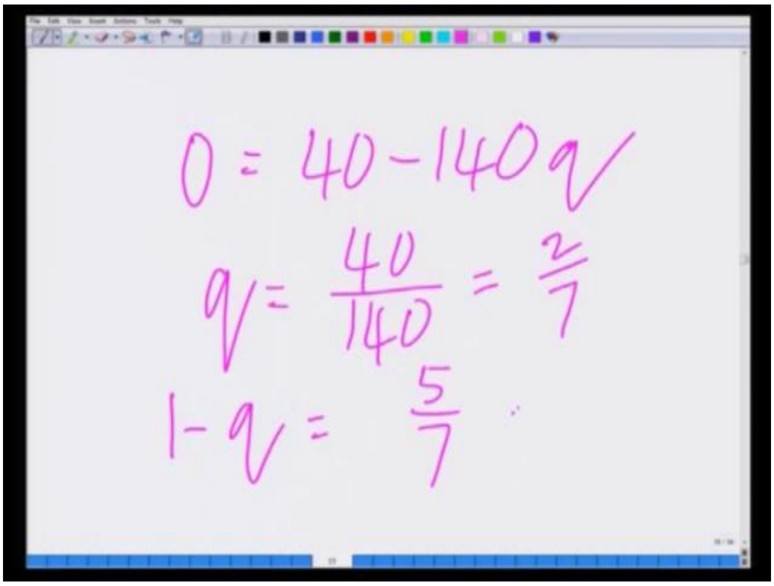

And he will randomly choose between the be honest and cheating only when both this payoffs are equal, which means 0 is equal to 40 minus 140 q, which means q is equal to 40 by 140, which is equal to 2 by 7, which means 1 minus q is equal to 5 by 7.

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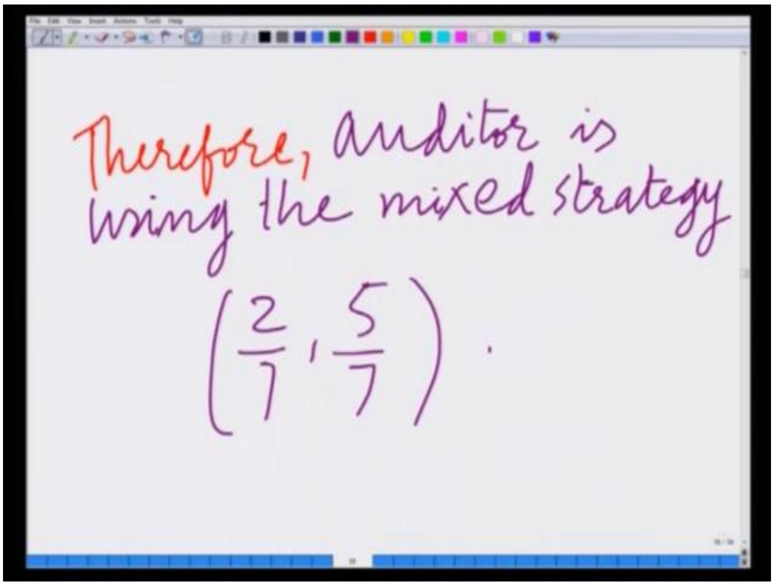

Therefore, the auditor is using the mixed strategy 2 by 7 or 5 by 7; that is randomly with probability 2 by 7 is auditing the form randomly with probability 5 by 7 is choosing to not audit a tax form at the Nash equilibrium.

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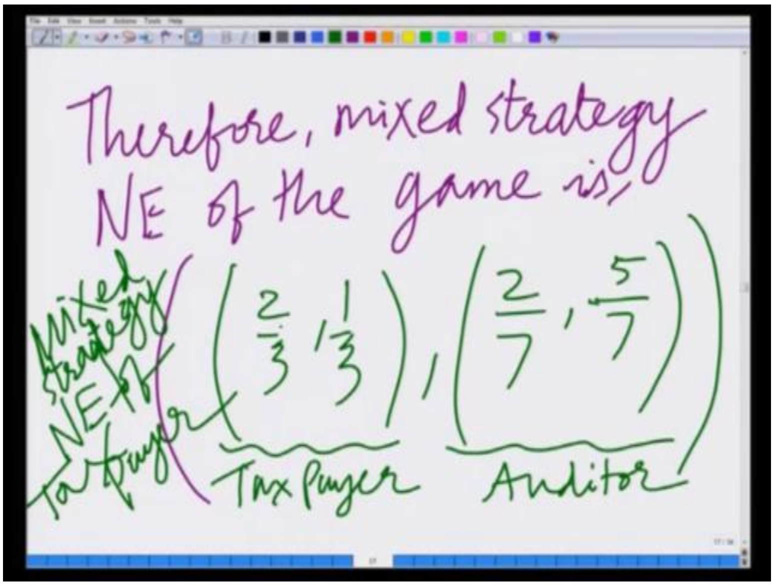

Therefore mixed strategy Nash equilibrium of this game is well for the tax payer, the mixed strategy is 2 by 3 comma 1 by 3. And therefore, the auditor the mixed strategy is 2 by 7 comma 5 by 7. So, this is the mix strategy of tax payer, this is the mix strategy of the auditor, which mean to say that art of a given population 2 3rds of individuals are honest tax payers, 1 3rds of the individuals are cheating or this honest with the taxes.

And the auditor is randomly choosing to audit 2 5ths or 2 7ths of the tax forms and is choosing to not audit 2 7ths or 2 5ths of the tax forms. This is the mixed strategy Nash equilibrium of the taxpaying game. This is the mixed strategy NE of the tax, this is the mixed strategy Nash equilibrium of the tax payers game. Hope you enable to appreciate this interesting application of mixed strategy Nash equilibrium to the population of tax payers. So, we will stop this module here and continue with the next module.

Thank you very much.