Applied Statistics and Econometrics
Professor Deep Mukherjee
Department of Economic Sciences
Indian Institute of Technology, Kanpur
Lecture-12
Hypothesis Testing (Part II)

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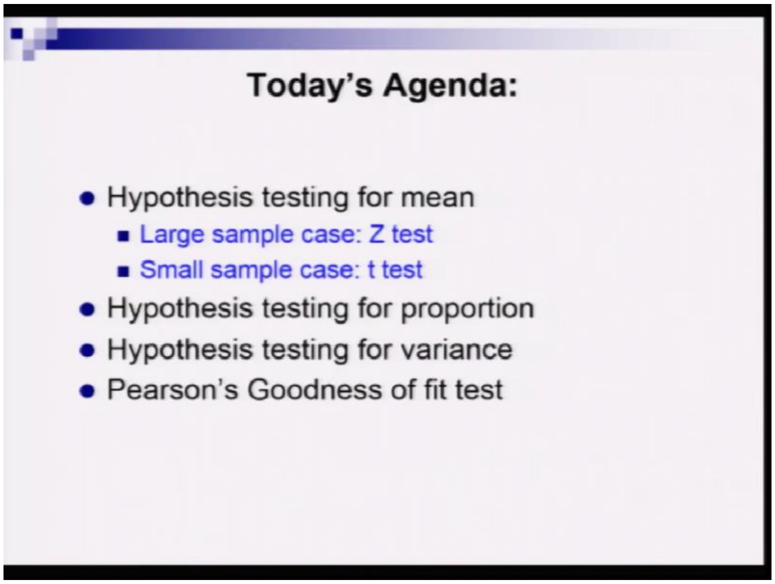

Welcome back to the lecture series on Applied Statistics and Econometrics. So today, we are going to continue our discussion on hypothesis testing. And specifically, we are going to look at a few 1 sample tests. So, let us have a look at today's agenda items. So, in today's lecture, we are going to cover hypothesis testing for population mean. And there we are going to cover 2 types of tests; one for large sample and that is z test and 1 for small sample case and that is the t test. And then, we are going to continue with testing procedure for population proportion and population variance. And we will end today's discussion with Pearson's Goodness of Fit Test.

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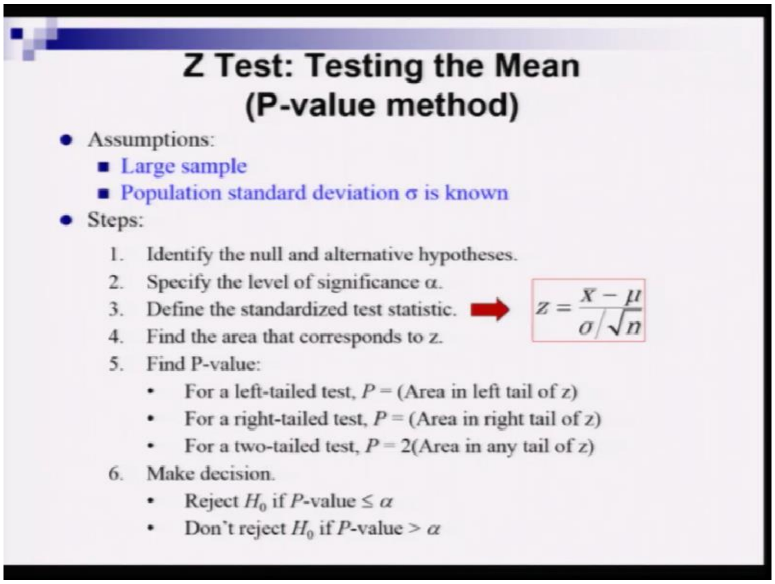

So first, we are going to talk about Z test and that is basically tasting the population mean. And if you remember, in the previous lecture, we have spoken about 2 different methods; 1 is the traditional method and the other 1 is p value method. So, we will first start with the p value method and then, we will go to the traditional method.

And I would like to remind you that when we are conducting as a z test, it is a large sample test. So, you can only apply this test when number of observations in a sample is greater than 30. And there is another assumption that population standard deviation sigma must be known.

So here, I am listing 5 or 6 steps that must be followed, if you want to conduct a z test. And note that here I am going to show you the steps for p value method. So, in the next slide, I am going to show you the steps for the traditional method. So, of course, the first step is to, you have to frame the null and alternative hypothesis for the test that you want to conduct. Then in step number 2, you have to specify the level of significance alpha. And if you remember the previous lecture, that is basically the probability of committing a type I error.

So, if I want to explain here a bit more, so you here fix a particular value of alpha, so that you can say that you are 1 minus alpha percentage confident about the outcome of the test. So, if you fix alpha equal to 0.1, then you can say that, well, I am 90 percent confident about the findings from the hypothesis testing.

So here, the third step says that you have to define the standardized test statistic and here that is called z score. And that is defined as the difference of a sample mean x bar and population mean mu divided by sigma over root n, where sigma is the population standard deviation and n is sample size.

And then in the fourth step, you have to find the area that corresponds to z. So, you have to get the probability values. Now, here note that, the probability values will differ on the nature of the test that you are conducting, in fact, on the nature of the alternative hypothesis that you have framed.

So, if you remember from the last lecture, there could be 3 potential scenarios. One is left-tail test, the other 1 is right-tail test. And the third 1 is called two-tailed test. So, if you are conducting a left-tail test, then you find out p as the area in the left-tail of the z distribution or the standard normal distribution.

And if you are conducting a right-tailed test, then basically you have to figure out the area in the right-tail of the z or standard normal distribution. And if you are conducting a two-tailed test, then the probability is going to be twice the area in any tail of the z. So, you can choose either left or right but, you have to multiply it by 2.

And those who do not remember what is left-tail, right-tailed and two-tailed test, you can wait for a couple of minutes in this lecture only, I am going to show you some examples and illustrations. So that discussion will remind you about the left-tail and a right-tail test. And in the last step, step number 6 you have to draw some conclusion, you have to make decision. We are going to reject our null hypothesis if the p value is less than or equal to the level of significance alpha that we have chosen in step 2. And we will not reject null hypothesis if p value is greater than the alpha, the level of significance.

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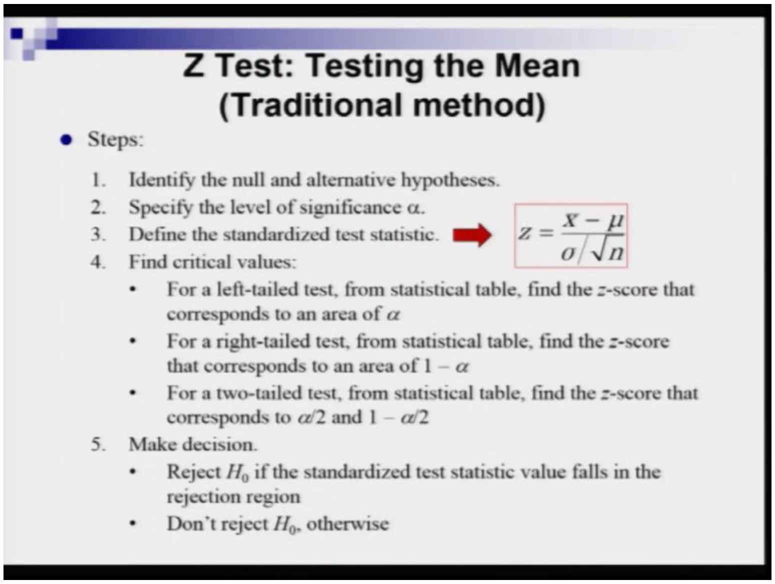

Now quickly, we have a look at the traditional method and then, we will discuss those steps involved in conducting a hypothesis test following the traditional method. So, you see that first 3 steps are same and I am not going to explain them again. The difference appears from step number 4. So here also, you will see that finding critical values depend on the nature of the alternative hypothesis and the rule changes, as you have a left-tailed test or you have a right-tail test or you have a two-tailed test.

So, why do we need to find critical values? Because in the traditional method, what are we doing? We are basically computing that standardized test statistic. And then, we see whether the standardized test statistic actually falls in the rejection region or not. If it falls in the rejection region, then we say that, well, we reject the null hypothesis.

So, how do I define my rejection region? That is basically the question that must be answered and the critical values actually help us in finding the answer. So, for a left-tail test, you have to find out the z score from the statistical table or from software that corresponds to an area of alpha that you have chosen previously.

Now, for a right-tail test, you have to find the z score that corresponds to an area of 1 minus alpha because this is basically a symmetric distribution. And if it is a two-tailed test, then from the statistical table or software, you find the z scores that correspond to the levels alpha divided by 2 and 1 minus alpha divided by 2.

And finally, you have to make a decision. Well, we have already spoken about the decision rule in the middle of the discussion, but let me repeat again. So here, you have to reject the null hypothesis, if the standardized test statistic value falls in the rejection region, and you do not reject the null hypothesis, if the standardized test statistic value does not fall in the rejection region.

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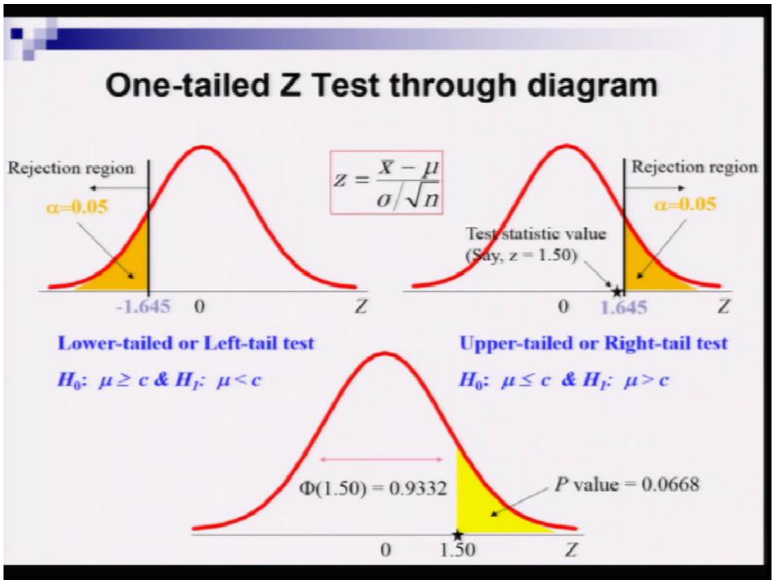

Let us now look at the one-tailed z test through some pictures and these illustrations will help you to understand some critical concepts well. We will save the pictorial depiction of the two-tailed test for numerical example. So there also I am going to show you graph but there I am going to deal with a particular problem. So, we are going to discuss the two-tailed problem later, separately. So, let us now look at the one-tailed test problem.

So, note that the one-tailed z test problem can also be of 2 types, right, so depending on the nature of the alternative hypothesis, so 1 is called lower-tail or left-tailed test and there you find the alternative hypothesis take the kind of form mu less than c, where mu is the unknown value of the population parameter and c some constant.

And the other type of the 1 tail z test possible is called the upper-tailed or right-tailed test. And in that case, alternative hypothesis takes a form mu greater than c. And I have already defined mu and c. So here, in the first top panel of the slide, I am going to show you the diagrams, which are going to explain you in detail how we actually make decision if we go by the traditional approach.

So, let us first look at the diagram at the left-hand side and that is the case of lower-tail or left-tailed test. So, that normal bell-shaped curve is drawn in red and that is the standard normal PDF. So, corresponding to the probability values I have the Z values, measured or marked along the horizontal axis. And of course, as you know by this time that 0 is the mean for the standard normal distribution.

Now, note that here let me explain the story by fixing a particular level of alpha because that helps. So let me fix the alpha at 0.05 or 5 percent level of significance because that is the mostly used by statisticians and econometricians. Now, if I fix alpha equal to 0.05, then basically I am talking about a 95 percent confidence level. Now, let me interpret that selection of alpha in terms of some area under the standard normal card, what you have to do? You have to go to the standard normal table and then, you have to figure out that for this corresponding value of alpha equal to 0.05, what is the value of the z?

Now, we are talking about the left-tail or the lower-tail. But if you remember, the standard normal table gives the values only for the other side of the standard normal curve or it gives the values, it shows the values for the right-tail. So basically, whatever value we observe there, you have to put a negative sign in front of that and then that is basically the critical value that you have obtained from the statistical table.

So here, you see that I have plotted that critical value minus 1.645, remember that it is a very important number. It can also be called a magic figure. So, for hypothesis testing, if you were selecting alpha’s value at 5 percent and if you are conducting, left-tailed or right-tailed whatever, if you are conducting a one-tailed z test, then 1.645 is the magic figure. That is basically the value of the, the critical percentage point on the z axis.

So, basically what will happen if I shade this area which is bounded by a vertical line drawn at minus 1.645 and red standard normal curve, and if I mark that area or shade that area by orange color, then basically that area's value is 0.05. So, that is what alpha tells me. So alpha is basically the probability of committing a type I error.

So basically, if I now reduce the alpha value to 0.01, it implies that I am talking about a 99 percent confidence limit, then what will happen? So, it will have an impact on the area under the standard normal curve bounded by the critical value, so of course, the area is going to be less so that means that the critical value now is going to move towards the left direction. And it is going to be even smaller number and corresponding area would be smaller area.

So, now let me move to the case of upper-tail or right-tail test. And here also, you have that same standard normal PDF in red color bell shaped curve. And everything is same, but the only difference is that as we are doing a right-tail test here, the critical value, I have to take 1.645 from the standard normal table and we do not have to assign the negative or minus sign in front of the critical value. So, at that critical value, I have again, erected a vertical line.

And then the area which lies between the red curve, the standard normal PDF and that black vertical line that is marked in orange color again and that basically is giving me now the area of 0.05 unit. So, that is basically the area and that is called the rejection region. And I am saying that the probability of committing type I error here is also 5 percent. So, for a left-tail test or lower-tail test rejection region lies at the left of the vertical line that you elect or draw at the critical value.

And if the test statistic value z falls in that region, it implies its value is less than minus 1.645 then you can actually reject the null hypothesis and you say, well, the sample evidence is such that it is the sample statistic that I calculate, say sample mean is far from the true value of the anticipated value of the population mean and hence, I declared that I reject my null hypothesis.

So, you now look at the other side of the story, which is an upper-tail or the right-tail case. So here, if the test statistic value, the z score is higher than 1.645 it lies in the rejection region, which is marked in orange color. And then you reject your null hypothesis.

And if by chance if you get a test statistic value which is lower than 1.645 as I am showing here, one such value 1.5 of course, it is just for illustration and I have marked that particular value with an asterisk on the z horizontal line, you see that these asterisk actually is not falling in the rejection region, which is marked orange. And hence, you can say that, my sample gives me or generates a statistic value, which is lower than the critical value. And hence, I am failing to reject my null hypothesis.

So, what will be the case if I now try to adopt a p value approach, so, that is what is being displayed or illustrated in the last or the bottom part of the slide. And here you see, I am again showing you that standard normal PDF in red. And there is difference in from the top diagrams here.

Here, I have not plotted any critical value, because critical values are used when you are taking a traditional approach. But when you are taking the p value approach, then what you have to do? You have to calculate the value of the standardized test statistic or z score. And as I assumed in the previous case, let me stick to that example or illustration or numeric value of 1.5.

So, then you have to go back to the standard normal table and then, for the value of 1.5 you have to now figure out the phi value, capital Y value or the cumulative probability value for 1.50 from the table. And if you go back to the previous lecture, where I have shown you the standard normal table or you can use any software also, you can calculate the value of capital Phi at 1.50 and that basically is 0.9332.

So, if you are conducting a left-tailed test, then what will happen you have got some number says z prime and then, you plot that z prime on the z axis, you go back to the standard normal table and then you will find out the probability again, the capital phi value from the standard normal table.

So, you have to draw a vertical line at the calculated value of the z score or the z statistic and that area which is lying at the left of that vertical line and that is bounded by the standard normal PDF that gives you the p value. So here, let us come back to the upper-tailed or right-tail test case. And here you see the test statistic value is 1.5 as I have assumed in the previous right-tail test example, so I have marked that point with an asterisk here.

And if you draw a vertical line at that point, then basically what happens the area that lies at the right hand side of that vertical line at 1.50 and bounded by the standard normal curve, that area actually gives you the p value. And here, you see the p value is 0.0668. So, if you deduct 0.9332 from 1, you are going to get these numbers 0.0668.

So here, you see that I have shaded that area with yellow color. Now, how to make a decision in this p value approach? So here, I already had told you that you have to compare the p fixed significance level or the value of alpha. And if the probability value or the generated probability value is higher than the fixed value of alpha, which is the case in this case, because here I have started the story by fixing alpha at 0.05. Then you do not reject the null hypothesis.

And if the p value you obtained by doing all these exercise, if it is less than 0.05 say 0.04 in case, then you will reject the null hypothesis. So, that is the way you make a decision in the p value approach.

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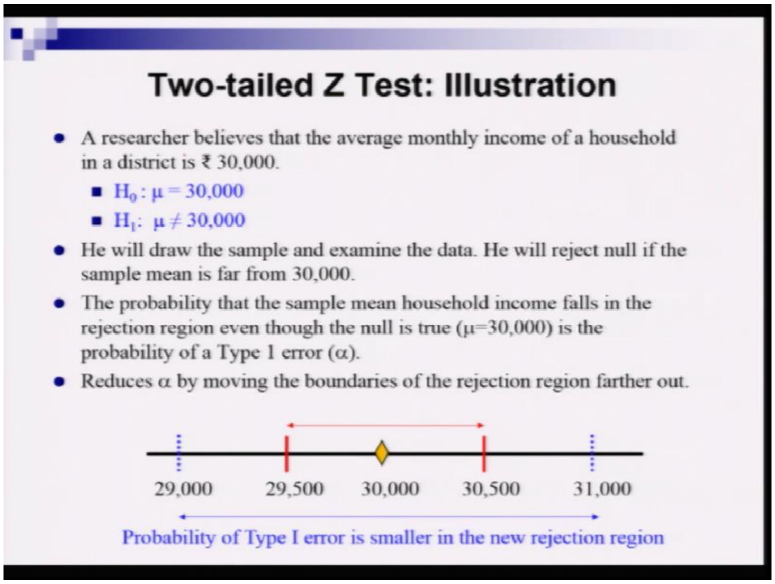

So now, we are going to look at the two-tailed z test through an illustration. So, let us assume a hypothetical story or case. So, a researcher believes that the average monthly income of households in a district is Rs. 30,000. And of course, needless to say that this is hypothesis, because it is a belief and the researcher wants to test that belief from real life data.

So, the researcher can start by framing the null hypothesis, where he says that the mu the unknown population parameter mean is 30,000. So, the probability that the sample mean household income falls in the rejection region, even though the null is true is the probability of committing a type I error or alpha. And you want to reduce these alpha as much as possible.

So, generally, in standard practice, alpha is set at 0.05. Some people want to know set alpha equal to 0.01. But whatever story we are going to tell that will not depend on the choice you make at this moment, because I am going to give a very general picture here. So, what is the duty of reducing alpha? So, reducing alpha actually tells you that you have less chance of committing a type I error, which is a very bad thing to do in statistical investigation.

Now, how can we reduce alpha? We can reduce alpha by moving the boundaries of the rejection region. So, let us look at the diagram that I am showing here at the bottom of this slide. So, I am assuming here that the unknown population mean is at 30,000. And that point I have marked with the diamond, golden color diamond. And the researcher can assume that, if the sample leads to a sample mean of some number in the range 29,500 and 30,500 then he is going to be sure that well, indeed the monthly average household income is 30,000 for the population.

So here note that the researcher is actually creating some confidence interval of width 1,000, so 500 on the left hand side and 500 on the right hand side. And that, of course, will lead to some decision making. But what if the researcher decides that, let me expand the interval. And then, I am happy if I find a sample mean is between 29,000 and 31,000. So, he has actually widened the confidence interval. And basically, he is now saying that error margin has increased, and he does so then basically, he is reducing alpha.

And note that, as probability of type 1 error is smaller in the new rejection region, which is basically the values less than 29,000 and above 31,000 the chance of not failing the null hypothesis or loosely speaking, accepting the null hypothesis has gone up, because the interval or the acceptance region has been widened.

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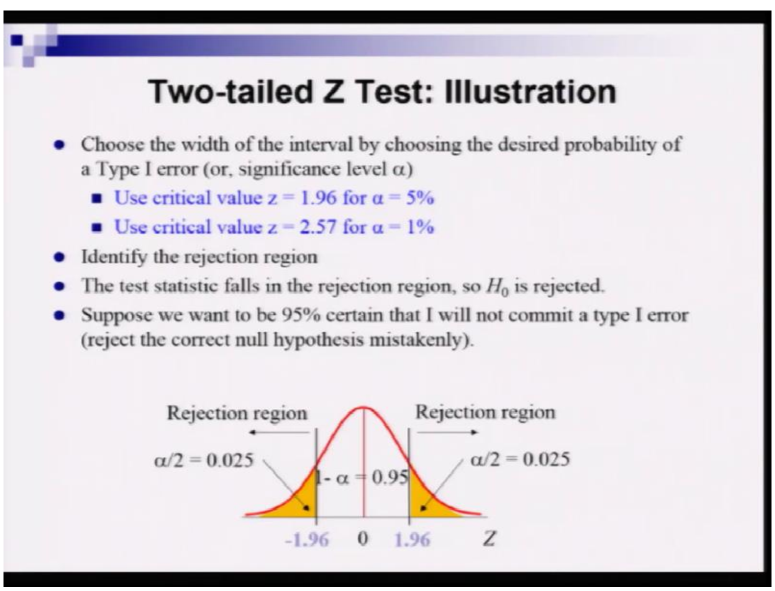

So, as I have already said, you have to choose the width of the interval by choosing a desired probability of committing a type I error. And generally, people go by alpha 1 percent or alpha 5 percent, mostly it is 5 percent. So here now we have to identify the rejection region. And if the test statistic falls in the rejection region, then you reject your null hypothesis.

So basically, what happens that you look at the diagram that I am showing at the bottom here. So, the confidence limit is 1 minus alpha that is 95 percent. So now, you have 5 percent of the mass as residual and as you are doing our two-tailed tests, actually, you will have to look into both tails, both right and left and the rejection region will appear in both tails.

So basically, now you have to split equally the residual mass which is 0.05. And why you have to split equally because simple standard normal curve is a symmetric curve. So, you have to split it half half and you see that here on the z axis I have marked the critical values minus 1.96 in the left-tail or for the left-tail and plus 1.96 for the right-tail.

And if you draw no vertical lines at these 2 critical values, then rejection regions are basically clearly marked. And that is basically an area which is marked orange and that is basically an area bounded by the standard normal PDF and the vertical line drawn at the critical value. And area in both cases or in both tails is 0.025, it is basically 0.5 divided by 2 or alpha divided by 2.

So here, if you now find out a test statistic value, which is say 1.25 then what do you decide? So, that value 1.25 actually is not falling in either of the rejection regions. So, that actually is in the acceptance region in this diagram and you can say that okay, I fail to reject my null hypothesis.

But suppose, you get a test statistic value, which is a minus 2 then what will you conclude? Then the minus 2 test statistic value actually falls in the left-tail rejection region and you say that okay, I am rejecting the null hypothesis. And what if you find a test statistic value of 2.3, then that value will actually fall in the rejection region in the right-tail and you will decide that I am going to reject my null hypothesis. So, I hope that these pictorial depiction of two-tail z test is helpful to you.

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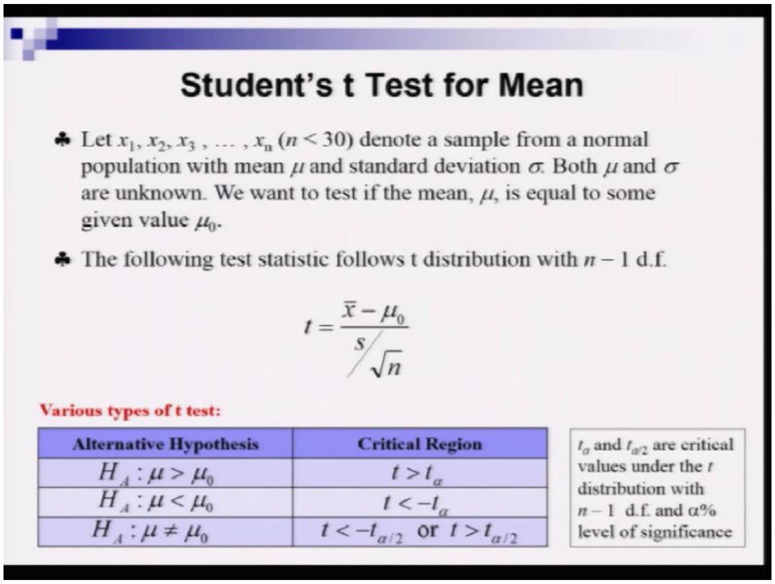

So far, we have discussed the case when we are dealing with large samples. So, we have n greater than 30. But what if we do not have so many observations in our sample? If we are dealing with small samples, can we go for a test for population mean? Yes, we can. And in that case, we have to perform our student's t test. We have already spoken about student's t distribution in the previous lectures. So, now, you are going to consult t table and t distribution to perform a t test in the case of small sample.

So here, we assume that we have a random sample from a normal population with mean mu and standard deviation sigma, both mean mu and sigma are unknown. And it is a very important thing that I would also like to notify here that in the previous case, when we conducted the z test for large sample, then we assumed that sigma is known, only mu is unknown.

So, when sigma is not given to you and you deal with small sample then you should not actually base on the sample variance and proxy unknown population variance by the sample variance. So, when you have sigma unknown you should apply a student t test.

So here, we want to test whether the mean mu is equal to some given value mu naught or naught. And we have to now come up with a new test statistic, the z test statistic will not work, because we are dealing with small sample. So, we now define a new test statistic which is defined as t and that is basically x bar minus mu naught divided by s over root n.

But you note that, although it is a new t statistic, it is very similar look wise if you compare this test statistic with the z test statistic, because last time in the z case, I just want to tell you that s was not there it was sigma because sigma was known. But here, as sigma is unknown, we replace it by s. So, now, this test statistic will follow a t distribution with the n minus degrees of freedom. And we have spoken about why degrees of freedom shall appear in this case. So, I am skipping that discussion again.

So, now, there are 3 types of t tests, which are possible in this context. So, of course, right-tailed test, left-tailed test and two-tailed test. So, let us go through 1 by 1. And here, I have summarized everything in form of a table. So, in column 1, you are going to see the alternative hypothesis because null hypothesis is common for all of these cases, and that is basically h naught colon mu equal to mu naught. And for the three different types of alternative hypothesis, I have shown the critical region that is there in the second column.

And, they are very similar to the z test that we have just performed and explained. So, to save time, I am not going to repeat why, you see 1 particular critical decision against 1 particular alternative hypothesis. Just try to we know link this with the discussion we had on the z test. And it should be no clear to you.

So, here t alpha and t alpha divided by 2 are basically the critical values under the t distribution with n minus 1 degree of freedom and alpha percentage level of significance. So, that is the only difference that you have to remember that you are not going to use the z critical values, you are going to consult t table and come up with t critical values. That is the only difference. Otherwise, the test works exactly in the similar fashion as we have studied the z test.

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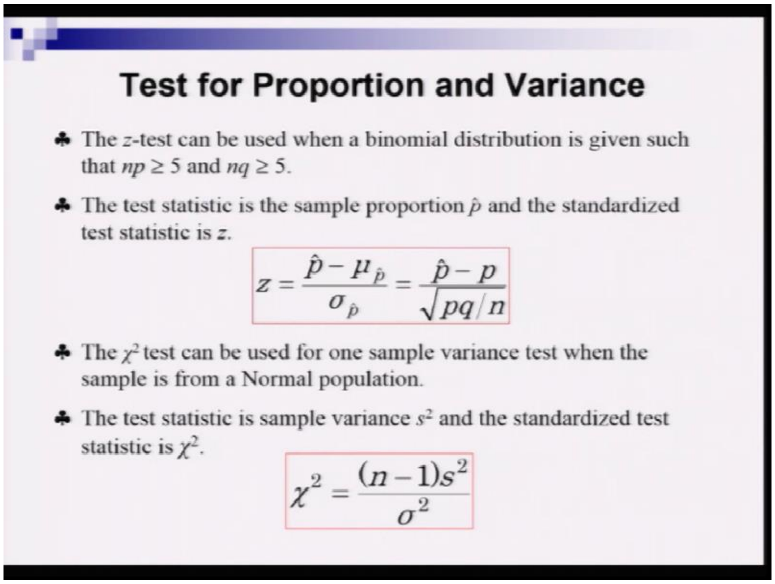

Now, quickly, let us go through the testing procedure for population proportion and population variance because sometimes these 2 also could be of interest of the researcher. So, the tasting procedure is very similar. So, I am not going to bother you by showing same old steps and approaches. You fix which approach you want to follow, the traditional approach or the p value approach and then, you start conducting the test. But here, I am going to give you the test statistics because test statistics will differ from the z test and the t test that we have done for the testing purpose of population mean.

So here, we will first discuss the case of population proportion. So, z test can still be used, when a binomial distribution is given such that our data is telling np is greater than equal to 5 and nq greater than equal to 5. So, we have spoken about the normal approximation or binomial several times in this course, and you see that this is such a useful thing that we are again coming back to that result in hypothesis testing section also.

So here, the test statistic is basically the sample proportion p hat, which is x divided by n, I know I have had a discussion regarding this in the previous lectures, I do not want to repeat. And the standardized test statistic z is now going to look very different, although it is the difference between the sample mean and the population mean divided by the standard deviation.

But now, everything is being told in terms of the proportion. So, expression is a bit different, the final expression looks different. And then you conduct your z test by following either approach, p value approach or traditional approach. So, to test for population variance, we have to make use of the chi-square test and we have not done chi-square test before. So, it is going to be of some interest to you, because it is a new thing that we are going to learn now.

So, actually, what you have to do, you have to frame the test statistic first as usual and then that is defined as the n, the degrees of freedom, n minus 1 times the sample variance s square and that is divided whole by the sigma square the population variance. So, we have discussed this chi-square statistic previously. So, that is basically the focal point in this slide, that if you want to conduct a variance test, then we have to make use of a different test that is chi-square and the test statistic will be completely different and that is a chi-square test statistic.

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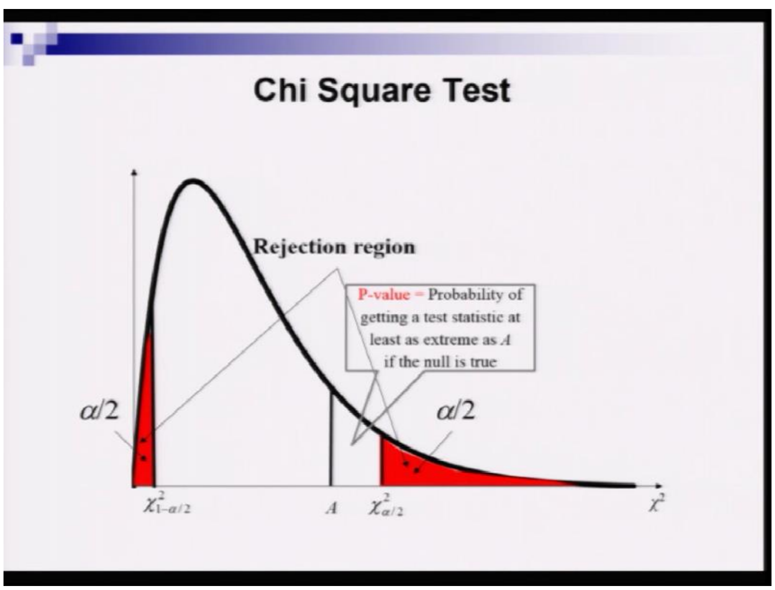

So, in this slide, we are going to show you the working of chi-square test through a simple diagram. So, as the horizontal axis is measuring the chi-square random variable, the y axis or the vertical axis is measuring the probability of chi-square random variable. And as I have told you previously that chi-square is an asymmetric distribution. So here. I have plotted 1 of such shapes and why I am seeing 1 of such shapes, because the shape of the chi-square distribution can vary depending upon the degrees of freedom.

So here, I am showing you just 1 possible shape of the chi-square distribution. And now, if you want to conduct a chi-square test, there could be 2 different approaches. 1 is the traditional method and the other 1 is p value method. So first, I am going to show you graphically, how to conduct a traditional method-based chi-square test. So, for that approach, you have to find out the critical values. And here, I am showing you the case of a two-tailed test, because that is the most complicated 1. So, for two-tailed test, you have to get 2 critical values, 1 for the left-tail or the lower-tail and one for the upper-tail or the right-hand tail.

So, if you want to find the critical value, first, you have to set the level of significance. So, let us assume that that is alpha and alpha could be 0.1 or 0.05. It does not matter; it is up to you. But some alpha for finding the 2 critical values for chi-square, what you have to do? You have to basically divide that alpha by 1 and then, for that alpha divided by 2 level of significance, you have to go to the chi-square table and then you have to figure out the critical value chi-square alpha by 2. And that is basically the value of your right-hand tail.

Similarly, you can also figure out the chi-square value for the lower critical limit or lower critical value. So, let that be chi-square 1 minus alpha by 2. So, I have denoted these 2 critical values both for lower tail and upper tail. And then, if I focus on the right-tail now then the area which is bounded by the vertical line at chi-square alpha by 2 or the upper critical value and the chi-square PDF, that is marked in red, and that is basically my rejection region and that has the probability value alpha divided by 2.

And for the left tail, I see that the area which is bounded by the chi-square PDF and the vertical line drawn at chi-square 1 minus alpha 2 that is also marked in red color and that also has probability value alpha by 2. So, you see that level of significance alpha is split into 2 equal parts half, half for both left-tail; and right-tail.

Now, how to set a decision rule? So, you set a decision rule that if the calculated value of chi-square statistic falls in either of these 2 rejection regions, you reject your null hypothesis. So, your chi-square statistics then has to be either very small as it falls in the left rejection region or it has to be very high. And in that case, it will fall in the right rejection region. And if you get chi-square calculated value somewhere between these 2 critical values, so then in that case, actually, you fail to reject the null hypothesis. So that is basically the decision rule or the working of a chi-square test, as per the traditional method.

Now, if you want to conduct the chi-square test by following the p value method, then what to do? So, in that case, you concentrate on the obtained value of the chi-square statistic and let us assume that value is a; and here, I have shown an arbitrary value a on the horizontal axis; and then, you draw a perpendicular vertical line at a and then, it will partition the area below the chi-square PDF into 2 parts, 1 in the left-hand side of it and 1 on the right-hand side of it. So, the area that is basically in the right-hand side of it has the interpretation of p value.

Now, let me remind you again here, what is p value? So, it is the probability of getting a test statistic at least as high as a if the null hypothesis is true. So basically, here, that area which is bounded by the vertical line at a and chi-square PDF, gives you the probability of extreme the test statistic values, which are higher than the calculated test statistic value a. And if this probability value or the mass or density, whatever you want to call it or probability is higher than the p fixed level of significance alpha, then you can fail to reject the null hypothesis. And if the p value is less than the previously fixed alpha value, then you reject the null hypothesis. So that is basically the working of chi-square test under the p value method.

So, I am not going into the theoretical details, telling you step by step in words or sentences, because we have already laid the scheme very clearly at the beginning of the lecture. And now, I think it is not a bad idea if I go through the same steps, but with the help of some examples.

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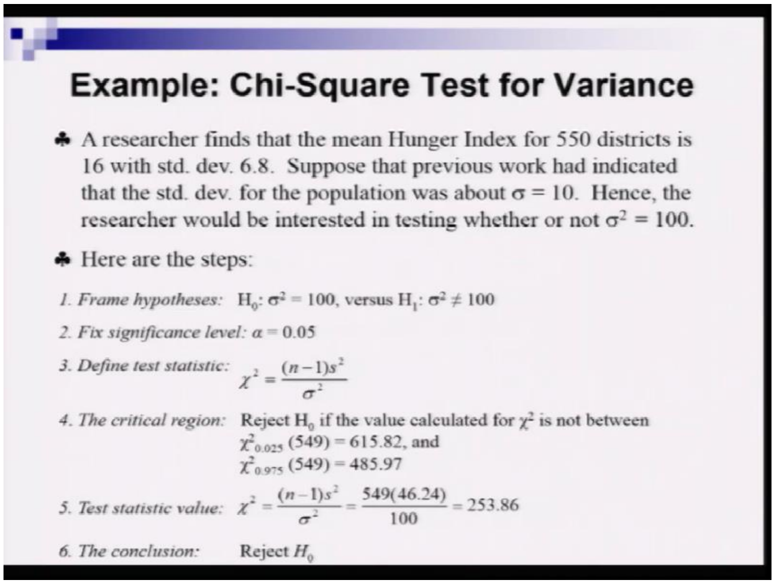

So here, I am going to show you a hypothetical example to show you how to conduct a chi-square test for population variance. Some of you may know that there exists something called a global hunger index that is published by international research organizations annually and they actually rank countries.

So, we take that hunger index, in our case, but of course, all fictitious numbers and this is totally hypothetical. So, suppose there is an economics researcher, who actually finds that the mean hunger index for 550 districts in India is 16. And the standard deviation of the hunger index is 6.8.

Now, also assume that, even before these guy did this job or work, even before this researcher came with his results, maybe 5, 6 years before another group of researchers have conducted some research on the same topic, and these previous literature indicates that the standard deviation for the population was about 10.

So, sigma used to be 10 in some other study. So, of course, the researcher actually is interested to know, whether the standard deviation, he has indeed gone down or not or is it just by chance, he has got 6.8. But actually, it is still you know, 10. So, the researcher would be interested in testing whether or not sigma square is equal to 100.

So, you start by framing the hypothesis and the null is sigma square equal to 100, the square of sigma and you test that versus the alternative hypothesis, which says the sigma square is not equal to 100. And as I told you, you how to fix the significance level, so, you set at 0.05 following the standard practices.

Now, you define your test statistic. I told you that it is chi-square that will be used in this case and so, you define your test statistic, the chi-square random variable. And then, you actually, let us assume that we will follow the traditional approach, the critical region approach, we are not going to follow the p value approach.

So, if we are doing so, we have to basically figure out the critical values and these critical values will come from the chi-square distribution, right. And you note that these number test statistic value is not falling in the range created by these 2 critical values that came from the chi-square table, namely, 485.97 and 615.82. So, you decide that I am rejecting the null hypothesis, based on the sample evidence, of course. So, in today’s lecture, we could not finish the Pearson’s Goodness of Fit. So, in the next lecture, we are going to start from there and then, we are going to continue with more interesting hypothesis testing problems. So, see you soon. Thank you.