**An Introduction to Microeconomics**
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**Lecture - 94**
**Cost Minimization: Few Examples**

So, now let us solve some mathematical we have looked at it graphically, some problem we will solve graphically and then we will derive the mathematical way to solve the this cost minimization problem.

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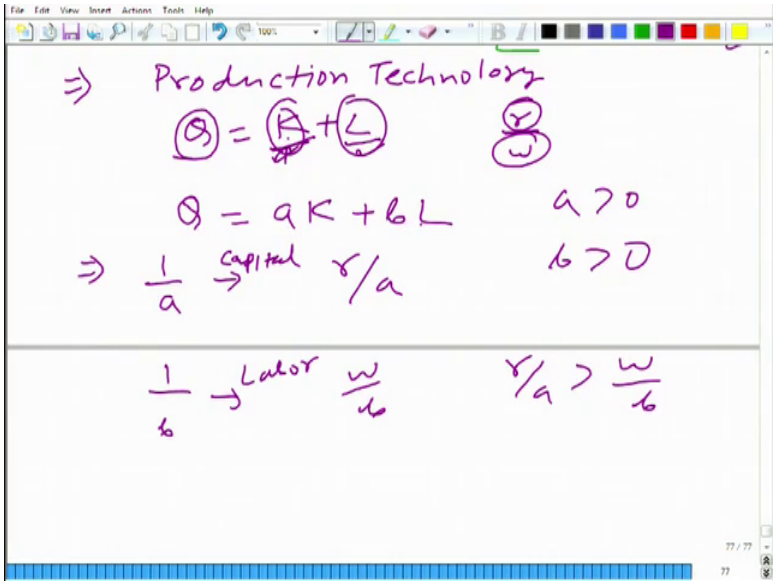


What we have done? Let us say where production technology is, let us say production technology is given by Y or Q that is what we have been using Q is equal to K plus L ok, price of capital is r; rent for capital is r and wage for worker is w. The unit should be corresponding to each other. If you are using capital here per day then here r should be in per day term. If L is here in the per hour term we cannot have these two in the different terms first of all, if it is in per day this should also be in the per day term then only we can add fine.

So, how can we solve? We have solved similar problem while when we discussed consumer theory.

Student: Straight line Isoquantizer straight line.

Isoquantizer straight line.

Student: So, this must come on the end pl ane, it is either when the x axis on the y axis.

Ok, that is one way to think about it and right, you are right you are not wrong. But I will say the more intuitive way to describe and what is more intuitive? What you are interested in? You are interested in Q amount of output and how you can produce? Using capital and labour and what it does not matter, how many units of capital you are using and how many units of labour you are using. What matters is that the sum of cap units of capital plus unit of labour is equal to the amount of output that you need ok.

So, now it is very clear that if rent is if capital is expensive than labour, then it is good idea to use only labour and if labour is more expensive than capital then it is you use only.

Student: Capital.

Capital fine. So, you will use only one of these two inputs in the rare case where capital and labour both have equal price. Then only you can use the combination of two ok, otherwise you will use the cheaper of these two.

So, let me make this little more complicated. How about when Q is equal to a k plus bL, where a is where a is a positive real number and b is also positive real number, rent is still for capital r and wage for labour is still w. What will you do? Is it clear? Are you guys following me?

Student: Sir, if first we would check on, we have to check on both simultaneously on prices as well as output (Refer Time: 03:28).

The simple thing that you should.

Student: W r by a is greater than w by b.

Student: Then we use all labours.

So, right logic let us see here, what is happening? You are in the right direction to produce 1 unit of quantity, let us say if you are using only capital. If you are using only

capital to produce 1 unit of quantity, 1 unit of output how many units of capital do you need? 1 by a.

Student: Q by a Q by a.

I am saying Q is equal to a, so you need 1 by a unit of capital and to produce this output using only labour, how many units of labour do you need? 1 by b. How much is the cost of this capital?

Student: R by a.

R by a. How much is the cost of this labour?

Student: R w by b.

W by b. Now it is very clear from here. If r by a is greater than w by b; it means it is expensive to produce this output using only.

Student: Capital.

Capital. So, you will use only labour. So in this case what will be your cost? Your cost is to produce units of output is going to be.

Student: W by.

W by b by multiplied be Q or Q naught, if you whatever you want to see. Why w by b? Because, 1 unit of labour would produce b units of output.

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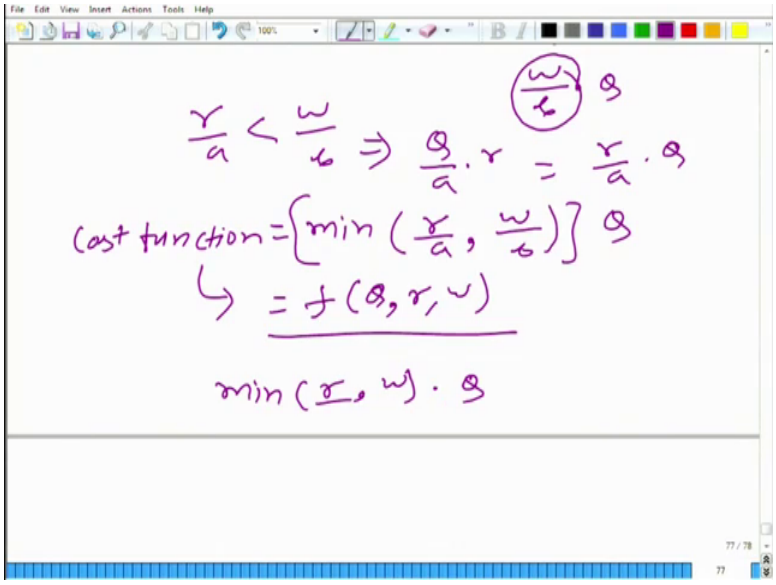


So, total if you are using L you will produce w by b and that is for 1 multiplied by Q. So, what you are going to get? Q b by b and if r a is less than w by b, then you are going to use only capital and to produce Q naught amount of output how many units of capital do you need? Q by a and what would be it is price?

Student: R.

Multiplied by r. So, in we can write it like this r a multiplied by Q. So which one will you use? Minimum of these two, so, cost to produce Q amount of output is going to be minimum of r by a comma w by b whole multiplied by Q and this is called cost function; this is cost function. What is cost function? When cost function is not any cost to produce Q amount of output, cost function gives the minimized the minimum cost that you would incur to produce Q amount of output.

So, cost function is always going to be a function of how many units you want to produce, let me write it here. How many units you want to produce? Rate of rental rate of capital and wage rate for workers. So, it is always going to be a function of Q r and w.

The earlier problem that I gave you where we a and b where equal to 1, what would be the cost function? It will be simply minimum of r comma w multiplied by Q. Let me change the problem little bit. Now let us say both inputs are used to produce some output and these two inputs are perfectly complement each other. In other way what it means is that Q; to produce Q amount of output what you need is; a amount of capital; a units of capital and b units of labour. Can you give me the production function first.

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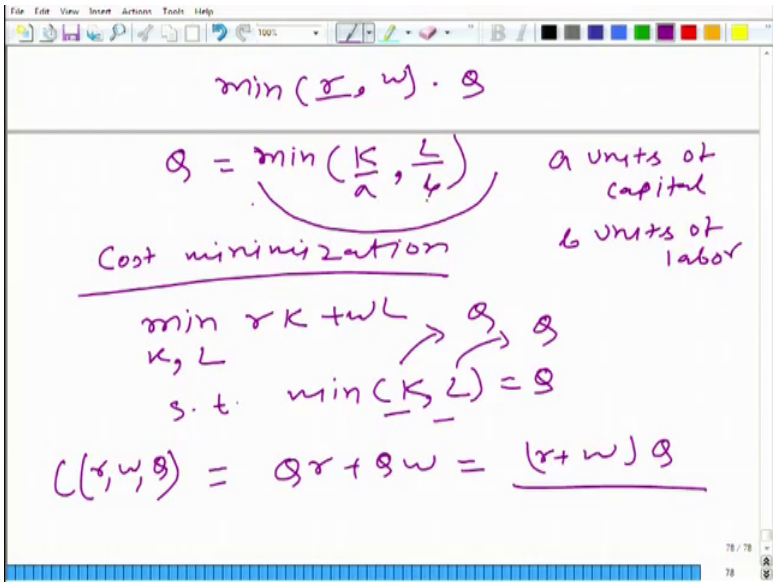


Student: (Refer Time: 07:38) Minimum of.

Think about it, think about it for a moment.

Student: Q by comma a by b.

Yes.

Student: Minimum of.

So, minimum of minimum of.

Student: K.

K by a comma

Student: L by b.

L by b. This is your production function,

fine. It is very similar to what we have done in the consumer theory and we had derive we had derived it there in detail. So, let us not waste time about it. So now tell me the cost function.

Student: Minimum of k by.

First let me solve make this problem easier for you where a and b are equal to 1.

Student: Into minimum of k comma L plus b r into.

Think about it again, we are not using any mathematics, any calculus. A simple that, you have to think about, you are little bit in the right direction.

Student: R and.

What we are doing here? Let me write the what is our purpose here. What we are doing? Cost minimization, let me write the complete problem cost minimization. In other word what we have is minimize rk plus wL with respect to k and L, such that minimum of k comma L is equal to Q, that is what we are doing.

Student: W plus r into Q.

W plus r into Q that is fine; that is you do not have any minimum or something there.

Student: Same thing minimum of.

[FL] you may be right, I did not pay that much attention but say that is what will happen. Now, to produce let us say to produce 1 unit of output what do you need? You need at least 1 unit of capital and 1 unit of labour. If you have more than 1 unit of capital and just 1 unit of labour, what you will have? Some unit of capital you would be wasting. So, we are talking about minimizing the cost, so we should not be wasting anything. So, what is the most the cost minimizing way to produce it to have the exactly 1 unit of capital and 1 unit of labour and how much is the cost of 1 unit of capital and 1 unit of labour?

Student: R and.

R plus.

Student: R plus w.

W and of course, if you are producing Q then what do you need rather than 1 unit of capital, and 1 unit of labour, what you need is Q amount of capital and Q amount of.

Student: Labour.

Labour. So, cost is going to be Q multiplied by r plus Q multiplied by w or it is going to be r plus w multiplied by Q. So, C of let us say write it here r w comma Q is r plus w multiplied by Q, you can solve the more difficult problem on your own.