An Introduction to Microeconomics

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Lecture – 85
Returns to Scale

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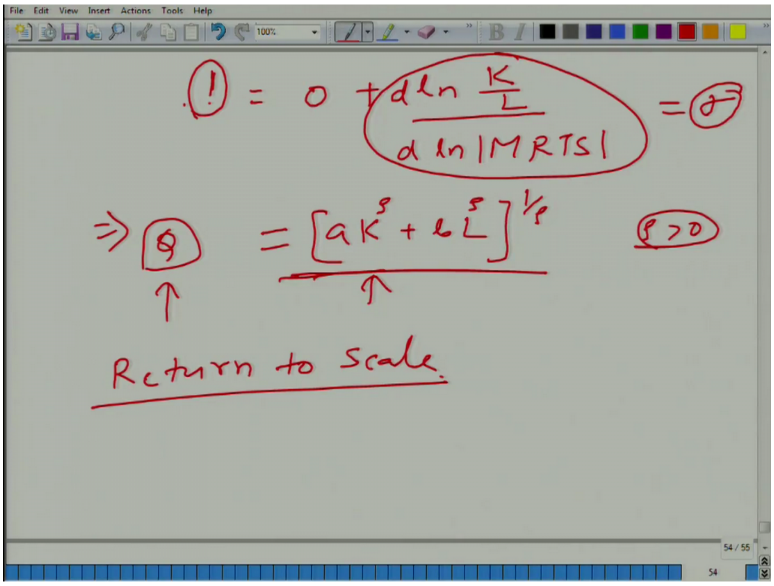

Now, let us talk about return to scale, and then we will talk about elasticity of scale. What is return to scale?

Student: The increasing the inputs by a fixed factor, how much the output increases.

So what we have been doing so far, if you pay attention that what we did earlier that we took a production function, a simple production function F of K and L fine; and what did we say; that let us talk about production function either in the sense, that we are talking about production function in one variable or in short run one variable or short run and that is how we fixed the value of K to K naught and we varied only L that is, what we did that is one thing we did.

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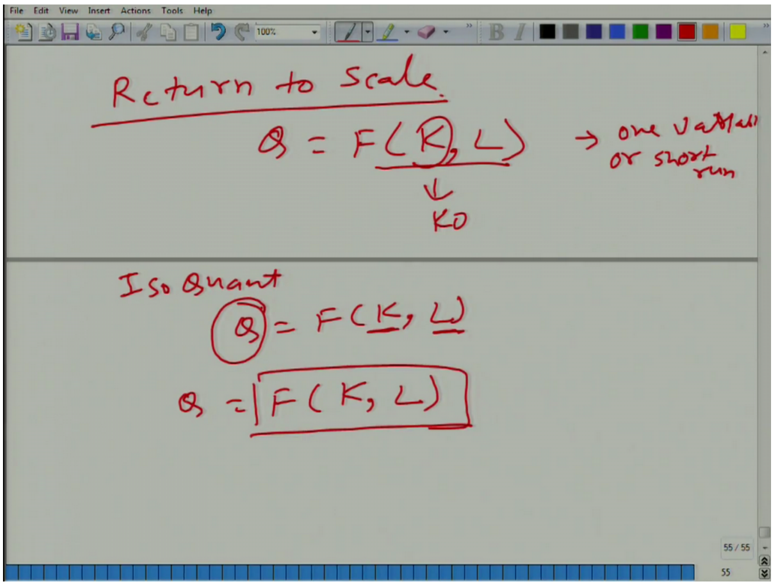

Second when we talked about ISO Quant, what did we do; we started with this production function, and we say that we will allow K and L to vary, but only in a particular way, so that; and what is that particular way; that Q remains fixed ok. So, something that we have been fixing, third way we are going to now again with the same production function we will take and, what we will say that these factors of productions are increased in the same proportion and we want to see what is it is effect on output.

So, let us say one thing that we can do starting with this let us say right now, Q is equal to 10 just for example, and we are producing with some combination of K and L ok.

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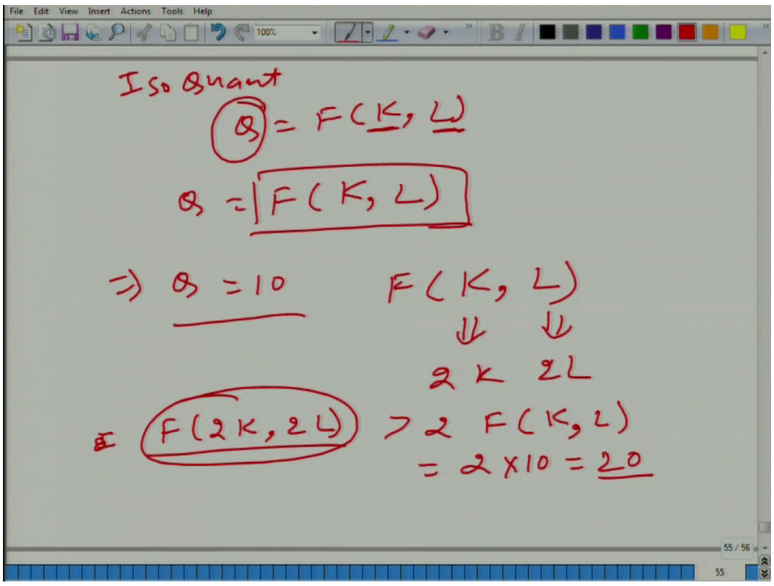

Now, let us say we double the K and we double the L ok; we double the K and we double the L fine. So, there are three possibilities, when we do that and of course, this is this is the increased you know that the new the production at the new label. There are three possibilities, that this is either greater than and what does it mean that this is greater than two multiplied by 10. So, when we double the capital and double the labor at the same time output is more than.

Student: Double.

Doubled fine this is one possibility.

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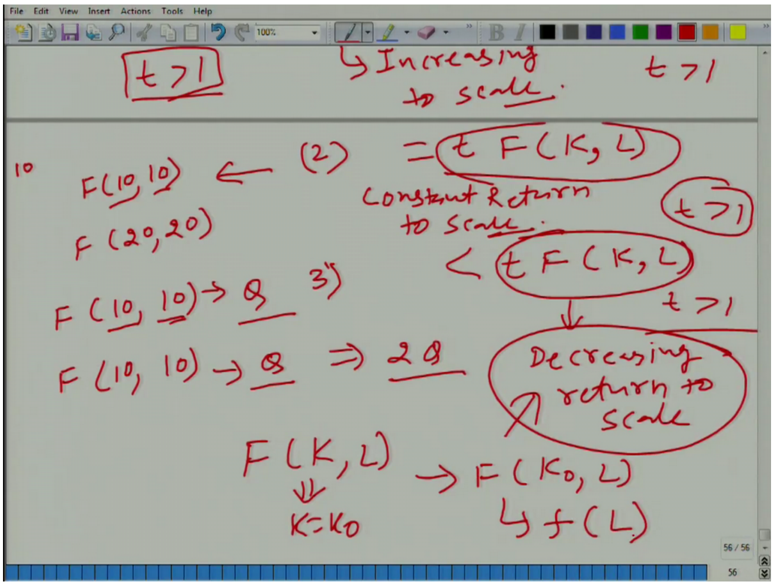

The second possibility is that output is equal to twice the earlier output, that is 20 in this particular case and third is that it is less than and that it means that it is less than 20. Now, see what is happening; we are increasing capital and labor in the same proportion.

Now, what I am going to do instead of using a particular value, I am going to make it more general. We are scaling up the operation, what does it mean; that we are increasing the input and output in the same proportion here; let us say that factor is t and of course, we will take t greater than 1, because we are talking about scaling up of the operation, we are increasing it. And instead of you are using 2; we can use here t and get rid of this. There are three scenarios and of course, keep in mind that at all place t is greater than 1 fine.

So, three things can happen and one of those three things, we have we have described all these three options. So, let us start x; start with number 2 ok, what is happening; let us say again, if we go back to let us say Q we are using with 10 capital and 10 labor and we are producing 10 units of output ok. Now, what we are doing; we are taking 20, 20. So, what we can do; if we have capital and labor as factor of production at least this much we can do we can start with 10 comma 10 means 10 units of capital and 10 units of labor and produce the output.

And since after using these 10, 10 units we will have 10 more units of capital left and 10 more units of labor left; we will again use and what we will get Q. So, we end up getting 2 Q. So, if we are able to replicate the production process, what will happen; we will get it equal to P multiplied by F of.

Student: K.

K comma L and this is the case called.

Student: Constant.

Constant return?

Student: To scale.

To scale fine; is it clear pay attention to this what I am writing; t is greater than 1, I will come back to this t greater than 1. Now, what more can happen is that; now you are using more labor and more capital; you know sometime it happens that when you have a bigger a bigger scale of operation some new production technique you can use and that may increase the output you know in more than this scale and if that is the case we call it.

Student: Increase (Refer Time: 06:48).

Increasing return.

Student: To scale.

To scale can you give me an example, where you know simple example where we can observe increasing return to scale.

Let us say wherever one simple example that comes to my mind is wherever you are using let us say pipe, you know for either air conditioning duct or getting the oil from the earth, if you double the size of the if you take the twice the amount of steel the size of pipe would go up more than twice is it clear. So, in that case output will increase and of course, what we are assuming that here production production depends only on the size of pipe that is the assumption.

Because remember we are not saying that K is being increased by 10 percent and L is being increased by 20 percent. We are increasing K and L in the same proportion. So, just a made up example you know or probably in study, when you study when a person solves you know you can say the input function input variable is mind power. You know let us say that two people are off exactly same mind, when they come together probably the output is twice more than twice collaborative effect. So, that is increasing return to scale.

But, probably if you move from two to three output will go up by more than three times, but if probably there will be a level where beyond that, if you bring more mind power probably the output would start decreasing, but mind you similar example I gave you earlier, but what is the difference here we are increasing all the inputs in the same proportion, but for the example sake we are taking that we only one input for this production is it clear. And when that happens that the output is less than the t times of the original label, then it is called decreasing returns to scale.

So, when do we hit decreasing return to scale, can you tell me from example, because let us say theoretically speaking we our production function gives the efficient level of you know if efficient it assumes the efficient uses of all the inputs. So, the example that I am giving you that you have eight minds and then probably the output would not be eight times as much as the single mind.

So, what we can do rather than using these eight together’s, what we can do we can separate these into eight different groups and then again we will get at least eight times as much as the earlier output and we will get the constant return to scale, then why do we hit this decreasing return to scale; you got my question?

Student: Yes sir.

No; see production function, what is production function? It gives the maximum amount of output possible.

Student: Yes sir.

So, we are talking about efficient uses of all the inputs. So, one thing in this particular case, because here the assumption is that a firm is doing; remember we are taking the production function view of the form. So, what would be more efficient; that by putting rather than putting the eight minds together I can separate them into eight different groups? And in that case assuming that they all have the equal mind power equal capacity to solve problems then you get the eight times as much as output as earlier.

So, then what is the need of talking about decreasing returns to scale. See typically what happens; remember the example we talked about the short run, what we said; we started with this F of K comma L and we can take K is equal to K naught and then instead of using F of K naught comma L we typically end up using F of L.

Similarly, it is not just like for example, in agriculture we talk about you know you need you need seed, you need water, you need manpower, you need sunshine to grow some crop; typically we because land is given as fixed. So, we miss this land we did not account for this land, but when we are talking about doubling everything, we may double over everything, but land there is a limit to it, you cannot keep on doubling it all the time ok. So, but in your production function typically this land is missing; in that sense you hit this decreasing return to scale is it clear.

Student: Also a also like for example, (Refer Time: 12:09) we can double the amount of (Refer Time: 12:14) machines.

So, some.

Student: (Refer Time: 12:17).

But, so if; so typically I am talking about those factors of production, which are not explicitly mentioned in the production function; sometime it is impossible to double them and when we are talking about increasing the scale, those factor of production become significant and that is why we end up getting decreasing returns to scale, if truly speaking; we if we can replicate the production process, at all level at the fractional level remember the additivity and divisibility, if we can replicate it and we can scale it you know.

So, then what happens you will never get decreasing return to scale you will always get constantly turned to scale, but the assumption is that you should be able to replicate the production process and you should have all the inputs available in available; but that may not happen some of the times and that is why you end up getting decreasing return to scale.

In fact, you can you can show that decreasing return of scale to scale is basically, because there is a limit to increase of one particular factor of production at least one. You can have more than one factor of production.

Student: Sir, but in normal ISO Quant maps, what we see that first; first at some point there is increasing returns to scale, then it is constant return to scale.

That is what I am coming back to that. Now let us let us look at the some of the mistakes that you make.

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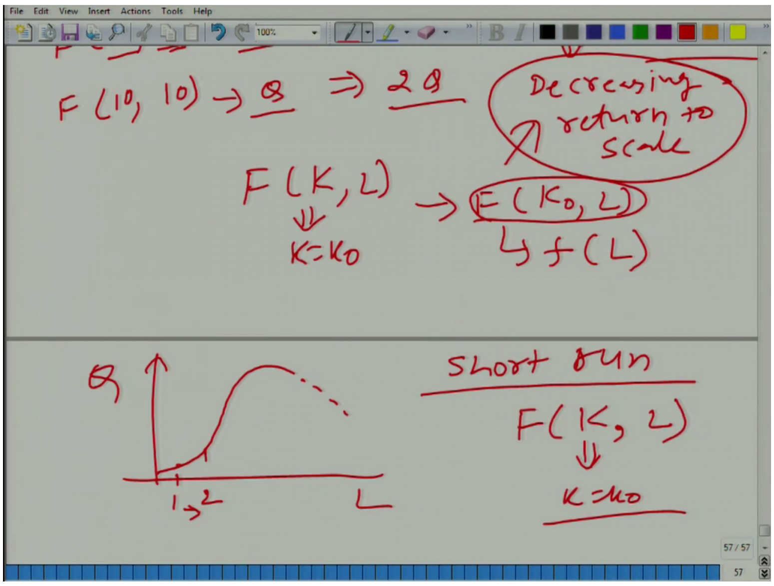

Let us say we are talking about here F comma L ok, oh sorry here we have output Q.

Student: Ok.

And here we have L, and this is what we have talked about here we should have instead dot fine and here we are talking about short run ok.

And of course, we know the factor of productions are K and L. The only thing is that K is fixed, K is equal to K naught from moving from here, let us say 1; 1 to this 2 we are not talking about scale, why; because capital is fixed at K naught and we are increasing only labor we are increasing only labor. So, we are not talking about return to scale here in this particular case.

Student: In the long run.

In the long run yeah.

Student: In the long run the ISO Quant maps I am talking about.

Ok I get your question hope; I think I got your question. So, let us discuss it a little bit more, then you can ask it again [FL]. So, what is happening is; that we are talking, we are what we are doing is; we are talking globally.

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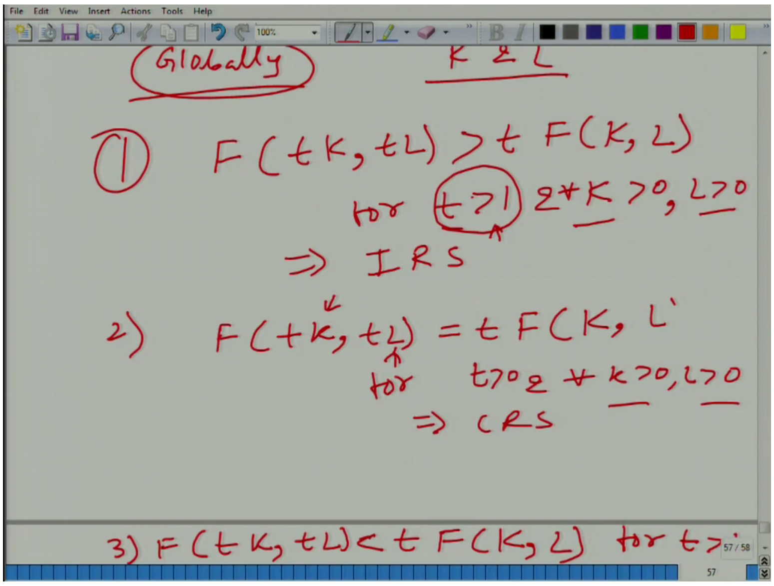

We are talking about all the levels; we have not specified the level of K and level of L.

We are talking about globally ok, but that is really you know typically what happens sometime production function exhibits increasing return to scale, some time constant return to a scale, and some other time decreasing return to scale same production function ok. Here, if you go back to the definition here we are talking about at all level of K and L this should be true let me write it here in the shorthand that F of t K comma t L is greater than t of F of K comma L for t is equal to t for t greater than 1 and K grateful; and for all K greater than 0 and L greater than 0, then what we get is; increasing returns to scale fine.

Now, let me say here is equal to fine, for t greater than or equal to 1 for all K greater than 0, L greater than 0, this is CRS.

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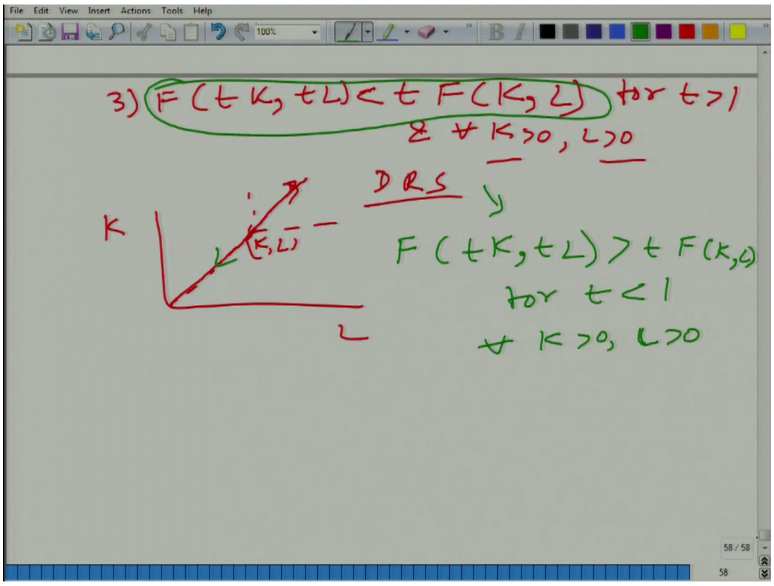

And the third is F of t K comma t L is less than t of F of K comma L for t greater than 1, and this is true for all K greater than 0, L greater than 0, we get decreasing returns to scale. So, here we are talking about this should be true for all the case and else fine, but that is not always true sometime a production function exhibits. Local return to scale you know rather than having true for the whole range of K and L, but before we do that; we talk about the local level rather than the at global level let us talk about this the value of p.

Student: And sir one more thing sir like the example you give off agricultural land that if it curve are covering the whole agricultural land, then it is a very long process.

No even.

Student: (Refer Time: 18:08) in short.

Even for an individual person the amount of land is limited ok; he cannot keep we cannot keeps on doubling the amount of land, keep on increasing the amount of land that he has.

Student: but sir, if we assume that the land prices are not increasing, then he can [FL] at till significant amount he can doubled his land.

So, as long as then it would not be; then at till that level it would at least exhibit constant return to a scale probably it would.

Student: (Refer Time: 18:41) is not able to buy the land then we would not talk about increasing scales [FL].

No, but we do; that is the problem, that we do that the reason is simple that some time we do not mention that factor of production explicitly in the production function, that is why I said that is the reason.

Student: (Refer Time: 19:05).

So, theoretically speaking, what we are the way we define it theoretically speaking we would not have any decreasing return to scale you understand.

Student: Sir time is time one of the function.

At if there is a production process, where time is because time is always a factor, input factor we do not mention it the time.

Student: And it is fixed.

Ok so remember by the way one thing that I should mention here we are mentioning K and L of course, these are flow variable, if you hear the first lesson or second for one of the first lessons, we have talked about flow and stock variable. So, time intercept, but sometime other than here, time is annex on independently also a factor of production, but we do not mention it typically; if it is not significant fine ok.

So, but before we go into go into discussion at local level, let us talk about this t; why we are talking about t greater than 1. Particularly here, what if we do t less than 1? So, when g is less than 1, what is happening; rather than scaling up the operation.

Student: (Refer Time: 20:21).

We are scaling down the operation. So, for constant return to scale, we can take t is equal to greater than 1 or less than 1 does not matter or you can say for all t, but we cannot do for all t or for t greater than.

Student: 0.

0 ok, but here we cannot do for t less than 1.

Why; because t less than 1, if we are defining; we are decreasing the operation basically let us say let us say here we are here we are K and L and we are moving in this direction ok. When we are decreasing all the input in the same proportion, so we are; let us say if we are at this point K and L, so we are moving on this line fine; if t is greater than 1, we are moving in this direction. And t is less than 1, we are moving in this direction. So, here what would be the definition of increasing return to scale, if you think if we want to define for t less than 1, what should be the definition of increasing return to scale? This one think about this.

Student: (Refer Time: 21:49).

This should be the definition of the increasing return to scale remember. So, rather than getting confused about it, what we do; we keep we define IRS and DRS for t greater than 1, otherwise if you want to define for t less than 1, then for DRS let me define here for DRS what would be the definition tK tL, it should be greater than t of F K comma L for.

Student: T less than 1.

T less than 1 for all K greater than 0 and L greater than 0 fine and similarly you can define for the other one fine.