An Introduction to Microeconomics
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Lecture – 84
Elasticity of Substitution

Now, what we have is the elasticity of substitution? What we have done; let us look at MRTS.

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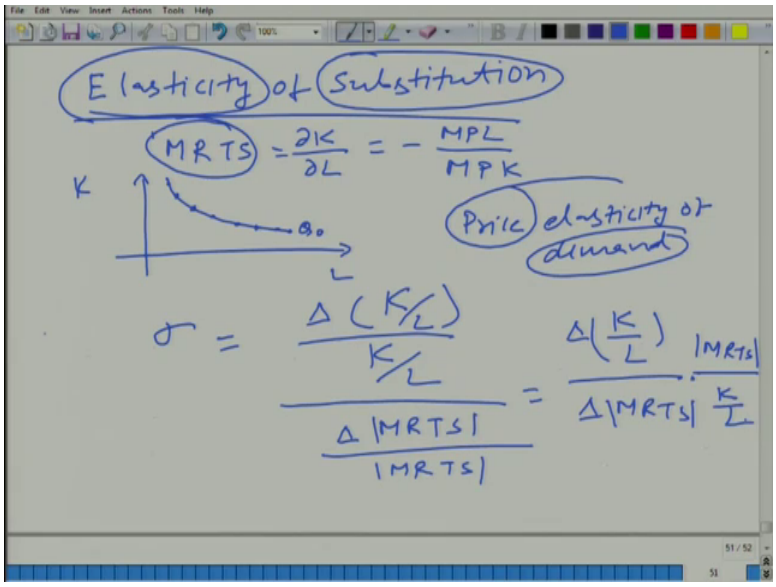

What is a MRTS? MRTS is basically slope of ISO Quant and this is MPL divided by MPK. Although, we have not done, but look at this ISO Quant ok; when you want to produce we will do this in more detail later on that the concept that I am talking about, what we are talking about is here; that to produce Q naught amount of output all these combination of inputs are fine.

All these combination of inputs are efficient; in the sense that you would not be wasting any input to produce Q naught amount of output, but it does not mean that we can pick any one of these; these are technically feasible we are talking about technical constraint, but how about the economic motive; you want to maximize your profit or you want to minimize your cost, to produce the same amount of output. We have not talked about cost minimization or profit maximization that we will do shortly; but this is very simple

that you should be able to understand. So, a point you will pick a point such that the cost of producing Q naught amount of output is.

Student: (Refer Time: 01:49).

Minimized; so how will you pick; at that point this MRTS will play a very important role. If you remember the concepts from consumer theory, what did we do? There the counterpart of MRTS; that is marginal rate of substitution should be equal to the ratio of.

Student: (Refer Time: 02:14).

Market prices here it is the same thing ok. So, what we are talking about is that it has something to do with the MRTS should be equal to the price ratio at the optimum level.

Student: Of level (Refer Time: 02:31).

Fine; although, I have not discussed it in detail ok, but you do not need to know this. So, MRTS can proxy for the price ratio at optimal level at all the optimal level fine and what is elasticity? I am going to talk about it in little differently also without using the concepts from profit maximize, what is elasticity?

Student: How much of the value deflect corresponding to (Refer Time: 02:57) something else.

When we say go back to, if what we have learned price elasticity of demand. what did we talk about?

Student: The percentage change in the quantity due to 1 percent change in price.

Price that is what we have; or it is rate or proportional change in demand with respect to proportional change in.

Student: Price.

Price that is; what we have talked about? So, what we are talking about is elasticity of substitution. We are not talking about; we are not saying here price elasticity of substitution, but what we are talking about is; some sort of elasticity of substitution. So, how the substitution, the proportional substitution changes as the proportional price; proportional price change in the changes in the market, and we do not have proportional

price, what do we have; to proxy for proportional price we have MRTS. So, what we can say in the proportional change in substitution with respect to proportional change in MRTS and that is elasticity of substitution.

Student: (Refer Time: 04:08).

So let me write it here in this particular context it is; this is proportional change in the inputs that you are using, this is instead of using this partial derivative sign, what you can do; you can write delta proportional change in the ratio of KL with respect to proportional change in MRTS. And instead of taking MRTS, I am taking the absolute value of MRTS, because MRTS is negative. So, does not matter I am taking the proportional value of a MRTS. So, in a sense it is very similar to the; what we had learned earlier; what we can; if we can rewrite it, what will we get; you can rewrite it like this K by L divided by fine; that is one way to write it.

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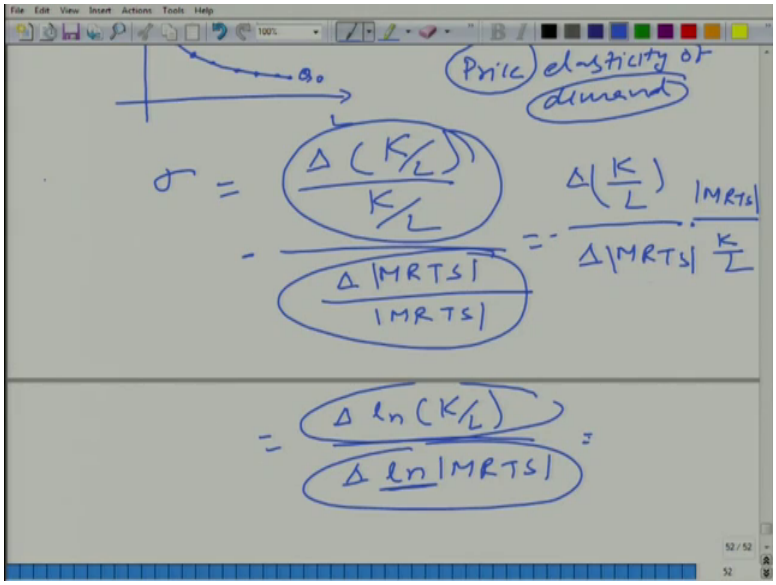

Another way to write it is take log, and you put here and of course, we are missing the minus sign typically ok, what we have here is basically delta ln K by L divided by delta ln MRTS what is ln?

Student: Natural log.

Natural log ok. So, this can be written in the form of natural log and that is what we get. And the denominator here in this part can be written as this fine ok. Let me let me not

use the concept of profit maximization, because we have not used it yet ok. Without using the concept of profit maximization, I will again try to explain; what is the elasticity of substitution; and let us look at it here graphically.

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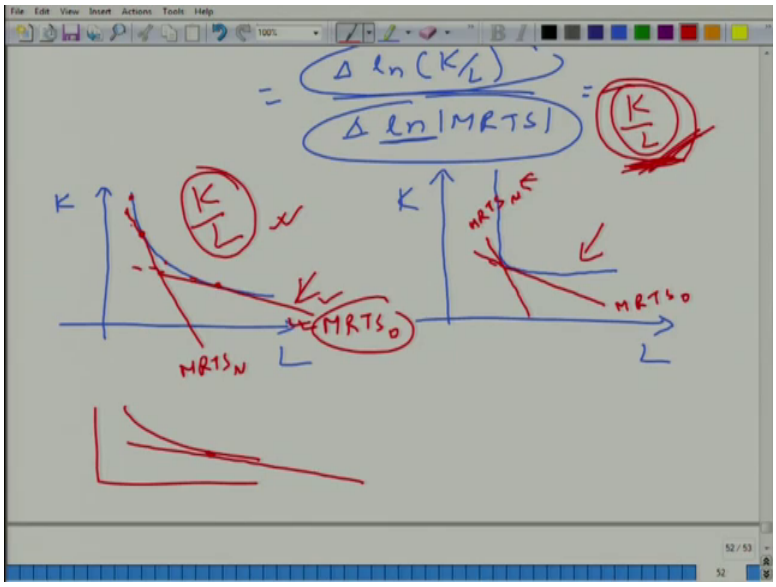

We have two graphs fine and let us take one, where we have Cobb Douglas function this is ISO Quant in the case of Cobb Douglas function. And let us take here ISO Quant; in case of perfect complement K inputs. Now, let us say this is the MRTS at this particular point, which point will you choose again we are again using concepts from profit maximization, but not explicitly used to you would not choose a point here, why; because anyway you are wasting this much of labor and labor is costly. So, you do not want to use that point. So, you will always produce here at the corner, because you are not wasting any of the inputs.

Student: So, how can we define a MRTS over there it will be any line.

Good point I should not say it is MRTS. What I should; let me change it little bit let me change it a little bit, what we have instead of instead of steep corner sharp corner what we have is a.

Student: Curve.

Curve good point my mistake fine. Now, we do not have a sharp corner ok. It is differentiable everywhere fine. Again, roughly your whenever you figure out where you

want to produce you will probably prove you will be producing probably here in this zone fine. Now let us say this is the MRTS here fine, if MRTS changes this is MRTS old and this is MRTS new.

Again K by L will not change significantly; in this particular case; how about here. In this case let us say the change is same, I am trying to you know old MRTS new; it is not (Refer Time: 09:18) it is this is here K by L would change significantly in comparison to the previous case, because from here you will move to this point. Again one thing that I am silent about; why I am choosing this particular MRTS? Why this MRTS?.

Because what I am talking basically talking about; that earlier the price ratio was this, because MRTS is in your control you change the point the combination and your MRTS will change is not it; here you can say that I will produce here I will produce here and accordingly MRTS will change, but wait which where you will produce actually where MRTS is equal to the price ratio.

So, again I have gone back to the previous concept that, we have just talked about fine. So, in that case you are producing here, because this MRTS represents the price ratio in the market. So, if MRTS you need to change, because of price changes in the market and this is the new MRTS, what will happen; the K by L will change significantly. Here, K by L will not change significantly because we are talking about substitution remember these two are per near perfect compliment ok.

So, substitution is not taking place that much, but here substitution would take place fine is it clear. So, basically if we go back and look at the geometry, what basically happening; when we are talking about MRTS, we are trying to measure the slope of this ISO Quant, but when we talk about elasticity of substitution, what we are trying to measure the curvature. Curvature of this ISO Quant, it represents the curvature of ISO Quant, how the different combination of how the combination of input to produce the amount of output would change with changing prices in the market and that is what we talk about in the elasticity of substitution is it clear.

If you want we can come back to it again, when we talk about profit maximization and there we will figure out that MRTS is nothing, where the production will take place it is equal to P what would it be equal to.

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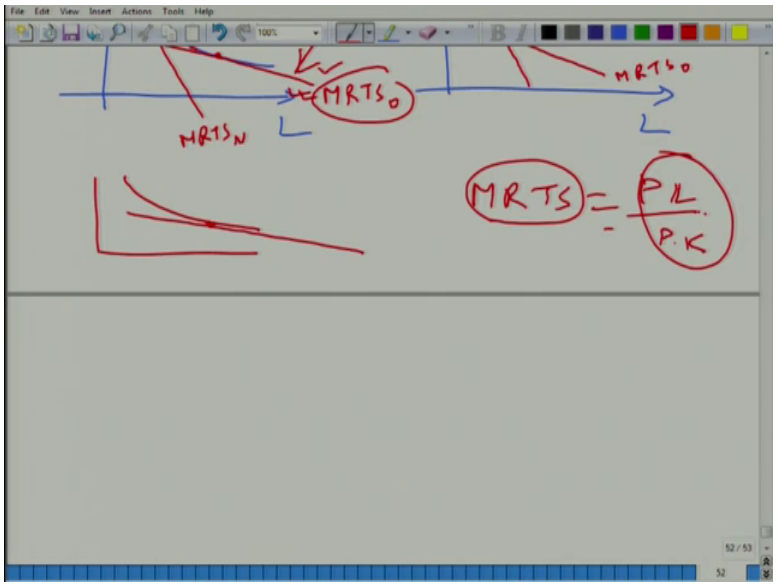

Student: P.

L by PK.

Student: (Refer Time: 11:56) minus 1.

Fine ok. So, it is doing this the proxy MRTS is a kind of a proxy for PL by PK and that we will see shortly. So, now, let us calculate MRTS, we had calculated MRTS for a particular case of Cobb Douglas function and do you remember what did we get?

Student: Minus.

Minus of course, it is always minus MPL by MPK, but what did we get in case of Cobb Douglas function check.

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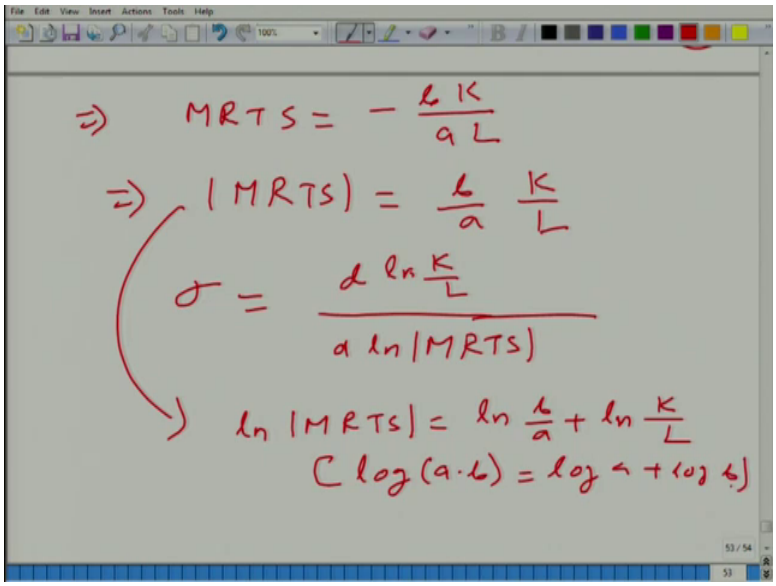

Student: (Refer Time: 12:36) K K by b L bK by a L.

BK by a K.

Student: (Refer Time: 12:42).

Fine; can we calculate now elasticity of substitution, how can we calculate the elasticity of substitution? You take here first the absolute value, what do we get? B a by KL and what we have learned that elasticity of substitution is nothing, but.

Student: (Refer Time: 13:07) delta p by L then (Refer Time: 13:13).

This is what we have learned fine. So, we have everything, we can calculate and let us see what do we get. From here, if we take log both side we get log b by a plus log K by L and this is the property of.

Student: Log.

Log a multiplied by b is equal to log a plus log b fine, and that is what we have used. We differentiate both side with respect to log of absolute value of MRTS, what do we get here we get one this, if we differentiate with respect to log of MRTS, what do we get; 0 and then here what do we get l n K by L.

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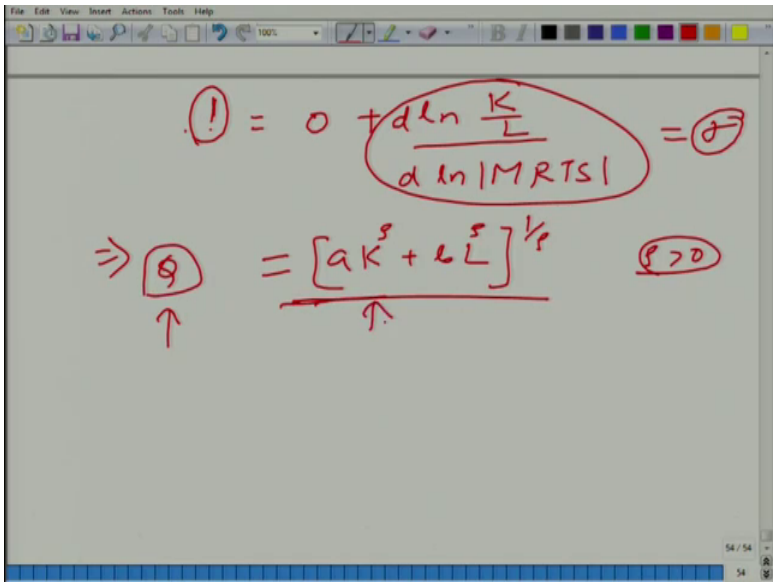

Student: MRTS.

MRTS; so what we have got; and this is our elasticity of substitution. So, in the case of Cobb Douglas it is equal to.

Student: 1.

1 and of course, based on it we can figure out a new production function where; because this is quite useful, that a special class I am not going to discuss it in detail a special class of production function for which, elasticity of substitution is always constant not just one always constant some constant value. And of course, then Cobb Douglas would be a special case of that kind of production function, because it for Cobb Douglas also it is a constant value 1, fine.

So, that that particular class of production function let me write it here, why do not you change it; why do not you check it; calculate let me write it here a K b L or let me write it this way this is of course, a production function Q is equal to a K to the power rho plus b L to the power rho and whole to the power 1 by rho and of course, there should be some restriction on rho, what should be that restriction?

Student: Greater than 0.

Rho is greater than 0, probably check rho is greater than 0 and what you need to do now; calculate the elasticity of substitution in this particular case. And then; so that Cobb Douglas functions is nothing, but a special case of this production function; that is the homework you need to do fine ok.