

An Introduction to Microeconomics
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Lecture – 70
Slutsky Equation

(Refer Slide Time: 00:14)

$$x_1^M(p_1, p_2, I) \equiv x_1^H(p_1, p_2, u_0)$$

$$\Rightarrow \Rightarrow \frac{\partial x_1^M}{\partial p_1}(p_1, p_2, e(p_1, p_2, u_0)) \equiv \frac{\partial x_1^M}{\partial p_1}(p_1, p_2, e)$$

$$\frac{\partial x_1^M}{\partial p_1} + \frac{\partial x_1^M}{\partial I} \cdot \frac{\partial I}{\partial p_1} = \frac{\partial x_1^M}{\partial p_1}$$

Now, what can we do, we have already talked about substitution effect and income effect in graphical sense. So, we have already talked about you know, this is basically we are going to get the Slutsky equation. An equation that gives relationship between compensated demand and Marshallian demand. So, we have already done it qualitatively, not quantitatively ok. So, we have already looked at it through graphs, but now we are going to do it mathematically.

So, what we can do here, because we are talking about vary P 1. How do we get the x 1, x 1 is quantity demanded, either in Marshallian sense or in compensated sense and to get generate the demand function what do we need? We need to get quantity demanded as function of P 1.

So, basically P 1 is changing and we are studying its effect on quantity demanded. So, here; of course, we will take P 2 u naught you know everything.

Student: Constant

Constant u naught constant in this case, but u naught here will, here u will change the utility achieved will change, but basically we are varying P_1 . And if we differentiate both side with respect to P_1 what do we get? Remember here $P_1 u$ naught is given; that is given to the system, its per, its you know you P changing P_1 will not change u naught, but changing P_1 will change the expenditure to achieve the u naught.

Is it clear. So, changing P_1 will affect the quantity demanded in direct way and here in.

Student: Indirect

Indirect way. So, how can we write, this is going to be, this is the direct effect and also what we will have here is.

Student: Indirect effect.

Di basically this is I ok.

With respect to the third argument and this is going to be.

Student: dP_1 .

With respect to.

Student: P_1

P_1 .

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$$\frac{\partial x_1^M}{\partial P_1} + \frac{\partial x_1^M}{\partial I} \frac{\partial e(P_1, P_2, u_0)}{\partial P_1} = \frac{\partial x_1^M}{\partial P_1}$$

$$e(P_1, P_2, u_0) = P_1 x_1^H + P_2 x_2^H$$

$$\frac{\partial e}{\partial P_1} = x_1^H(P_1, P_2, u_0) \equiv x_1^M(P_1, P_2, e(P_1, P_2, u_0))$$

$$\Rightarrow \frac{\partial x_1^M}{\partial P_1} + x_1^M \frac{\partial x_1^M}{\partial I} = \frac{\partial x_1^M}{\partial P_1}$$

Fine, and how about here, this is what we will get of course, P 1 will not change P 2 and P 1 will also not change u naught.

Fine and here rather than using I, what we are doing basically here is we have P 1 comma P 2 u naught, is it clear ok. Now if you pay attention what is this. This is the optimal level of expenditure or minimum level of expenditure to reach, to have at least u naught level of.

Student: Utility.

Utility. So, basically what it is; of course, this is not perfectly correct, but we havent discussed that much of mathematics in this class. So, I am taking a kind of, you know that doesnt look completely right, but what this is basically P 1 x 1 plus P 2 x two.

Or here it is. This is what the total expenditure is, and this is not for any amount, this is for the optimal amount.

So, and how did we get this of course, from the minimization problem that I discussed earlier. So, if I differentiate e with respect to P 1 what will happen.

This is, let me tell you, again this is not perfect way, but this is going to be equal to x 1 H.

Of course you can say that x two is also a function of P 1.

Student: P 1.

But at optimal level it does not matter ok, that is what its little bit advance ok, but not that advanced. You can get this using the equation and that will be your one of the homework problem ok. Fine, you can get it from here. And this we already know, this we already know of course, P 1 P 2 and here we have u naught. This is we already know is equal to x 1 M P 1 P 2 e of P 1 P 2 u naught fine. So, we can put it back there what do we get ? dx 1 M del, this is partial derivative of x 1 with respect to P 1, what is x 1? Its Marshallian demand.

So, what we are trying to get here is, the slope of Marshallian demand function with respect to P 1.

Student: P 1.

What is it equal to?

x 1 M, this is x 1 M and this is the optimal amount, I am writing in shortcut. This is the Slutsky equation, this is called Slutsky equation.

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The image shows a digital whiteboard with handwritten mathematical notes. At the top, the Slutsky equation is written as:

$$\Rightarrow \frac{\partial x_1^M}{\partial P_1} + x_1^M \frac{\partial x_1^M}{\partial I} = \frac{\partial x_1}{\partial P_1}$$

Below this, the text "Slutsky Equation" is written. The equation is then decomposed into two parts:

$$\Rightarrow \frac{\partial x_1^M}{\partial P_1} = \underbrace{\frac{\partial x_1^M}{\partial P_1}}_{\text{Substitution effect}} - \underbrace{\left(x_1^M \frac{\partial x_1^M}{\partial I} \right)}_{\text{Income effect}}$$

At the bottom, there are two conditions for the income effect term:

$$\frac{\partial x_1^M}{\partial I} > 0 \Rightarrow \text{Normal Good}$$
$$\frac{\partial x_1^M}{\partial I} < 0 \Rightarrow \text{Inferior Good}$$

We will spend some time talking about what does it mean. Although we have already studied, but again it will be a revision math. Is clear to you, how we got this. So, I can rearrange it, I can rearrange it what I can write this is, this has to be equal to and what

does this represent? This is substitution effect. Fine? What is substitution effect, what we are doing.

We are changing P 1 here and we are studying its effect on x 1 M. How do we get the substitution effect, that we change the P 1, but we do not change the utility level, we remain on the same utility label. So, this represents x 1 H represents, you know its compensated demand in the sense that utility remains the same. So, this is giving us substitution effect. And this term is giving us income effect. Let us spend little more time and let us look at, let us pause this income effect side x 1 M is a non negative number ok, it is quantity demanded, it cannot be negative, quantity demanded cannot be negative.

And what is this?

Student: (Refer Time: 07:12).

This we have already studied if it is greater than zero then it is.

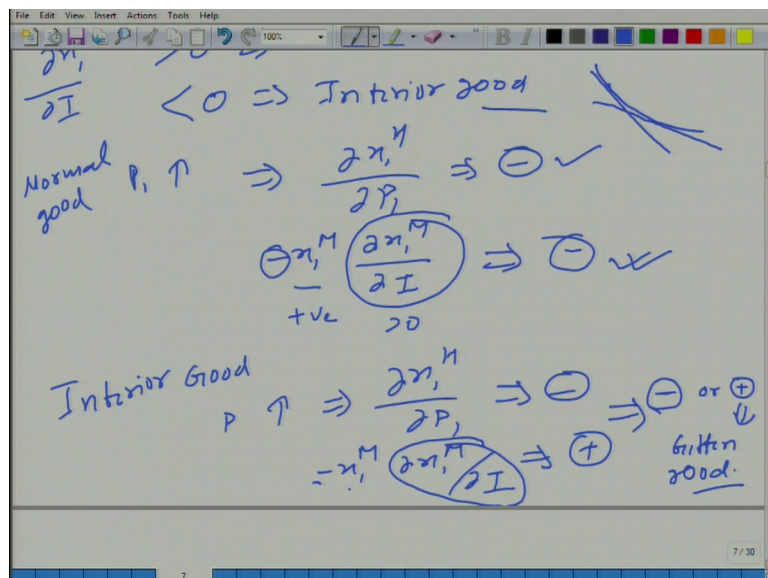
Student: Normal good.

Normal good and if it is less than zero.

Student: Inferior good.

It is inferior good. Fine. So, if P 1 goes up.

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If P_1 goes up, this we know this is so, we are certain about, because of convexity of the preferences, this is shaped like this. So, if it is rotation P_1 is increasing, what is happening if P_1 increases, then it will rotate like this.

Budget curve will become steeper.

Because what is the slope of budget curve minus P_1 divided by P_2 .

So, if P_1 goes up it will become steeper and if it becomes steeper, because this particular shape, the consumption of good one even in the compensate, even in the compensated sense would come down. So, P_1 goes up, this is negative, as P_1 is increasing x_1 is decreasing.

Now, let us look at this part x_1 M dx_1 M del x_1 M with partial derivative of x_1 M with respect to i . What is happening here? This is negative, this is positive or non negative and what is this for normal good, for normal good what is this, for normal good this is.

Student: (Refer Time: 08:50).

Positive. So, overall this is negative. So, for normal good substitution effect is negative and income effect is.

Student: Negative.

Negative. We have already learned this, this is just a repetition of the same fact and how about inferior good, this is for normal good.

P_1 goes up, just we have already established this is negative and minus x_1 M this is.

Student: (Refer Time: 09:30) 0.

Positive.

Student: (Refer Time: 09:31).

Because this is negative, this is positive and this is.

Student: Negative.

Negative. So, negative positive and this has two possibility that either you get minus or.

Student: Plus.

Plus if you get minus fine, but if you get plus.

Student: Giffen good.

Then you get Giffen good. So, one requirement for Giffen good is.

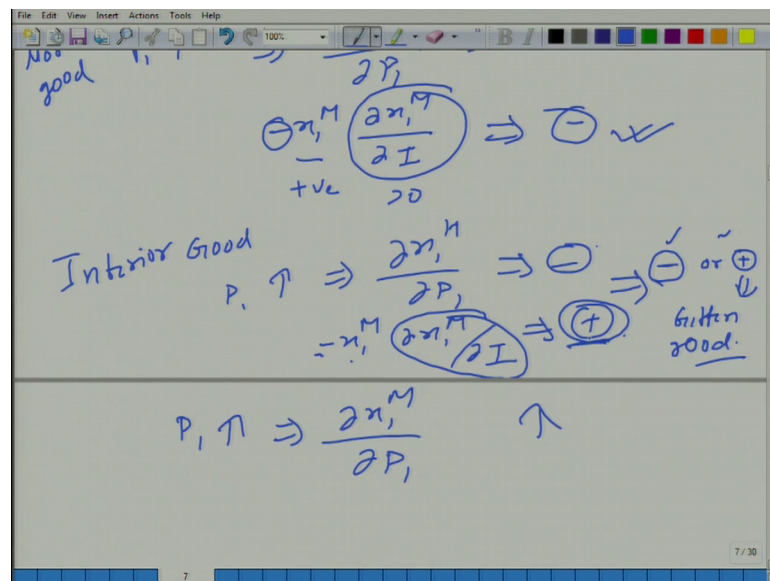
Student: Inferior good.

That the good has to be inferior, the second requirement is that income effect has to be large.

Student: Large.

Student: Sir can you (Refer Time: 10:08) Giffen good.

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Giffen good is a kind of good. So, now, what is happening, if it is plus then what is happening P_1 is going up and Marshallian demand of that good is also increasing. So, here demand curve is an upward sloping curve. Why it is happening let us think about it substitution, we have already seen, substitution effect is always if price goes up ok, compensated demand would definitely go down.

Now, we have to look at the income effect. Income we as already we discussed that income it is not necessary that when a income goes up you consume more of.

Student: (Refer Time: 10:52).

A particular good.

One example could be that you know potato example that I gave you earlier, that income goes up what would happen? You will decrease the amount of potato that you would consume, because potato people, people consume potato, because they do not have enough money, they will probably substitute it by meat or some similar product or milk ok.

So, income goes up consumption of a good may come down.

Student: Yes sir.

Ok.

Fine. So, here it can be, it can move in any direction, either negative direction or positive direction and for inferior good income effect is, a price, price goes up what would happen that the purchasing power.

Student: Comes down.

Comes down, purchasing power comes down, it means real income is coming down, but that is why you will consume more of that good, because if your income is coming down you can no longer afford milk, mutton or some other products. So, you have, you will have to consume.

Student: Potato

More of potato. So, that is what is happening here. So, now, the scenario here is, that for normal good that we have figured out that substitution effect and income effect they work in the same direction, but for inferior good they work in the opposite direction. So, there is a theoretical possibility that for inferior good that not only, you know for that that income effect is larger than substitution effect. And in that case what would happen, the price would go up and consumption of that good will also go up.

And that sort of goods are called Giffen goods.

It is very difficult to get Giffen goods in the real life, because the condition that income effect has to be large.

Student: Large.

And it is difficult to have large income effect, because you know you do not spend really significant amount of income on one particular good.

You distribute your income over a large number of goods ok. So, that is why its difficult to get Giffen good. Its clear, Slutsky equation is clear and substitution effect an income effect with help of substitute Slutsky equation is also clear. So, we have done it using graph, graphs and also mathematically, fine.