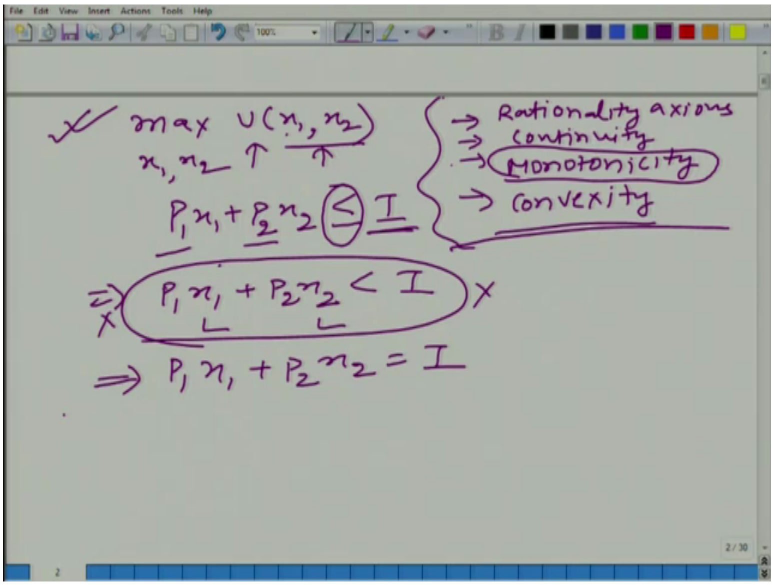
An Introduction to Microeconomics
Prof. Vimal Kumar
Department of Economic Sciences
Indian Institute of Technology, Kanpur

Lecture – 68
Expenditure Minimization as a Dual Problem of Utility Maximization

So just to revise; what we have done in this chapter; we have started with the consumer, we have talked about how he selects a bundle that would give him maximum level of satisfaction given all the constraint this consumer has. And of course, the maximum level of the satisfaction we use utility to represent that satisfaction.

(Refer Slide Time: 00:44)



So, mathematically, I can see what a consumer is trying to do? Consumer is trying to maximize his utility.

And again I am taking only 2 goods in this world one can take in goods as long as n is finite we are fine with it and we maximize it with respect to x 1 and x 2. And of course, what we have we have started with we have taken some axioms that this utility should satisfy and those axioms are rationality axioms, continuity, monotonicity, and convexity. Again I am not saying that all of us our preferences satisfy all these axioms, but what I am talking about is true when these axioms are satisfied.

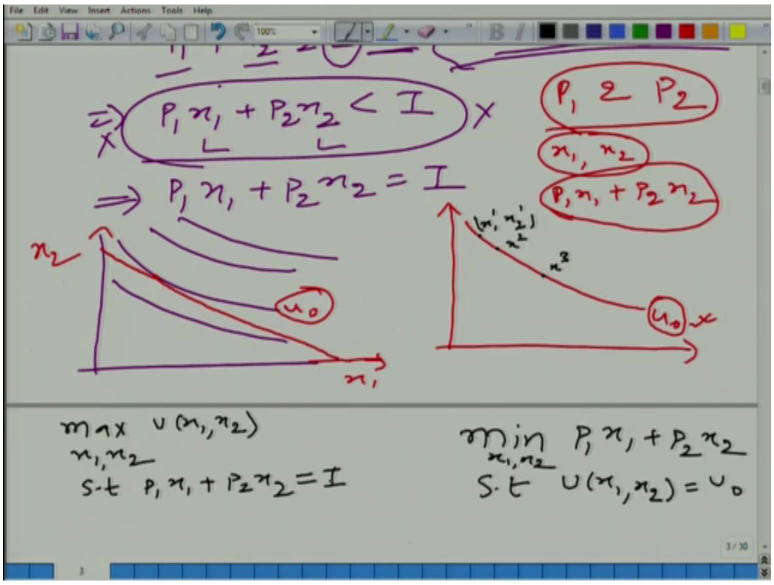
And of course, what I am talking about may be true even if some of these axioms are not satisfied, but I am talking about only the cases where these axioms are satisfied. And this utility maximization has to be done with respect to some constraint and the constraint the budget constraint we take P 1 x 1; P 2 x 2 should be less than or equal to I; P 1 is the price of good 1, P 2 is the price of good 2, x 1 represents the amount of good 1, and x 2 represents amount of good 2 and I is the income of this person.

And here we have sign less than n equal to, but remember here what we said that these axioms should be satisfied monotonicity should be satisfied it means more is better. So, we cannot have a situation in the optimal level where P 1 x 1 plus P 2 x 2 is less than I.

Why it means some of the income is left this person does not derive any satisfaction from leftover income. In real life it is it is possible that a person derives some satisfaction from having some money left in his pocket, but the way this problem has been framed here the persons satisfaction depends only on his level of consumption of good 1, and good 2.

So, he does not get anything from keeping some money left in his pocket. So, there is no point having considering this situation ok. Because by increasing x 1 and x 2 this person would not lose anything, but will be able to increase his level of satisfaction or that is utility so that is why in optimal level this has to be true with the equality sign. So, right now we are going to consider a problem where the utility maximization problem is given here under the constraint this ok.

(Refer Slide Time: 03:53)



In other word; graphically what we are saying? Graphically what we are saying that here we have our indifference map, and here is our budget constraint fine and this is the optimal bundle this is what we have done.

Now, let us look at this problem from different little bit different angle rather than taking indifference map and what is an indifference map it is set off some indifference curve so that we get clear picture of this persons utility or this persons test. So, let us take just one indifference curve and this is this let us say this is the u naught, and let us say we are taking u naught here.

Now, here what we are talking about we are talking about that we have certain budget constraint and under this budget constraint what is the maximum level of utility that can be achieved by this person. Now let us turn this problem little bit you know in the opposite direction, we say that this person wants to achieve this u naught level of utility which is same as this u naught ok. How much what is the minimum level of income he needs to get; this u naught level of utility.

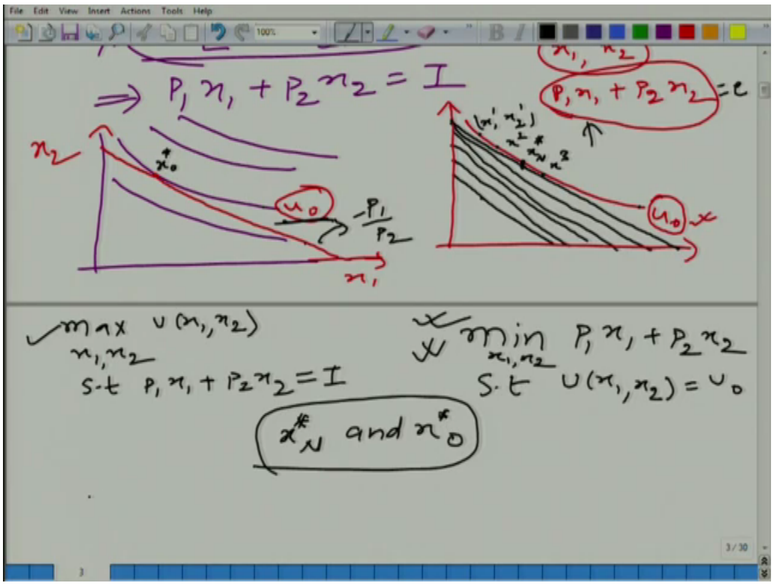
So, what is happening P 1 and P 2 these are market determined that so we are taking because we are talking about consumer P 1 and P 2 is its from the market and individual let us say that so far we are taking that individual is not able to influence P 1 and P 2, we are talking about that scenario. So, the budget the cost of having let us say x 1, and x 2 x 1 amount of good 1 and x 2 amount of good 2 is going to be P 1 x 1 and P 2 x 2.

And what we are trying to say that this has to be minimized. This what we are saying that we should be able to achieve u naught now how can we achieve u naught we can take any bundle let us say if we take a bundle here that will give us u naught utility, a bundle here that will also give us u naught utility, a bundle here will also give us u naught utility ok. This bundle is let us say x 1 1 comma x 2 1 or it is a 0.1.

Similarly, what we have here is x 2 here is x 3 ok. And how can we what is the idea to minimize the expenditure I should achieve let us say if I am that consumer I should be able to reach to this utility level by spending as less as possible. So, what we are trying to do basically we are trying to minimize let me write it to this side. So, I will write the earlier problem this side what we are saying minimize P 1, x 1 plus P 2, x 2 such that u of x 1 comma x 2 is equal to u naught.

Did we talk about earlier that maximize u of x 1 and x 2 with respect to x 1 and x 2 again here also it is with respect to x 1 and x 2 because x 1 and x 2 a consumer can decide the amount of x 1 and x 2. So, what we have here is such that P 1 x 1 plus P 2 x 2 is equal to I.

(Refer Slide Time: 07:56)



Now let us look at these 2 problem; what is happening is so what we can do here; let us look at this problem the new problem, what we can do here P 1 plus x 1 plus P 2 plus x 2 is equal to let us take K, where K is any number. So, for different number of K, if we try to draw this because this is a line how did it look like?

Student: Downward sloping.

Downward sloping slope is again going to be equal to minus P 1 by P 2 here also is slope is minus P 1 by P 2 ok. And similarly, we can draw for different values of K and this is the optimal bundle, this is the bundle; let us say x star and here is let us call it also x star in the old problem just to distinguish x star in the new problem.

What we have done here we have taken the indifference map and then we have taken the budget constraint and what we have done we have started checking with the what is the highest level of indifference curve that can be achieved under this budget constraint and we have figured out that x does not or x naught 0 x star 0 is the optimal bundle.

Now, what we are doing we are from here we figure out that x star dot x star 0 the utility level is u naught we take that utility level and we draw only one indifference curve where utility level is u naught fine. Now what we are doing we are drawing a set of a family of this line P 1 x 1 plus P 2 x 2 of course, different line corresponds to different value of K.

And what is this K? K better representation would be e because it is expenditure on x 1 and x 2 these are also expenditure lines I can say. We take any bundle here the expenditure is going to be the same in both of these cases that is what it represents.

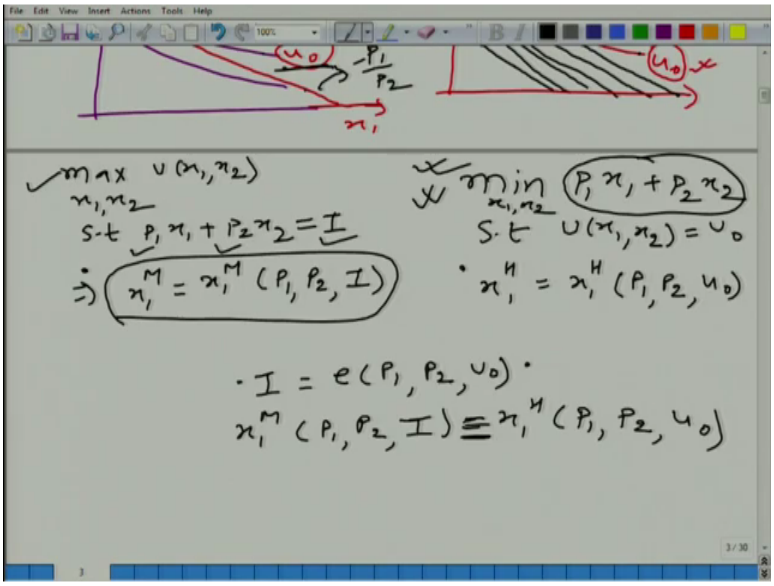
So, can you say can you say any relation between x star n and x star naught or x star is 0 any relation they are going to be the same they are going to be the same. Why they are going to be the same that this this line is going to be the same as this line and this indifference curve is the same as this indifference curve again here it is very clear that in both the condition both the scenarios the tangency condition have to be satisfied.

So, again it will be tangent at the same point. In fact, we can overlap what we are doing here we are varying the indifference curve and keeping the budget line fixed and what we are doing here we are varying the budget line in the other word expenditure line, budget line is also an expenditure line. We are varying the expenditure line while keeping the indifference curve.

Student: Fixed.

Fixed. So, basically these two these two are dual of one another. If we solve these two we reach to the same consumption bundle; it is clear ok. So, if we solve it what do we get?

(Refer Slide Time: 11:41)



Here let us say for x 1 instead of using star let me use term call M I will explain what does this M me this is x 1 m is a function of P 1 P 2 and I here u naught is not fixed ok. What are the parameters P 1 P 2 and I so x 1 the optimal level of consumption of good 1 should be given as a function of parameters in this system and the parameters are P 1, P 2 and I is it clear?

Similarly let me write here x 1 h again I will explain what is h only thing that you should remember at present that this is the optimal level of consumption of good 1 and this is the optimal level of consumption of good 1 here in this system and this should be a function of what are the parameters here P 1 P 2 and u naught.

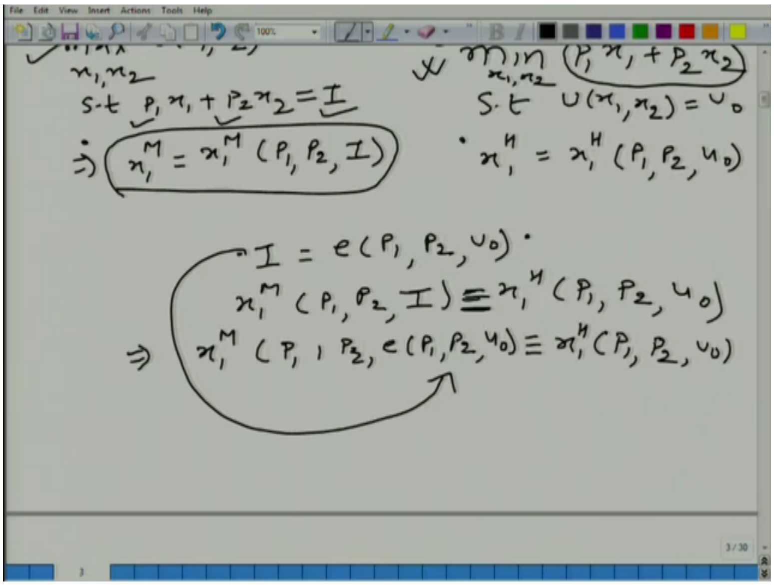
Student: U naught.

Because here u naught is fixed, here u naught is fixed, and also what we can say I; I is equal to e of P 1 P 2 and;

Student: U naught.

U naught, why what is I? I is P 1 x 1 plus P 2 x 2. What is e? E is nothing but P 1 x 1 plus P 2 x 2. We are checking we have set up the problem such that this I is equal to this u naught and what we have learned is that x 1 m P 1, P 2 I is equal to x 1 h P 1 P 2 u naught. This is what we have learned this is an identity not just equal to this is an identity this is always true.

(Refer Slide Time: 11:53)



I can write it further if you pay attention to this that x 1 m P 1 comma P 2 and I is e of P 1 comma P 2 and u naught nothing, but I have used this equation here; is it clear.

Student: Yes.