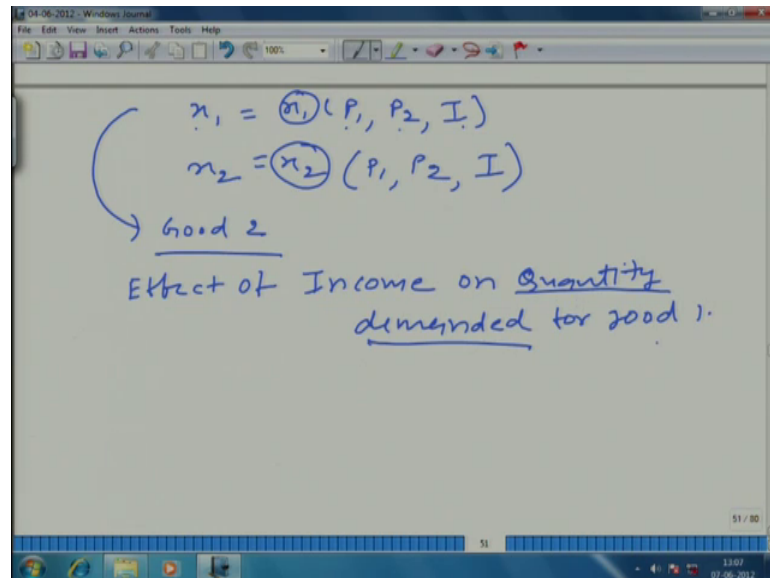


An Introduction to Microeconomics
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Lecture - 64
Effect of Income on Quantity Demanded

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Now what we are going to do if again whatever we are discussing is true for n dimensional world or a good populated with n different goods but I am going to describe a world which has only 2 goods.

So, here basically what we have x_1 is a function of P_1 , P_2 and I and x_2 is a function of again not necessarily the same function that is why here x_1 and x_2 these are different x_2 is again a function of P_1 , P_2 and I .

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The image shows a handwritten derivation of a consumer's utility maximization problem. It starts with the objective function $\max_x U(x)$ and the budget constraint $P_1 x_1 \leq I$. The constraint is then written as a summation $\sum_{i=1}^n P_i x_i \leq I$ and expanded as $P_1 x_1 + P_2 x_2 + \dots + P_n x_n \leq I$. The solution is given as $x = x(P, I)$, with individual quantities $x_1 = x_1(P_1, \dots, P_n, I)$ and $x_n = x_n(P_1, \dots, P_n, I)$.

$$\begin{aligned} \max_x U(x) \quad & x = (x_1, x_2, \dots, x_n) \\ & P_1 x_1 \leq I \\ & \Leftrightarrow \sum_{i=1}^n P_i x_i \leq I \\ & \text{or } P_1 x_1 + P_2 x_2 + \dots + P_n x_n \leq I \\ & \rightarrow x = x(P, I) \\ & \leftarrow x_1 = x_1(P_1, \dots, P_n, I) \\ & \quad \vdots \\ & \quad x_n = x_n(P_1, \dots, P_n, I) \end{aligned}$$

So, what we are going to do? We are going to study the effect of let us say let we will do it only for good 1 of course, good 2 will become clear to you. But we are going to study the effect of change in I on x_1 , then we are going to study the effect of change in P_1 on x_1 .

And then also we are going to discuss the effect of change in P_2 on x_1 3 things we are going to discuss fine. So, the first thing first let us start with the income the effect of change in income on the quantity demanded of good 1, effect of income on quantity demanded for good 1.

This is what we are going to describe of course, you should know this quantity demanded how a consumer is buying any of these goods depending on his optimization problem he has certain test, he has certain preference that we have translated into utility function and indifference curves and he has some budget constraints and based on these 2 he figures out that how much at the to maximize his utility to get the maximum level of satisfaction how much of good 1, he should consume.

And that optimal level is his quantity demanded for good 1, it is not just any quantity demanded. So, let us take an example rather than doing a general case let us take an example what we will do we will take a specific kind of utility function and this is probably the most important utility function because you will encounter it again and

again in the economic theory and that utility function is the type is called Cobb Douglas function.

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Effect of Income on Quantity demanded for good 1.

$$\begin{aligned} \max_{x_1, x_2} & \quad x_1^a x_2^b \\ \text{s.t.} & \quad P_1 x_1 + P_2 x_2 \leq I \end{aligned} \Rightarrow \log x_1 + \log x_2$$

$\Rightarrow \log(x_1^a x_2^b)$

$$= \log x_1^a + \log x_2^b$$

$$= a \log x_1 + b \log x_2$$

Again we are talking about 2 good world x_1 and x_2 you maximize it with respect to x_1 and x_2 such that $P_1 x_1 + P_2 x_2$ should be less than or equal to I ok.

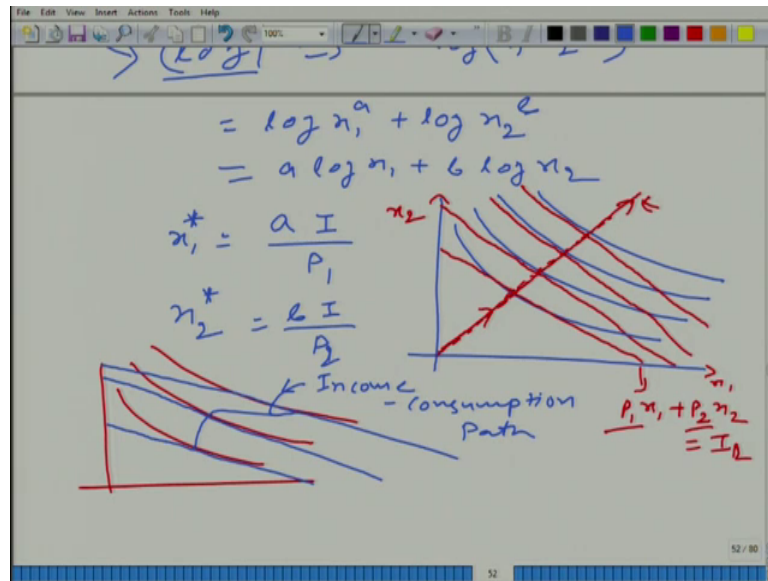
If you solve it remember a similar problem we have solved in the class what was that problem we had solved $\log x$ plus $\log y$ we have solved here of course, $\log x$ is x_1 and law of y \log of x_2 ok.

Just look at it look at this problem we know that form the hour of the optimal level of consumption of good 1, and good 2 does not depend on the form it depends on the preference that we have already talked about it. So, if we take any monotonic transformation that would also work to get the optimal level of x_1 and x_2 .

So, let us take monotonic transformation \log is of course, we if we take \log of this function we will get the monotonic transformation of this particular function and what we will get a \log or let me write it step by the step this is going to be $x_1^a x_2^b$ and here its multiplication.

So, what we can write it we can write $\log x_1^a + \log x_2^b$ this is property of \log . And then this can be written as $\log x_1^a + b \log x_2$. Now I consider you sufficiently familiar with this technique. So, I am not going to solve it completely.

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But if you solve it what you will get? x_1 star is equal to aI divided by P_1 and x_2 star is equal to bI divided by $P_1 P_2$.

Student: (Refer Time: 05:08).

Thank you bI divided by P_2 . So, what is happening here let us look in the graph if you draw the Cobb Douglas it will look like this ok? Now let us say what we have here is let us do this what we have is this is the budget line touching it here of course, my poor drawing and then what we have here is;

Student: (Refer Time: 05:41).

This is the next budget line. How did we get the other budget line here let us say this is $P_1 x_1 + P_2 x_2 = I_1$. Now what has happened here these 2 lines are parallel.

So, what we are doing we are keeping P_1 and P_2 fixed, because what we are interested in we are interested in the effect of change in income on.

Student: Quantity.

Quantity demanded for good 1. So, now, we are changing it to I_2 and when we change it to I_2 what happens there is a parallel shift in the budget line. And that is what we get and similarly let me draw it just like this touching and then this is the third, and this is the

fourth ok. These are parallel to each other and of course, all the optimal bundle lies on bundles, lie on this line that I know; how do I know?

Students: By getting tangency (Refer Time: 06:52).

By getting the tangency points ok.

Student: (Refer Time: 06:58).

Student: if we are putting P 1 constant.

Student: Then x_1 is a 1, so it is a linear function and I (Refer Time: 07:05) linear function in I. So, if we increase the income, but it would be a linear transaction only;

X linear; it will be linearly increasing. So, what we are getting this path basically this line which passes through all the optimal bundles when income is increased or decreased or in other word when income is varied is called income expansion path.

And of course, income expansion path is drawn on graph giving the indifference map. So, what is important we are drawing remember for income expansion path, we are drawing the indifference curve and we are finding the optimal bundles.

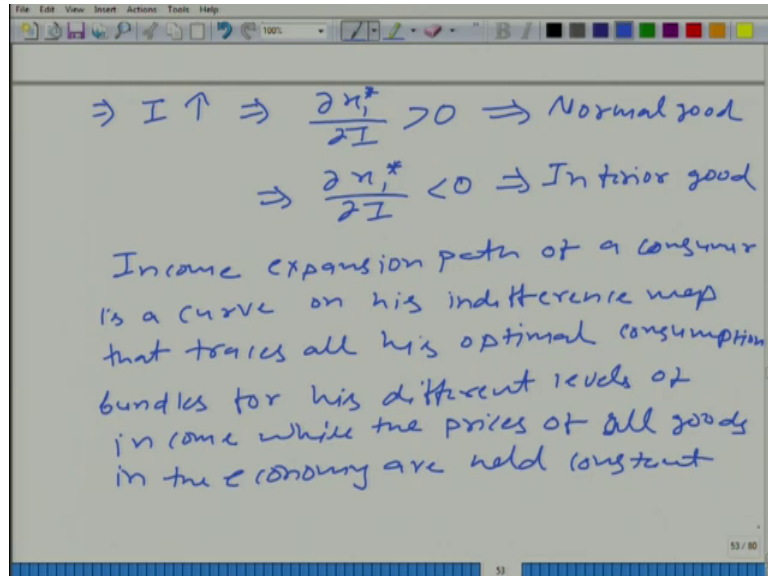
And when then we are drawing a curve that passes through all the optimal bundles that curve is called income expansion path or income consumption curve fine or income consumption path fine.

In other word it is very much possible that we have this kind of this is the indifference map and then let me draw the budget line what we have here is. So, it is like it is going like this not necessarily it is a straight line this is the curve giving all the optimal bundles. So, this is this curve is called income.

Student: Consumption.

Consumption path; fine.

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If I look at it I can say there are to possibly at any level that let us say when income goes up either it goes up x 1 star goes up. Let me write it here right what is happening income is going up and d x 1 star d I is;

Student: Positive.

Positive. It what does it mean that when income increases.

Student: X 1 (Refer Time: 09:34).

The consumption of good 1 at the optimal level increases. So, in this case we call these goods this.

Student: (Refer Time: 09:42).

Good is.

Student: Normal good.

Normal good ok; and when it leads to decrease in consumption of good 1.

Student: Inferior good.

Then it is called inferior good. At some places normal good is give a name of superior good that we will stick to these names. Normal good and inferior good fine is it clear.

Student: Yes.

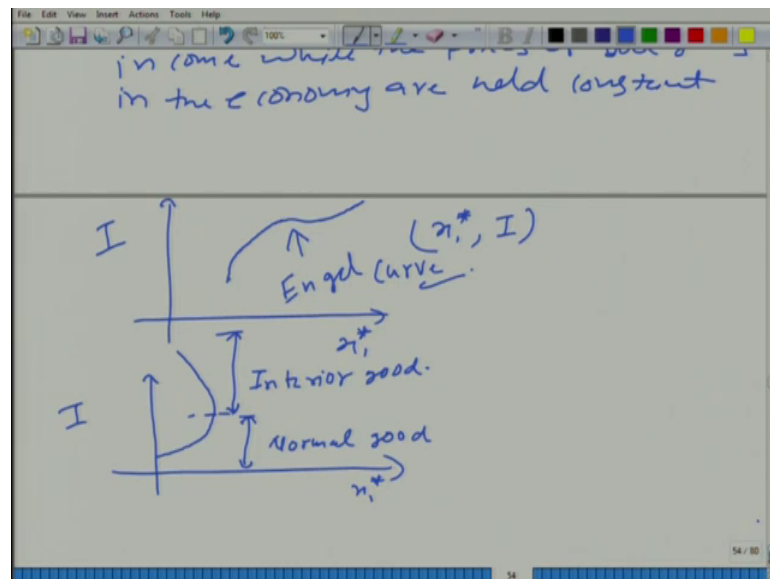
So, now, what can be do that rather than drawing this curve let us look at the definition first for a moment that just what we understood about income expansion path that income expansion path of a consumer of a consumer is a curve on his indifference map. This is very important that this curve is represented on.

Student: Indifference.

Indifference map on indifference map that traces all his optimal consumption that traces all his optimal consumption bundles for his different level of level levels of income; while the prices of all goods in the economy are held constant fine ok; so, what I am trying to say that you should not forget this term that this curve is traced on this indifference map.

Now, what we are going to do? We are going to trace these just the component of optimal bundle on different curve.

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What we have here is let us say that x axis gives us the optimal level of consumption of good 1 on y axis we have income. So, what we are taking let us look at it that this here this is x_1^* this x_1^* corresponds to a particular level of income. So, we can figure out that how much is x_1^* and how much is the;

Student: Income.

Corresponding income and we plot it here ok. And similarly for all the optimal bundles what we plot x_1 star and respective I and then we will plot depending on the different problem we plot it will look like this ok. It can be any curve and then we can of course, if we take more curve more optimal bundles we will have better.

Student: Continuous curve.

Continuous curve fine and this curve is called Engels curve. Now let us look at it on Engels curve if Engel curves is like let us say this there is a possibility that consumption of good 1 increases as income increases up to certain level and that is till this point and then it starts decreasing. What does it mean? It simply means that in this zone this good is;

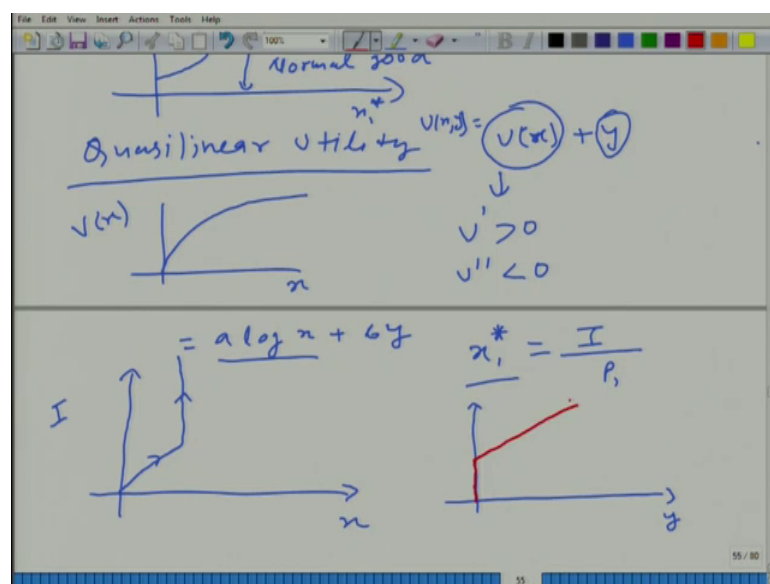
Student: Normal good.

Normal good and in this zone.

Student: Inferior good.

It is inferior good fine is it clear ok. Now let us take 1 or 2 examples we have obtained the expression of x_1 star in terms of P_1 and P_2 and I earlier. Let us try to draw the angle curve for those particular cases. So, let us take one case of quasi linear utility.

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What did we say that quasi linear utility will be of this form v x plus.

Student: Y.

Y.

Student: (Refer Time: 14:52).

Where it is linear in y and $v x$ where of course, we should put in some assumption here that v dash is increasing in x , but the rate of increase is decreasing as x is increasing. What it means is that any fixed level of y if we change x the utility will increase, but at decreasing.

Student: Decreasing.

Rate.

Fine; can you tell me what will be the income expansion path we have taken a special case where $v x$ was log function.

Student: (Refer Time: 15:37).

What we had I do not remember exactly, but we had something like a $\log x$ plus $b y$. Something like this we had. So, can you tell me what will be the income expansion path for x remember we had solved it that first it linearly increases in income ok. Because what happens that up to certain level this person spends is income only for.

Student: Yes good (Refer Time: 16:06).

Good one. So up to that level x 1 star is basically I by.

Student: P 1.

P 1; a function of I by P 1, fine. So, it is linearly increasing and after a certain level is reached.

Student: He stops by next one.

He stops he known he does not buy anymore of good 1. So, it becomes independent of income. So, it will be like this. So, Engle curve looks like this. ow about for good 2?

Student: first 0 till;

Up to certain level it will be 0 let me use this colour. So, it is like this 0.

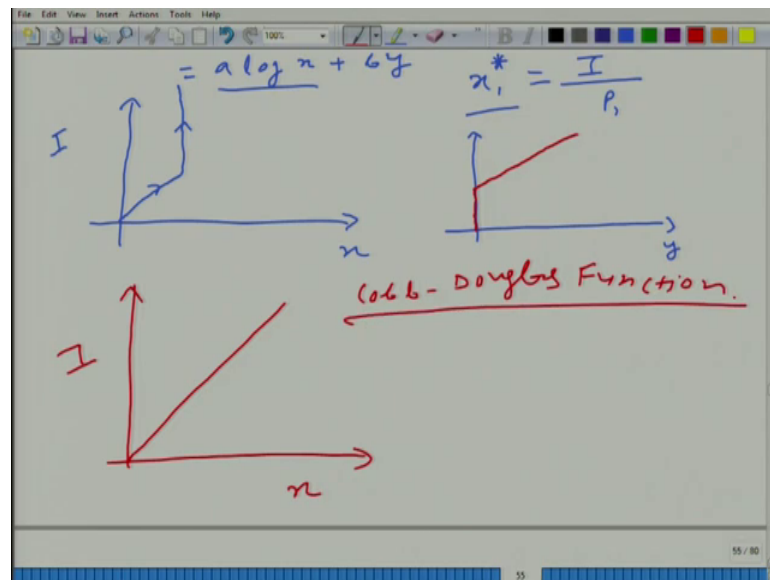
Student: Increasing.

And then it starts increasing.

Student: linearly.

Linearly fine is it clear. So, now, we have studied the effect of change in income on quantity demanded. Of course, we have done it for only good 1; we can do it for good 2 also.

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So, let us look at Cobb Douglas function what happens in for the Cobb Douglas function it is linearly.

Student: Increasing.

Increasing.

So, this will be the Engels graph Cobb Douglas, fine.