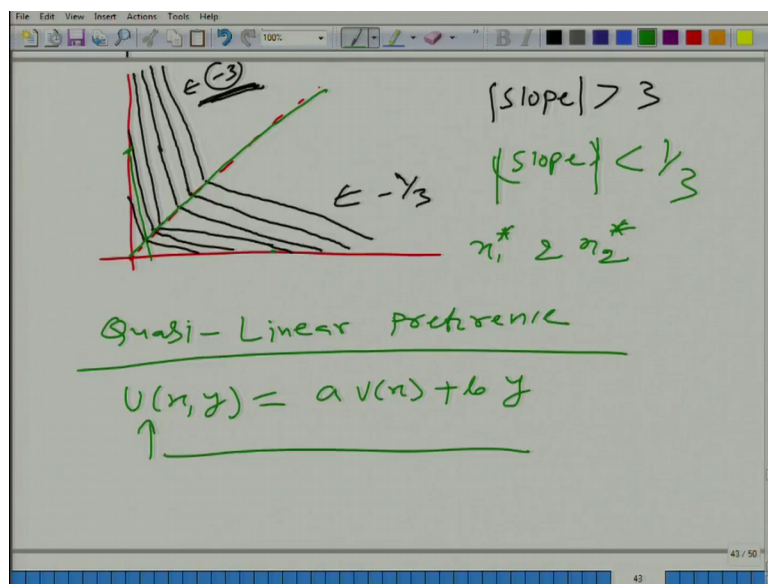


An Introduction to Microeconomics
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Lecture - 62
An Example with Quasi Linear Preferences

Now, I am going to give you another problem where we will use mathematical, some mathematical concept that we have learned to solve, that one is also difficult.

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Now, we are talk, going to talk about something called quasi linear preferences, let us say in 2 good world preference is called quasi linear if utility function representing this preference can be written as all and b by. What it means is that utility function is linear in one of the arguments, are not linear in the other argument then we say the utility function the utility function represents quasi linear preference fine, is it clear.

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Quasi-Linear Preference

$$U(x_1, x_2) = a \ln(x_1) + b x_2$$

log

$$\max_{x_1, x_2} a \ln(x_1) + b x_2$$

$$p_1 x_1 + p_2 x_2 \leq I$$

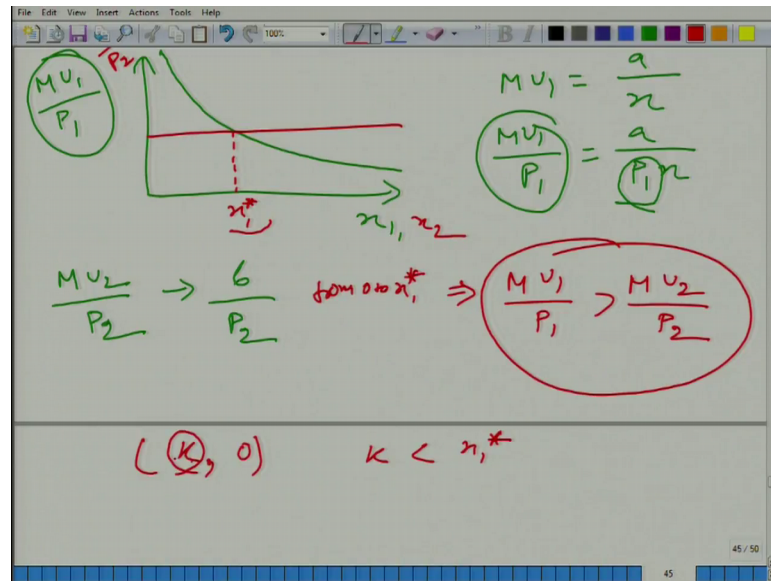
So, now let us take a problem where, what we have is maximise a lets instead of vx just for simplicity let us take log, a log x plus by or if we are using x 1 sorry .

Student: (Refer Time: 01:34).

V is a function it is not it is not same v as the earlier one, if it is may confusing you take it as w that it is a function of x, what I mean to say an individual is said to be having quasi linear preference if the utility for function representing his preferences can be written in this particular form and what is this particular form that it is linear in y and also this is additively separable in x and y. That x and y are not appearing as in a multiplicative term, they are additively separable then the preference is called quasi linear preference fine is it clear.

So, let say its x 1 and it is x 2 and here we have maximise this is the last example we are talking about and then we will move to next topic x 1, x 2 a l n x 1 this is natural log ln is natural log and let say b x 2 fine and the budget is p 1, p 1, x 1 p 2, x 2 and it should be less than or equal to r. How will you proceed, indifference curve draw indifference curve how would it look like of course, you can solve it using indifference curve later on we will see that, but right now I want to give you flavour of mathematical techniques. So, let us solve it using the concept of marginal rate of substitution the concept that we have learned earlier ok.

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So, what we can see here. Let say here we have amount of x_1 and what we should put here is $\frac{MU_1}{P_1}$ and what is this $\frac{MU_1}{P_1}$ the gain in utility if you spend 1 rupee on x_1 . So, to do this what we need we need to get MU_1 and what is MU_1 a by x and MU_1 by P_1 will be $\frac{a}{P_1 x}$ is not it. So, how will it look like P_1 is of course, given to you, you do not have any control over P_1 it is parameter that does not change in short period you can change it as modular you can play with it by you can study the effect of change in P_1 on this entity, but typically in its given to you ok.

So, how would it look like $\frac{1}{x}$ graph something like this fine and how about let us calculate $\frac{MU_2}{P_2}$, again this is for good 2 and what do we get how much is $\frac{MU_2}{P_2}$ b divided by P_2 . So, how will it look like it is the same straight line horizontal line.

So, what we are doing is here we are drawing first with respect to $\frac{MU_1}{P_1}$ with respect to x_1 and then $\frac{MU_2}{P_2}$, let me use the different colour $\frac{MU_2}{P_2}$ and this is this is with respect to x_2 and deliberately I have put it on the same graph. So, what it means is that up to this level lets I am calling in it x_1^* why it will become clear immediately from 0 to x_1^* , $\frac{MU_1}{P_1}$ is greater than $\frac{MU_2}{P_2}$, no matter what is the level of the second good it is independent of the second good fine.

What does it mean? What does this equation mean what will be your interpretation for this equation.

Student: He would buy more of first good.

More of first good he will buy only the first good up to this level, unless this level is achieved we will not buy the second good because let say right now he has k comma 0 and k is less than x_1^* . So, it makes sense for him to spend one more rupee on good 1 rather than on good 2. So, he will keep on buying good 1 you can think you can stretch it little further you can say he is starting from 0 comma 0 .

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Handwritten notes on a whiteboard:

$(k, 0)$ $k < x_1^*$
 $(0, 0)$ $x_1 > x_1^* \Rightarrow \frac{MU_2}{P_2} > \frac{MU_1}{P_1}$
 I \rightarrow he starts buying good 1
 \rightarrow till he reaches x_1^*
 $\frac{MU_1}{P_1} = \frac{MU_2}{P_2} \Rightarrow \frac{a}{P_1 x_1^*} = \frac{b}{P_2}$
 $\Rightarrow x_1^* = \frac{a}{b} \frac{P_2}{P_1}$

And of course, first unit he should buy of good 1 why? Because mu one divided by p 1 is greater than m u 2 divided by p 2 and he will keep on buying good 1 until he reaches the level of x_1^* and once it reaches the level of x_1^* and he is still has some money left because there is a possibility income is again a parameter, income is given to him there is a possibility that given his income is not able to buy x_1^* of good 1. If he is not able to buy x_1^* of good 1 in that case is optimal level of consumption would be the maximum amount of good 1 he can buy and 0 amount of good 2 is it clear.

But the second condition, second situation is that he has let us say he has already bought x_1^* amount of good 1 and he still has some money left, now what is happening that in after beyond this beyond x_1^* , greater than x_1^* m u 2 divided by p 2 is always greater than m u 1 divided by p 1. So, it means that once he reaches to the level of x_1^* he would no longer buy any more of good 1 and he will start buying good 2. So, what is happening here this is a very nice situation, if you understand it that he has

income what he do he starts buying good 1 and he keeps on buying good 1 till he reaches, till he reaches x_1 star and how can we get the x_1 star we have any way to figure out x_1 star. How that at this level its $m u_1$ divided by p_1 is equal to $m u_2$ divided by p_2 and we have value for both of these entities and how much is $m u_1$ divided by p_1 , a by p_1 x and $m u_2$ divided by p_2 is b by p_2 . So, it is independent of y .

So, x_1 this gives us x_1 star and x_1 star is a divided by b p_2 divided by p_1 . So, unless he accumulates this amount of good 1 he will not buy good 2 and to buy this much of good 1, how much income does he need?

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The whiteboard contains the following handwritten equations:

$$\frac{MU_1}{P_1} = \frac{MU_2}{P_2} \Rightarrow \frac{1}{P_1} x_1^* = \frac{1}{P_2}$$

$$\Rightarrow x_1^* = \frac{a}{b} \frac{P_2}{P_1}$$

$$P_1 x_1^* = \frac{a}{b} P_2$$

$$0 < I < \frac{a}{b} P_2 \Rightarrow x_1^* = \frac{I}{P_1}$$

$$x_2^* = 0$$

$$\frac{a}{b} P_2 < I \Rightarrow x_1^* = \frac{a}{b} P_2$$

$$x_2^* = \frac{I - \frac{a}{b} P_2}{P_2}$$

$P_1 x_1$ star and that is a by b p_2 . So, what we are saying that if income is less than a by b p_2 and of course, greater than 0 then he will buy in this case x_1 star is going to be equal to I buy p_1 because he will buy only good 1 and x_2 star is going to be equal to 0 fine is it clear.

What if he has, what if it is more than a by b p_2 what happens in this case, x_1 star is going to be equal to a by b p_2 and how about x_2 star because this much is going to buy good 1. So, it is going to be I minus divided by p_2 .

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The image shows a whiteboard with handwritten mathematical equations. The top equation is $0 < I < \frac{a}{b} P_2 \Rightarrow x_1^* = \frac{I}{P_1}$. Below it, the quantity x_2^* is set to 0. The second main equation is $\frac{a}{b} P_2 < I \Rightarrow x_1^* = \frac{a}{b} P_2$ and $x_2^* = \frac{I - \frac{a}{b} P_2}{P_2}$. The terms x_1^* , x_2^* , and $\frac{a}{b} P_2$ are circled in red. The whiteboard interface includes a menu bar (File, Edit, View, Insert, Actions, Tools, Help), a toolbar with drawing tools, and a status bar at the bottom right showing '46 / 50'.

$$0 < I < \frac{a}{b} P_2 \Rightarrow x_1^* = \frac{I}{P_1}$$
$$x_2^* = 0$$
$$\frac{a}{b} P_2 < I \Rightarrow x_1^* = \frac{a}{b} P_2$$
$$x_2^* = \frac{I - \frac{a}{b} P_2}{P_2}$$

So, can you say anything can you make any comment about this scenario that x_1 is kind of a necessary good, you know this person does not consume any of good 2 unless he has sufficient amount of good one, fine that is the situation described by this particular kind of utility function is it clear.