

An Introduction to Microeconomics
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Lecture - 59
Marginal Utility Vs. Marginal Rate of Substitution (MRS)

Now, let us use the concept let us learn about marginal utility, although, we talked about marginal utility earlier also, but just what is marginal utility?

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The image shows a digital whiteboard with handwritten notes. At the top, there are three red vertical dashed lines with dots at the bottom, labeled '10', '0', and '10' from left to right. Below this, the text 'Marginal Utility' is written in orange, followed by '(x1, x2)' in black. A horizontal line is drawn under 'Marginal Utility'. Below the line, the utility function is written as $U(x_1, x_2) \rightarrow U(x_1+1, x_2)$. An arrow points from the x_2 in the second function back to the x_2 in the first function. Below this, the formula for Marginal Utility of good 1 is written: $MU_{x_1} = \frac{U(x_1+1, x_2) - U(x_1, x_2)}{x_1+1 - x_1}$. A checkmark is next to the denominator. Below that, the formula for Marginal Utility of good 2 is written: $MU_{x_2} = \frac{U(x_1, x_2+1) - U(x_1, x_2)}{x_2+1 - x_2}$. A checkmark is next to the denominator. The whiteboard interface includes a menu bar (File, Edit, View, Insert, Actions, Tools, Help) and a toolbar with various drawing tools. The page number '27 / 50' is visible in the bottom right corner.

So, it is the amount of amount by which total utility increases every increase one good in the bundle by 1 unit while keeping all other goods in the bundle fixed. And then it is marginal utility with respect to that particular good which amount has been increased ok.

So, let us say what we mean here is we have utility some utility let say for a bundle let say this is of course, if you are using notation x comma y that it donates it represents a bundle; what is happening and if we can use our earlier notation also x_1 comma x_2 what we have here is x_1 comma x_2 ok.

So, what we are saying that we keep x_2 fixed while we increase x_1 by 1 unit. So, what is happening x_1 plus 1 this will be the new utility and let us say if it satisfies monotonicity then of course, in the new bundle we have same amount of good 2, but more of good 1. So, of course, here utility will be higher. So, the increase in utility is let say that change in utility is denoted by ΔU then it is going to be $U(x_1+1, x_2)$

x_2 minus U of x_1 comma x_2 . And denominator although we do not have anything in the denominator or we have 1 in the denominator; I can write it like x_1 plus 1 minus x_1 that is what we have and this this is basically defined as marginal utility with respect to x_1 fine.

But here what we are doing we are changing x_1 by one unit what is the marginal?

Student: Sir that could be ΔU upon Δx_1

No wait we will we are talk here basically is again if you have you are talking about ΔU by Δx_1 let me explain it to you; how we reach there. Basically what we are talking about here is ΔU is in the numerator and in the denominator we have 1; 1 we can express as x_1 plus 1 minus x_1 and that is the that is that is equivalent to 1 and this is marginal utility with respect to x_1 ; I will come back to the calculus the definition that we gave earlier using calculus.

Now, let us look at the marginal utility with respect to x_2 ; what it means? That we keep x_1 fixed and we increased x_2 by 1 unit and here we have and of course, denominator we can leave it as it is does not matter; this is marginal utility with respect to x_2 , but now what we are doing? We are changing x_1 by 1 unit; what we can do that rather than changing x_1 by 1 unit.

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The image shows a digital whiteboard with handwritten mathematical derivations for marginal utility. The derivations are as follows:

$$MU_{x_1} = \frac{\Delta U}{x_1 + 1 - x_1} = \frac{U(x_1 + 1, x_2) - U(x_1, x_2)}{x_1 + 1 - x_1}$$

$$MU_{x_2} = \frac{\Delta U}{x_2 + 1 - x_2} = \frac{U(x_1, x_2 + 1) - U(x_1, x_2)}{x_2 + 1 - x_2}$$

$$\frac{\Delta U}{\Delta x_1} = \frac{U(x_1 + \Delta x_1, x_2) - U(x_1, x_2)}{x_1 + \Delta x_1 - x_1}$$

$$\lim_{\Delta x_1 \rightarrow 0} \frac{\Delta U}{\Delta x_1} = \lim_{x_1 \rightarrow 0} \frac{U(x_1 + \Delta x_1, x_2) - U(x_1, x_2)}{x_1 + \Delta x_1 - x_1} = \frac{\partial U}{\partial x_1}$$

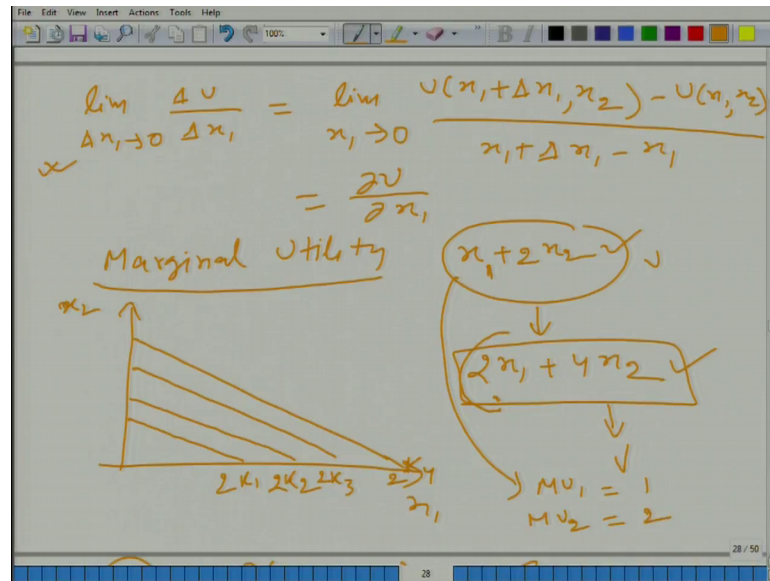
Let us look at the change in change in utility if we change x_1 by Δx_1 unit very small unit and then what we have here is $x_1 + \Delta x_1, x_2$ minus $U(x_1, x_2)$ fine.

And what we have this ΔU is now change because of Δx_1 unit change in amount of good 1 in the bundle fine so, but Δx_1 can be anything. So, rather than talking about absolute change; we should talk about rate of change and how can we get the rate of change? If we divide it by Δx_1 then this is the rate of change. And here also we can divide it by we will have to divide it by Δx_1 and Δx_1 can be written as $x_1 + \Delta x_1$ minus x_1 .

And now if we can take limit where Δx_1 is going to 0 what will we get? ΔU by Δx_1 is equal to limit Δx_1 is going to 0; $U(x_1 + \Delta x_1, x_2)$ minus $U(x_1, x_2)$ divided by $x_1 + \Delta x_1$ minus x_1 . And this is nothing, but the partial derivative of U with respect to x_1 . So, both definitions are fine this is more precise; here we are talking about rate of change in U with respect to x_1 while keeping x_2 fixed in the bundle.

This gives us marginal utility with respect to x_1 ; here we are taking approved way because sometime we do not know calculus, then we use this if we if we do not know calculus then we can use this definition and then we have one in the denominator because Δx_1 is 1 in that particular case fine and this is marginal utility, but here is the problem we have learned about marginal utility.

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Let us look at the one particular problem that marginal utility leads to. Let me just draw this problem and then I will come back to the problem that we intended to solve right in the beginning of this topic ok. So, what we will do? We will come back to that topic and we will solve it using some other technique, but we should also learn the problem with the marginal utility concept.

Now, what is happening let us draw the indifference map for x plus $2y$ or if we want to convert it if we want to denote it by x_1 and $2x_1$ the problem would remain the same its the same problem; does not matter. So, you can change the variable because x_1 and x_2 they are just representing their the name; so, does not matter. So, here we have x_2 and here we have x_1 ; when we draw it how would it look like? Downward sloping with slope minus; minus 1 by 2 something like this it would look like fine and let say let us start if we can say this is we have here K_1, K_2, K_3, K_4

Now, instead of using this notation K_1, K_2, K_3, K_4 can I use this notation it is $2K_1, 2K_2, 2K_3$ and $2K_4$ or in other words rather than using x_1 plus two x_2 can I use $2x_1$ plus $4x_2$; will it represent the same preference nothing would change why because utility is ordinal in nature it is about order fine

So, now what we are saying that this these two utility functions; they represent the same preference nothing different. Now get the marginal utility for both of these utility function; marginal utility with respect to x_1 ; what will you get?.

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$$MU_1 = \frac{\partial(x_1 + 2x_2)}{\partial x_1} = 1$$
$$MV_1 = \frac{\partial(2x_1 + 4x_2)}{\partial x_1} = 2$$

Cardinal in nature

Marginal utility in case of let us say in shortcut M U 1 represent with respect to first argument we have also written it as M U x 1 and what we have is d x 1 plus 2 x 2 with respect to x 1 and we get here 1. Or what we are saying is in other word; if we do not use the calculus definition; what we can use our non-calculus definition. So, if we increase x 1 by 1 unit how much will be increase in total utility? 1 same as this fine

Now, how about M U 1? In the let us denote this utility function as U and this utility function as V. So, what will be M this is U and this is V; what will be the marginal utility in this case? It is 2 again use non-calculus definition if you increase x 1 by 1 unit; how much increase will you get while keeping x 2 fix? How much increase will you get in the total utility 2; according to this.

Now, what is happening in one case we are getting 1, in another case we are getting 2. So, it seems that marginal utility is related to it somehow cardinal in nature cardinal in nature; it assumes that the value attached to a particular rank is can be doubled can be halved ok.

So, remember earlier we discussed that these; these were quite important when we studied utility function as cardinal in nature, cardinal utility function, but now we have figured out that utility we do not need cardinality of utility function; ordinality will work well. But when we are talking about ordinality we should not be moved by the value of M U 1 or M V 1 because they are cardinal in nature. So, be very vary of using M U 1 and

M V 1 marginal utility in your practical problems because you will reach to wrong place if you do not know fine.

So, what is the solution? The solution we will see immediately again let us solve this problem using the technique that we have learned in the class ok. Earlier we solve it using just description and then table; now let solve it using the techniques that we have learned.

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\rightarrow MRS = slope of budget line
 \rightarrow slope of indifference curve
 $2x_1 + 4x_2$

$$\text{MRS} = - \frac{\partial U / \partial x_1}{\partial U / \partial x_2} = - \frac{1}{2}$$

$$= - \frac{\partial U / \partial x_1}{\partial U / \partial x_2} = - \frac{2}{4} = - \frac{1}{2}$$

And what did we learn? That we learned that M R S should be equal to the slope of the budget line or in other words M R S is nothing, but the slope of slope of in different curve fine.

Now, let us calculate M R S in both the cases what does it equal to? It is equal to the exchange rate that you have in your mind at that particular bundle of course, in some of the cases your exchange rate that you are comfortable with in your mind would change as you have different bundles, but in this particular case what is happening? Exchange rate remains the same; it does not depend on how many units of good 1 or good 2 you have fine

So, how much is M R S? If we use calculus to calculate M R S is equal to this is what it is equal to I have used earlier x for x 1 and y for x 2, but let us take to that this fine and

how much is this? Minus for the first utility case for the first utility case from here what we can get M U 1 is 1 and M U 2 is 2 fine.

And for the second case for the second case that is 2 x 1 plus four x 2; 2 and 4. So, what we get in the first case its half and if you take another utility function that is what we have is minus 2 by 4 and that is half. So, M R S; M R S is independent of the particular selection of the utility function; what we have learned let me just emphasise this point once again why we are getting something like this?

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$$Q \succsim R \Leftrightarrow u(Q) \geq u(R)$$

$$\Leftrightarrow \forall Q, R \in X \quad u(Q) \geq v(R)$$

$$Q = (x_1, x_2) \quad \underline{v(Q)} = \underline{g(u(Q))} \quad \frac{g'}{g'} > 0$$

$$MRS = \frac{-\frac{\partial v}{\partial x_1}}{-\frac{\partial v}{\partial x_2}} = \frac{-g' \frac{\partial u}{\partial x_1}}{g' \frac{\partial u}{\partial x_2}}$$

So, what we learned earlier that if we have preference such that this then it you will be able to represent of course, it should satisfy some axioms that we have discussed; that this and then any monotonic transformation of this utility function would also work. This is what we have of course, this symbol says if and only if and only if. So, both ways fine for all x_1 and x_2 in the consumption set.

So, in other word in other word what is $V \times 1$; $V \times 1$ of course, right now let us choose some other because it will lead to confusion we are using x_1 and x_2 for not different bundles for to denote the amount of a particular good in the bundle. So, rather than using x_1 and x_2 ; we can take it here P and Q we have R Q; let us take Q and R sorry because P again is what? Price is it clear? This is for all Q and R in x

So, now when we take V of Q of course, V is monotonic transformation of U . So, what how can we write it? This is nothing, but g of U of Q and where g dash is greater than 0 by our definition; we have discussed it in the class fine. So, now, let us calculate MRS using this particular function and what is this equal to minus dV with respect to x_1 because remember we are talking about two good world Q has two goods x_1 and x_2 divided by partial derivative of V with respect to x_2 fine.

And when we use this; what we will get? Minus g dash dU partial derivative of U with respect to x_1 minus again here g dash partial derivative of U with respect to x_2 .

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The image shows a digital whiteboard with the following handwritten content:

- At the top: $V(Q) = U(Q)$ with a note $\forall Q, R \in X$.
- Below that: $V(Q) = g(U(Q))$ with the condition $g' > 0$.
- The Marginal Rate of Substitution (MRS) is defined as:

$$MRS = \frac{-\frac{\partial V}{\partial x_1}}{-\frac{\partial V}{\partial x_2}}$$
- The derivation shows the substitution of the composite function:

$$= \frac{-g' \frac{\partial U}{\partial x_1}}{-g' \frac{\partial U}{\partial x_2}}$$
- The final simplified result is:

$$= -\frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}}$$

So, this will get cancelled and we are back to the MRS ; which we calculate we calculated with the first utility function representing the preference of this particular person. So, MRS is independent of monotonic transformation of utility function. So, it does not depend on the particular valuation; it is x_2 yes fine it is clear?