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An Introduction to Microeconomics
Prof. Vimal Kumar
Department of Economic Sciences
Indian Institute of Technology, Kanpur

Lecture - 57
Utility Maximization: Example

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Let us take one particular example where utility is given by log xy.

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Fine and budget constraint is given by of course, x has to be greater than or equal to 0 y has to be greater than or equal to 0 how can we solve it? First what we will do? From here we can get the indifference curve for the fix level of utility we will get different indifference curve. And what will be the slope of that indifference curve? Or in other word what would be the marginal rate of substitution it is equal to Ux by Uy and what is Ux?
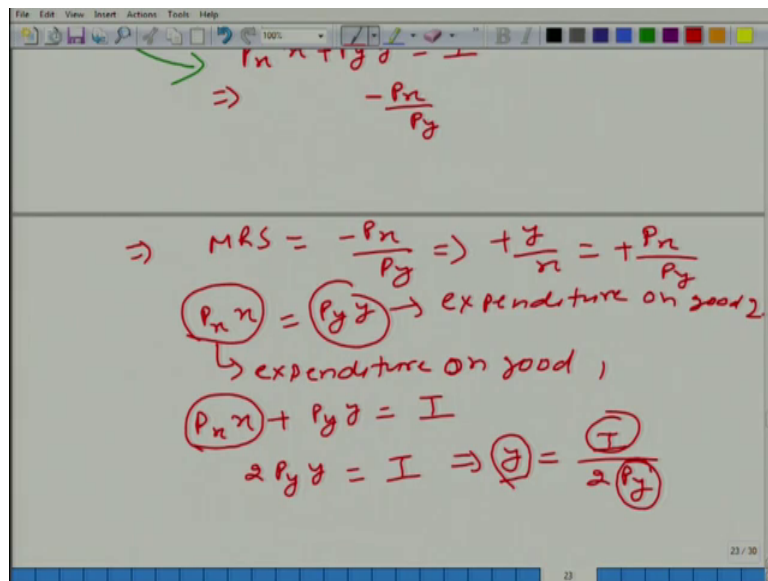
Student: Log x

Let us let us calculated it what we have is u of x comma y is equal to log xy; if we use the log rule it can be written as log x plus log y. And marginal utility from x and what is marginal utility? The change in utility if we increase one of the goods by one unit. While keeping all other goods at the same level and what we will get 1 by x. And similarly, del u by del y is y.

So, in other word if we, this from here we will get minus y by x. From here, what we get because of monotonicity, we do not have to worry about this inequality part. What we have to worry about Px x plus Py y should be equal to I and because of monotonicity it is increasing in x. The utility is increasing in x and y we do not have 2 worry about these 2 constraints also.

So, what is the slope of this indifference curve, this budget line? It is.

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Minus Px by Py. So, MRS has to be equal to minus Px by Py and how much is MRS is equal to minus y by x. In other word, Px x should be equal to Py y and what is Px x? Expenditure on on good 1 on good 1. And this is expenditure on good 2 mind you, this is not always true. This is because of a specific utility function that we have selected. It depends on utility function and from the budget line what do we know? Px x Py y has to be equal to I.

So, this can be replaced by the other what we get 2 Py y should be equal to I. So, y is I divide by 2 Py mind you I is parameter income given to that person person typically does not have control over it. Similarly, Py is the market given and y he has to choose how much y will he choose to maximise his utility? That is going to be I divided by 2 by Py and similarly x is going to be I by 2 Px.

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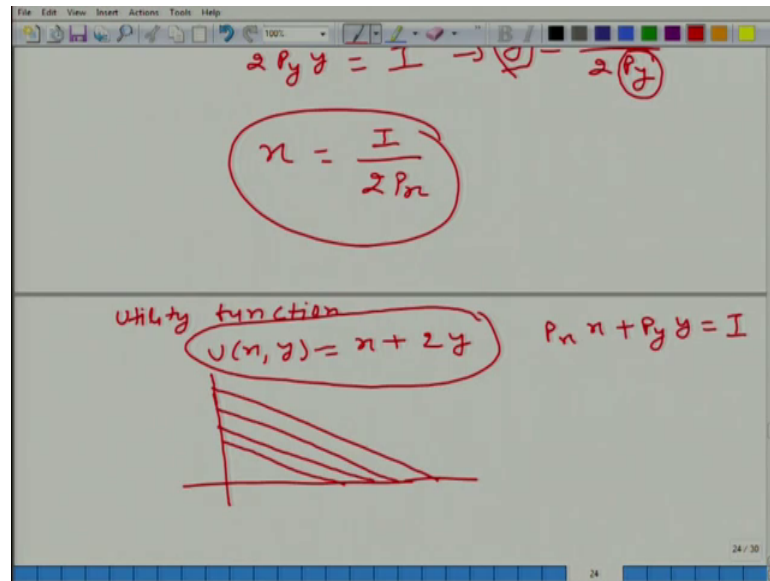
Student: (Refer Time: 04:20).

Now, let us look at different utility function. Let say utility function is given by x plus 2 y. What it means? Is that this person utility is a straight line that we have already discussed. How would it look like? How it is going to look like? Minus 1 by 2 like this. Let say, the price of course, the budget line is going to be this. Let us not worry about the inequality sign, because we have already discussed if monotonicity satisfied, monotonicity satisfied. We do not have to worry about inequality in the budget set.

The problem here is, that this utility function does not satisfy strict convexity axiom. It is convex, but not strictly convex. What would be the bundle that would lead to the maximum utility for this individual.
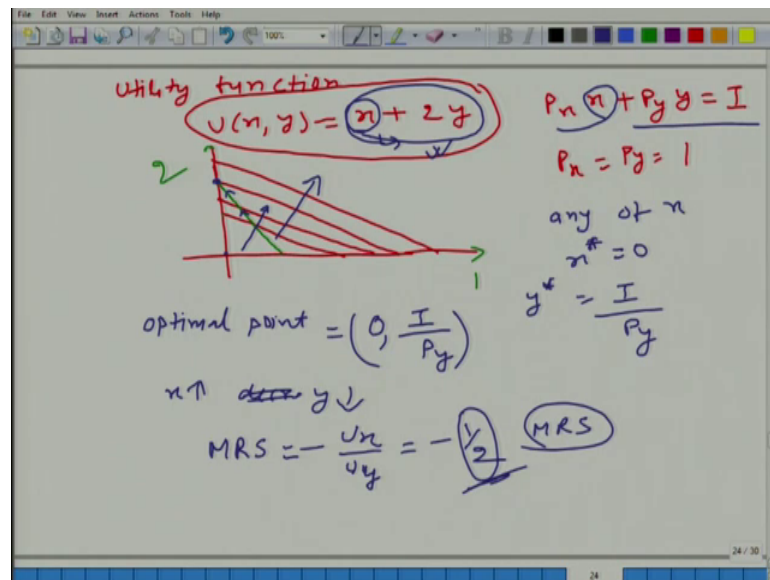
Let us say Px is equal to Py. Let us say it is equal to 1 think about it. So, in other word, how it is going to look like? A budget line is going to look like this something like this. So, from graph it is very clear good 1 and good 2 utility is increasing in this direction. So, what this person will try to do? Try to reach the maximum possible indifference level.

So, from let us say if he starting with x equal to 0, y is equal to 0 he will move in this particular direction. And keep on moving and after he hits here let say the budget line what he would realise that, if he moves in this particular direction, his utility increasing and he would end up at this point it means he will not consume.

Student: x

Any of x x star is equal to 0 and then this problem is very simple. How much y he would consume? Everything if x star is 0, then everything it is would be all his money would be put for to purchase y. So, it is going to be I and let us say Py is not equal to one then I by Py and if I say I give you more general problem.

Now, let us look at it mathematically also what is happening at this point? He has here is the point optimal point that we have figured out is 0 comma I by Py, I by Py fine. Now, let say if we increases x and decreases y and of course, with the aim to remain on the same indifference level. So, what does it say? What is his marginal rate of substitution?

Marginal rate of substitution marginal is, remember is minus Ux divided by Uy. So, minus 1 by 2 or if you just look at here what he will be what he wants to maximise? Is the number that he obtains from x plus 2 y.

So, to remain on the same number remain on the same indifference curve and he wants one more unit of x. How many units of y he will have to give up? Half unit, so that is why it is equal to MRS is equal to minus half. Minus is minus only indicates that x and y are moving in the opposite direction that is it. So, he will be he will have to give up half unit of y to get one more unit of x that is his marginal rate of substitution any doubt about this?

Now, what is happening? Look at it what is the market price the market price of good 1 and good 2 are equal it means in market, you can exchange one unit of good 1 by giving up one unit of good 2 or you can get one unit of good good 1 by giving up one unit of good 2. I think I said the same thing, but does not matter the ratio is one is to one fine.

Now, for you to satisfy what does it mean? What is your ratio of exchange in your mind? 1 is to 2, 1 unit of good 1 is 2 unit of good 1 is equivalent to 1 unit of good 1 good 2. 1 unit of good 1 let us pay attention here one unit of good 1 is equivalent to.

Student: Half unit of.

Half unit of good 2 and similarly one unit of good 1 good 2 is equivalent to.

Student: 2 units.

Two units of good 1 fine so, of course, in your mind the exchange rate is bad for bad for bad to get good 1.

Student: Good on.

Bad to get good 1 market is giving better price. So, what you will do? You will keep on decreasing the consumption of good 1.

Student: Good 1.

With the hope that to increase your utility, that is what would happen when you move in this direction, but you keep on doing it and this the indifference curves are straight lines.

So, MRS is same everywhere MRS is not decreasing even if you have more of good 2. MRS is not decreasing. It is not strictly convex remember, we talked about diminishing marginal rate of substitution. In case of strict convexity, here we do not have a strict convexity. So, marginal rate of substitution remains the same. So, you keep on decreasing and market is always giving you better deal for good 2. According to your utility function. So, where will you end up you will give up all of your good 1 and you will keep consume only good 2 fine is it clear?

Student: Yes sir.

Ok.

So, at the optimal level, the rate of exchange in the market should be same as rate of exchange, that you have in your mind. Otherwise, if these 2 are not equal you would always trade in the market that will make you better off fine.

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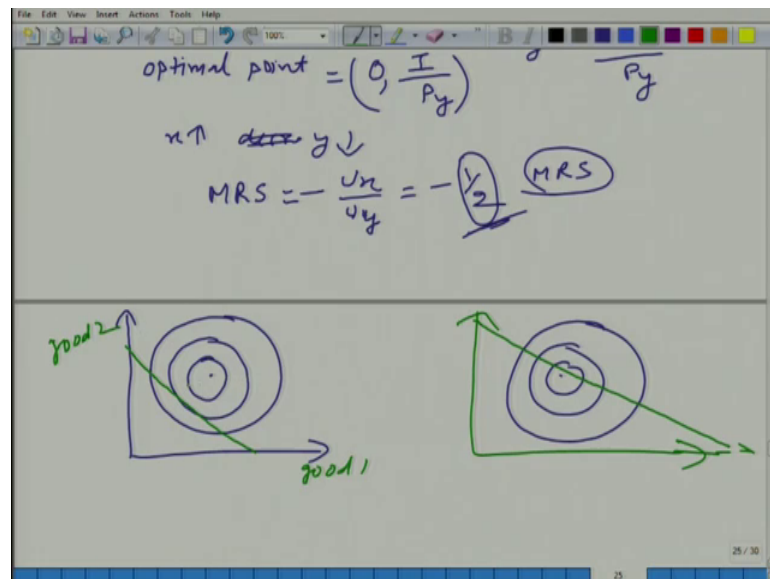
Now, let us look at another utility scenario rather than giving you a function and solving it completely just. So, that you understand let say this person gets satiated at 1 point. So, his indifference level will be given as this something of course, this is bad and here is his budget line. There are 2 possibilities 1 is this good 1 good 2 and second is this possibility and your budget line is this these are 2 possibilities.

So, far we have been talking about that we should have the optimal bundle on the budget line. How about this case? We would not have because remember what is happening in this case? Monotonicity is violated more is not better for better than less, but how about this case here? Is still it would be satisfied why because we are dealing with in the zone where more is still better than less.