An Introduction to Microeconomics
Prof. Vimal Kumar
Department of Economic Sciences
Indian Institute of Technology, Kanpur

Lecture - 55
Utility Maximization: Tangency Criterion

Now, let us interpret what does it mean, we did it mathematically, what does it mean?

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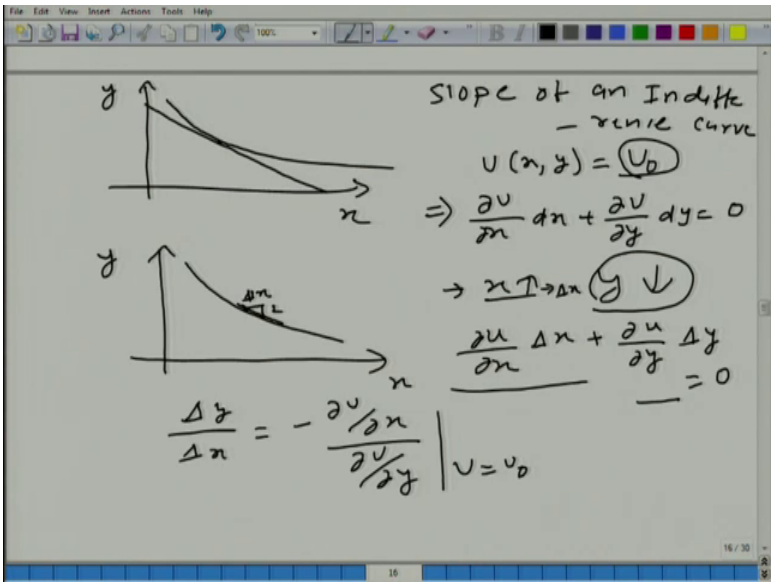

This is the point of course, this is bad drawing, but this is the point. What is the special property of this point? It is tangent. Point of tangency; in other word if we get the slope of indifference curve at this point. It is equal to the slope of budget line at that point.

Now, what how do we get the slope of just to interpret, how do we get the slope of an indifference curve? Very good, but let us see how do we reach there. What do we, how can we denote the indifference curve mathematically? It is basically uxy of course, I am talking about 2 good world, and is equal to some constant. Let us say k 0, or we can say u 0, does not matter, because it is constant. And we will differentiate it totally, total differentiation. What we will get? Equal to 0, why? Because this is a fixed number, if we differentiate with respect to any variable what do we get? 0.

So, what do I mean by total differentiation? Let us look at it mathematically. We are going back and back in the description, but it is important to understand this. Slope of indifference curve, we want to get at a point. Let us say we want to get at this point.

What we can do? That we start at this point and we pick any other point. Let us say 1 and 2. How can we move from 1 to 2? 1 and 2 what is happening? X is going up and.

Student: y.

y is coming down. So, so x is going up, x is going up if, x is going up, how much increase in the utility you will get?

What is del u by del x? Del u by del x is nothing but rate of change in u, with respect to x, fine? So, du by dx is rate of change in u with respect to x. And of course, we are keeping y constant. So, x is let say x is going up by delta x. So, total change in utility will be du by dx del u by del x, that is the rate of change in u with respect to x multiplied by delta x.

And similarly, how much we will get here? Del u by del y multiplied by delta y. Not minus, it will take care of itself of it on it is own we do not have to put. So, total change is going to be sum of these 2. And this is has to be equal to 0. If I want him to if I want this person to bring back to the same utility level. So, when I move from here to here by increasing delta x utility goes up. And to bring him back, what we have to do is; We have to decrease y by delta y amount. And so, these are the changes and this is equal to 0.

So, what would be the slope? Let us say this is the slope at this point, this is the slope. Delta y by delta x is the slope. So, this is equal to from this formulation, this is equal to del y as u equal to u naught, fine? And of course, what we are doing? This is we are picking this point that is little further from the original point. What we can do? We can keep on decreasing the change in x; that is, delta x, we keep on decreasing the change in x. Accordingly change in y will also decrease. Because see ultimate aim is to bring this person back on the same utility level. So, ultimately this is what we get. This is what we will get, and same thing you can get from here. This is mathematical root, this is a mix root that graph as well as mathematics. Nothing fancy from here, what we are doing we are taking here limit delta x goes to 0, fine?

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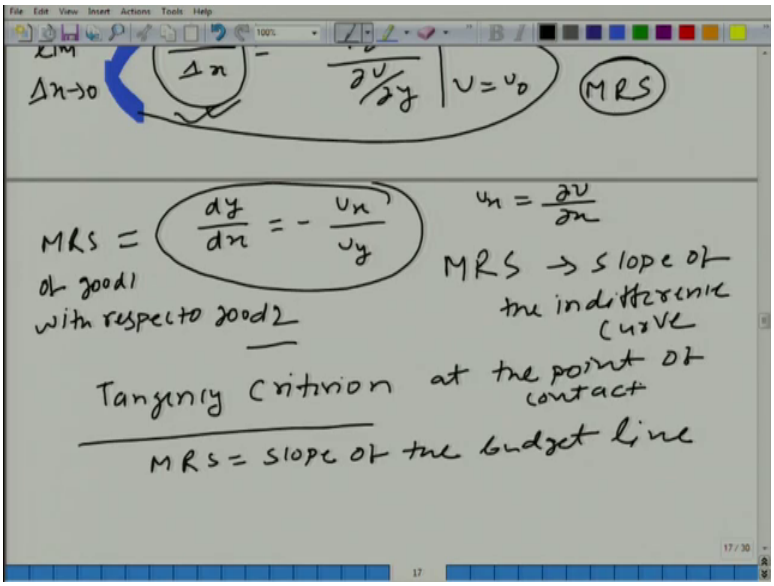

And we are obtaining the slope, fine? And what we are getting? Delta y by delta x is equal to, and that the sort of this is u of x divided by u of y, u of x is nothing but del u by del x. And what is this? How did we define it? Earlier MRS this was earlier defined as MRS. Of course, we talked about a special case of MRS we are change in x was given by one unit. But this is more general definition of MRS. So, this is a MRS marginal rate of substitution of good one with respect to good 2. So, in other word MRS is nothing but the slope of the indifference curve fine.

So now, what we are talking about that slope of indifference curve should be equal to the slope of budget line. That is the tangency criteria, because if it is not true then, indifference curve would intersect the budget line at 2 point. They have to be equal, then only you will get that u that indifference curve is just touching the budget line, fine?

So, what we have talked about? That MRS has to equal to slope of budget line at the point of contact. Fine, is it clear? We will do just, we will do this thing again using just argument so that we not only learn the mathematics, but also economic intuition behind it why we are doing it something like this? But one thing at a time.

And what is the slope of budget line?

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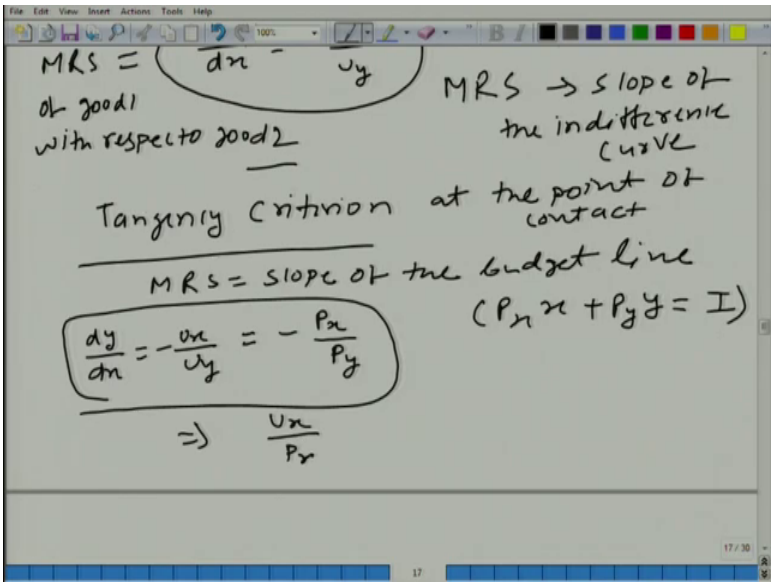

Minus P minus Px by Py, because remember the line is Px x plus Py y is equal to I. Remember, I started with less than or equal to I, but we figured out because of monotonicity what we need? The constraint is no longer this, but constraint becomes un equality. And how much is; this is what we have figured out.

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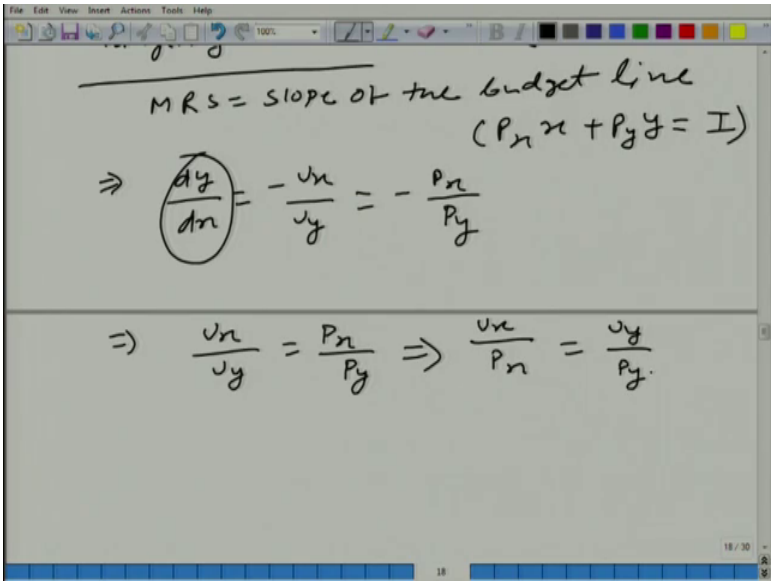

So, what we have figured out is, dy by dx is equal to minus ux by ux by uy, and this should be equal to that is the tangency criteria? That this is equal to minus Px by Py. Or

if we leave this part, what we get is; ux by uy should be equal to Px by Py. Or in other word what we are saying is, ux by Px should be equal to uy by Py ok.