An Introduction to Microeconomics
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Lecture – 44
Defining Utility Function

Now, here you have 2 dimensional world then you have just 2 elements in a consumption bundle.

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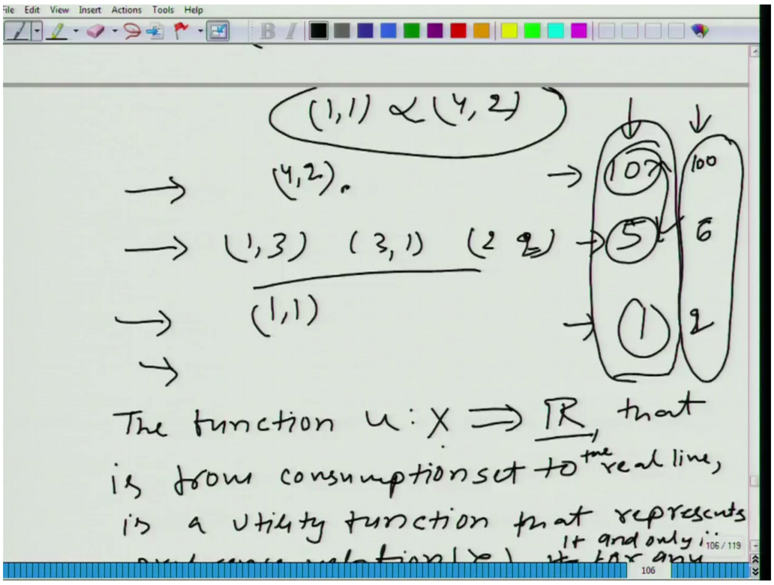

If you are talking about n dimensional world we will have n different goods in a consumption bundle. So, is there any way if that we translate this problem from n dimensional problem to one dimension, what we are interested in that whenever we compare 2 bundles we should able to rank them and those rankings should be consistent over the whole consumption set.

So, how can we rank, do we need you know we can rank then just like first, second, third, fourth and we are not really interested in first second third fourth we are just interested can we say here let me say just here that this has value 10.

Student: Hm.

This has value 5 and this has value 1.

Student: Hm.

Student: Association with (Refer Time: 01:12).

So, we compare these 2 numbers since 10 is higher than 5.

Student: Sir.

Its rank higher.

Student: Hm.

5 is higher than 1 its rank higher. So, instead of dealing with these things dealing with 10, 5 and one because these are one dimensional while these are 10 dimensional it is not easier.

Student: Hm.

So, what we can do, we can define a function.

Student: Utility function.

We can define a utility function and what is the utility function what does this utility function does?

Student: A function describes a utility.

Its represent.

Student: Utility.

A persons preferences.

Student: Hm.

It represents a person preferences.

Student: (Refer Time: 01:51).

So let me give you the definition of utility function.

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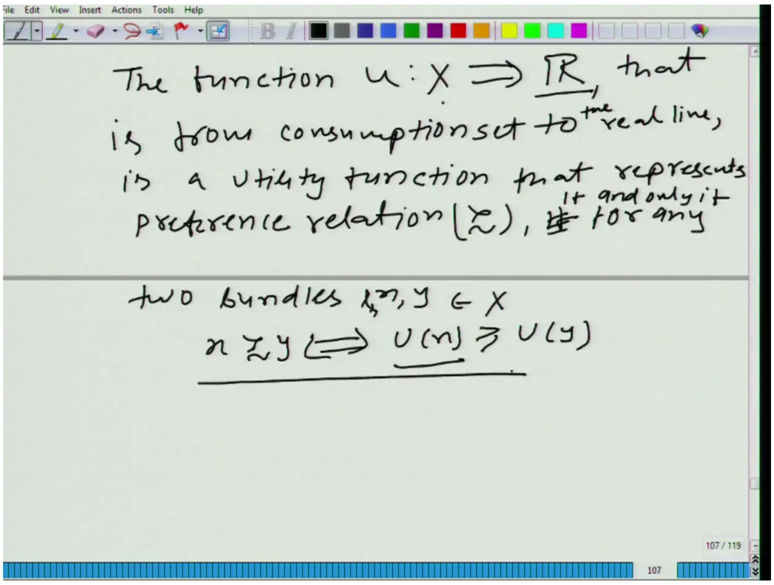

The function u, that is I am talking about the function u, function u is from set x to r and this set x is consumption set.

Student: Consumption set.

From consumption set and what is this r represents.

Student: Real number set.

Real numbers, real number set from consumption set to the real line. So, the function is a utility function ok. So, the utility function that represents (Refer Time: 03:00) preference relation denoted by this, this is a relation if for any 2 bundles x and ys in consumption set what we have let me let me change into if and only if what we have is.

So, the function u from the consumption set to the real line is the utility function representing the preference relation if and only if between any 2 bundles from the consumption set and what we get. If x is at least as preferred as y then we get ux is greater than or equal to u of y other way around that if u of x is greater than or equal to u of y then what we get x is at least.

Student: (Refer Time: 04:14).

As preferred as y.

Student: y.

So, now, instead of dealing with a bundle which has n goods we are dealing with a particular number assigned to that bundle and comparison becomes lot easier just to compare the 2 number. Just let us check that will it satisfy the 3 properties it will because whenever you take 2 bundles of course, a number will be assigned to a particular number will be assigned to a particular bundle. So, both the bundles will have their own numbers you can compare those numbers and when you compare there are 3 possibility that 1 is greater than the other or the second 1 is greater than the first or these 2 are equal.

Student: Equal.

So, completeness is satisfied, similarly if you take a bundle again compare it with itself a number is at least as big as itself. So, reflexivity is also satisfied how about transitivity, transitivity is also satisfied.

Student: Satisfied.

Because Take 3 bundles if one is greater than.

Student: (Refer Time: 05:19).

The second and second is greater than the third then of course, first is greater than the third.

Student: Third.

Implying that first is preferred over the last. So, all these 3 properties are satisfied fine.

Student: Yes sir.

So, you can prove it, I am not proving it rigorously and how about preferences, which do not satisfy the 3 axioms can they be represented by a utility function.

Student: Hm.

Think about it again it is good idea to try to prove it mathematically of course, they want let us take up person who has cyclic preferences.

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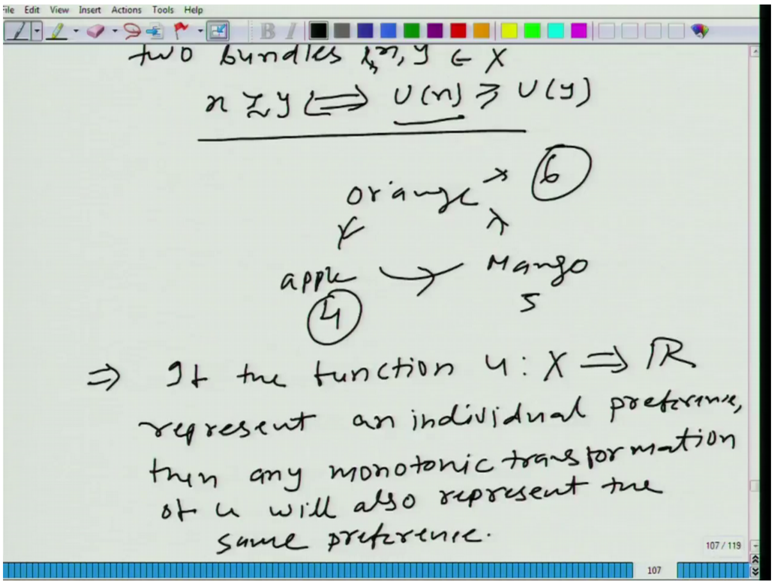

Means orange, apple and mango that we used earlier ok we will not have any such utility function describing this cyclic rotation, why? Let us say assign it 5.

Student: Each of.

The assign 6 what would you do here.

Student: (Refer Time: 06:33).

Student: each of 3 utility root converts to infinity.

Each of.

Student: 3 uti 3, each of the each of the products utility would converts to infinity because one is greater than other one is greater than other one is greater than other.

Huh that is one way to look at it , but what I am saying just look at assignment the way we need to assign that orange is preferred over mango. So, then let us say you give it here 6 and you get to mango 5, a mango is preferred over apple strictly prefer.

Student: Sir.

So, here you give 4, but apple is preferred over orange, but 4 is not greater than 6.

Student: 6 6.

So, we cannot have this sort of assignment. So, I am showing it you through an example , but what I am saying is much more broader, that any preference which does not satisfy the 3 axioms cannot be represented using a utility function fine.

Student: Function.

Now, let us look at it one more, lets go back to the discrete example that I gave you where we ranked and we said 10 5 and 1, if I say it is 100 it is 6 and it is 2.

Student: Hm.

We are still this these numbers and these numbers, these numbers is coming from a different utility function and these numbers of course, coming from a different utility function , but let us compare these 2 in both you get the same order the complete order that you get. It is the same as the previous one and when we are talking about rational preferences what we are worried about the complete ordering that you take any 2 element you should be able to order them rank them and that is true for any bundle ok.

So, that does not matter whether you use this scheme or you use this scheme you are describing the same preference. So, what does it mean, that the utility function that we have talked about is not unique for a particular person preferences ok.

Student: (Refer Time: 08:55).

It is not if you get a utility function you can definitely find another utility function, let us say you just add one in then that utility function you get another one as again one fine.

Student: Hm.

So, it is not unique. So, let me give you again more general result and the result is the function u.

Student: (Refer Time: 09:36).

If the function u represents an individual preference; So, of course, this individual has to have rational preference otherwise it cannot be represented by a utility function.

Student: Utility function.

Then any monotonic transformation I will come to it what is monotonic transformation any monotonic transformation of you will also the represent the same preferences, same preference fine is it clear.

Student: Yes sir.

Let me give you an example let us say that again its good you know it would be simpler if you imagine a consumption bundle with the finite number of bundles.

Student: Bundles.

A consumption set with finite number of consumption bundles.

Student: Hm.

And let us say you have ranked them and you have a rubber that you can stretch you can shrink you know and you can stretch you know for a different part different and on the rubber you have marked the position of different bundles.

Student: Hm.

Now, you stretch that rubber what you will get let us say the original position will be given by a particular function utility function when you are stretching you will get another function, but it would not change the rank. So, that is what monotonic transformation does, that monotonic transformation preserves the rank it does not change the rank. If 1 is greater than the second and we do a particular kind of transformation where this with the; new valuation 1 remains greater than the second then we do not have any problem and that is what we are talking about.

So, what if you have little knowledge of mathematics?

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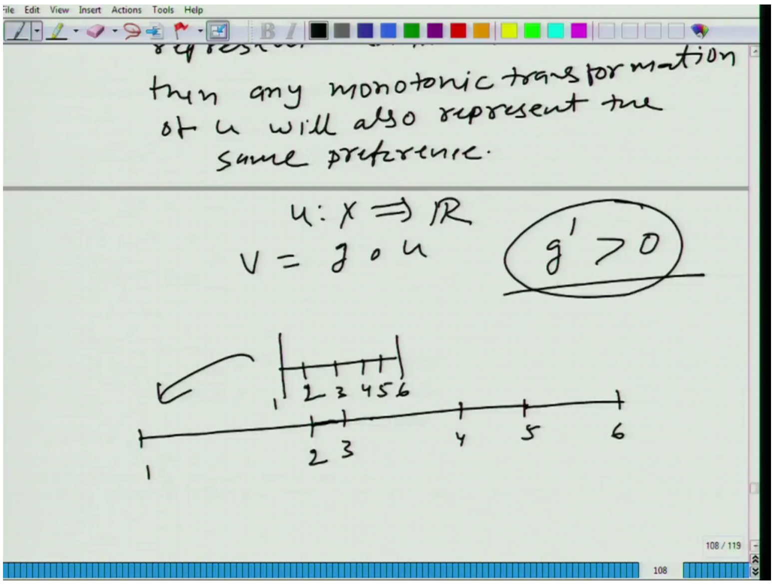

Let us say u is the function, utility function and then monotonic transformation will be presented by v what is v v is nothing, but g of u.

Student: Hm.

Where g dash is greater than.

Student: 0.

0 at all fine is it clear.

Student: It can be less than 0.

It can be less than 0 mathematically speaking it can be less than 0 , but in economics by convention we use for monotonic transformation v dash greater than 0.

Student: (Refer Time: 12:26) less than (Refer Time: 12:27).

So, for, so, what happens if it is less than 0 from the whole range then what would happen it would inverse the ranking?

Student: Yes sir.

So, in mathematics when we talk about monotonic transformation what we mean g dash is greater than 0 at all points or dash is less than 0 at all points, but in economics when we say monotonic transformation what we mean is that g dash is greater than 0 at all points fine.

Student: (Refer Time: 12:53).

G dash less than 0 is also monotonic transformation. So, what we are doing let us say this is the rubber and we are stretching it here are this. So, we can stretch it in the different manner that this part remains the same this part let us say here 1 1, 2, 3, 4, 5, 6 bundles. So, what we can do one we can stretch here 2 we are keeping there 3 we are keeping there 4 we are stretching 5 we are stretching less 6 we are stretching what we are talking about this that order we are keeping the order preserved and then we get another new utility function this is fine with you.