

An Introduction to Microeconomics
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Lecture – 24
Elastic, Inelastic and Unit- elastic demand

Now I gave you the definition of price elasticity of demand when do you think the demand is elastic enough? And when do you think the demand is not elastic enough or in other words, inelastic that is the opposite of elastic.

Student: sir when we have many substitutes.

No, I am not talking about the factors affecting the price elasticity of demand. I am talking about mathematically. So, what I am saying, let us say, I do not have those many products, but here I put little bit force and I am able to stretch, let us say if I am putting lot of force and I am not able to stretch it or if I am able to stretch it little bit, when can I what should be the cut off, that then in this case it is elastic and in the other case it is not elastic.

Student: vertical then it is not elastic and horizontal then it is elastic.

You are ahead of you know where we are in the class right now, but how about just to continue with your thought, how about when it has some slope.

Student: mathematical number.

So, mathematical number. So, how do we get the number?

Student: if it is less than 1.

So, one how do you know my point is not that I am going to describe in a minute that why we take one? As that number why not 2? Why not 3?

Student: sir because if we apply a force like you gave an example if it stretches the same like rubber; that means, it is responding 100 percent with the means 100 percent in inversion proportional with the price.

Student: So, we take one as.

Good. So, what you are saying is that if there is a 25 percent change in price and if we get at least 25 percent change in quantity then we call it elastic, it means what we are saying is the price elasticity of demand, is at least equal to one and mind we are using the minus sign in the definition and when we put 25 percent change in the price and we observe less than 25 percent change in the quantity demanded then we have in elastic demand so one is that cut off point one comes out as a natural number, but also there is a reason.

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$$\text{Total Revenue} = \text{Price} \times \text{Quantity}$$

$$= PQ$$

$$\begin{array}{l|l} P \uparrow & Q \downarrow \\ \hline P \uparrow & Q \uparrow \rightarrow \text{Inelastic} \\ P \downarrow & Q \downarrow \rightarrow \text{Elastic} \end{array}$$

$$TR = PQ$$

$$\frac{\partial TR}{\partial P} = Q + P \left(\frac{\partial Q}{\partial P} \right) = Q \left[1 + \frac{P}{Q} \frac{\partial Q}{\partial P} \right] = Q(1 - \epsilon)$$

Let's look at the total revenue and what is total revenue price multiplied by quantity. It is the revenue earned by a seller, if he sells a particular quantity at particular price. So, total revenue is PQ and now I am going to use calculus little bit to describe. Because, we already talked about in word why we are doing it, but here using calculus we will do it more precisely.

Now, let us say, that the cut off would come from here if P goes up if seller has increased it is price and PQ also goes up then do you call this product elastic or inelastic?

Student: inelastic

Inelastic and when PQ has come down why? It involves certain bit of logic let us look, at it P has gone up and what is the effect of P on Q? Q decreases P goes up Q decreases. So, if P goes up by little bit, and Q does not decrease as much as the P has decreased. Then what is the impact on total revenue? Total revenue would go up, but that is case when p goes up by 25 percent and Q goes up by less than 25 percent that is the case of inelasticity. So, this is

what we get here inelasticity and when P goes up Q comes down, but overall impact on revenue is that total revenue comes down it means, it is elastic why it is elastic? Again, P has gone up if I continue with the same example, P has gone up by 25 percent and Q has gone up by more than 25 percent what would be the end result?

Student: PQ will decrease.

PQ will decrease, that is why it is elastic not because PQ has decreased, but because.

Student: Q has decreased.

Q has decreased more with respect to.

Increase with proportional increase in P that is why it is elastic.

Now, let us look at it using calculus let us denote total revenue by TR if you could not understand calculus you can skip. Total revenue is P multiplied by Q. Now let us look at it what happens to the total revenue if we change P, and how can we get that change using differentiation partial differentiation of this expression both side of this expression with respect to P. So, what we get? And this will be equal to Q plus P $\frac{dQ}{dP}$ first, we are keeping Q fixed and differentiating with respect to P. So, when you differentiating P with respect to P you get one. So, one multiplied by Q, Q now we are keeping P fixed and differentiating Q with respect to P what we get $\frac{dQ}{dP}$. So, if you take, Q common $1 + P \frac{dQ}{dP}$ and what the definition we put? Minus sign.

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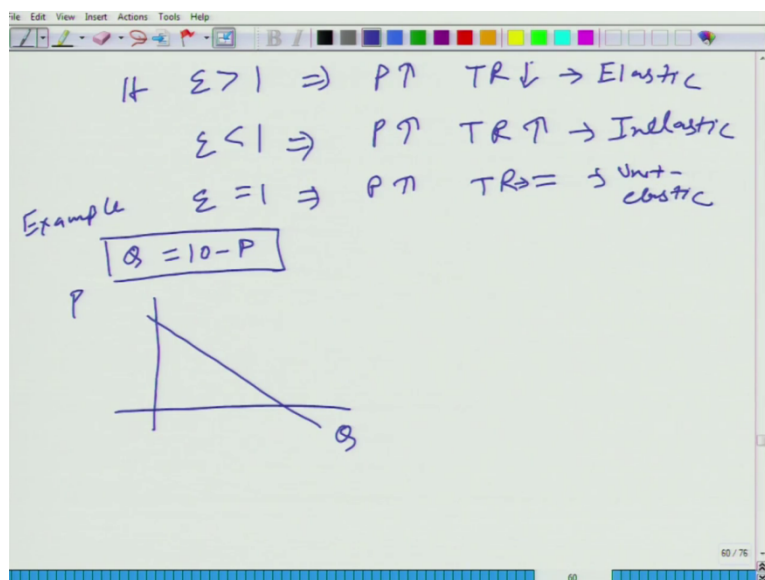
Handwritten derivation showing the relationship between price change and quantity change:

$$\begin{aligned}
 & \text{Initial state: } P \rightarrow x \\
 & \text{New state: } P + \Delta P \rightarrow x + \Delta x \\
 & \text{Change in quantity: } \frac{x + \Delta x - x}{x} = \frac{\Delta x}{x} \\
 & \text{Change in price: } \frac{P + \Delta P - P}{P} = \frac{\Delta P}{P} \\
 & \text{Elasticity (Epsilon): } \epsilon = \frac{\frac{\Delta x}{x}}{\frac{\Delta P}{P}} = \frac{P}{x} \frac{\Delta x}{\Delta P} \\
 & \text{Using midpoint formula: } \epsilon = \frac{\frac{\Delta x}{\frac{x + \Delta x}{2}}}{\frac{\Delta P}{\frac{P + \Delta P}{2}}} \\
 & \text{As } \Delta P \rightarrow 0, \epsilon = - \frac{P}{x} \frac{\partial x}{\partial P}
 \end{aligned}$$

So, what is this? This is equal to 1 minus epsilon. If by the way, I forgot to mention although I have been using it. That epsilon is elasticity is denoted by epsilon

So now let us look at it, if epsilon is greater than one, what happen?

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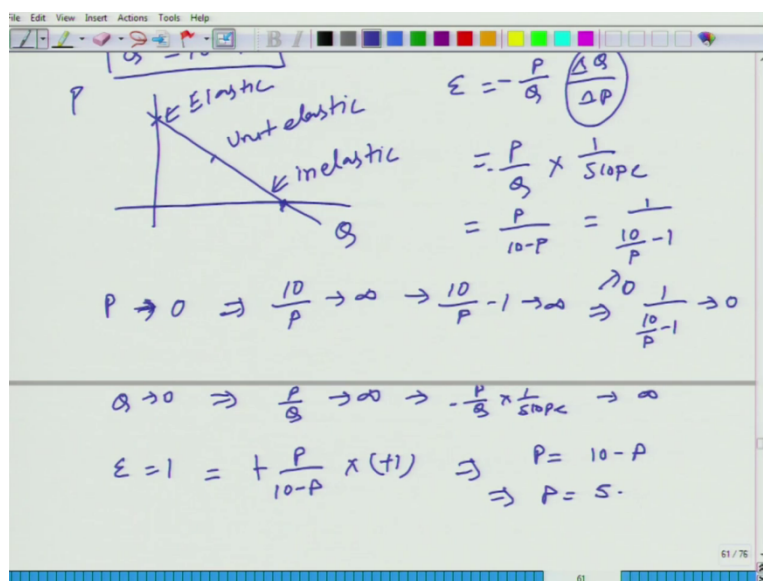


P goes up, total revenue goes up no, total revenue goes down So, in this case we have already discussed this is elastic.

And when epsilon is less than one, P goes up, total revenue goes up. Inelastic and we should also discuss the case when epsilon is equal to one, this is called unit elastic P goes up TR remains the same unit elastic.

Now, let us look at it, the demand function that we discussed Q is equal to 10 minus P example we are talking about Q is equal to 10 minus p can you tell me? That this demand function is elastic or inelastic? it is unit elastic someone is saying it is unit elastic why do not you draw?

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So, 2 3 ways now we have to look at it. One way to look at it, if we use the definition what is the definition of elasticity? P by Q delta Q by delta P here we were using x in place of Q it does not matter this is just a variable and delta Q by delta P what is this?

I will give you through this example; I will give you one more method. What is this delta Q by delta P? For this not just minus 1 for this it is minus 1, but in general, what is this? Delta Q by delta P it is inverse of the slope. So, also it can be written here that elasticity can be given as, one divided by slope. So now, how much is the slope? It is minus 1 and P divided by how much is Q? 10 minus P or in other word, what we can write? 10 by P minus 1 is it true? So, when P is equal to 0 what do we get? P is equal to 0 when put here P is equal to 0 what you get in the denominator when P is equal to let us write it here in detail P is equal to 0 it implies 10 by is infinity. It implies 10 by p minus 1 extending to infinity. So, what is happening here?

This is extending to 0. So, here, at this point what is it is inelastic. Fine how about here? At this point?

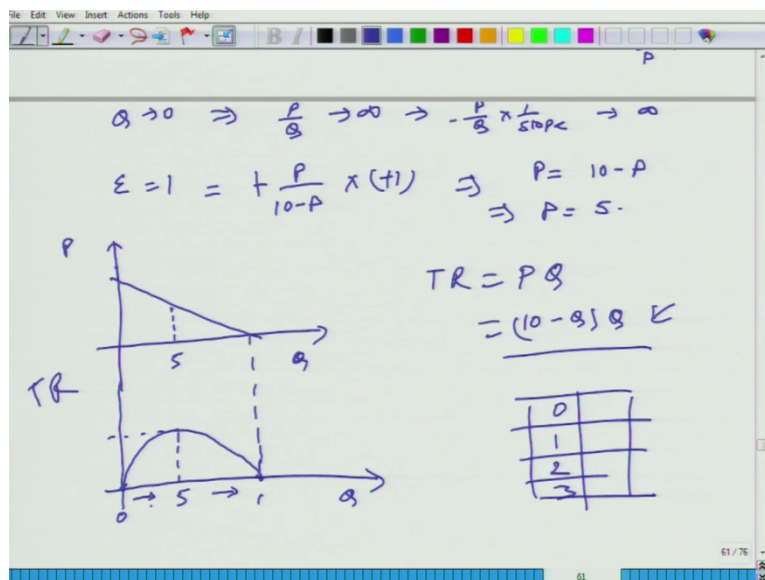
Student: elastic

It is at this point, what do we have Q is equal to 0. So, P by Q is tending to infinity. So, P by Q multiplied by 1 by slope, is also tending to infinity. So, here it is elastic. So, similarly if you continue with this thought, at some place in the middle you will get where elasticity is equal to one because, it starts with inelastic and becomes elastic and somewhere you will get unit elastic.

Student: unit elastic.

What is that point can you tell me where? now we know we are trying to find a point where elasticity is unit elastic; it means, that epsilon is equal to one and this should be equal to minus P by 10 minus P multiplied by minus 1 or in other word P should be equal to 10 minus P. So, P should be equal to 5 ok.

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This is the one way we have used to calculate.

We have also have another way to calculate it. How can we do it? Here we have the same line. Here we have P, here we have Q on the same scale here we can draw Q on the x axis and total revenue on the y axis. When Q is equal to 0 how much is the total revenue 0 and when at

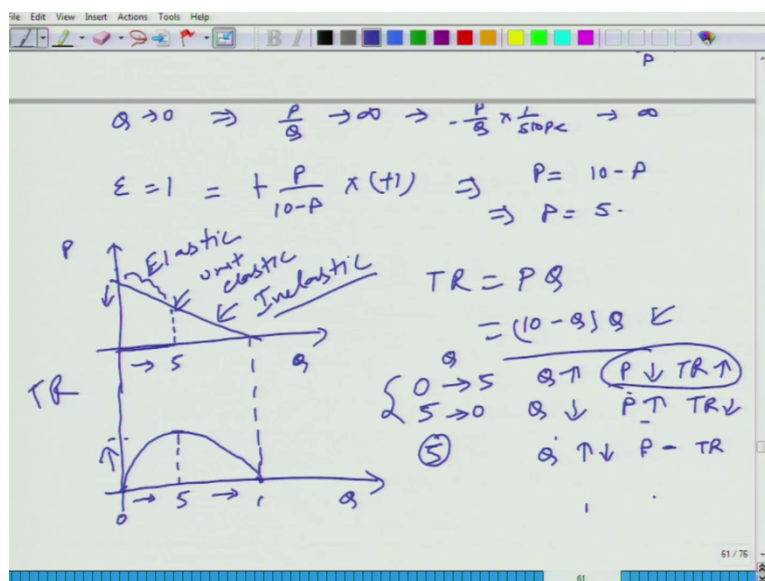
this point when P is equal to 0 how much is the total revenue when P is equal to 0 total revenue is 0 what is the total revenue P multiplied by Q. So, one of these 2 is 0.

Student: is 0.

Then the whole thing would be equal to 0. So, the total revenue is equal to 0. So, at this point it is 0, at this point this is equal to 0, how about in the middle? PQ or in the other word what we are saying because we are writing with respect to Q. So, P is equal to 10 minus Q and this is if you use little bit of algebra, if you do not know the algebra then create a table and try to draw it, but if you know the algebra, this is an equation of it is an equation of quadratic type. So, what will you get? It is also an equation of if you use coordinate geometry, this is an equation of parabola. You do not need this much of information to draw. I am just saying.

In case you know you can also identify. The simple way to draw it is that you start, you make a table Q is equal to 0 Q is equal to one 2 3 and draw more point you take better your drawing will be, but another way to draw is to understand that this is a quadratic equation and this is also the equation of parabola and then it would be much simpler it is going to look like this. Here you have 5 here you have 5. So, what is happening in this zone? In this zone from 0 to 5 quantity you increase the quantity let us let me erase this simple thing that you should look at it, that 0 to 5 range for quantity. Q is going up it means P is coming down any way look at it here.

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In this zone if you move in this direction p is moving in the downward direction. So, Q is going up P is coming down and what is happening to TR ? TR is going up TR is going up. So, if you look at this, that price is going down or TR is going up. Similarly, just to understand you move from 5 to 0 direction; what is happening? Q is decreasing, P is increasing and TR is decreasing. So, simply P goes up TR is coming down, it simply means it is elastic zone. So, this zone is elastic zone.

How about at 5 little bit of change because, here it is flat or in other word, this total revenue is maximise at this point. So, little bit of change in price will not bring any change in the total revenue. So, here Q goes up or down while; P remains the same TR remains the same. So, unit elastic. So, at this point it is unit elastic fine now in this zone what is happening? Just reverse of this, and here you get simply what is happening p is going up and TR is also going up. When TR is coming down TR is also coming down. So, here it is clearly inelastic. So, in the zone where price and total revenue moves in the same direction then what do we get? Inelastic zone. When price and total revenue move in the opposite direction we get elastic zone.