An Introduction to Microeconomics
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Lecture – 138
Cournot Model of Duopoly

So, let us talk about Cournot model of duopoly.

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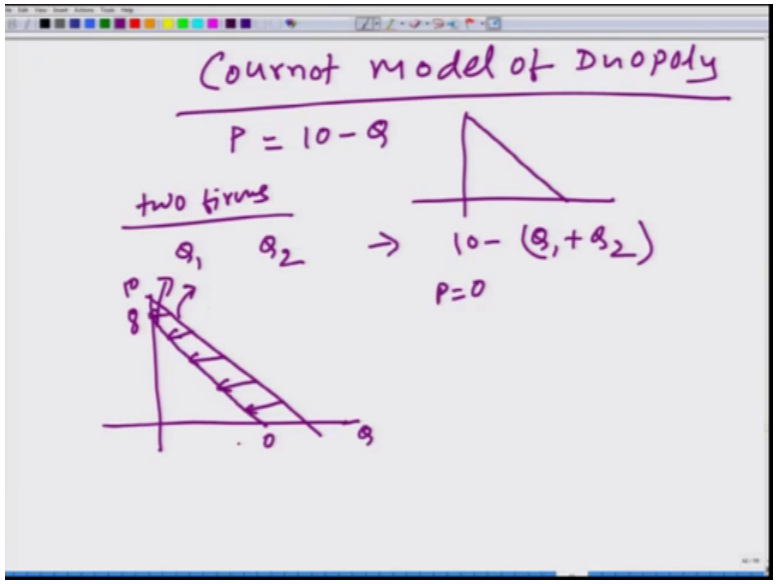

What we have here is, let us say that the market demand is given by 10 minus Q. This is linear demand function that we have been using. Now what happens that we have 2 firms operating in the market. Now both the firms independently decide how much quantity to produce, and price automatically gets determined through the quantity. So, if firm 1 decides to produce Q 1 and firm 2 decides to produce Q 2, the market price is going to be 10 minus Q 1 plus Q 2, as long as Q 1 plus Q 2 is less than 10; otherwise, the market price is going to be equal to 0, ok.

. So, let us say we will talk about residual demand function. What is residual demand function? Firm 1 and firm 2 they make their decisions simultaneously. What it means? That they do not know how much the other firm has decided to produce. Firm 1 is going to produce 8 units, then the price will become equal to 0, ok, hold on I will talk about it once again so, edit [FL], ok. So, let us look at the market demand function, ok.

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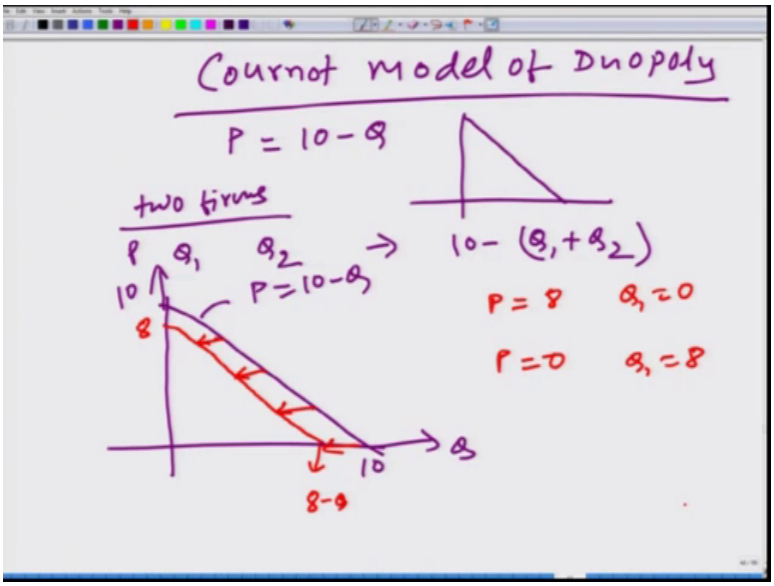

This is 10, this is 10, it means this is P is equal to 10 minus Q, here we have Q, here we have P.

Now let us say firm 1 thinks, that firm 2 is going to produce 2 units, again firm 1 does not know how much firm 2 is actually going to produce. This is what firm let us say if this is what firm 1 believes in that firm 2 is going to produce 2 units. Then if it produces 0 unit, then the price in the market is going to be 8.

And if it is if Q 1 happens to be 0, and if Q 1 happens to be 8, P is going to be equal to 0. So, what I mean to say that this particular firm would see the residual demand in the market, which is nothing, but this particular line which is given by 8 minus Q. How did we obtain this line? We sifted this line by 2 units everywhere. And that is how we obtain the residual demand function.

So, let us say what is the profit maximizing problem for this particular firm now?

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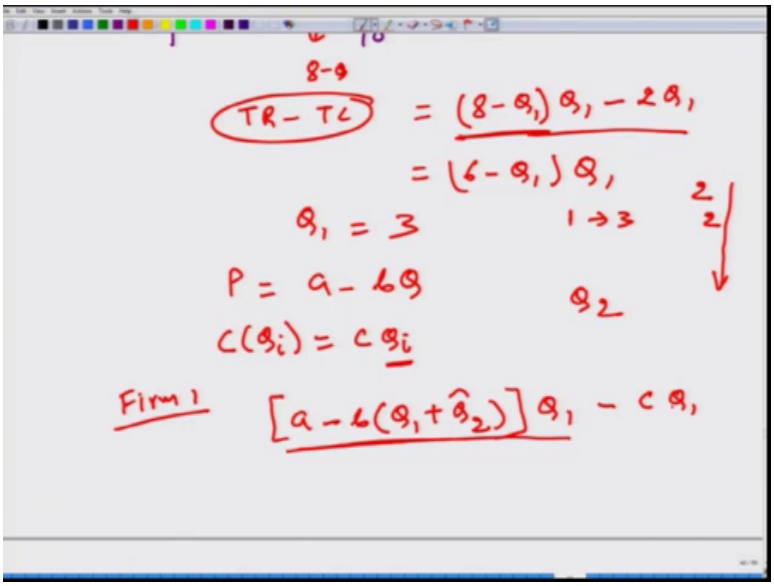

So, TR minus TC, this is what this firm would like to maximize. So, TR is going to be 8 minus Q 1 multiplied by Q 1. Remember, this is not the market demand, this is the residual demand, that this firm for c for the future, here we will have the total cost.

And let us say just for example, the cost is 2 Q 1. So, we will see how much this firm should produce, the firm would maximize this particular equation, and from here we can get this is 6 minus Q 1 multiplied by Q 1. So, when the firm maximizes Q 1 will come out to be 3.

So, what we figured out when firm 1 thinks that firm 2 is going to produce 2 unit then firm 1 should produce 3 units. And what we can do we have to calculate this for all the different values of Q 2. So, rather than proceeding it like this, we will do it for the general case.

So, let us say that market demand in this general case is a minus b Q which we are familiar with we have done it many times this is the linear demand function in our previous problem a is equal to 10, and b is equal to 1. And let us say firm 1 thinks that firm 2 is going to produce Q 2. And let us say the cost function are both the firms are cQi. So, they have exactly the same cost. Qi is the amount this particular firm i produces, ok.

So, let us look at how much the firm 1 should produce for firm 1 a minus b Q 1 plus Q 2. This is not actual Q 2, this is the Q 2 that from one thinks that firm 2 is going to produce multiplied by Q 1 because this is the price which is determined by Q 1 plus Q 2 amount.

So, even before this production has taken place, the firm one would think, this is what would be my revenue. And minus cQ 1. Now the firm 1 would maximize this particular equation as a function of Q 1.

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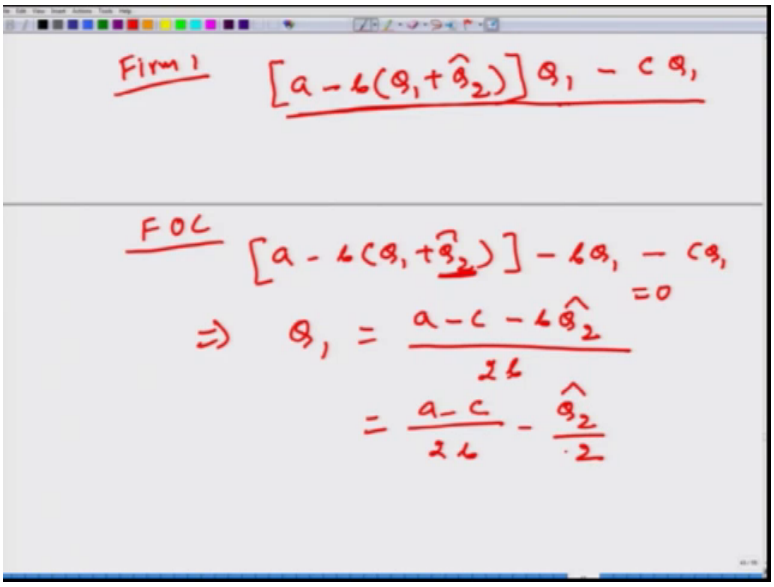

So, what we are going to get if we do this maximization? We are going to get our first order condition as a minus b Q 1 plus Q 2 hat minus bQ 1 minus c Q 1 it has to be equal to 0. So, from here, firm 1 can decide how much Q 1 to produce, and it is going to be the function of firm once belief about level of production by firm 2. So, we can write a minus c minus bQ 2 hat divided by 2 b. Or we can write it like this a minus c 2 b minus Q 2 hat by 2.

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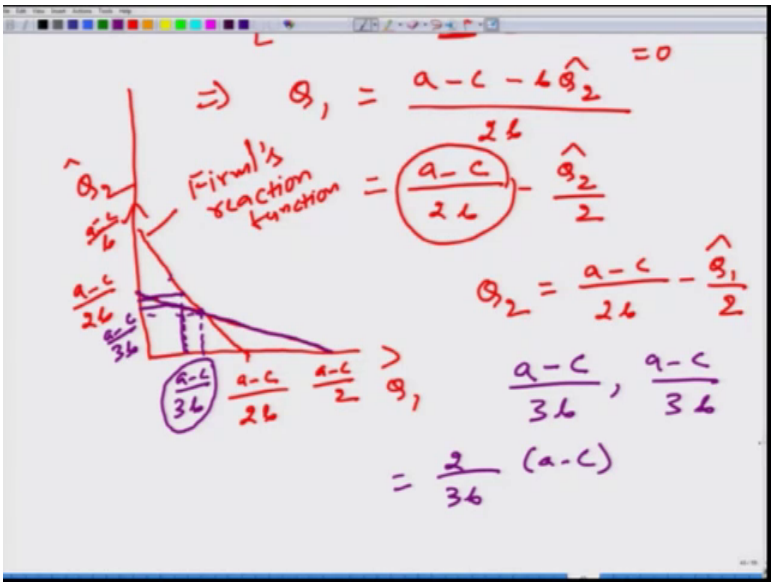

In other word, we can draw here let us say we have Q 1 and here we have Q 2, let us say Q 1 and Q 2 hat. So, we can draw, a firm 1 thinks that firm 2 is not going to produce anything the optimal amount for firm 1 is a minus c divided by 2 b. So, if Q 2 is equal to 0, this is a minus c divided by 2 b.

So, Q 2 hat has to be equal to a minus c divided by b, for firm 1 to not to produce anything so, if we draw this line. And this is a minus c by 2, sorry, a minus c by b, right? This is what is called firm one ones reaction function. This is reaction or response function we can say, this is the response that firm 1 would gave according to it is belief about the level of production from firm 2.

Similarly firm 2 would calculate remember that firms are not deciding these things simultaneously. So, firm 2 will decide how much to produce which would come out to be if you do it is will going to come out to be this particular equation. Also, you should understand, the problem is symmetric. So, how did we do it? That instead of using Q 2 here we are using Q 1 hat here. And then we can plot it and it is going to look like here we have a minus c by b, and here we have a minus c by 2 b. If we want to change the color, I think it is good idea to understand this is for firm 2, ok. And this is the special point that we would pay attention to, ok.

What is this point equal to in these 2 lines they intersect here. So, at this is equal to a minus c divided by 3 b and here also a minus c divided by 3 b. What it says, that if firm 1

thinks that firm 2 is going to produce a minus c divided by 3 b, then firm 1 would produce a minus c divided by 3 b.

And same is true for the firm 2, and you should think you should understand that this is the Nash equilibrium in the problem. We cannot have any other equilibrium, because let us say that firms are producing here.

So, if let us say the production is taking place here to begin with, it is taking place here. What it means that sorry this violet color is the reaction or the response function of firm 2. And the red color here gives the response function of firm 1.

So, what it says that given that firm 2 is producing this much how much is the optimal for firm 1 to produce? More so, firm 1 should increase it is level of production. And if firm 1 increases the level of production to here, how much is the optimal for firm 2 to do? We can go back to this and this much is the optimal for firm 2 to do.

So, we see that in this zone firm 1 and firm 2 both have incentive to increase their level of production. And here when they reach to this particular point they will not have any incentive to change the level of production. And therefore, this is the equilibrium level.

So, in Cournot competition in this particular setting, they will produce a minus c 3 b and a minus c 3 b, how much is the total production? 2 by 3 b multiplied by a minus c.

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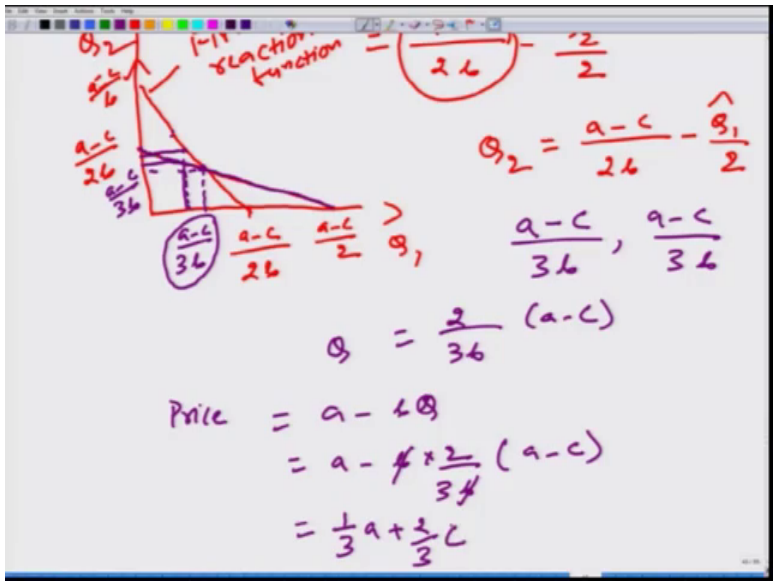

What would be the price? The price is going to be a minus b, this is what is Q a minus b q. So, from here we can get a minus b multiplied by 2 by 3 b multiplied by a minus c. So, from here we get 1 by 3 a plus 2 by 3 c. So, this is going to be the price in the Cournot competition. So, that is it about the Cournot competition, we would come back to the Cournot competition, when we compare Cournot competition with Stackelberg competition.

Thank you.