An Introduction to Microeconomics
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Lecture – 126
Profit Maximization for the Monopolist

So, we are going to look at the monopolists profit maximization problem, and we are going to do it 3 ways. First, we are going to do we are going to take a simple example and we are going to use a table. We will also do the same problem graphically, and then we will do a more proper generic problem using calculus.

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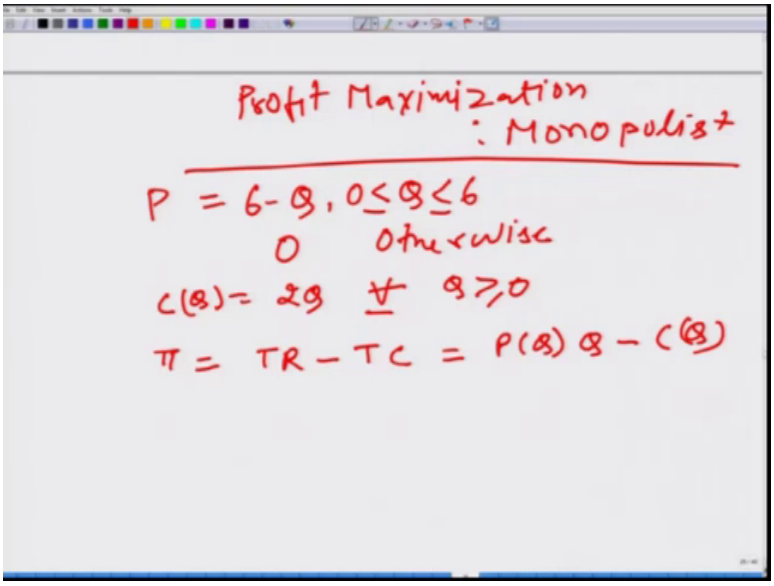

So, profit maximization for monopolist. And let us say that the market demand is given by of course, here I am writing inverse demand function, but as long as we understand what it is there is no harm in writing it like this. So, then Q is between 0 and 6 and 0 otherwise.

So, this is the linear demand function, ok. Let us assume the cost is a linear function of the quantity produced and let us say it is 2 Q, for all Q greater than or equal to 0, ok. So, this sign indicates for all so now, when we want to do the profit maximization first we write what is profit, profit is total revenue minus total cost. And how much is the total revenue, P as a function of Q multiplied by Q minus C of Q so, what we are going to write?

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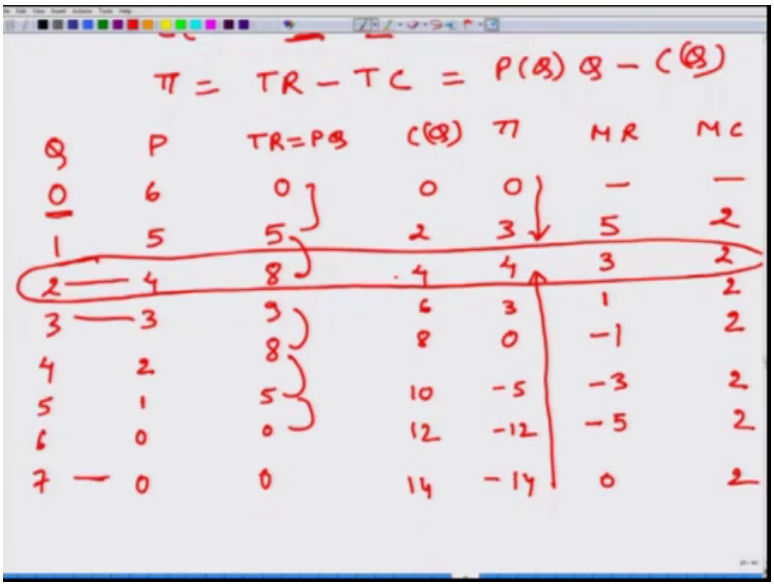

We are going to make a table, and here we are going to write Q, then we are going to write the corresponding P, remember as firm wants to sell more output in the economy, the price the firm will have to bring down the price. Because price and quantity are related to each other through the market demand function and which is a downward sloping curve.

So now here is total revenue which is PQ, then the total cost. And here is profit, how do we get the profit? Total revenue minus total cost and here we are also going to write the marginal revenue as well as marginal cost. So, if Q is equal to 0, P has to be equal to 6, why? The formula is P is equal to 6 minus Q. So, plug Q is equal to 0, you would get P is equal to 6. And how much is the total revenue in this case? 0 because if you multiply anything with 0 you get 0 and how much is the cost? As you are producing 0 output as this mono policy is producing 0 output C of Q is going to be 2 multiplied by 0. So, 0 profit turns out to be 0, we cannot calculate marginal revenue and marginal cost.

Now, let us say if Q is equal to 1, here it is going to be equal to 5, the simple rule is as long as Q is between 0 and 6 P plus Q is going to be equal to 6. And it should be clear from the formulation here 6 minus Q if you bring Q to the left-hand side what you will get P plus Q is equal to 6. So, let us use this and then the total revenue is 5, the cost is going to be 2, because here is 2 multiplied by 1. And the profit is going to be 3, 5 minus 2 is equal to 3, and how much is the marginal revenue? As quantity increased by one unit

the marginal revenue is now 5 minus 0, that is 5 and marginal cost is going to be constant in this case, which is 2 all the time.

So, let us write 2 3 4 5 6 as well as for 7. And here P is going to be 4 3 2 1 0 0. So, if you follow as up to Q is equal to 6, P plus Q is equal to 6, but if Q is greater than 6, then as we have defined that in that case also the price is going to be equal to 0. So, if price becomes so high no one in the market would demand any output and that is why you know no one and that is the reason that we take P is equal to 0 so, let me repeat this. So, if Q if Q becomes higher and higher, what is happening, that if monopolist want to sell, these more and more output the monopolist would have to decrease the price, and if the output that it wants to sell becomes really really high, then those many outputs can bought only at 0 price and that is what is happening here, ok.

So, let us say it calculate the total revenue, and that is very simple just multiply these 2 number and we will get the total revenue, and here 5 here 0 and here 0. And the CQ is simple; we have to multiply Q with 2. So, we are going to get here 4, 6, 8, 10, 12 14. And profit is just the difference between these 2 number; so, 4 3 0 minus 5 minus 12 minus 14. And how much is the marginal revenue? From here we had calculated marginal revenue earlier also; it is exactly the same example. So, marginal revenue is 8 minus 5 which is 3 9 minus 8 which is 1, 8 minus 9 which is minus 1. 5 minus 8 that is minus 3, and 0 minus 5 is minus 5 and then 0 onwards. And marginal cost happens to be always equal to 2, ok.

So now let us pay attention where the profit is maximized. As output is increasing first profit is increasing from 0 to 3, 3 to 4, but beyond this the profit starts decreasing, ok. So, it is clear from this graph that this row is gives us it gives us the optimal level of output the optimal price and so on. What is happening that in this case Q is equal to the Q star is equal to 2.

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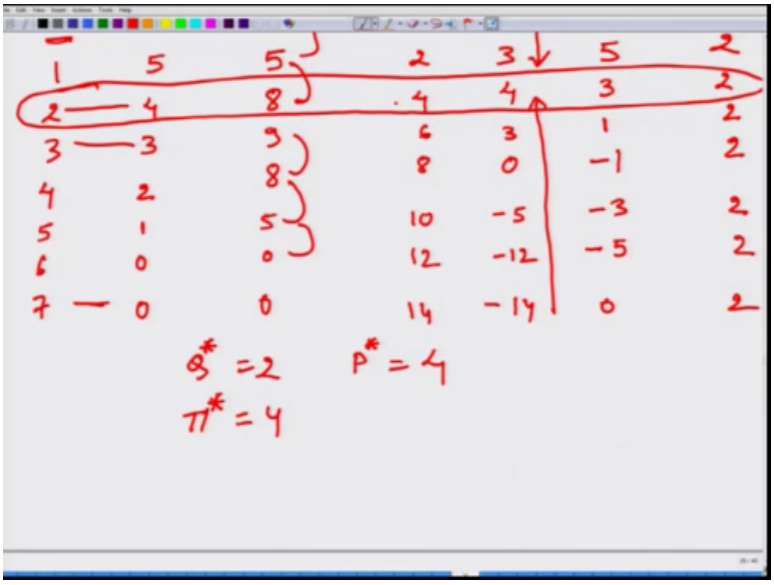

And P star also happens to be equal to sorry P star happens to be equal to 4, ok. So, the profit and how much is the maximize profit? Pi star is equal to pi star is equal to 4, ok, fine now what is happening, let us try to understand the same problem using graphical technique.

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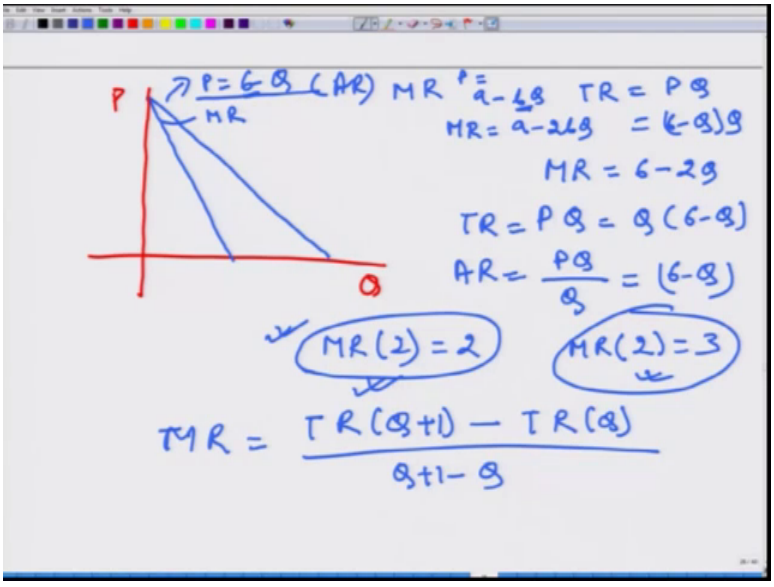

So, what we have here is, let us say here we have P, here we have Q, and then we draw this; this is the market demand curve P is equal to 6 minus Q.

Now, we also need to draw the marginal revenue curve. And to draw the marginal revenue curve, we have to we can do one of these 2 ways, that we plot these points and

get the marginal revenue curve. Or the second way is to write the total revenue which is P multiplied by Q. So, 6 minus Q multiplied by Q, and as we had obtained earlier the marginal revenue came out in the case of in the earlier lecture, we had talked about what if the market demand function is a minus b Q P is equal to a minus b Q, and we got marginal revenue is equal to a minus 2 b Q. So, what is the value of a here? The a is clearly 6 and b is 1. So, what we get? 6 minus 2, ok.

So, we can draw this and this is the way it would look like. So, this is market demand curve, and I should also emphasize this is same as the average revenue curve. Why it is average revenue curve? Because if we write the total revenue, what do we get? P multiplied by Q. So, Q multiplied by 6 minus Q, and how much is average revenue? PQ divided by Q. So, we get back this 6 minus Q curve. So, this is also average revenue curve, and here we get the marginal revenue curve. So, let us say, let us calculate; how much is the marginal revenue at 2 when Q is equal to 2; according to this formula, the marginal revenue is 2. But if we pay attention to the table, we get marginal revenue is equal to 3, what is happening? Why are we getting different numbers? That is very, very important to understand, although it is not related to directly related to or a specific to not related, it is specific to profit maximization. Because we are using 2 different techniques, and we are getting marginal revenue equal to 3 or 2, which one should we take? Should we take this particular number or should we take this number.

Please note that the marginal revenue the way we define, there are 2 definitions that we have been using. One definition is that increase in revenue if output increases by one unit that is one definition and the second definition that we have been using the rate of change in the total revenue with respect to the quantity. So, when we are using the rate definition this is what we get, and when we are using change in revenue when we produce one more unit then this is the value. We get which one is more accurate. Of course, this is a better definition when we are used the rate concept. And that should be clear that.

Let us look at it the 2 different definition let us look at it clearly although we are digressing from profit maximization, but it is important to understand that according to 2 definition this marginal revenue is total revenue Q plus 1 minus t r Q divided by Q plus 1 minus Q, which turns out to be one the denominator turns out to be equal to 1.

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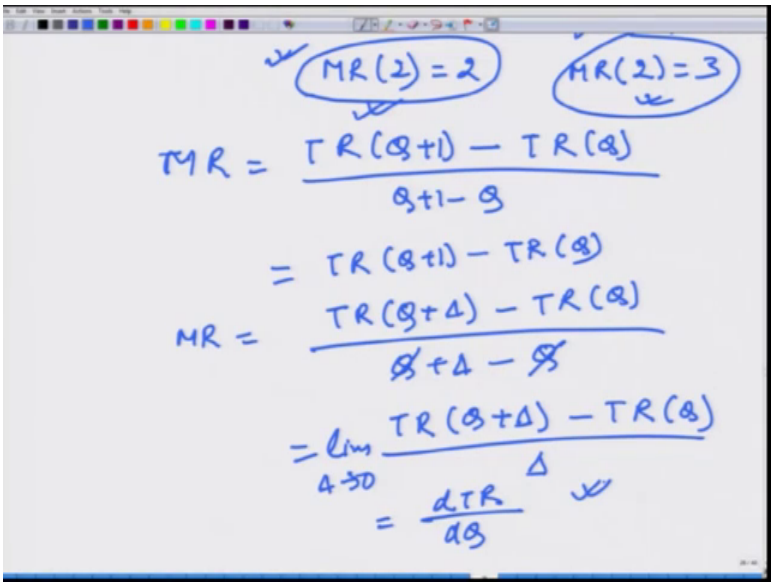

So, we can write it like this, TR Q plus 1 minus TR Q. The other definition that we are using is this, the TR Q instead of saying the change of one unit; we say the change of a delta unit, minus TR Q and here Q plus delta u delta minus Q. So, this Q-Q gets cancelled, and what we get is and divided by delta. And then we further add that this delta is very, very small, and that is how we get that marginal revenue is rate of change in total revenue with respect to Q.

So, of course, this definition is more precise more accurate, but if you have difficulty in understanding calculus then we can use this definition. Of course, sometime we get not so accurate result when you we use this definition, ok. So now, let us get back to our graph and we were doing the profit maximization for the monopolist before we started digressing. So, let us use what is the marginal cost curve? No matter which definition we use, the marginal cost is 2.

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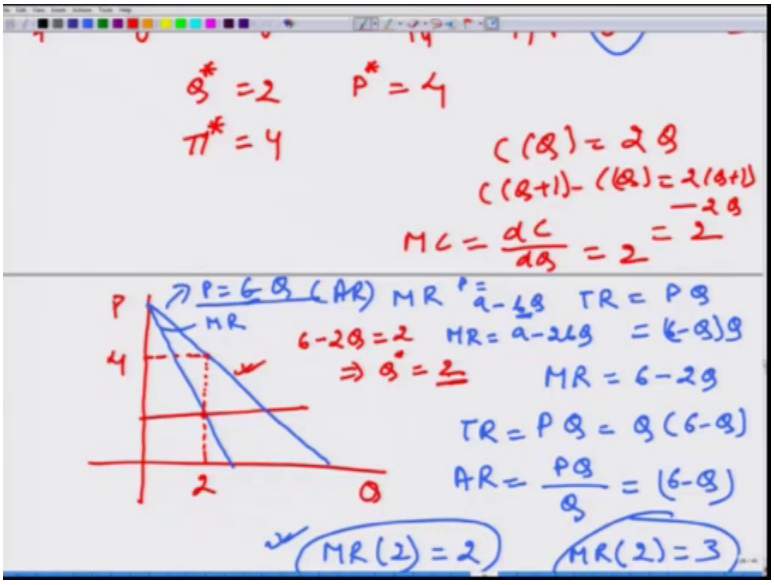

Because here if I write C of Q is equal to 2 Q, then C of Q plus 1 minus C Q is going to be equal to 2 plus 1 minus 2 Q which turns out to be equal to 2, and we use the rate definition then the marginal cost is dC, that is rate of change in C with respect to Q and again we get 2 so, in this case there is no issue.

So, in other word, the marginal cost curve is a horizontal curve and it looks like this, ok. And they intersect at where that we can obtain the marginal cost where do they intersect this is the marginal revenue curve. So, marginal revenue curve is 6 minus 2 Q if it is equal to 2. So, it intersects at Q is equal to 2, and this is where it would intersect. Now if this firm this monopolist would like to sell 2 units in the market, then the it has to price it according to the downward sloping demand function.

And how much should be priced? We should extend it up, and because this line which is the demand function gives us the different price and quantity combinations which is which would be demanded in the market. So, from here we can read how much is the price. So, to read the price we have to plug this number 2 in the demand function. So, when we plug it P we want to know P. And so, when we plug this 2 into 6 minus Q so, we get 6 minus 2 is equal to 4. And this is what we had obtained even though the table was the approximation, but this is what we had obtained.

So, this is the profit maximization for the monopolist. Let us do graphically a more generic case so, here it is.

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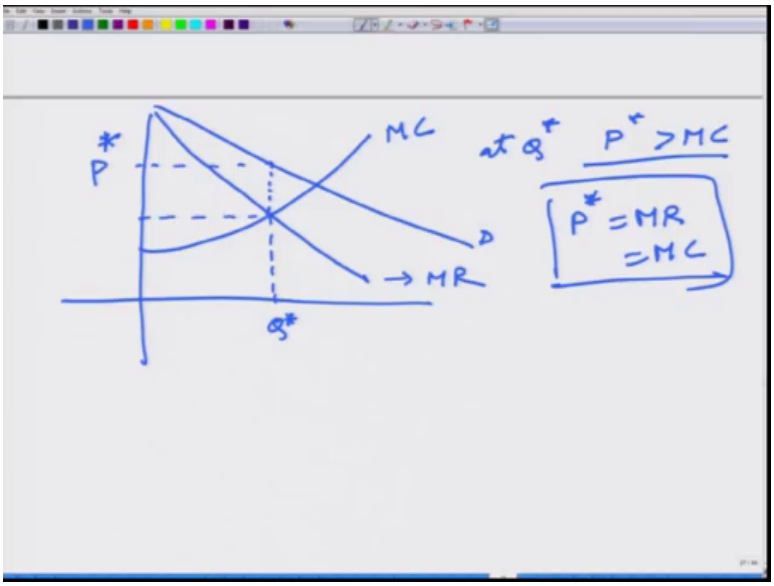

Let us say this is the way we are going to do it graphically. So, here is a demand function, then we will have a corresponding this is demand function we will have corresponding marginal revenue function. And let us say this is the marginal cost function. And from here, wherever this marginal cost function intersects with the marginal revenue function that would determine the quantity that this monopolist would supply in the market.

So, we get the quantity supplied in the market or bought in the market. Both are going to be equal in the equilibrium, and we can calculate the price per failing price in the market when we see that to sell Q star units in the market; how much should be the price and we can read it from the demand curve, and this is what is going to be the price in the market.

Please note that at Q star P star is greater than MC. If you remember and I will keep on coming back to perfectly competitive market, if you remember what we derived in the case of perfectly competitive market, and at the equilibrium P star is equal to MR is equal to MC. And this is what we had talked about when we had just discussed the profit maximization for any firm, ok. So, of course, this is slightly different than perfectly competitive market. So, let us do this problem using calculus.

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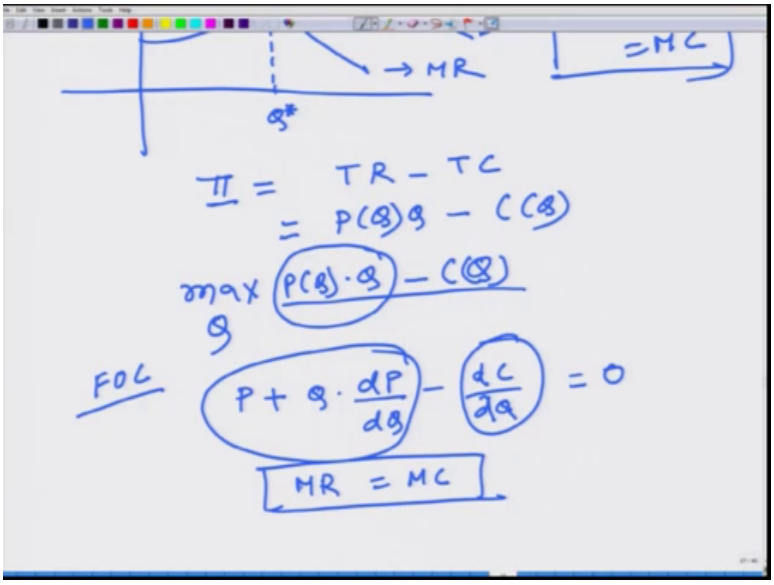

So, pi which is profit is equal to TR minus TC, TR is total revenue, and TC is the total cost, again we can write total revenue as a function of a quantity and price, but price itself depends on quantity, and here is the cost total cost. Now what is profit maximization? Select Q such that this pi is maximized. So, what we want to do is maximize PQ multiplied by Q minus C of Q with respect to Q. So, the job is to select Q so, of course, the first order condition is to differentiate this expression with respect to Q, and what do we get? And we had done this part of definition earlier when we were talking about marginal revenue.

So, what do we get? We get P plus Q multiplied by d P by d Q minus d C by d Q. This is marginal cost, and this is marginal revenue, this has to be equal to 0. So, here also we are getting marginal revenue has to be equal to marginal cost. And this is universal whenever a firm wants to maximize the profit firm has to operate at the level of output where MR is equal to MC, which is universal.

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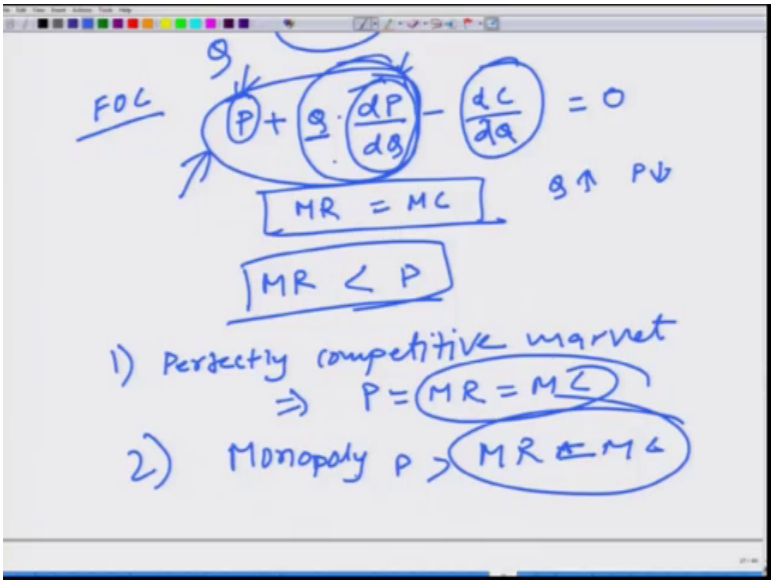

How it is different? Because if firm wants to sell one more unit here, then firm will have to if Q has to be increased firm will have to decrease P. So, to sell if let us say firm wants to sell one more unit, let us try to interpret this equation. So, if firm wants to sell one more unit. If it sells one more unit of course, the revenue generated is P, but at the same time to sell one more unit, firm will have to decrease the price, which would come how much that decrease should be it would come from the demand function.

So, this is what it indicates that decrease, because price is a downward sloping function of Q. So, d P by d Q has to be a negative number. So, price for each unit would decrease so, earlier this firm was selling Q, and now price has decreased for each unit so, how much would be the decrease? Because of this decrease in price as the if we can give that by multiplication of these 2 terms.

So, this is the gain from selling one more unit, and this is the loss because price has decreased and this loss is coming from all the units, ok. So, this is the marginal revenue, if marginal revenue happens to be greater than marginal cost. Of course, the monopoly should produce more, because it would increase the profit. And if marginal revenue happens to be less than marginal cost, then the monopolist should decrease the level of production to increase the profit.

So, at the profit maximization level, MR has to be equal to MC. What is new here is that very, very clearly that MR happens to be less than P that I already discussed. So, what we learn? 2 things, and it may sound repetitive, but it is important that in case of perfectly

competitive market, market the profit maximization condition is P is equal to MR is equal to MC. And in the case of monopoly, the first order condition that we have obtained is that MR is equal to MC, which is always the case with a profit maximizing firm. No matter whether it is a perfectly competitive market or monopoly, what is important to note here is that in case of monopoly P is greater than MR, ok.

Let us continue with the profit maximization. And remember when we talked about perfectly competitive market, we just didn't talk about first order condition, but also, we talked about second order condition ah. Because condition for profit maximization condition for maximization and minimization are the same when we just look at the first order. So, we already talked about something similar in case of perfectly competitive market so, here I am going to talk about the same issue using graphs.

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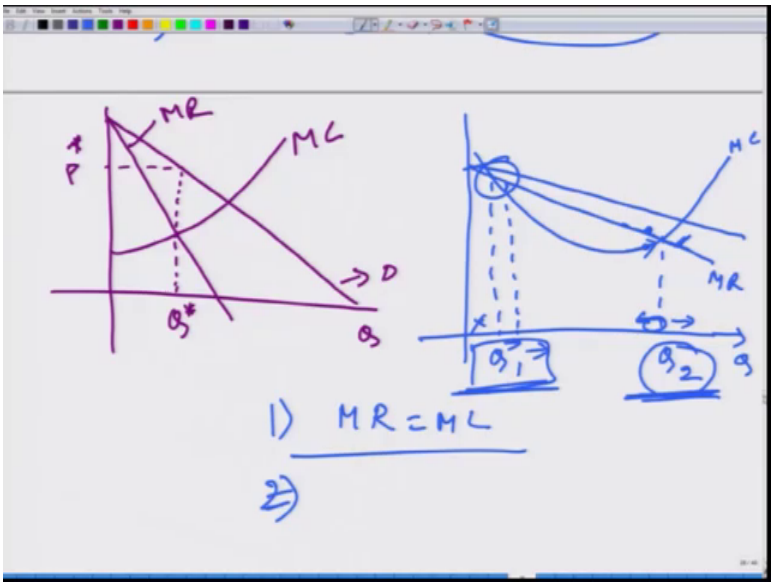

So, what we did earlier was that we had this particular problem in which we took a linear demand downward sloping demand function, and then we had from demand we got the marginal revenue function. And this is the marginal revenue function, and we also looked at the marginal cost function. And we said wherever the marginal revenue function, and marginal cost function, they intersect; it determines the optimal quantity. And by looking at optimal quantity and with help of the demand function, we can figure out what should be the monopoly price of this product in the market. This is the optimization we did

earlier. But now of course, here very conveniently, I looked at a problem, in which marginal cost function and marginal revenue function they intersect only once.

What if marginal revenue function and marginal cost function; they interview more than once? So, let us look at another example and here let us say this is the marginal cost function, and here what we have is let us say this is the market demand function, and corresponding this is the marginal revenue function. So, we see that these are the 2 points of intersection Q 1 and Q 2. Which one is the profit maximizing level of output is it Q 1 or Q 2? Let us see what happens when this firm increases the production beyond Q 1.

It is clear from the graph that let us say at certain higher level, how much is the marginal cost? The marginal cost is coming down, ok. So, very clearly the marginal cost lies below marginal revenue function. So, this if production is increased here, marginal revenue is going to be larger than marginal cost for the next few units. It means that if firm increases the production the profit is going to go up. So, it is very, very clear that profit maximizing is maximization is not taking place at Q 1.

While at Q 2, what is happening that marginal cost curve is cutting the marginal revenue curve from below. This is exactly the second order condition. So, what is happening if you increase the production beyond Q 2? For the higher level the extra unit the marginal cost is going to be higher than marginal revenue. It means that producing little bit more would incurred a loss to the monopolist, and thus profit would decrease. How about if we decrease the output from Q 2? Of course, we see that marginal revenue happens to be greater than marginal cost, it means if this unit is produced, this monopolist would increase it is profit.

And that is the reason the Q 2 is the profit maximizing level of output. Again, the key is as we have seen that MR has to be equal to MC, that is the first order condition, and the second order condition is that the MC curve should be cutting the marginal revenue curve from below like here. This is the marginal revenue curve, and here if you pay attention this is the marginal cost curve. So, it is cutting the marginal revenue curve from below as opposed to here, where marginal cost curve is cutting the marginal revenue curve from above, and that is the reason it is not profit, but the profit maximization is not happening at Q 1.

So, that is one thing that we have to be careful about. The second thing that we have to be careful about is what if the firm is not able to recover the variable cost. Remember, what we had learned earlier that even though what we had learned earlier, that if firm is not able to recover the variable cost then firm would like to shut down, ok. So, here also we can see that let us say the profit is profit the first order condition says that MR is equal to MC at Q star.

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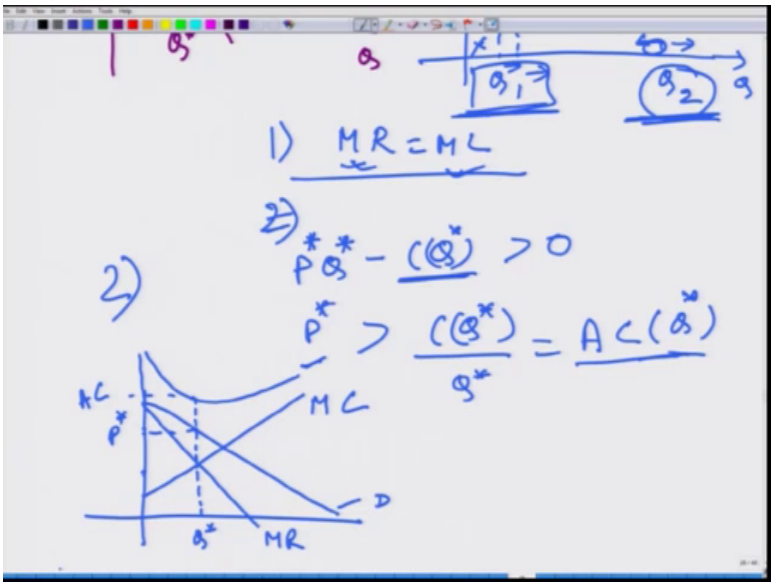

So, of course, corresponding to Q star we will get P star, and off here we have to look at the C Q star. And let us say that if the production does not take place and of course, here I am mixing short run, and the long run if you want to be careful about this, then in short run you will divide it into 2 part, that is the fixed cost part and the variable cost part. And of even if you shut down the process, you will have you will incur the fixed cost, but let us not worry about it, here just look at it colloquially, that it has to be greater than 0 otherwise profit maximization is not happening.

What it means is, that P star has to be greater than C Q star divided by Q star. Which is nothing but the average cost at Q star so, where we obtained let us say at Q star MR is equal to MC what we have to check other than that that at Q star other than all the factors that we have to check that P star happens to be greater than ac Q star. So, let us look at it here graphically, this can very well be the case.

Of course, again this is standard demand function, this is marginal revenue function, and this is the let us say this is the marginal cost function. And it so happens that average cost comes out to be something like this. Then what happens look at Q star what is happening ? P star this is what this firm can charge in the market, but firm is not able to recover it is cost, because average costs turns out to be greater than price. In this case, firm is better off by not producing anything.

Thank you.