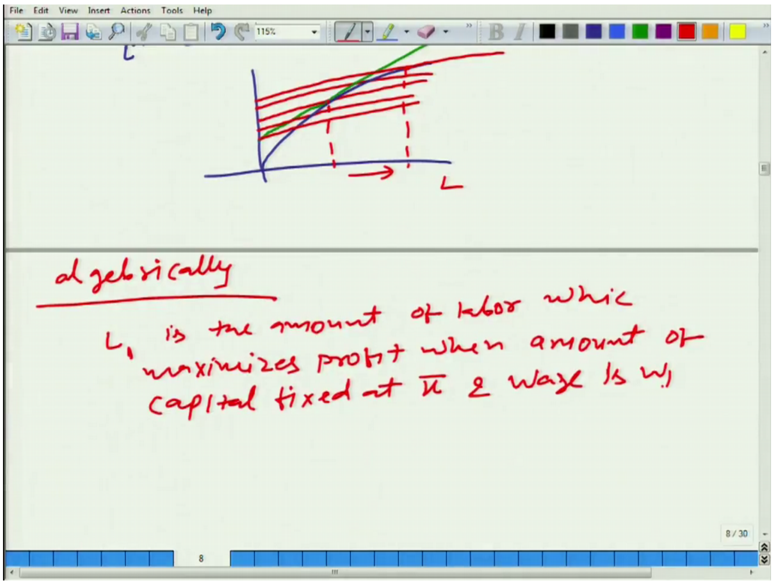
An Introduction to Microeconomics
Prof. Vimal Kumar
Department of Economic Sciences
Indian Institute of Technology, Kanpur

Lecture – 112
Profit Maximization in Short Run Through Algebra and Calculus

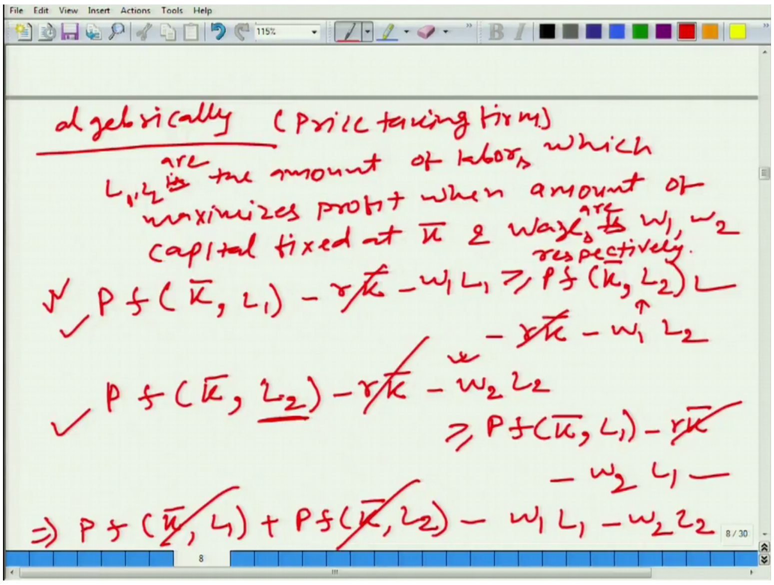
Now, what we can do? We can look at it algebraically also, it is good idea to understand it algebraically what is happening algebraically.

(Refer Slide Time: 00:23)



So, we can say that remember L, let say L 1 is the amount of labour of labour, which maximizes profit, this profit when amount of capital is fixed at just fixed at k bar and wage is w 1 fine.

(Refer Slide Time: 01:11)



Now let us see what happens. If what it means is that p of f, k bar L 1. This is the profit when you are using k bar amount of capital and L 1 amount of labour. It is going to be more than, at this label of capital and at this wage rate it will more than any other profit which you get if you use any different amount of labour. So, let me put a different amount of labour as, is it clear, fine.

Similarly what we can do instead of taking L 1, let say at L 2 is the labour that maximizes the profit when fixed amount of capital is k bar and the wage rate is w 2. So, what we can write is that p of f k bar L 2 minus r k bar w 2 L 2 will be more than L 1. I am putting any other labour I am using this labour, but that is any other labour minus r k bar w 2 L 1, this is clear. This we can cancel both side, fine. And we add up this side with this side and this side with this side. What do we get if you do?

Student: Sir, but advantage (Refer Time: 02:53) the profit that will.

The second statement, look at the first statement, we can say it that here we can change L 1 and L 2.

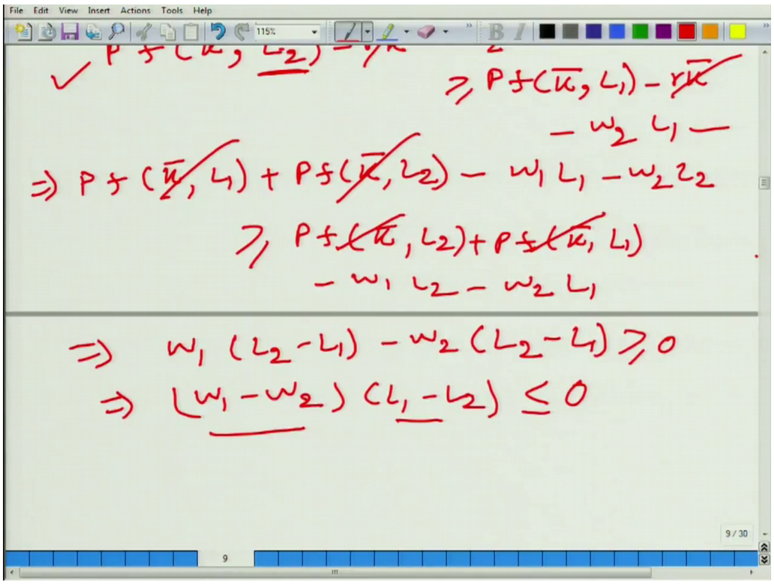
Student: (Refer Time: 03:06).

Are the amount of labours which maximizes profit when the amount of capital is fixed at k bar and wages are w 1 and w 2 respectively; now it should be clear.

Student: But how can we compare between.

We are not comparing see here we are using L 1, wage we are using is w 2 and as we have said we have already obtained that, when wage is w 1 and this is the production function and amount of fixed capital is k bar, then L 1 maximizes the profit. So, whenever we use any other amount of labour we can always get in this case more than this. Similarly here we are doing we are reversing it. We are using that here we are using w 2. So of course, for w 2 L 2 is maximizing the profit ok.

(Refer Slide Time: 04:08)



Now let us add it up, what do we get? P f k bar L 1 plus P f k bar L 2 minus w 1 L 1 minus w 2 L 2 and this is greater than or equal to f, here from here we get L 2, from here we get P of f k bar L 1, and from here we get minus w 1 L 2, and from here we get minus w 2 L 1. These 2 will get cancelled on both side.

Student: Sir, but here price is the function of (Refer Time: 04:46).

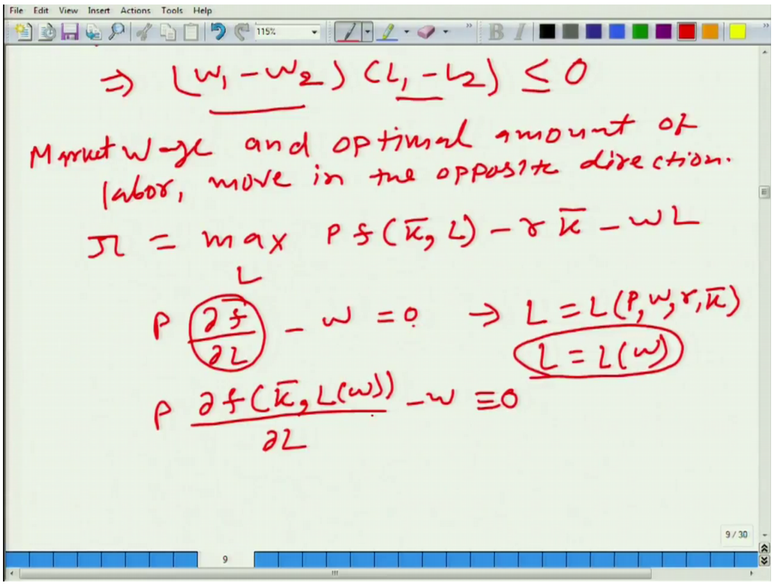
Price is oh, price is not here we are not using the price as a function of quantity. Sorry let me say here, that price we are not; till here it is clear? Here in this case we are using for price takers, price taking firm. We are using for price taking firm that price is not changing fine, ok is it clear?

Now, we can rearrange it and what we will get; w 1 L 2 minus L 1 again we take it on the same side what do we get there.

Student: (Refer Time: 05:30) w.

Minus w 2 L 2 minus L 1 should be greater than or equal to 0. Or we can write it w 1 minus w 2 multiplied by L 1 minus L 2 is less than or equal to 0. What it means is, that w the wage an optimal amount of labour they move in the opposite direction, they cannot move in the same direction.

(Refer Slide Time: 06:08)

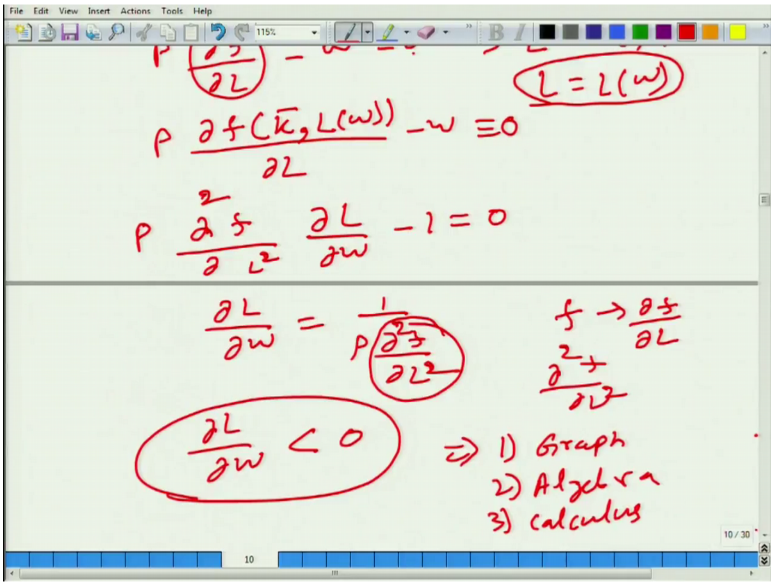


They move, wage, market wage again this is fixed, this a firm is taking as given; market wage an optimal amount of labour move in the opposite direction. And this is what we got from the diagram ok. This is exactly what we learn from the diagram fine. Now we are going to do this using calculus. Exactly same thing we will get using calculus. This is quite important that you understand diagram, you understand the algebra, of course these all have origin in different techniques, but right now we are not learning the techniques we are learning about profit.

So, let us use calculus. And of course, we will say that the firm is acting as price taker, firm is not able to rig the price. So, what is the profit? Profit let me write the small pi; a small pie is nothing but maximize amount of profit that this form can obtain; so maximum of this is profit. And from here what can we get is, we can obtain the first order condition, and what is the first order condition? P; and what is this? This is marginal productivity of labour minus w is equal to 0. We can solve it, ok.

When we solve it what we will get? L as a function of optimal amount of labour as the function of price of output in the market, also wage rate, also rent of capital and fixed amount of capital. But right now we are interested only in L as a function of wage rate we will take other things fixed. So if we bring it here, what we can write? And now it will convert into identity, it will always be true because, how did we get this by solving this equation. So, by plugging it back this is always going to be true and if you differentiate it with w, what we will get p, because p is independent of w p does not depend on w. What does depend on w only L.

(Refer Slide Time: 09:00)

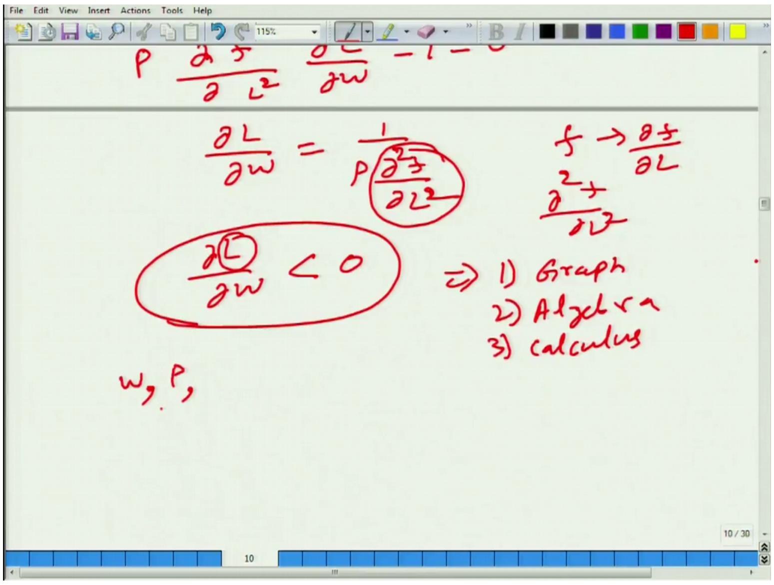


Student: (Refer Time: 09:11)

L depends on w and thus function also depends on w fine. So, if we differentiate this again with w P we will get as it is, here k bar also does not depend on w. So, 0 is not a function of w so when we differentiate 0 with respect to w this is what we get fine. From here we can write; and this we already know what does what do we know about this, it is negative not decreasing it is what does it mean? We start with f from f we get partial derivative of f with respect to L, that is marginal productivity of labour.

And then we again differentiate, what do we get that marginal productivity of labour decreases with increases labour. So, diminishing marginal productivity and this is negative. So, what do we get, that d L by d w is negative, fine. And this is what we did we derive earlier, did not we?

(Refer Slide Time: 11:03)



So, we have solved it using three different techniques: graph, algebra and calculus. And this is quite important what I would be interested in that you also see what happens the impact? What happens to the optimal amount of L? This is not just any L this is optimal amount of L; what happens to the optimal amount of L, when you change w that we already derived? What happens to the optimal amount of L when you change p? What happens to the optimal amount of L when you change r?