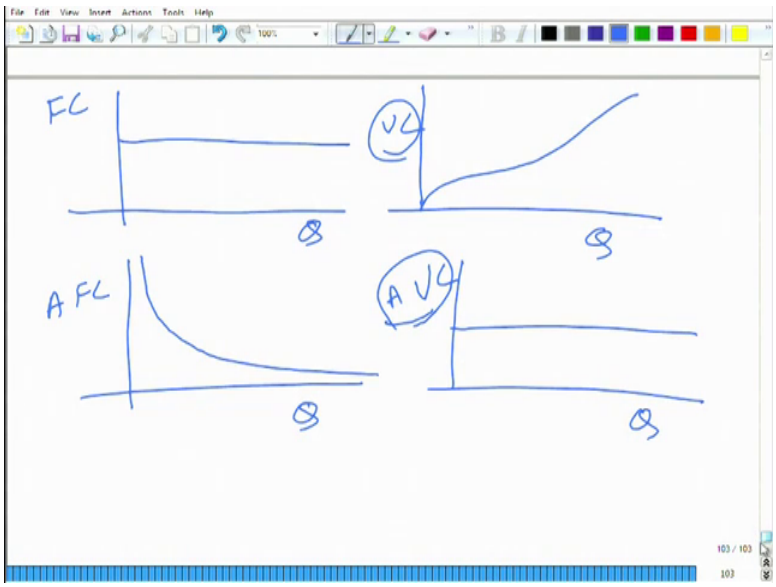
An Introduction to Microeconomics
Prof. Vimal Kumar
Department of Economic Sciences
Indian Institute of Technology, Kanpur

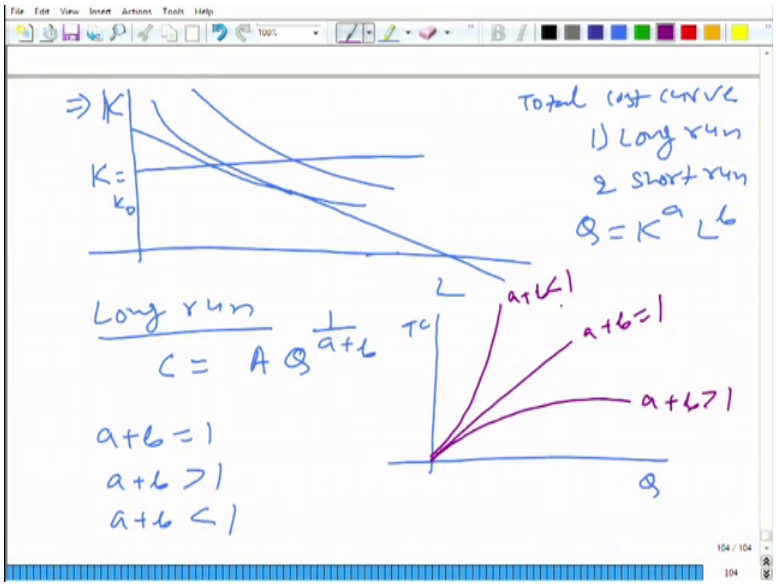
Lecture – 104
Cobb-Douglas Function: Cost and Returns to Scale

(Refer Slide Time: 00:15)



Now, let us look at it just one simple example what we had in the in the class we took 3 examples one example of Cob Douglas function, cost minimization of Cob Douglas function, cost minimization of a production function when inputs are perfect substitute of one another and when they are perfect complement of one another.

(Refer Slide Time: 00:24)



So, let us see what we have here is Cob Douglas function fine. We are right now talking about. We have not decided whether its long run or the short run we are just talking about the way we have drawn here it is of course.

Student: Long run.

Long run and let us say in the short run you have something like K is fixed is it K is fixed at K naught. Now, what you should do? You should derive the total cost curve for long run and short run how it would look like fine ok. Let me let me give you how will it look like in the long run we the cost curve when we have production function as Cob Douglas function as production function, the cost function remember what we had Q is equal to K to the power L K to the power.

Student: a.

a, L to the power.

Student: b.

b, fine. So, cost function is going to be a function of it is going to be a function of if you get here some let me say a is the constant and you get it as a function of this is what you will get if you derive fine. So, now, let us draw the total cost curve with respect to output. Of course, it will depend on value of a and b. Let us say let us derive it for three

cases when a plus b is equal to 1, a plus b is greater than 1 and a plus b is less than 1. When a plus b is equal to 1 what do we get?

Student: Straight line.

Straight line when a plus b is greater than one what do we get.

Student: (Refer Time: 03:04).

Total cost increases at decreasing rate total cost increases at decreasing rate and when a plus b is less than 1. What happens to total cost?

Student: (Refer Time: 03:18).

So, here a plus b is equal to 1, a plus b is.

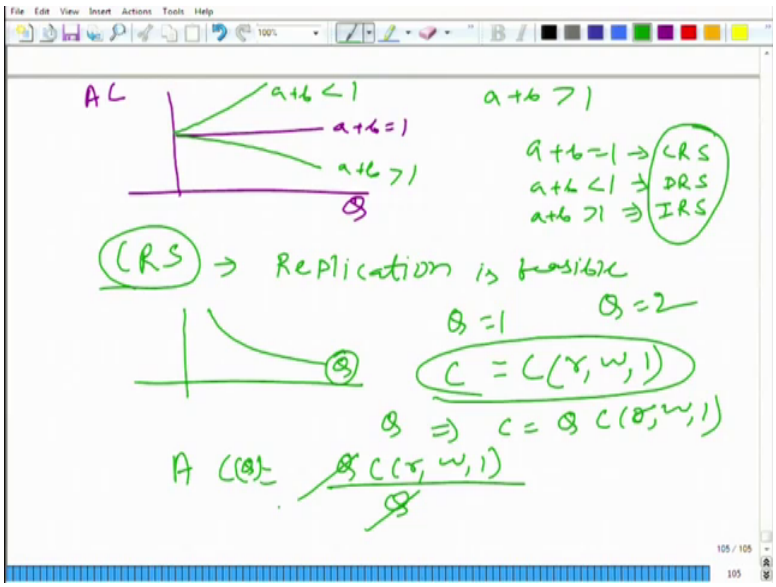
Student: Greater than.

Greater than 1 and a plus b is.

Student: Less than 1.

Less than 1. These are the three condition ok, fine.

(Refer Slide Time: 03:34)



Now, let us look at the what happens to the average cost here we have talked about total cost here we are talking about now average cost average cost with respect to output. Well, what we get when a plus b is equal to one average cost is fixed is not it why it is fixed we take a point here and we take any point on this curve and when we draw a line from this point to origin what we get we get the same line again and again. So, average cost remains the same. What if a plus b is greater than 1, what do we get? It decreases.

Student: Decreases.

Something like this ok. I do not know how much it at this point.

Student: Decreases.

And this is.

Student: a plus b greater than 1.

And this is.

Student: Increases.

Increases. Again I do not know the value at Q is equal to 0 I am just drawing fine. Now, go back to what we discussed the topic of return to scale. Do you find any relation between return to scale and the average cost?

Student: (Refer Time: 05:00).

What happens when a plus b is equal to 1, what do we have?

Student: (Refer Time: 05:05) constant return (Refer Time: 05:06).

Its constant return to scale ok. And when a plus b is less than 1 what we have?

Student: (Refer Time: 05:18) decreasing (Refer Time: 05:18).

Decreasing returns to scale. And when a plus b is greater than 1 what do we have?

Student: Increasing.

Increasing return to scale. So, do we see any relation? When we talked about constant return to scale or decreasing return to scale what we are doing? We were talking about production technology. Now what we are doing? We are relating it to the average cost curve and average cost curve does not give the average cost of producing just Q amount of output it gives average cost of producing Q amount of output at the minimum.

Student: Minimum.

Possible cost. So, do you see any relation?

Student: Sir.

Think about it.

Student: Constant returns to scale average (Refer Time: 06:05) fixed (Refer Time: 06:05).

Let us go with the logic. What did we say? When we were talking about constant return to scale replication is feasible replication is feasible exact replication. So, now, you have a production technology and you let us say you have a production technology and its isoquant looks like this fine. And you want to produce this Q amount of output or let us start with the Q is equal to 1 amount of output ok.

So, cost of producing 1 unit of output of course, it will depend on the input prices and in input prices in our case our r and w and 1. Now, if the production technology exhibits constant return to scale then if we want to produce Q is equal to 2 what do we do? If this is the minimum cost of producing one unit then what would be the minimum cost of producing 2 unit.

Student: Q C.

Q C. So, what would be the minimum cost of producing Q? It is going to be C multiplied by Q of.

Student: (Refer Time: 07:29).

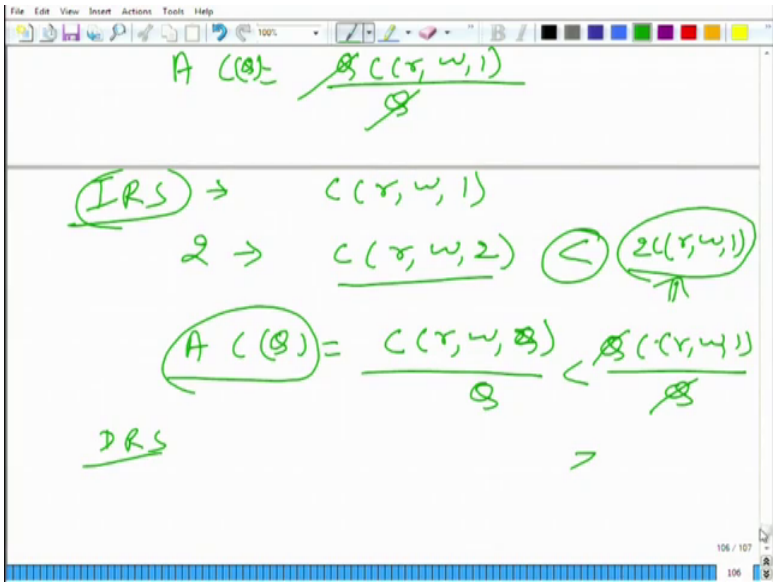
R w comma 1 is not it. So, what is the average cost going to be equal to?

Student: Q (Refer Time: 07:40) C.

Student: C.

So, it is going to be the same as cost of producing 1 unit of output considering that we are using cost minimizing combination of inputs, fine.

(Refer Slide Time: 08:00)



Now, let us talk about increasing return to a scale. What do we mean by increasing return to scale? So, when we increase the production we find better way to produce the outputs. So, what will happen or in other word when we double all the inputs output gets more than.

Student: Double.

Doubled. So, now, we start with one unit of production what we have the cost is C of r comma w comma 1. Now, if you want to produce two units of course, the cost is going to be r comma w comma 2, but can we say any relationship between this and 2 of r C of r comma w comma 1 is there any relation.

Student: Yes, sir it is greater or the.

Greater or this is going to be greater.

Student: Many that is going to be.

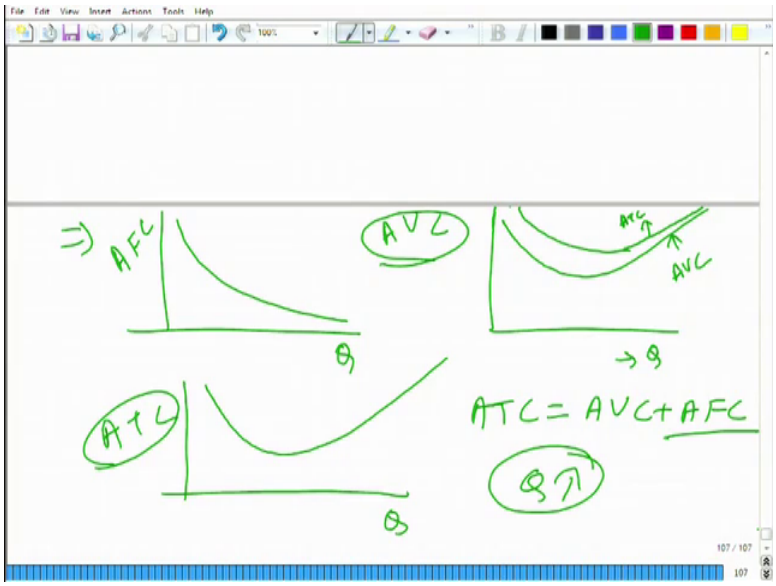
This is greater, this is greater why because we are not using any return increasing return to scale. What we are doing here we are just replicating the process, but what did we say when we have increasing return to scale we find better techniques. We double all the inputs we get more than double of the outputs.

So, we do not need to double all the inputs to produce the twice the amount the inputs all the inputs should be increased by less than 100 percent. So, of course, cost is not going to be the double it is going to be something less than double, so this is true. So, whenever you have increasing return to scale average cost let us look at the average cost this is going to be less than the cost of producing just one unit of output.

So, what further if we consider this logic and take it further what will happen to average cost it will keep on decreasing because we have better and better technique to produce, is it clear. So, that is why we get here look at here. That is what we get when we have increasing return to scale average cost is falling fine.

Now, let us look at the decreasing return to scale what will happen here it is going to be greater than the average cost will be increasing as Q is increasing. So, that is what we get here. It is not true just for the Cob Douglas function its true for depending on the return to scale fine, is it clear,. So, now, again we can look at it from some different angle.

(Refer Slide Time: 11:07)



What is happening with the average fixed cost its decreasing. How about the variable cost, it will decrease or it will increase?

Student: What.

As Q increases typically.

Student: First would decrease then would increase.

Why it should decrease first?

Student: Increase in returns to it is normally also that first it is increasing.

We are not talking about the any return increasing return to scale just explain it in using simple words.

Student: Ok, (Refer Time: 11:42).

Average fixed cost is decreasing with Q. How about the variable cost? Of course, we are talking about the long run.

Student: Average (Refer Time: 11:53).

A short run.

Student: (Refer Time: 11:55) able cost may increase or decrease.

May increase.

Student: The able cost or average within cost.

average variable sorry, average variable cost.

Student: May increase or decreasing.

May increase or decrease why? When will it increase and when will it decrease?.

Student: Sir, when it is increasing returns to scale.

Ha.

Student: (Refer Time: 12:13).

So, just what you are right, but just think about it what why does it happen?

Student: Sir, because if we are producing one more unit the cost where implying is less than tilde previous unit.

Student: Sir, sir it could be like one labour can produce 8 units and.

2 labour than produce more than 6 or 7 (Refer Time: 12:37).

Student: More than 16, but if there are more workers coming in then.

Hm.

Student: Their productivity would fall and then one labour could produced less than 8 units.

Student: Like 7 or 6, so variable cost would raise.

So, see simply what is happening a when we are talking about average variable cost variable cost of producing you know on average one unit. As we are producing more and more and more what would happen? Typically remember when we it is related to marginal productivity that I am going to show you little later how it is related to marginal productivity.

What did we see earlier from the table that in the beginning when we bring one more worker let us see that capital is fixed and we are increasing the labour. When we are bringing one more worker the output is increasing at increasing rate, but after some point of time what happens by bringing one more worker output starts decreasing. So, that will translate into the average variable cost.

So, it may happen I am not saying it is true it is possible that you get something like this, average variable cost is increasing linearly or it is also possible that average variable cost remains the same I just talked about. But typically what we get that average variable cost first decreases and then increases.

Student: Increase just (Refer Time: 14:00).

Fine, and if we bring it what will happen to the average total cost? It decreases and then its starts increasing.

Fine, is it clear.

Student: Yes sir, yes sir.

And also if you look at this graph average variable cost and average total cost if I bring the same graph here. What happens? That earlier look at it here average total cost is nothing but average variable cost plus average fixed cost. As Q is increasing average fixed cost is.

Student: Decreasing.

Falling. So, what happens as you move further and further? These two of course, this one is average total cost and this one is average variable cost. As you move the earlier the difference is going difference will gradually.

Student: Decrease.

Decrease.

Student: The difference is equals to the rate fixed cost right.

Huh ok. Again it is not perfect, but let us see fine. As we increase Q the difference will keep on decreasing, and for very very high Q average variable cost and average fixed cost are going to be equal of each other.

Student: (Refer Time: 15:22) variable cost and average should be.

Average total cost will be equal to each other yes, fine, ok.