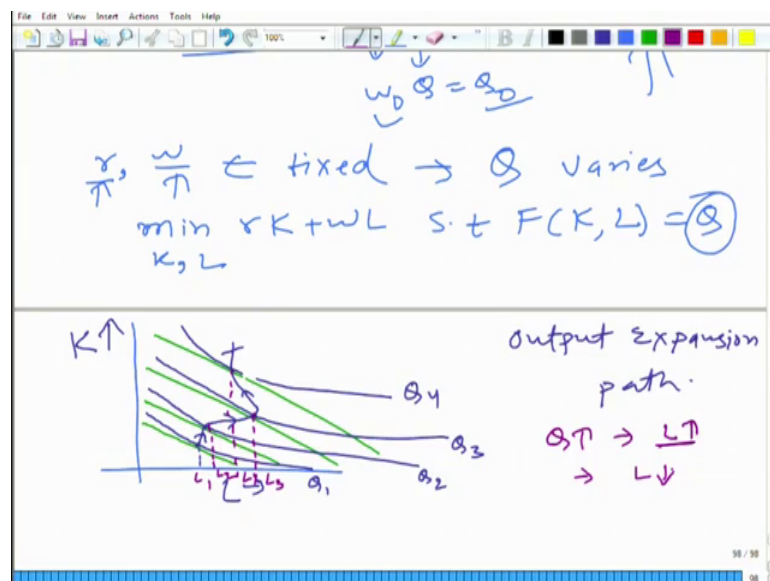


An Introduction to Microeconomics
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Lecture – 102
Output Expansion Path

Now, what we are going to do? We are going to keep r and w , that is r is again rental and w is wage rate we are going to keep these two fixed ok, and we are going to vary Q ok.

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What happens to the problem let us look at it graphically. In other words what we are trying to do again we are trying to minimize rK plus wL with respect to K and L such that F of K comma L is equal to.

Student: Q .

Q and how we are changing the problem? We are changing this Q ok. To represent it graphically what is happening our iso cost map does not change it looks like this let us say for particular value of w and L this is the way it looks like fine. And let us look at the isoquant map, this is the isoquant map representing different level of output let us say let us call it Q_1 Q_2 , Q_3 .

Student: (Refer Time: 01:42).

Q 4.

Student: (Refer Time: 01:43).

Fine, here we have capital and here we have labour fine. Of course, believe me that what I mean show is that they are tangent to each other although they it is not very perfect diagram fine.

Now, what we can do? Remember earlier in the consumer theory, again and again I am talking about consumer theory because if you have understood consumer theory its locked its very very easy to understand the producers theory that is why I am going back to what we have learned earlier. Remember earlier we talked about income expansion path, price consumption curve. So, something similar we are talking about here.

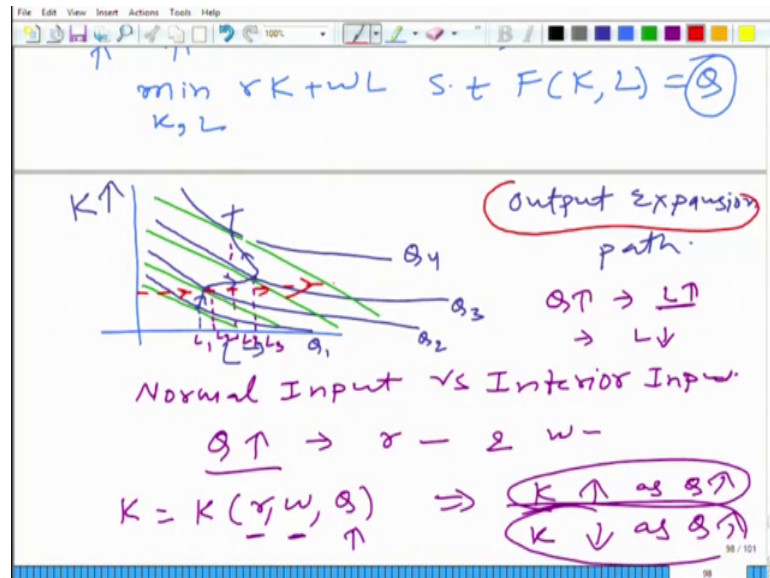
Student: (Refer Time: 02:30).

Here we are talking about output expansion path. And what is output expansion path? We can output expansion ka path is nothing, but a curve that passes through the cost minimizing combination of inputs as the output quantity Q varies while the input prices are held constant, ok.

So, let me just say again this is not perfect and let us say again we do not know, but I am just making it up, it is going like this fine ok. Now, if you pay attention what is happening? We should look at in this zone here at look at it here the labour requirement is L_1 here labour requirement is L_2 , here labour requirement is L_3 and in this case labour requirement here is L_4 .

What I am saying as Q is increasing for some zone L required the input required to produce Q optimally in which sense the in the cost minimizing sense L is increasing, but in some zone L is decreasing. So, earlier remember we define something called normal good and inferior good here also we can define normal input and inferior input; versus inferior input. And what is normal input?

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Student: Let me we have (Refer Time: 04:28).

When output is increasing while keeping the prices of all the inputs fixed and amount of input required to produce output at the minimum cost possible manner and the input that particular input increases; then that input is normally input.

Student: Sir, can you repeat it.

So what is happening? We are increasing the output while keeping K and not k , while keeping the.

Student: (Refer Time: 05:07) w .

R and w fixed we are not changing r and w fixed and we are increasing the output Q what we are talking about here is r w and Q . These two are fixed and we are increasing Q . Now, there are two possibilities, K value of K after putting here r w and Q it may go up as Q goes up or it may come down as Q goes up if this happens capital is the.

Student: Normal.

Normal.

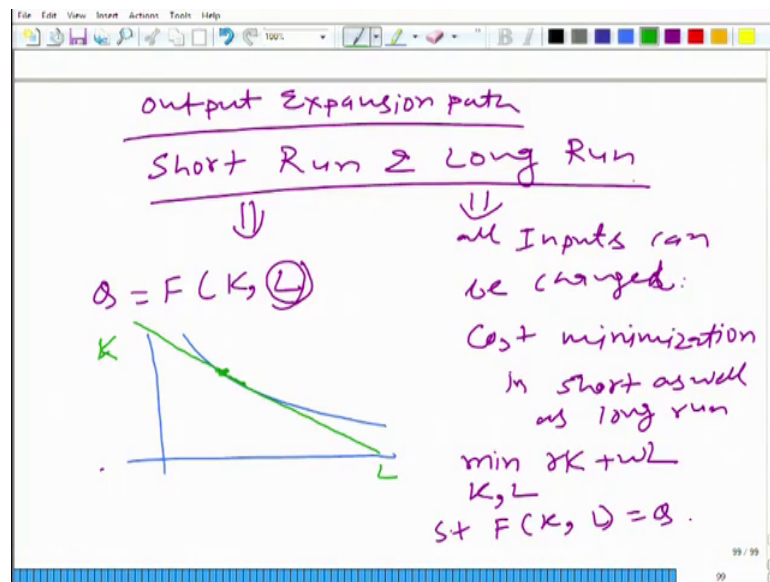
Student: Input.

Input if this happens then the capital is inferior.

Student: Inferior.

Input similarly one can talk about labour also; as, but remember two things we are talking about one that the prices of all inputs are fixed fine. And we are not talking about any combination of input to produce this output the combination should be such that it minimizes the cost of producing that amount of output. So, these two things are there, fine. So, now, we have learned about output, expansion path.

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Now, I want to bring the concept that we learned earlier short run and long run in this discussion ok. What is short run?

Student: A period where at least one input is fixed.

One input can.

Student: Cannot be.

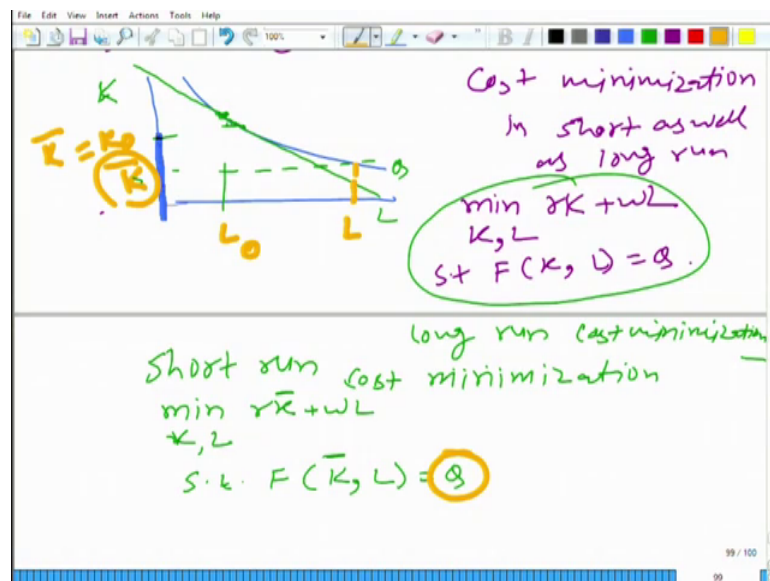
Cannot be varied and here all inputs can be varied, can be changed fine. Now, let us look at it here we are talking about a production where we have only two inputs and invariably we take that right we have been talking about it that L we can change even in the short run, but it is difficult to change the capital in the short run it is not true.

But this is the theoretical position we have taken in this course for illustration it depends on the scenario the legal framework and various other things, but we are not talking about that. Now, let us look at it the output expansion path that we have been talking let me go back to that earlier graph that we had. Here is the graph that we have talked let me just say here even before we talk about output expansion path.

Let us talk about cost minimization, cost minimization in short as well as long run. And what we have been talking about? We are talking about minimizing rK plus wL with respect to K and L such that F of K comma L is equal to Q and this is Q and then we get here this is what we have done we have let me just ok, this is the point of tangency ok.

So, what we are assuming basically that we can reach to this point by changing K and L ok, but that is feasible only in the long run. So, although we have not mentioned, but the problem the cost minimization that we have been doing that is long run cost minimization. Do you understand? This is is the long run cost minimization.

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What happens in the short run? Short run we have one more constraint. What is that constraint? The short run the position that we have taken in this particular course in this particular chapter that in short run K cannot be.

Student: Varied.

Varied, K cannot be changed. So, what we have the short run cost minimization problem is bit different, short run cost minimization let me write it what we need to do is basically minimize rK plus wL with respect to K and L such that F of K comma L is Q , and one more that K is equal to let us say \bar{K} this is also this is extra. So, we can rewrite this problem little differently how can we rewrite it rather than writing using two constraint what we can say what we have here let us get rid of this instead of K what we have to use is.

Student: \bar{K} .

\bar{K} and here we have to use \bar{K} and then we do not need to say here, but this is what basically we are doing. So, coming to this graph let us say \bar{K} is here this is the \bar{K} level can we reach to this point no in short run it is not possible to use the this particular cost minimizing combination of.

Student: Input and output.

Input in the cost minimization combination of K and L it is not possible because we are forced to use this particular \bar{K} we are forced to use this particular level of K and to produce Q output we cannot if we take L that is the earlier we derived that we need this L naught amount of labour and K naught amount of capital to produce Q at the cost minimizing level in cost minimizing fashion. But that is no longer possible because of this \bar{K} we end up using this particular amount of labour. So, do you think which one is going to be higher the which cost is going to be the higher let us say we want to produce Q .

Student: Short run cost.

Short run cost is going to be at least as great as.

Student: Short run.

The long run cost minimizing the long run cost of producing Q . Why because why as because this \bar{K} it depends on the \bar{K} what if we say that \bar{K} the \bar{K} that is selected is equal to K naught in that case the.

Student: Long run.

The cost required to produce Q amount of output in long run and short run are going to be the

Student: Same

Same, but the cost to produce in the short run is never going to be less than cost to produce the same amount in the long run.

Student: (Refer Time: 13:10)

Just think about it logically in long run you are allowed to vary everything capital and labour, but in the short run you are not allowed to vary capital. So, you have less degree of freedom, ok. So, best what you can do in the short run? To get to get the cost that is feasible in the minimum cost that is feasible even in the long run you cannot do better than that fine. So, if we solve this problem what we get? This is.

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Short run cost minimization
 $\min r\bar{K} + wL$
s.t. $F(\bar{K}, L) = Q$
 $\Rightarrow K = \bar{K}$
 $L = L(r, w, Q, \bar{K})$
 $C = C(r, w, Q, \bar{K})$
 $= r\bar{K} + wL$

So, if you solve it what will you get? K you have to use as K bar and L is going to be a function of r w. What else?

Student: (Refer Time: 15:53)

Q and also a function of.

Student: K bar.

K bar here, in fact, in this particular case the problem is very simple because you had two degree of freedom you can change r you can change K and L, but now in the short run you are not allowed to change K. So, there is only one particular level of labour that will give you this particular amount of output and that is this.

So, basically you are not doing any optimization you have only one point and out of one point you have to pick the best point. So, there is only one possibility fine. But imagine a scenario where you have more than two inputs, in that case you are not able to vary one input, but in that case you will have to do the optimization cost minimization and then you will get this particular form fine.

Student: (Refer Time: 14:52).

And what is going to be the cost? Cost is going to be a function of r w Q and K bar y because cost is nothing, but r K plus w L ok, fine, is it clear.

Student: Yes.

Any doubt about it.

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The image shows a whiteboard with handwritten mathematical derivations. The text is as follows:

$$\begin{aligned} & \min_{K, L} r\bar{K} + wL \\ & \text{s.t. } F(\bar{K}, L) = Q \end{aligned}$$

Then, it shows the substitution of $K = \bar{K}$ into the cost function:

$$\begin{aligned} L &= L(r, w, Q, \bar{K}) \\ C &= C(r, w, Q, \bar{K}) \\ &= r\bar{K} + wL \end{aligned}$$

At the bottom, it summarizes the cost functions for long and short run:

$$\Rightarrow \boxed{w \geq r} \Rightarrow \begin{aligned} c &= C(Q) \in \text{long run} \\ c &= C(Q, \bar{K}) \in \text{short run} \end{aligned}$$

And let us say one more thing I would like to say just for convenience. Let us say that w and r are determined by market it is not in the control it is in not in the firms control. So,

what we can say roughly we can write C is a function of Q in the long run and C is a function of Q and this particular level of.

Student: Capital.

Capital in the short run; fine. So, let us see what happens to the output expansion path. Let us go back to this graph let us say capital is fixed at this particular level. So, how the output expansion path will look like? A horizontal line this will also give us the output expansion path.

So, you should be clear when you are talking about output expansion part whether it is long run or short run you are concerned about, ok. Similarly for the cost minimization you have to worry about whether you are talking about short run or the long run, fine.