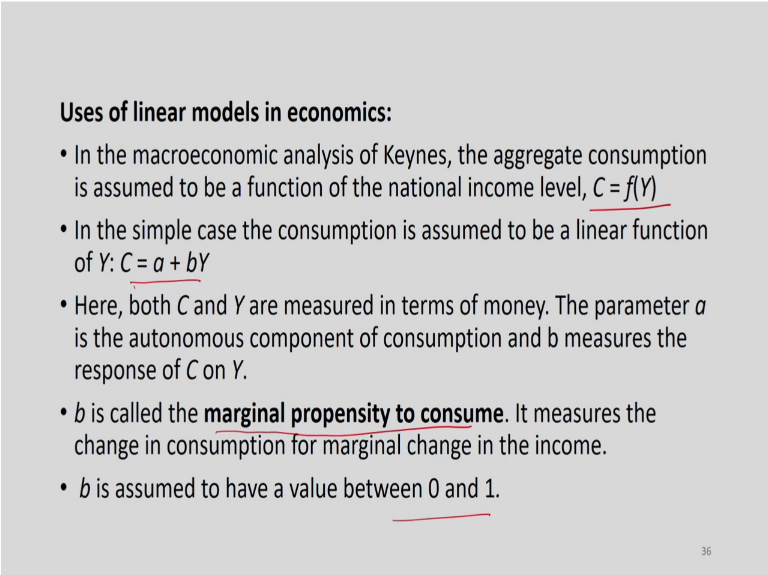
Mathematics for Economics 1
Professor Debarshi Das
Humanities and Social Sciences Department
Indian Institute of Technology Guwahati
Module: Functions
Lecture 7: Quadratic Functions

Welcome to the seventh lectures of this course, Mathematics for Economics part 1. So, in the last few lectures we have been talking about functions and we talked about in particular linear functions and how in the linear function there could be two components, one is the autonomous or constant component and the other is the component which varies with change in the argument or the independent variable.

(Refer Slide Time: 01:08)



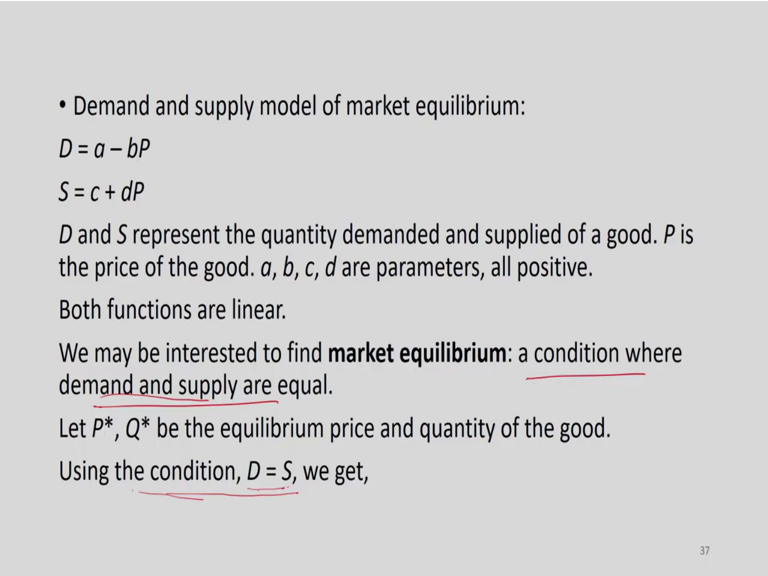
And I was talking about the uses of linear models or linear functions in economics. So, this is the example that I gave in the last lecture. This is a macroeconomic model and it was proposed by Keynes. Keynes was the founder of this particular sub discipline in economics called the macroeconomics and here what you have is this function $C = f(Y)$. C is the consumption, the aggregate consumption of the economy and you can see it is assumed to be a function of Y, Y is what?

Y is the national income and so, $C = f(Y)$which means that consumption depends on income, but you can take a more simpler form of this particular function, you can take

$C = f(Y) = a + bY$, where a and b are parameters and so C depends on Y, but it depends on Y through this particular route.

In this particular form of consumption function, b is called the marginal propensity to consume, it measures the change in the consumption for marginal change in the income and generally there is an assumption that the small b is going to lie between, within a particular interval the interval is from 0 to 1. So, this is an example of how linear functions are used in economics.

(Refer Slide Time: 02:54)



And this is another example, here you have demand and supply models of market equilibrium. So, first you have $D = a - bP$, a and b are small letters and $S = c + dP$, I am sorry, this is not Y it is P, P is the independent variable here. So, D and S are the quantity demanded and quantity supplied respectively of a particular good.

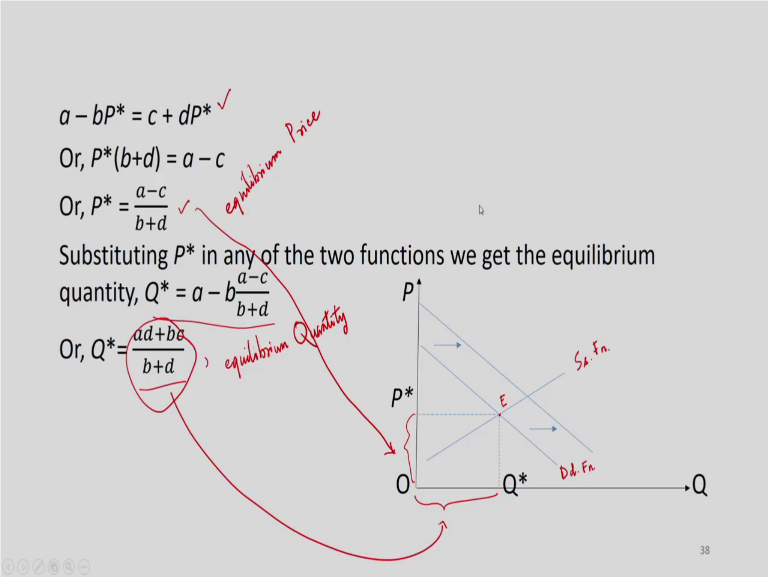
So, D is the quantity that people are demanding of a particular good and S is the quantity that is being supplied of that good. P is the price of the good, price means price per unit, like you know 20 rupees per ice cream. So, there 20 rupees is the price, a, b, c, d are all parameters and they are all positive numbers and you can see both these functions are linear function, demand function is a linear function, the supply function is also a linear function.

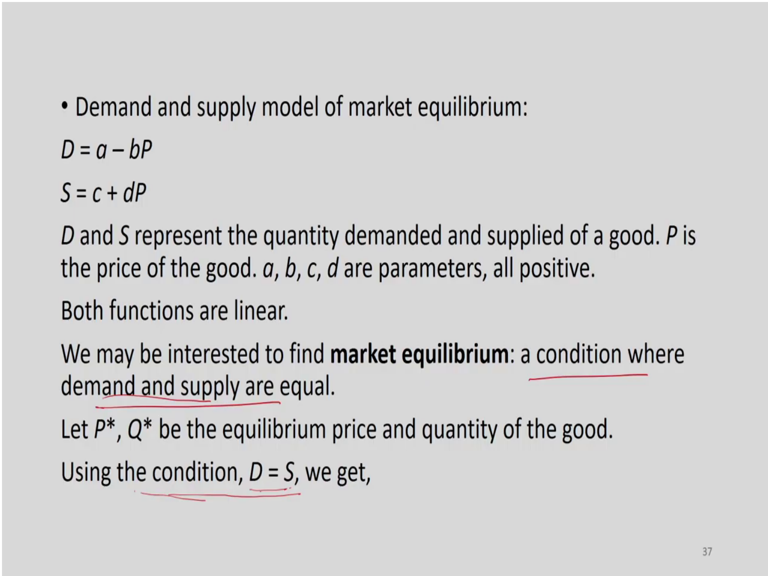
Now, what is the, what is the task? We may be interested to find market equilibrium and what is the market equilibrium? What is the definition? It is a condition where demand and supply are equal. So, quantity demanded is just equal to quantity supplied. So, neither demand is more than the supply nor is it the case that demand is less than the supply.

So, we assume that P^* Q^* are the equilibrium price and equilibrium quantity of that good, that means, if we have an equilibrium, then at that equilibrium there will be some amount of quantity and somewhat price and those quantity and price are represented by Q^* and P^*. So, these are particular values of these two variables P and Q.

Now, as we know the equilibrium is defined to be a condition where the demand and supply are equal. So, we have to use this use this relation and by using this relation or condition we get, this particular equation that D must be equal to S. So, now we use the fact that $D = a - bP$ and $S = c + dP$.

(Refer Slide Time: 06:08)





So, we use these two functions and we get this relation $a - bP^* = c + dP^*$. I am writing P^* instead of P because we know that in equilibrium, this price is taking that particular value P^* and then we simplify this a bit and we get $a - c = P^*(b + d)$ and so, $P^* = \frac{a-c}{b+d}$.

So, this is the equilibrium price. So, we have found what is the equilibrium price. This is given by $\frac{a-c}{b+d}$. The next task is to find out what is the equilibrium quantity that can be found by substituting this P^* in any of these two functions that is the demand function and the supply function and if we do that, we get this particular $Q^* = a - b\frac{a-c}{b+d}$.

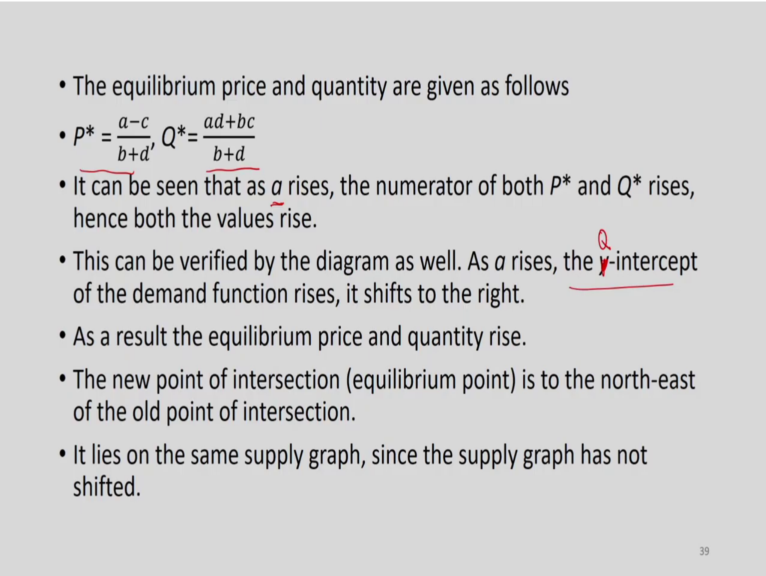
So, I am just using the demand function here and if I simplify this, I get this. So, this is my equilibrium quantity, $Q^* = \frac{ad+bc}{b+d}$, if I had used the supply function, rather than demand function to find the equilibrium quantity, I would have got the same sort of expression for Q^* and here was the mathematical solution P^* and Q^* and here you have the geometrical of the same thing.

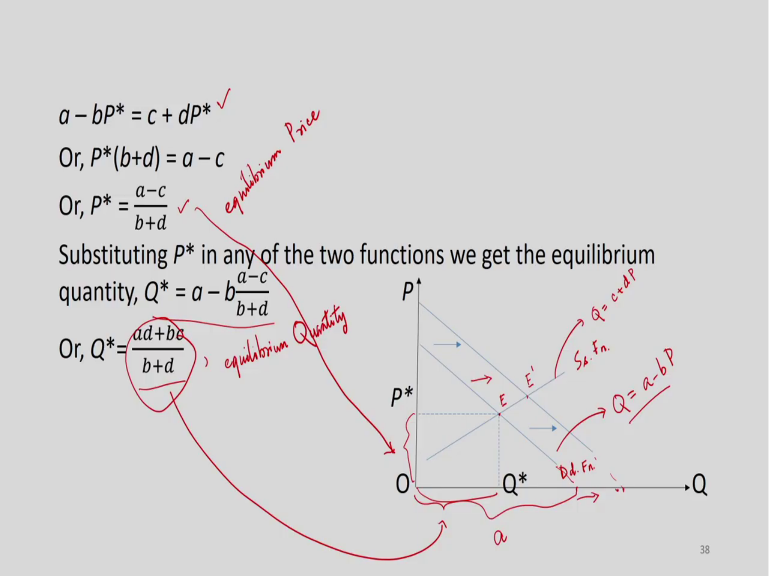
So, here this is the demand function and this is the supply function and you can see the supply function is an upward rising line, because you have this $c + dP$, this is S and we know it is going to be an upward rising line if D is positive and these are indeed positive and about the demand function, demand function is a downward sloping line, because what is the demand function is $a - bP$ and b is positive. So, minus b is negative, therefore, you have a downward sloping straight line.

So, they are intersecting at this particular point, we can call this point as E the equilibrium point. Now, this equilibrium point from this I draw two perpendiculars on the horizontal and the vertical axis. So, this perpendicular EQ^* on O Q. So, OQ^* this part, is your, this value $Q^* = \frac{ad+bc}{b+d}$ and as I draw the perpendicular on the vertical axis, I get OP^* that intercept and that is given by this quantity $P^* = \frac{a-c}{b+d}$.

So, this is the geometrical interpretation of the same thing, same solution the equilibrium, quantity demanded and quantity supplied, they are intersecting at a particular point at point E, E is the equilibrium point because at that point quantity demanded is just equal to quantity supplied and from E, I find out the coordinates, the coordinates are given by OQ^* and OP^* and which are given here.

(Refer Slide Time: 10:46)





Now, what happens in this story, if some of these parameters change. Suppose, this small parameter a, which is positive it rises and you can see that straight away from the expression of P^* and Q^*, what happens if a changes. So, in both P^* and Q^*, a appears as a positive entry in the numerator and so, if a rises then both P^* and Q^* rise and if the numerators rise then obviously, these numbers will also rise.

So, both P^* and Q^* will go up as a rises and this can be verified by the diagram as well, as a rises the y intercept of the demand function rises it shifts to the right, let us see that. So, what was the demand functions equation, it was given by $Q = a - bP$ and so as we know a is the intercept, a is the intercept, if P is equal to 0, then Q is equal to a.

So, basically it means this is a, because along the horizontal axis, your P value is always 0. So, if I put P is equal to 0, then from this equation, you can see that Q is equal to a. So, rise of a means that this intercept is rising and it is going to maybe this particular value and that basically means that the entire demand curve is it is not exactly a curve, it is a straight line.

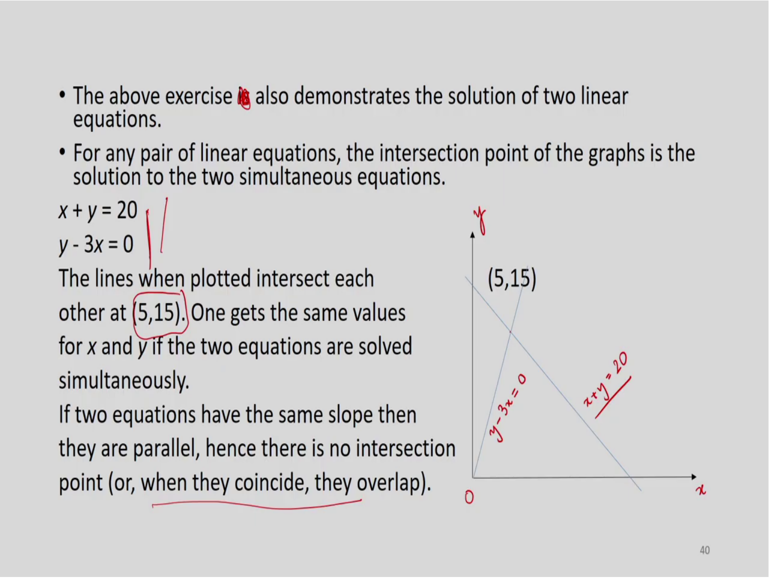
In this particular case, it is shifting to the right and if it is shifting to the right, then there will be a new point of intersection with the supply function and that point in this case, we are calling it E'. So, as a rises, the y intercept of the demand function rises, it shifts to the right. As a result, the equilibrium price and quantity rise.

A new point of intersection or the equilibrium point to the northeast of the old point of intersection, it lies on the same supply graph since the supply graph has not shifted. So, this is very clear, the supply function or the supply graph has this equation $Q = c + dP$. So, neither c nor d has changed, so the supply function remains the same, the supply graph remains the same.

The only thing that has changed is a has gone up, which means that the demand graph has shifted to the right and therefore there will be a new point of intersection, new point of equilibrium and here the new point is to the northeast of the old point, which basically means both equilibrium quantity and equilibrium price will rise.

Just small correction should be there, that this I have written as y intercept. This is not y intercept, automatically, a basically means the intercept on the, on the horizontal axis, like this a is the intercept on the horizontal axis. Now y intercept also rises because the curve is shifting to the right. But the interpretation of a is that in this particular case, it is the intercept on the x axis, that is that quantity axis. So, it will be correct to say that this is the Q intercept or intercept on the horizontal axis.

(Refer Slide Time: 15:24)



The above exercise also demonstrates the solution of two linear equations for a pair of linear equations, the intersection point of the graphs is the solution to the two simultaneous equations. So, instead of thinking of the demand function and the supply function, the model could be thought of as just mathematically two linear equations are given and suppose you have to find out what is the solution of the simultaneous linear equations.

And what we have found is that geometrically these two equations, the graphs of these two equations will intersect at some point and it is the intersection points coordinates, which are the solutions to these two simultaneous linear equations. So, here is an example. So, you have two linear equations $x + y = 20$ and $y - 3x = 0$. The lines, these two lines when plotted intersect each other at this point (5, 15).

One gets the same values for x and y if the two equations are solved simultaneously. So, for example, this downward sloping line as we know, if we have a downward sloping line, then what will be the corresponding equation of that line, this line is equal to $x + y = 20$, this is the equation of this line and this is the upward rising line it will be $y - 3x = 0$.

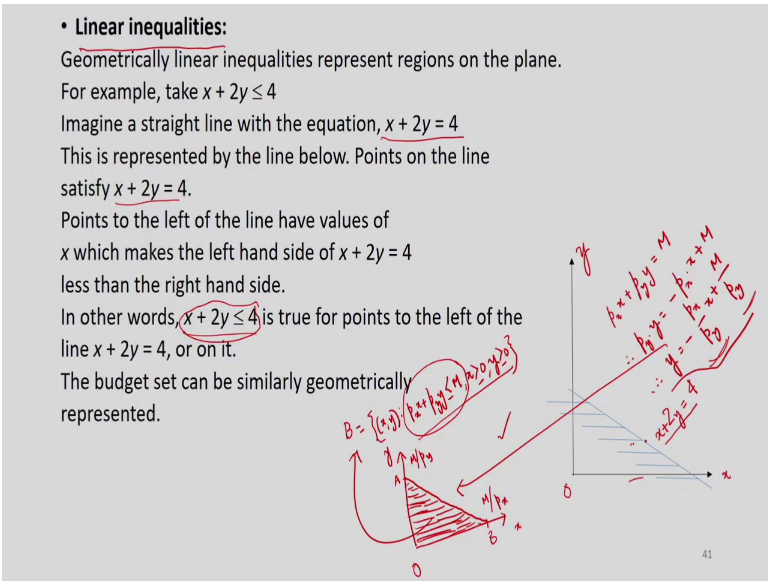
So, if you forget about the diagram and just solve these two equations from here, you will get a solution and that solution will be (5,15) and if you do not solve it mathematically, but just plot these two lines on a diagram, so this is the x axis and this is the y axis and if you plot them you get these two blue lines and they will intersect at some point and you will find that the coordinates of this intersection point is (5, 15).

If two equations have the same slope, then they are parallel. Hence, there is no intersection point. So, in this case these two equations did not have the same slope and they intersected. So, if you have two equations which do not have the same slope, then they will have to intersect at some point here in this case it is intersecting at the first quadrant, but it is not necessary that they will any two arbitrary equations linear equations, the graphs of them will intersect only in the first quadrant, but they will intersect at some point.

But, if the slopes are different, then these two equations will be parallel and two parallel lines do not have any intersection point, that is one possibility. The other possibility is that, it is possible that they are not parallel, but they coincide which means they are overlapping with each other, in that case there will be an infinite number of solutions.

So, these are the two extreme cases if you have two lines, which have the same slope either they have infinite number of solutions or they have no solution, these are the two extreme cases. But in general if there are two equations which have different slopes, then they will have a single point of intersection like this and that point of intersection gives us the solution.

(Refer Slide Time: 19:37)



Now, we from equations we come to inequalities in particular linear inequalities. So, if we have inequalities, we have talked about inequality before. If we have linear inequalities, then in terms of a graph, in terms of visual representation, then what does it mean? Geometrically linear inequalities represent regions on the plane. So, they are not straight lines, they are regions. For example, take $x + 2y \leq 4$.

So, this will represent a region, but which region? So, imagine a straight line with the equation, $x + 2y = 4$, so I am just removing the less than sign, I am just imagining that instead of less than equal to I am taking the equal to sign. So, if I do that, I get $x + 2y = 4$, this is represented by this line.

So, this is $x + 2y = 4$, points on the line will satisfy $x + 2y = 4$ that we have seen before, how to plot a straight line, if we are given a linear equation, we know how to plot that. So, points on the straight line will satisfy this particular equation, points to the left of the line have the values of x which makes the left hand side of $x + 2y = 4$, less than the right hand side.

Let us understand this. So, if you have a point on this line, then $x + 2y = 4$, which means you take the coordinate of this point and you substitute on the equation and then the left hand side that is $x + 2y$will be equal to the right hand side which is equal to 4. Now, if you take a point to the left of this line, a point like this, which means the value of x is slightly less.

So, in that case, the left hand side will be slightly less than the right hand side. In other words, $x + 2y \leq 4$ is true for points to the left of the line $x + 2y = 4$ or on it, because you know, you have this less than equal to sign. So, it allows for there to be an equality.

So, if you are picking up a point on the line, then this will be satisfied or if you are picking up points to the left of this line, again this will be satisfied because LHS will be less than RHS. So, what will not satisfy this inequality is if you take some point to the right of this line, then the LHS will not be less than equal to the RHS.

So, an example from economics is the budget set, can be similarly geometrically presented. Remember, I talked about budget set in a previous lecture. Now, I am saying now that that budget set can be geometrically represented like this. So, let me take you through what I mean by that. So, budget set, if I write it mathematically remember what it means.

So, this was the budget set that I talked about earlier, $B = \{(x, y): p_x.x + p_y.y \le M,\ x \ge 0,\ y \ge 0\}$. It is a set of combinations of x and y. So, any element in this set will have a pair of elements x and y and they will satisfy these conditions $p_x.x + p_y.y \le M,\ x \ge 0,\ y \ge 0$. So, this was the mathematical representation and here there are some parameters p_x, which is the price of x, p_y is the price of y and the money income which is M, all these are parameters given from outside.

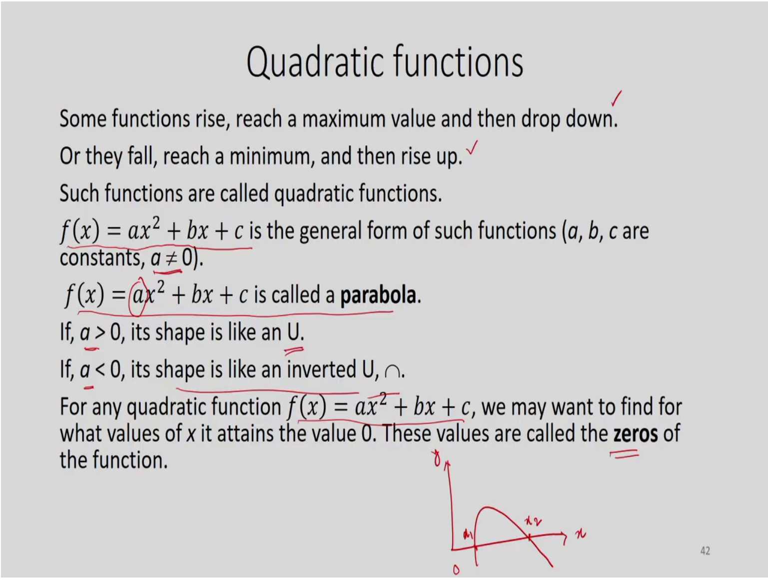
Let us see how we can represent this in terms of, in terms of geometry. Now, notice the $x \ge 0$ *and* $y \ge 0$ that means we are going to be in the first quadrant, only this quadrant. So, that has to be there in this other example, the first example that $x + 2y \le 4$, our x and y could have taken points from any of these quadrants, x and y could have been negative also, but here that is not allowed.

So, we have to be strictly in the first quadrant. Now, the main thing is this one. So, $p_x.x + p_y.y \le M$. Now, we adopt the same strategy that we adopted here, which is that we first convert this inequality into equality. So, $p_x.x + p_y.y = M$, let us take this equation and from this what do we get $p_y.y = -p_x.x + M$ *or* $y = -\frac{p_x}{p_y}.x + \frac{M}{p_y}$.

So, this is what we get and this is a straight line, it is a linear equation, it has a negative slope because you know the intercept, the coefficient of x is negative and there is a positive intercept on the y axis which is given by $\frac{M}{p_y}$. So, here is suppose $\frac{M}{p_y}$ and suppose here is $\frac{M}{p_x}$ and you join these two points and this line is nothing but the graph of this equation and you can verify the slope of this will be $-\frac{p_x}{p_y}$, the perpendicular divided by the base, so it will be $-\frac{p_x}{p_y}$.

Now, what about the region, our main purpose is to find out geometrically what region are we talking about. So, as we have seen before, the region will be to the left of this line, but not all points because we have to be in the first quadrant. So, therefore I actually have to take all the points from the first quadrant, but within the straight line. So, let us suppose this is point A and this is point B. So, this shaded region is my budget set.

(Refer Slide Time: 28:17)



From linear equations and linear functions, we now go to a little bit complicated functions and this is called the quadratic function. Some functions rise, reach a maximum and then drop down or it may happen these functions first decline, reach a minimum and then rise up. So, you can see that unlike linear functions which go only in one direction, these functions behave in a more complicated manner, such functions are called quadratic functions.

So, here is the general form of a quadratic function, $f(x) = ax^2 + bx + c$, where a , b, and c are constants, these are we have talked about them, we call them parameters and we have to make sure that this does not degenerate into a linear function.

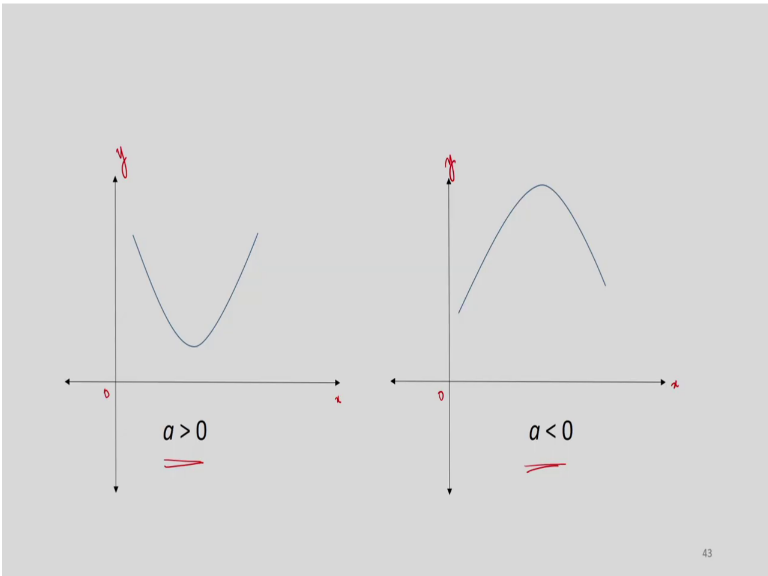
So, we have to have a to be non-zero, $a \neq 0$ because if $a = 0$, then you actually get a linear function. We do not want that, we want a proper quadratic function. Now, this function or the graph of the function is also called a parabola and we talked about two kinds of movement of this function, it can go down and then after reaching a minimum it can go up.

So, this will be like U shape, roughly U shape, but in that case you need the a to be positive. So, this a is crucial as far as the shape of the function is concerned and if $a < 0$, so, the second case, then, the shape will be like it rises first and reaches a maximum value and then comes down.

So, it will be like an inverted U, not exactly I mean, but you know roughly an inverted U and apart from this, it may be of interest to us that for any quadratic function which is of this form $f(x) = ax^2 + bx + c$, we may want to find for what values of x it attains the value 0, these values are called zeros of the function.

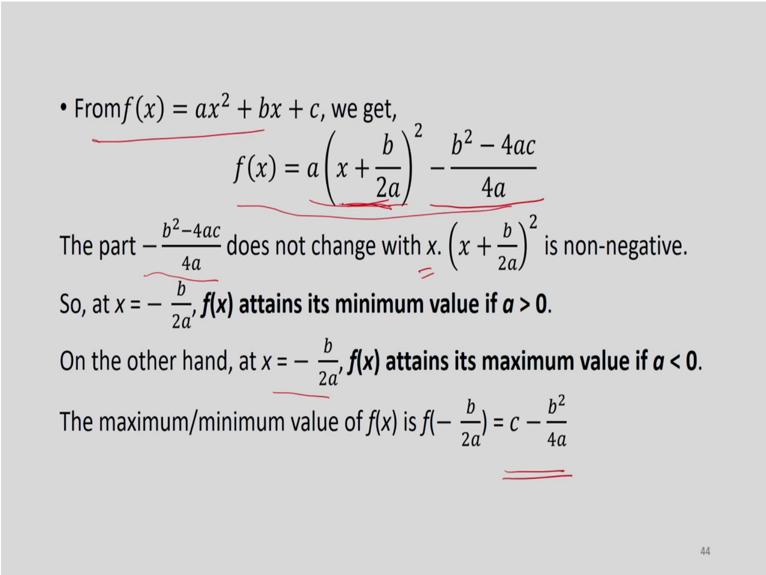
So, geometrically think about this. So, you have the x axis here, y axis here, here is the origin. So, suppose the function is like this. So, at two points this function is taking the value 0. The points of intersection with the x axis, suppose this is x_1 and this is x_2. So, in this particular example x_1 and x_2 will be called the zeros of this particular function.

(Refer Slide Time: 31:45)



Here are the shapes of two arbitrary quadratic functions, let me just write the access in the first case where you have roughly an U shaped quadratic function, you need the coefficient of x square to be positive. On the other hand if you have an inverted U shape like this, upturned U the a which is the coefficient of x squared in that equation has to be less than 0.

(Refer Slide Time: 32:28)



Let us see how this is coming out, why you need a to be positive and negative. Let us take the general form which is $f(x) = ax^2 + bx + c$ and we can write this in this form by taking whole square term, a is getting multiplied by a term which is a whole square term within that there is x and then you have this other part which is $\frac{-b^2-4ac}{4a}$.

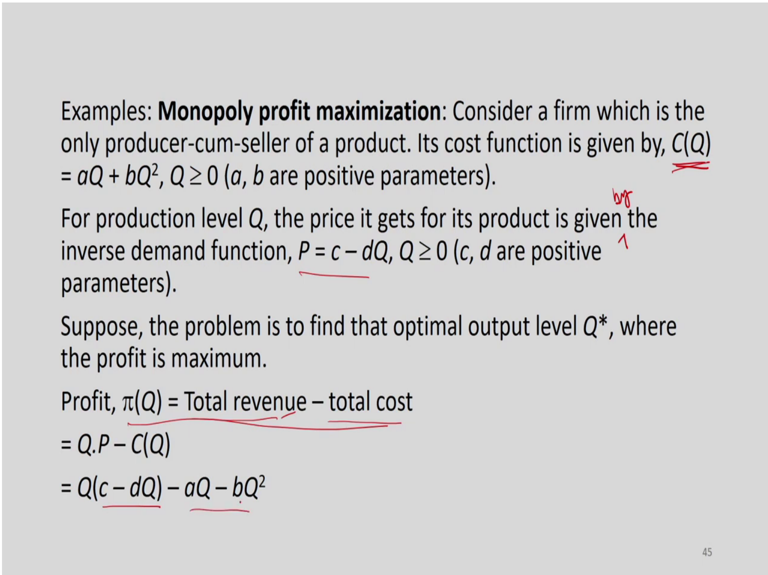
Now, the second part which is $\frac{-b^2-4ac}{4a}$, it is not containing this term x. So, it is independent of x. So, as x changes this part does not change, it is constant. On the other hand, if you look at this whole square term, since it is a square term then it is going to be a non-negative element.

So, from these two observations we can make the following conclusion that suppose $x = \frac{-b}{2a}$, in that case this part becomes 0. The first part becomes 0. In that case, the $f(x)$ attains its minimum value if $a > 0$. Why am I saying that because if a is positive, then this enter first term is positive.

So, this enter first term becomes 0, that is the least value of this first term, if $x = \frac{-b}{2a}$, that is why I have said that $f(x)$ attains its minimum if $a > 0$. On the other hand, if you have $x = \frac{-b}{2a}$ and a to be negative, then the centre term is negative, but if $x = \frac{-b}{2a}$, then the first part vanishes, so $f(x)$ becomes less negative and it becomes it attains its maximum value therefore, if $x = \frac{-b}{2a}$ and $a < 0$.

So this is the reason why we are getting this result that whether you have a maximum value of a quadratic function or a minimum value of a quadratic function, it depends on the value of a, now the maximum and minimum value of $f(x)$, what are they? We just have to put in $x = \frac{-b}{2a}$and if I do that I get that particular maximum or minimum value to be $c - \frac{b^2}{4a}$.

(Refer Slide Time: 35:26)



This is an example where we use the quadratic functions and try to see where the maximum is occurring or the minimum is occurring. So, this is called the monopoly profit maximization, consider a firm which is the only producer cum seller of a product, its cost function is given by $C(Q)$, we have talked about cost function before, but here the form is given, it is equal to $C(Q) = aQ + bQ^2 \ if \ Q \geq 0$.

Here a and b are positive parameters. Now, for production level Q, the price it gets for its product is given by, given by the inverse demand function capital $P = c - dQ$ where $Q \geq 0$, c and d are positive parameters. So, these two things are given to us, one is the cost function, which is $C(Q)$.

Cost function tells me if the firm produces different levels of output, positive or non-negative output, then what are the costs that it has to bear. On the other hand cost is not the only thing that the firm should be concerned about, it should be also concerned about the price that it is getting by selling its product and the price that it gets is given by this inverse demand function this is called the inverse demand function, where P is a function of Q.

In the demand function, Q is a function of P. But in the inverse demand function, you take the inverse of that, I am going to talk about what is an inverse function later on, but it is just the opposite. You take P to be a function of Q, P is equal to, in this case, a linear function $P = c - dQ$ where c and d are positive.

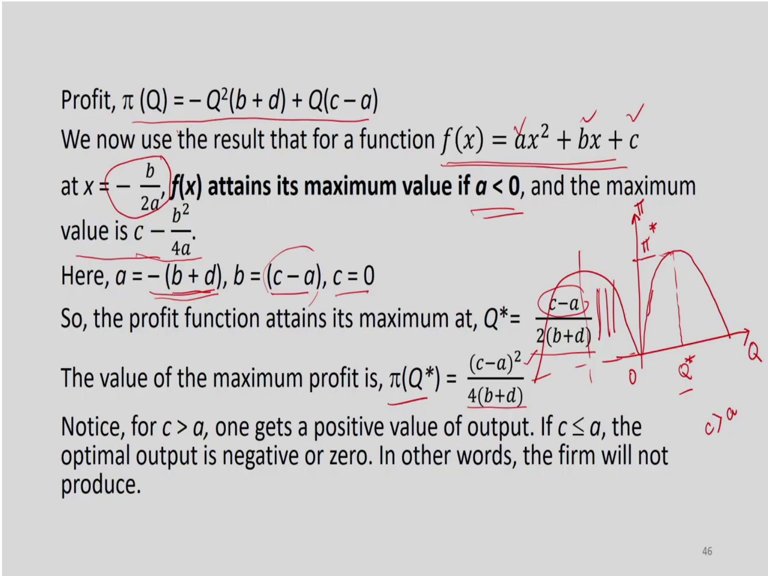
So, here you see in this problem, you have the combination of both a linear function and a quadratic function. The cost function is quadratic and the inverse demand function is linear. Now, what is the problem, the problem is to find the optimal output level Q^* where the profit is maximum.

So, this is again going to be a kind of constant assumption with us, that we are going to assume that firms mainly try to maximize their profit. Now, how do I do that, so first let us try to write the profit or the profit function. So, profit also is a function and it is going to be a function of the quantity, as we shall see. That is why I have written it as Π, $\Pi(Q)$.

Now, what is profit after all, profit is what money you are getting by selling your product minus the cost that you have to bear to produce that product. So, that is the first element, the money that you are getting is called the total revenue and the cost that you bear is called the total cost and so, this boils down to total revenue is $Q.P - C(Q)$.

In the next stage, I am just writing the inverse demand function. So, P I know is given by $c - dQ$, I am using that and I am writing the cost function, which is $C(Q) = aQ + bQ^2$.

(Refer Slide Time: 39:19)



And then, simplifying this a bit by taking the square terms in the first part and then I am taking the debt part where you have Q multiplied with some constants. We can now use the result for a function of this form which is the general form $f(x) = ax^2 + bx + c$, at $x = \frac{-b}{2a}$ we have found that $f(x)$ attains its maximum value if $a < 0$)and the maximum value is given by this particular things $c - \frac{b^2}{4a}$, that we have just found.

So, here, we have to compare how we can interpret the a and b and c here. This a, b and c, which are the a and b and c. Here $a = \; - (b + d)$ and $b = (c - a)$ and there is no c term, $c = 0$, there is no constant term. So, this is a case where a is negative, because b and d are positive, so $-(b + d)$ is negative. So, this function will attain its maximum value that we are sure.

Now, at what value of Q will it attain its maximum value that is given by this particular expression, $= \frac{-b}{2a}$. So, minus b, what is b? $b = (c - a)$ writing that divided by 2a, so a itself is negative. So, minus and minus sign will get cancelled. So, you are going to get $Q^* = \frac{(c-a)}{2(b+d)}$. So, this is the particular value of Q, which we are calling as Q^*, where the profit will be maximized and what is the value of that maximum profit?

The maximum profits value is given by this thing $c - \frac{b^2}{4a}$. Now, there is no c here, so that part is 0, what is $b^2 = (c - a)^2$ and divided by 4a, a itself is negative. So, again negative-negative terms will cancel you have $4(b + d)$. So, this is the solution, $\Pi(Q^*) = \frac{(c-a)^2}{4(b+d)}$ that we have found for this particular problem of profit maximisation, we did not use any calculus or anything, we just used the basic result that for any general form of this kind, there is a maximum or if a is negative and what is the maximum value.

Notice, if $c > a$, one gets a positive value of output. So, if this can be valid, if you have $c \geq a$, only in that case Q^* is positive. If $c \leq a$ then you are in trouble, because then Q^* will be negative or 0 and if you have Q star negative then that is something which is not feasible, you cannot produce negative amount of output which basically means, in that case, the output will be 0.

Let us look at the diagram how it looks like. So, you have Q here and you have profit here and look at the profit function, if there is no constant term, so it starts from 0 and it goes up like this and it comes down and you have this maximum here, which is Q^* and what is that value of profit at the maximum, let us call that Π^*.

So, this is this Π^* and this is that Q^*, this is the case where $c > a$, that is necessary for to have this diagram, if $c < a$, then actually the diagram will look like this. So, you have output which is negative and that is not meaningful in this particular context.

(Refer Slide Time: 44:01)

- If $d = 0$ in the above example, $P = c$
- Substituting it in the optimal output and profit expressions we get,

$Q^c = \frac{P-a}{2b}$, $\pi(Q^c) = \frac{(P-a)^2}{4b}$

These are valid if $P > a$. Otherwise, the optimal output, profit = 0.

- A market where the price is a constant, it is not dependent on the output produced by a firm, is called a **perfectly competitive market**.
- The firm is no longer the sole producer. Its output does not affect the price because its own production level is insignificant compared to the production carried out by all firms.

47

Examples: **Monopoly profit maximization**: Consider a firm which is the only producer-cum-seller of a product. Its cost function is given by, $C(Q) = aQ + bQ^2$, $Q \geq 0$ (a, b are positive parameters).

For production level Q, the price it gets for its product is given the inverse demand function, $P = c - dQ$, $Q \geq 0$ (c, d are positive parameters).

Suppose, the problem is to find that optimal output level Q^*, where the profit is maximum.

Profit, $\pi(Q)$ = Total revenue – total cost

$= Q.P - C(Q)$

$= Q(c - dQ) - aQ - bQ^2$

45

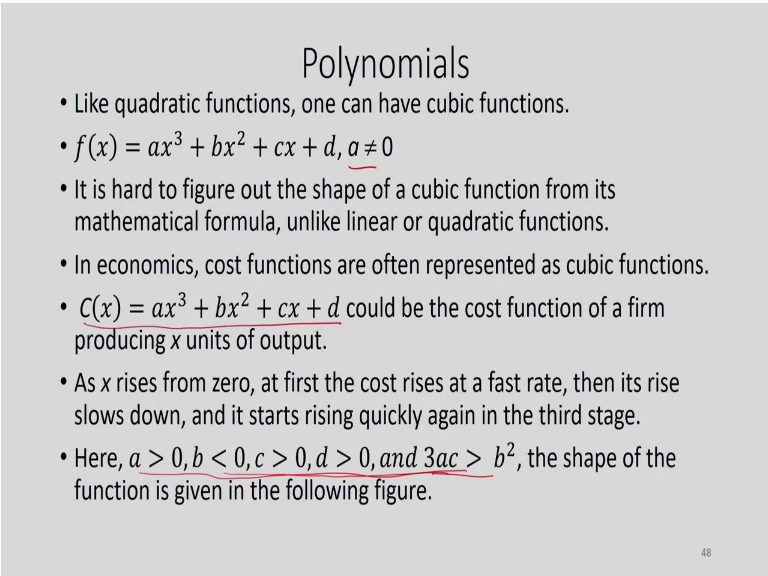
Now, we talk about another particular important result here, suppose d is 0 in the above example. So, what was d? Let us go back and see where d came from. d came from the inverse demand function. So, if $d = 0$,, then you get $P = c$, and c is a constant amount that means, that price becomes a constant thing, if $d = 0$.

Substituting this in the optimal output and profit expressions we get these two. So, instead of Q^* and $\Pi(Q^*)$ I am writing Q^c and $\Pi(Q^c)$. So, this is the special case where $d = 0$ and if we do that, just put $d = 0$ and you get these two expressions and obviously, these will be, these will be meaningful expressions if $P > a$, otherwise the optimal output and profit will be 0.

Now, what was the reason of taking this particular case of $d = 0$, a market where the price is constant, it is not dependent on the output produced by a firm, is called a perfectly competitive market, the firm is no longer the sole producer. So, that is the difference that is has to be pointed out, this is the case where you have multiple firms, actually we talk about in finite number of firms and all these firms are producing similar product and since there are close to a finite number of firms, any individual firms ability to influence the market price is nil.

So, it can produce more output, it can produce less output, but that is not going to affect the market price, market price is given at c. So, this kind of market is called a perfectly competitive market, its output does not affect the price because its own production level is insignificant, compared to the production carried out by all firms. So, that is why I use the notation Q^c . c stands for competition.

(Refer Slide Time: 46:30)



We talked about two quadratic functions. Now, we are going to talk about another kind of functions and I start from something called polynomials like quadratic functions, one can have cubic functions, first talk about cubic function and then we are going to proper polynomials. Here $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$, just to make sure that it is a cubic function, therefore a should be nonzero.

It is hard to figure out the shape of a cubic function from its mathematical formula, unlike linear or quadratic functions. So remember, in case of linear function, I just had to look at the slope of the function and from that, I could figure out whether the function is going up or going down by looking at the first term which is a and in case of quadratic functions depending on the coefficient of x square, I could find out whether you have a U shaped function or an inverted U shaped function.

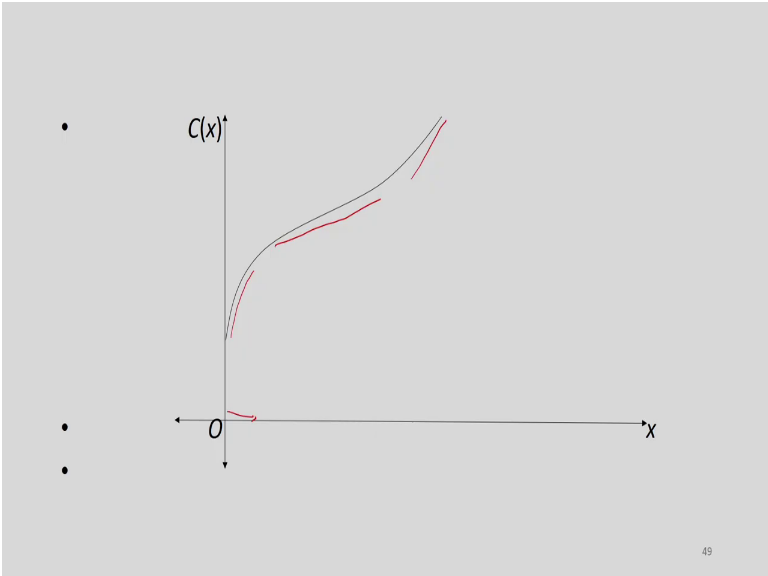
But in case of cubic function, the things are not so simple. Do we use cubic functions in economics, in some cases, we do, cost functions are often represented as cubic functions. So here you have a cubic function $C(x) = ax^3 + bx^2 + cx + d$, where a, b, c, d are all positive parameters, it could be the cost function of a firm producing x units of output.

As x rises from 0, at first, cost function rises at a fast rate, then its rise slows down and it starts rising quickly again, in the third stage. So, it is conceivable that cost function of a firm behaves like that. Firstly, it rises very fast, you do not have a lot of output to produce and there at that point of time, there might be some inefficiencies and because you are not using the plans properly.

So at that point, costs might be rising at a very fast rate, after a point of time, you might be reaching your optimal level of output. So, there the cost is rising, but not at a very fast rate and finally, there might be shortages after the point of time, there the cost will be rising at a very fast rate.

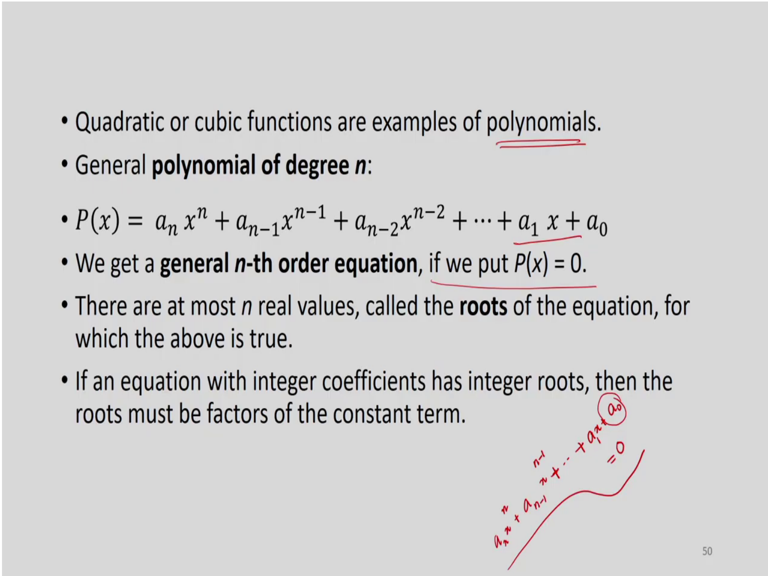
That can be represented by this, that kind of behaviour can be represented by this function. If we have this sort of pattern of coefficients, $a > 0,\ b < 0,\ c > 0,\ d > 0$ and you also need $3ac > b^2$. The shape of the function is given by the following figure.

(Refer Slide Time: 49:42)



So here on the x axis, you have the output levels of the firm and along the y axis, you have the cost and you can see that the function actually behaves the way I just described, initially the output as it rises, the cost is rising at a very steep rate, but here as the output rise, the cost is rising, but at a now slower rate and finally, it all again starts to rise at a very fast rate. So, this can be represented by a cubic function.

(Refer Slide Time: 50:25)



Now, quadratic or cubic functions are examples of what are known as polynomials. General polynomial of degree n is given by this, $P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_1 x + a_0$.

So, you can see that the subscript of a is always matching with the power of x. So, in fact the last term you can say that this is a 0 multiplied by x to the power 0 and we know x to the power 0 is equal to 1. We get a general nth order equation if we put $P(x) = 0$. So, $P(x)$ if we put, $P(x)$, what do you get? $a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_1 x + a_0 = 0$. So, this is called a general nth order equation.

Now, there are at most n real values called the roots of the equation for which the above is true. So, from this equation you can see this, this is an equation of nth degree or order. In this equation there will be at most n real values or the roots of this equation. So, for at most n real values of x, the left hand side will be equal to the right hand side.

If an equation with integer coefficients has integer roots, then the roots must be factors of the constant term. So, this is a very important result that suppose you have an equation like this and all the coefficients are integers and suppose you know that the roots are also integers, then those roots must be the factors of this constant term, this a_0. This is an interesting result and we are going to look at this result more carefully in the next lecture. Thank you.