Mathematics for Economics 1
Functions of one variable, graphs of function
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Module: Functions
Lecture 6: Linear Functions

Welcome to everyone. So, we are going to start lecture 6 of this course called Mathematics for Economics part 1. The topic that we have been covering is called Functions and its Functions of One Variable and we have covered a substantial part of this particular topic. So, today we are going to talk about something else about functions. So, what did we cover in the last couple of lectures. So, we defined what is a function.

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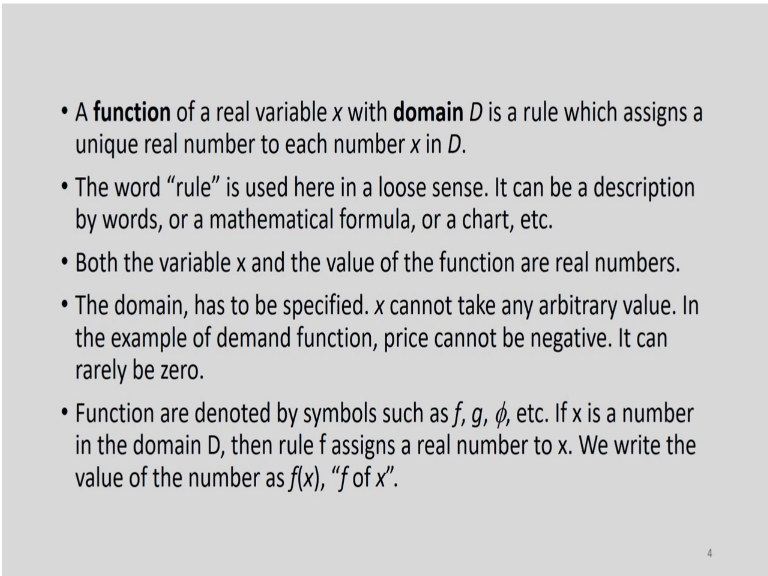

So, this is the definition function of real variable x with domain D is a rule which assigns a unique real number to each number x in D, a function can be specified as a mathematical formula, it can also be specified as a verbal statement and it can also be mentioned in the form of a chart.

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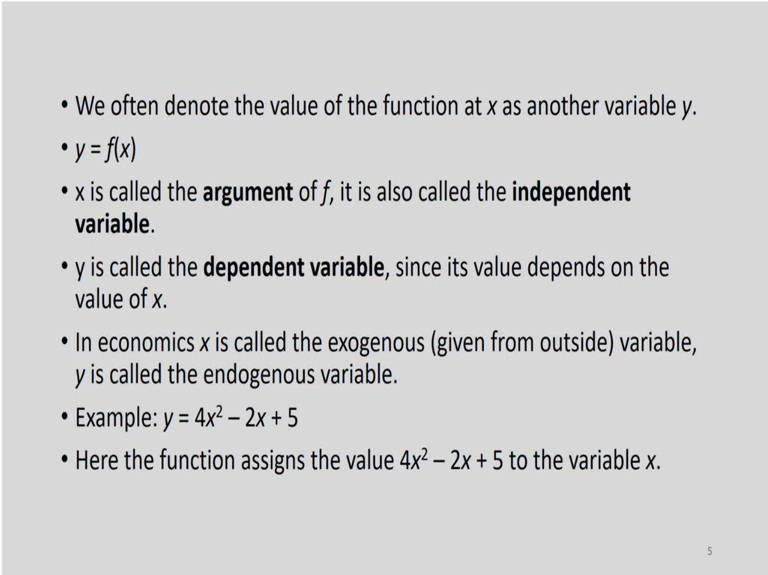

So, the variable which is the x variable is called the independent variable or the argument and the y which is effects is called the dependent variable.

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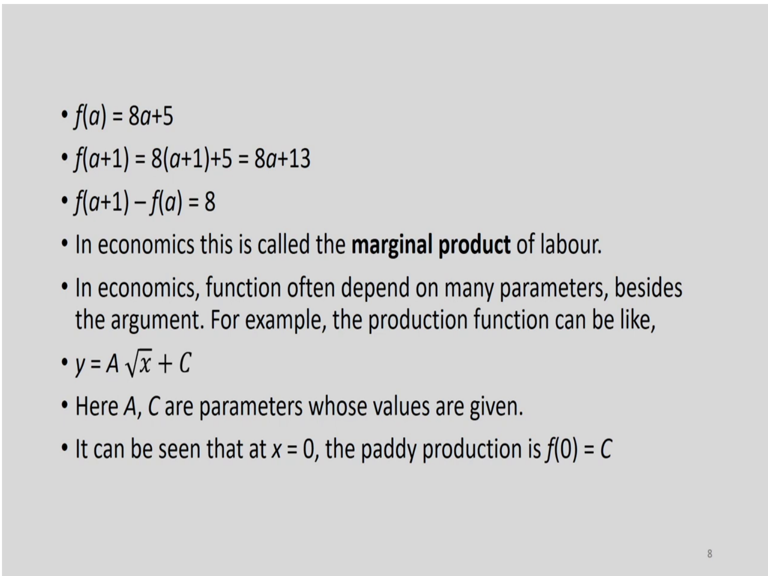

So, here are some examples of functions, we talked about production function which is an application of function in economics, related to the idea of production function there is something called the marginal product of labour and I gave some examples.

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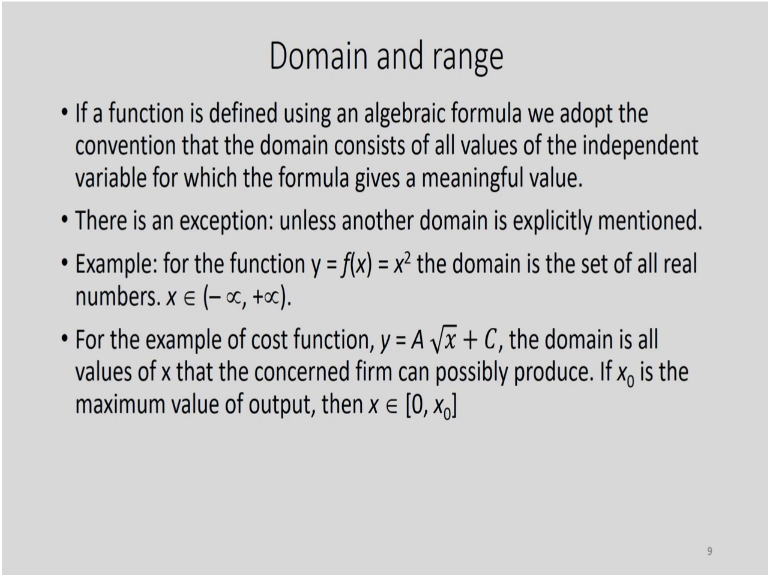

Then we defined what is a domain and what is a range? A domain is the set from which the x that is the argument is taking its value, the range is the set to which this maps to that is the set of values of the function.

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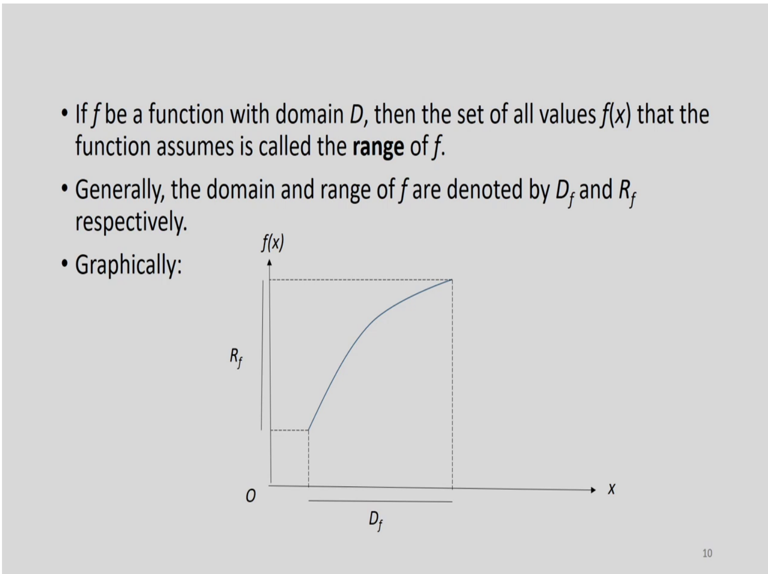

So, this is an example of D, D is the domain, R f is the range.

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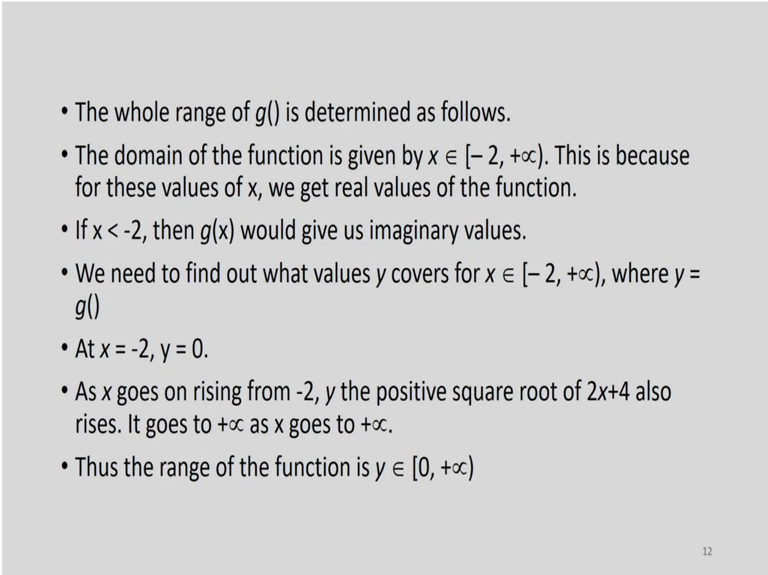

And we talked about different examples again, domain can go from 0 to infinity and so can the range.

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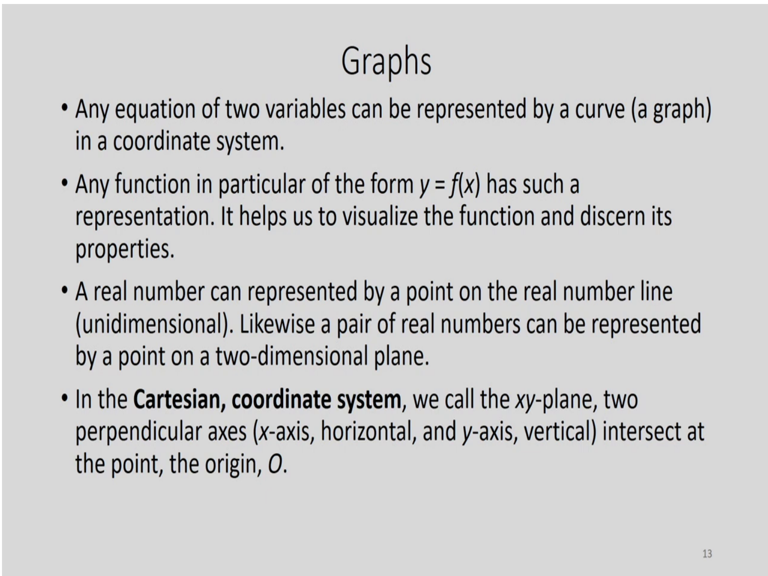

Then we talked about what is a graph of a function, a graphs are visual representations. So, you have to have a coordinate system, which is called a Cartesian coordinate system.

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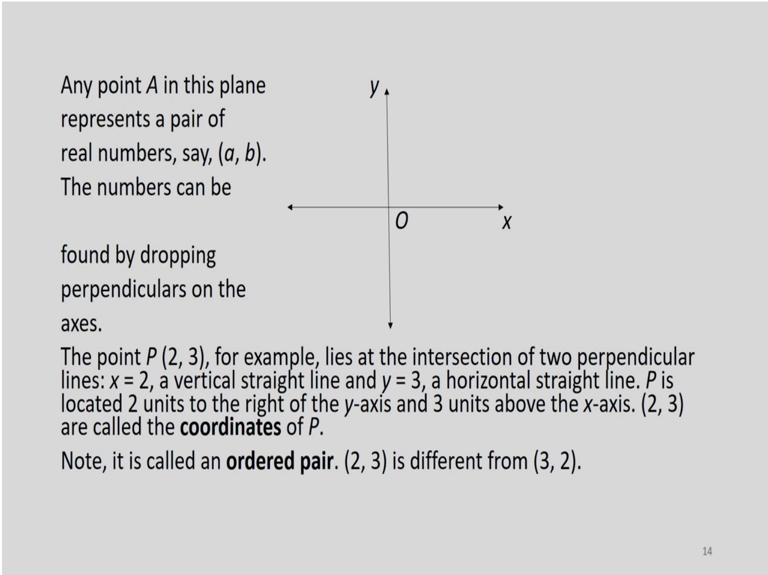

Where you have the x axis and the y axis, they are intersecting vertically; they are intersecting perpendicularly on each other at the point O that is the origin.

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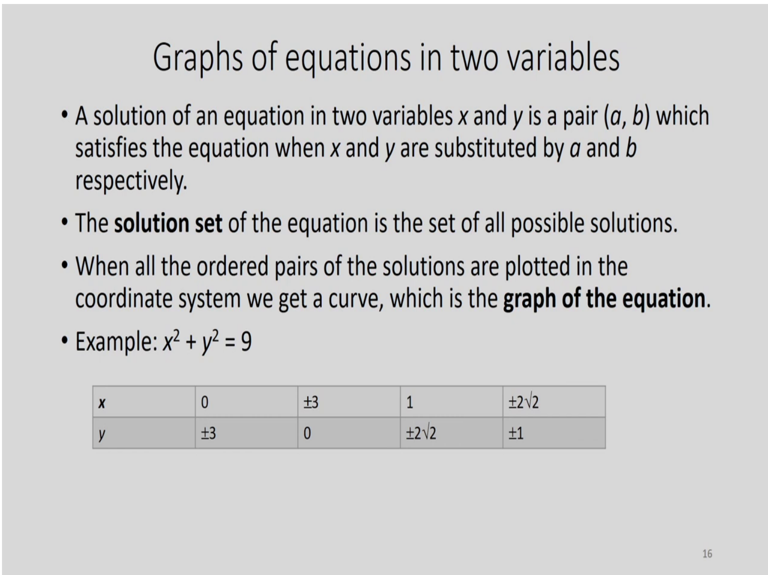

x	0	± 3	1	$\pm 2\sqrt{2}$
y	± 3	0	$\pm 2\sqrt{2}$	± 1

And every point on this coordinate system represents a pair of numbers.

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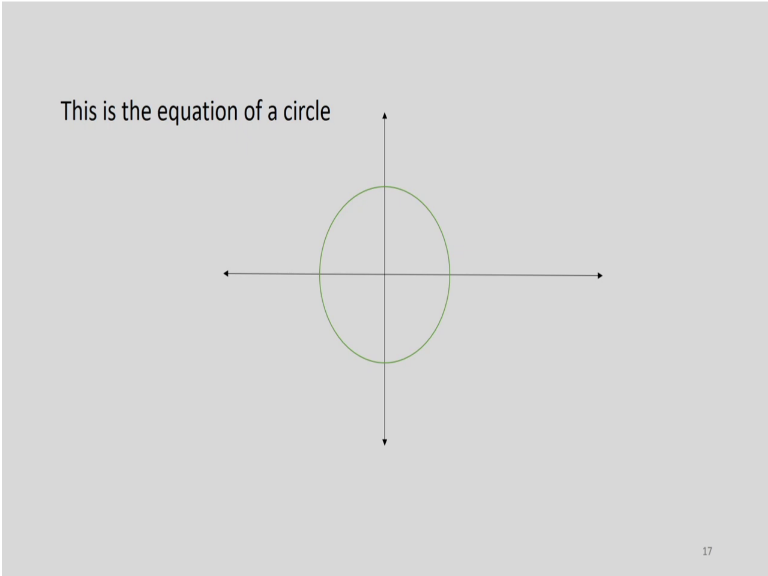

Now, we can have an equation in two variables which can be plotted and this is a graph of that equation called $x^2 + y^2 = 9$.

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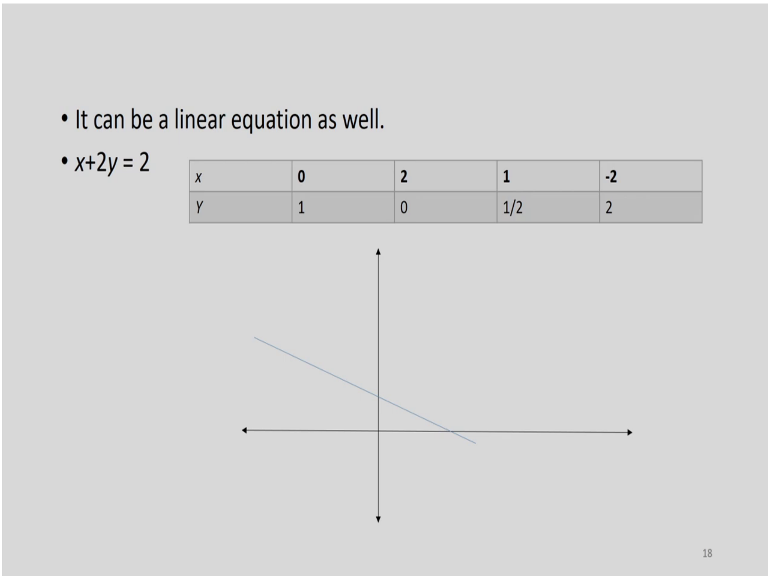

And it could be a linear equation also, $x + 2y = 2$ It is a linear equation and the corresponding graph will be a straight line.

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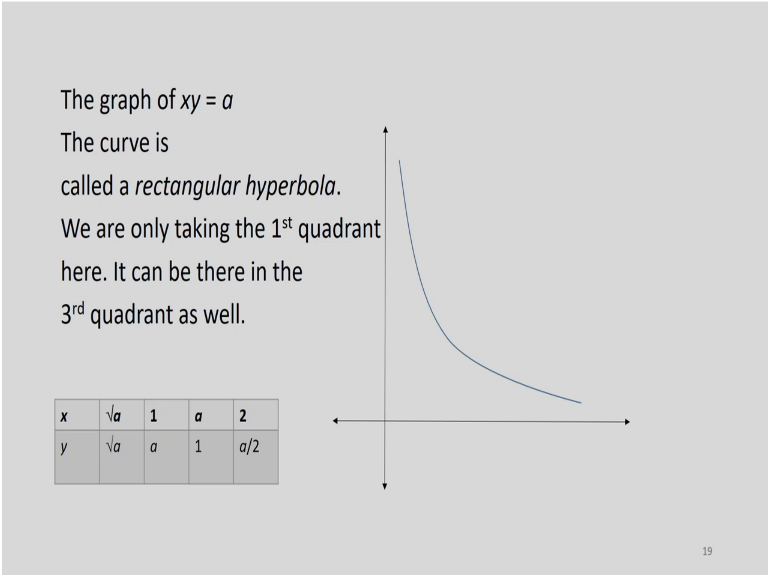

And here is another example $xy = a$, a is a positive number and here the graph is a rectangular hyperbola.

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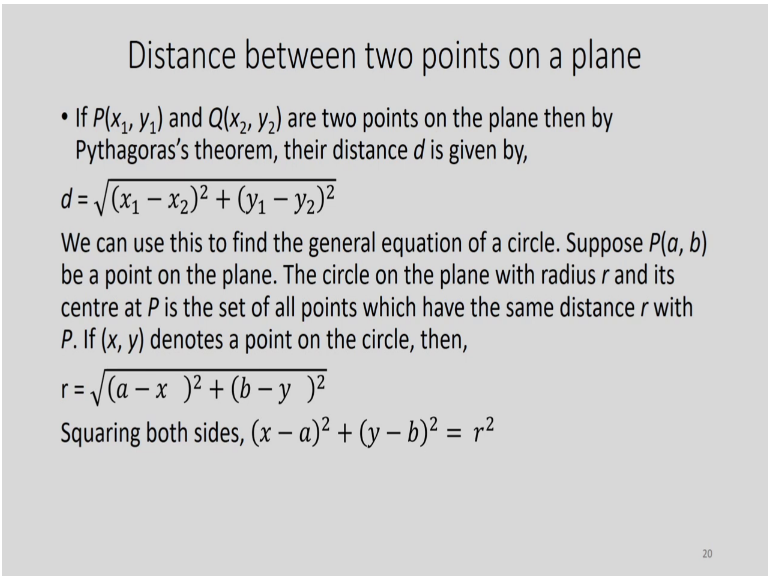

And then, we talked about how we can represent the distance between two points on a plane if these two points are P and Q with the coordinates (x_1, y_1) and (x_2, y_2), then the distance $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ and therefore, from this we get the equation of a circle and this is given by r square, r is the radius of this circle, $r = \sqrt{(a - x)^2 + (b - y)^2}$ and (a, b) is the centre of the circle.

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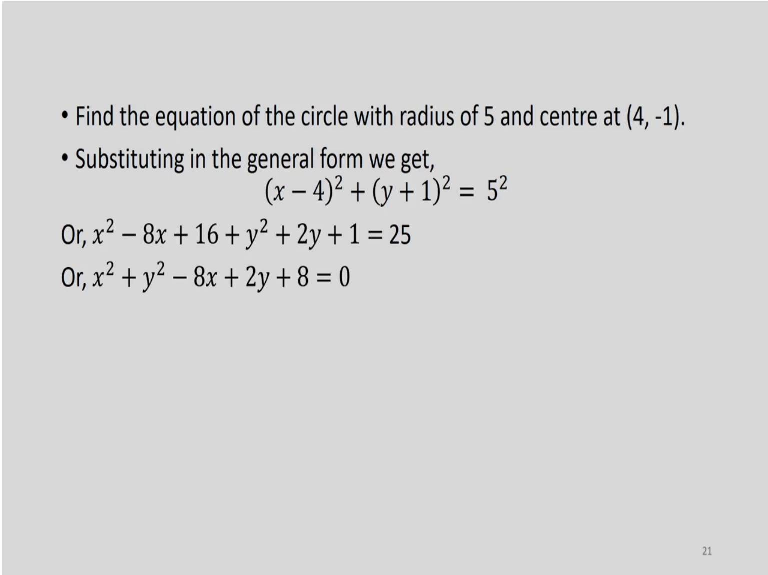

So, there we left in the last lecture. So, here is an example of how to apply this formula of equation of a circle. Suppose the question is this, find the equation of the circle with radius 5 and centre at (4,-1). The radius is given which is 5 and the centre, coordinates are given (4,-1). So, we have to find the equation of this circle, what do we do we just substitute these values here x - a, a = 4 and y - b and b = - 1.

So, you have $(x-4)^2 + (y+1)^2 = 5^2$, we simplify this and you get $x^2 + y^2 - 8x + 2y + 8 = 0$.

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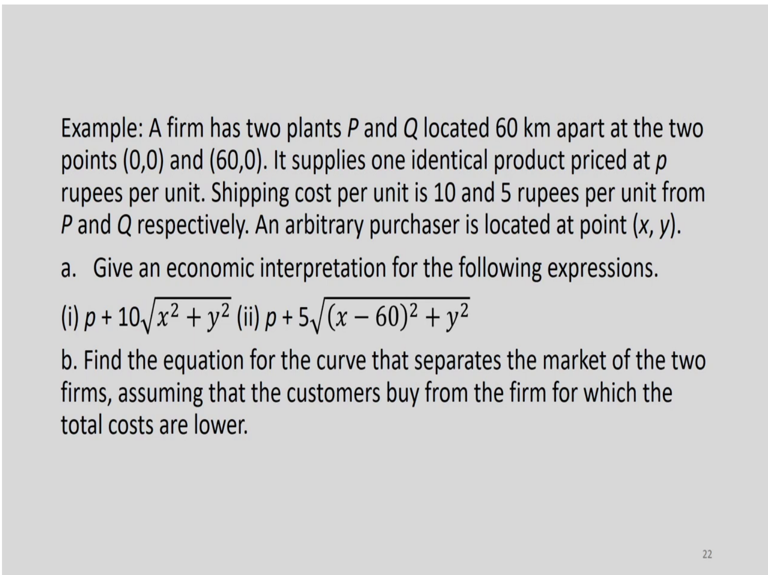

Here is another example, which uses this idea of distance between two points. A firm has two plants P and Q are located 60 kilometres apart at the two points (0, 0) and (60, 0), it supplies one identical product priced at p rupees per unit, shipping cost per unit is 10 and 5 rupees per unit from P and Q respectively, an arbitrary purchaser is located at point (x, y). Give an economic interpretation for the following expression. $(i)\ p + 10\sqrt{x^2 + y^2}$ and $(ii)\ p + 5\sqrt{(x - 60)^2 + y^2}$. Let us first try to answer this first one.

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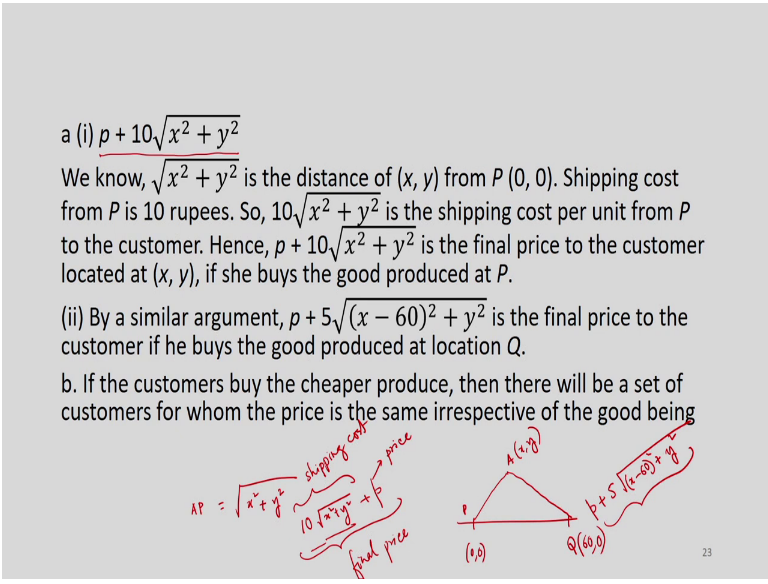

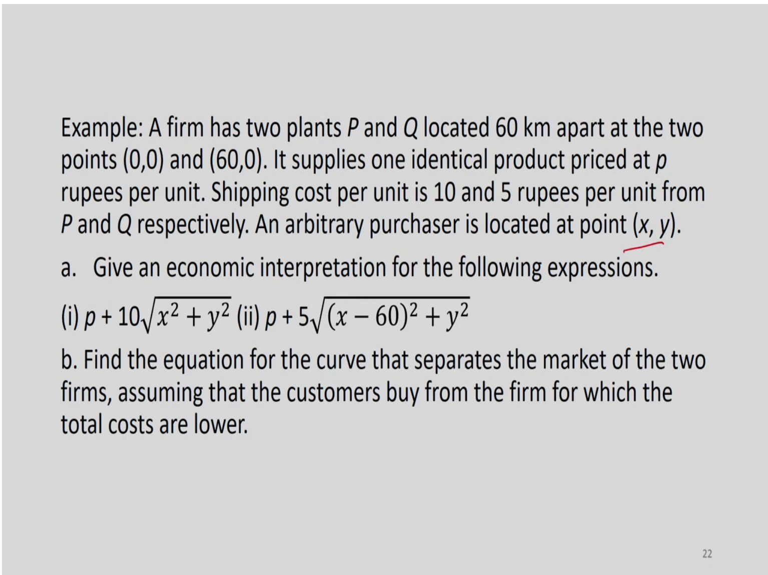

So, what is the economic interpretation of this expression? Let us visualize this. So, there are two points one is P(0,0) and other is Q (60,0). So, they are 60 kilometres apart, now there is this one customer, a purchaser whose coordinate is given by (x, y). So, suppose (x, y) is a point here, point A let us suppose.

Now, then what is the economic interpretation of this expression $p + 10\sqrt{x^2 + y^2}$. First note that this expression $x^2 + y^2$ this is the distance between P and point A. So, this is nothing but AP this distance and this is being multiplied with 10. So, 10 multiplied by $x^2 + y^2$.

What is the meaning of 10? 10 is the shipping cost, shipping cost from P to A, remember P is a production facility. So, from there the good is going to be shipped to point A and the cost of shipping per unit, let us say per kilometre is 10, total distance is this much. So, 10 multiplied by this is the shipping cost and then what is being added to this is this p, small p. What is small p? Small p is the price, price, which is the price you have to pay.

Suppose you are located at P itself, then this is the price you have to pay. But if you are located at point A obviously, with the price the shipping cost has to be added. So, this is the final price. This is the final price that any customer will have to pay for an unit of that good if he is located at A and if he is buying the good which is produced at P. So, this is the final price. So, that is the economic interpretation.

Similarly, what is the shipping cost from Q to A and this we shall see is given by $5.\ (x - 60)^2 + y^2$, because AQ, this distance is $\sqrt{(x - 60)^2 + y^2}$ and with that, you are multiplying 5, that is 5 is the cost per kilometre. Therefore, 5 multiplied by this total distance is the cost of shipping and with that you are adding small p, this is the price at Q.

So, the entire expression $p + 5\sqrt{(x - 60)^2 + y^2}$ is the final price that this guy has to pay if he decides to buy the good produced at Q. So, we have answered these two questions. Find the equation for the curve that separates the market of the two firms assuming that the customers buy from the firm for which the total costs are lower.

So, we have to find the equation for the curve which separates the market of two firms. So, one part of the market will buy from P and the other part of the market will buy from Q. So, we have to find that curve which separates these two markets and assumption is that any customer will buy the good from that firm, from that location where the price is lower. Now, then, there will be a set of customers for whom the price is the same irrespective of the goods being produced.

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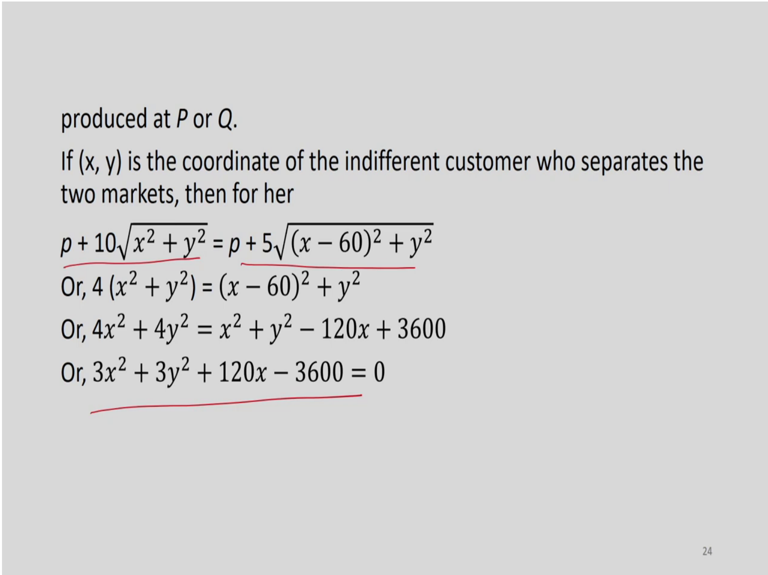

At P or at Q. So, there will be some customers for whom the price will be the same, they will be indifferent whether the good is produced at P or the good is produced at Q. Let x y be the coordinate of the indifferent customer who separates the two markets, then for her this $p + 10\sqrt{x^2 + y^2}$, this is remember the price he had to pay if he buys from P and on the right hand side is the price if he buys from Q.

So, for him these two prices are same. So, therefore he is indifferent that is the thing we have written here, that for him the two prices are the same and if we simplify this we get this expression, $3x^2 + 3y^2 + 120x - 3600 = 0$.

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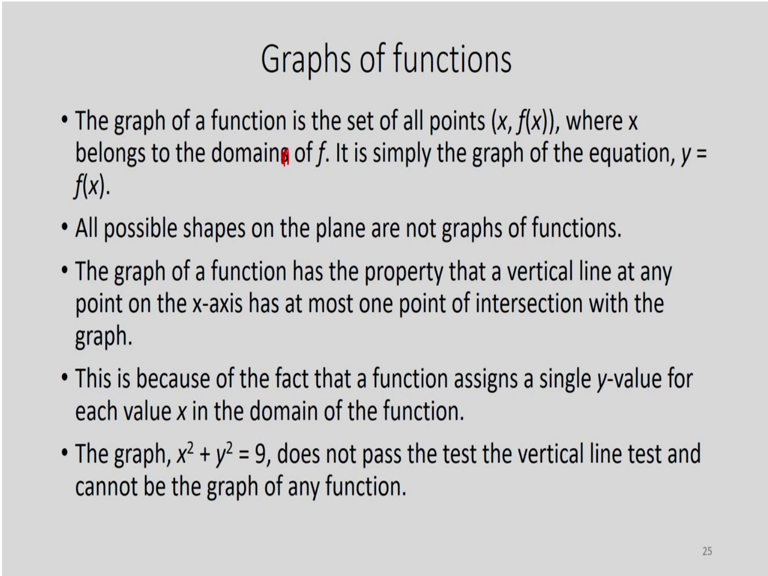

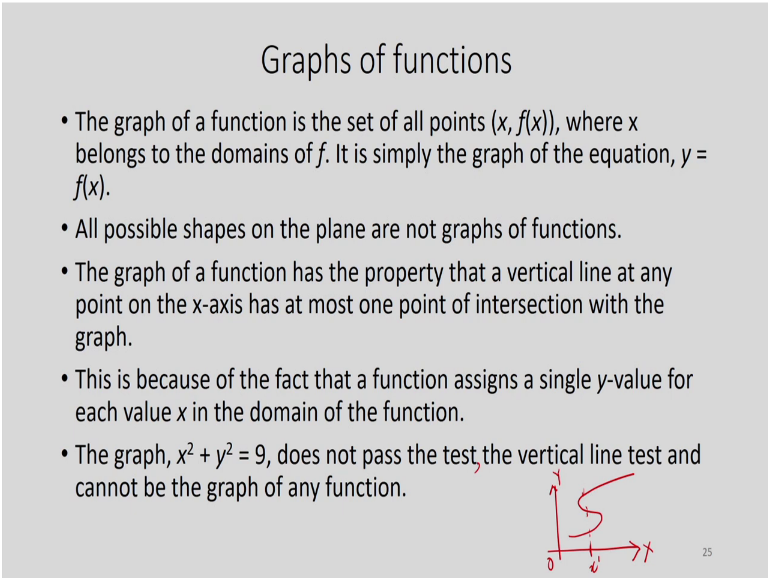

So, we have talked about graphs of equations. Now, we come to graphs of functions. As we mentioned in the last lecture functions and equations are not exactly the same, some equations could be or let us say some graphs of functions could be graphs of equations also. But, it is not true that all graphs of equations will be graphs of some function.

The graph of a function is the set of all points $(x,\ f(x)\ )$ where x belongs to the domain of x, this should be domain. It is simply the graph of the equation $y\ =\ f(x)$. So, in a coordinate system you have x on the horizontal axis and y on the vertical axis. So, suppose $y\ =\ f(x)$ and then we plot the graph of $(x,\ f(x)\ )$. So, that graph will be the graph of that function.

All possible shapes on the plane are not graphs of functions. The graph of a function has the property that a vertical line at any point on the x axis has at most one point of intersection with the graph. So, what I am saying is that suppose you take any shape on the coordinate plane. Suppose this is a particular shape you took.

Now, question is can this curve be the graph of a function and the answer in this particular case is no, this cannot be the graph of a function, because the function as we know has to satisfy a property that for any value of x, if it belongs to the domain, then the corresponding value of the function must be at most 1, it cannot be more than 1.

But here if you see if I take x here, this suppose this is the x I am taking x dash, then it has three values, we can read that off from the curve. So, with respect to one particular value of x, if you have three values of the function, so called function, then it is not a function actually. So, this cannot be the graph of a function.

The graph of a function has the property that a vertical line at any point on the x axis has at most one point of intersection with the graph. So, here, if this x dash belongs to the domain, then this cannot be the graph of a function, because I can see there are three points of intersection.

This is because of the fact that the function assigns a single y value for each value of x in the domain of the function, the graph $x^2 + y^2 = 9$ does not pass the test, the vertical line test and cannot be the graph of any function. So, what was $x^2 + y^2 = 9$, just remember how this shape looked like.

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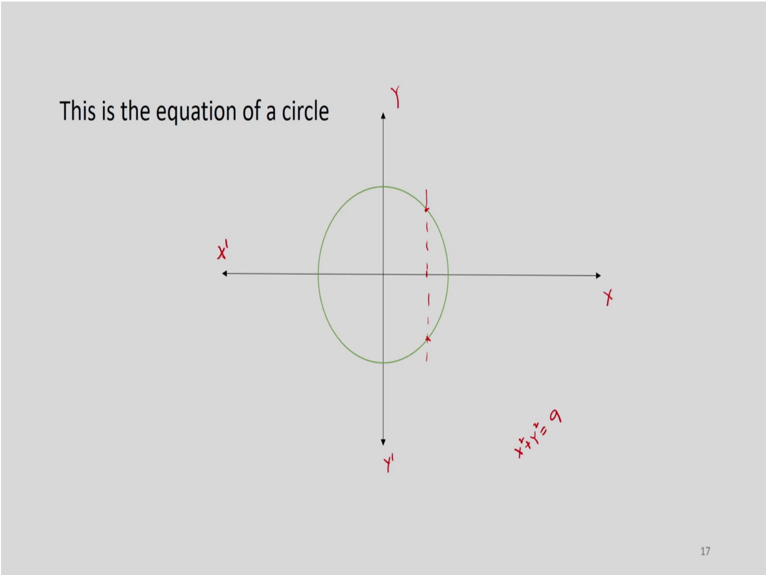

This is $x^2 + y^2 = 9$. Now, this does not pass that test that I am talking about, if you take any particular point on the x axis, let us suppose this point and draw any vertical line, then you have two points of intersection with the curve, therefore it is more than one and therefore, this cannot be the graph of any function.

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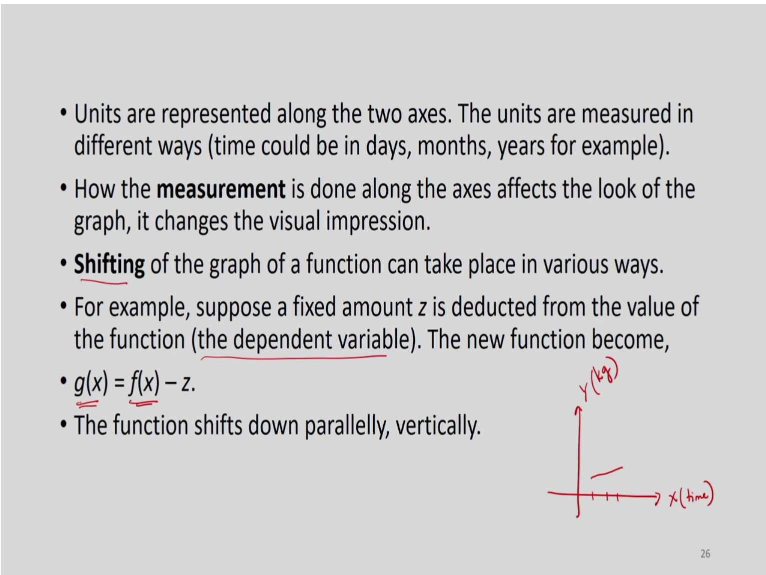

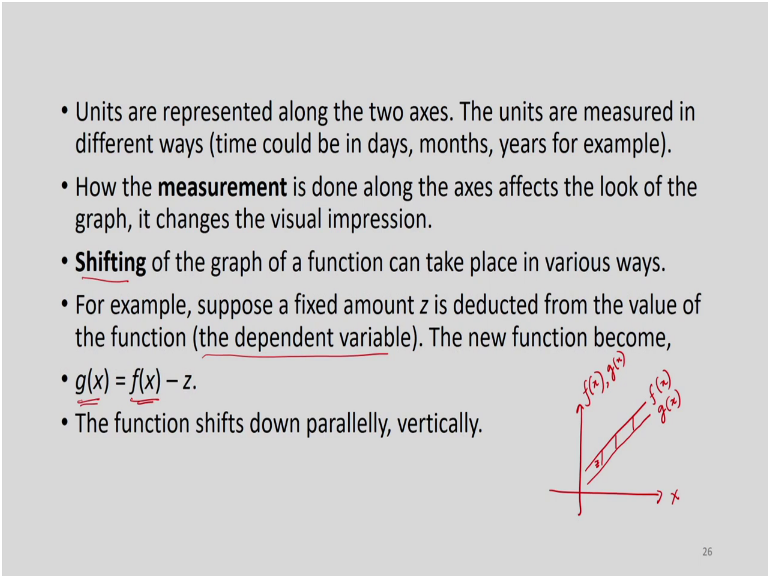

Next, we come to measurement units are represented along the two axes, if you think about the Cartesian coordinate system, you have x measured along this axis and y measured along the vertical axis. Now, when something is measured, obviously there are some units along these two axis. The units are measured in different ways for example, time could be in days, it could be months, it could be years or for example weight can be you can measure it in terms of pounds or you can measure it in terms of grams, kilograms, etc.

How the measurement is done along the axis affects the look of the graph, it changes the visual representation. Let me give you an example. Suppose you have this particular coordinate system and along the y axis you are taking weight, but you are taking gram. So, it is in terms of gram and here suppose you are measuring the weight of a child and along the x axis, you are measuring the months, so this is time.

So, weight of a baby. Now, if you are measuring in terms of gram then it could be rising at a very fast rate. Because you know gram is a very small unit. On the other hand, if you do not measure in terms of gram, but something else, let us say kg, then this graph will not be so steep, it will be quite flat, because in terms of kg, there is some rice, but the weight gain will be very small, if we measure in terms of kg, which is like 1000 gram is equal to one kg.

Therefore, the steepness will go down by a lot. So, the visual representation also changes or visual impression also changes a lot depending on how we measure the two variables along the two axis. There is another thing which is very important, which is shifting of the graph. The graph of a function can shift in different ways.

For example, suppose a fixed amount z is deducted from the value of the function that is the dependent variable. So, from the y you are deducting some fixed amount, let us suppose that amount is z and then you get a new function. So, the old function was $f(x)$, from $f(x)$ z is being deducted, z is constant.

So, this is important z is not a variable, z is a constant. Now, if you deduct the z from $f(x)$, then you get another value which is let us suppose g, remember f itself was a function of x, so g will also be a function of x, so $g(x) = f(x) - z$, in this case what will happen if I use the same coordinate system here.

So, here if you measure f first, suppose this is $f(x)$ then this line is let us suppose like this. Now, if you measure g, then it will shift parallelly below. So, along the y axis you are measuring two things and this vertical gap is z, it is shifting down parallelly and vertical.

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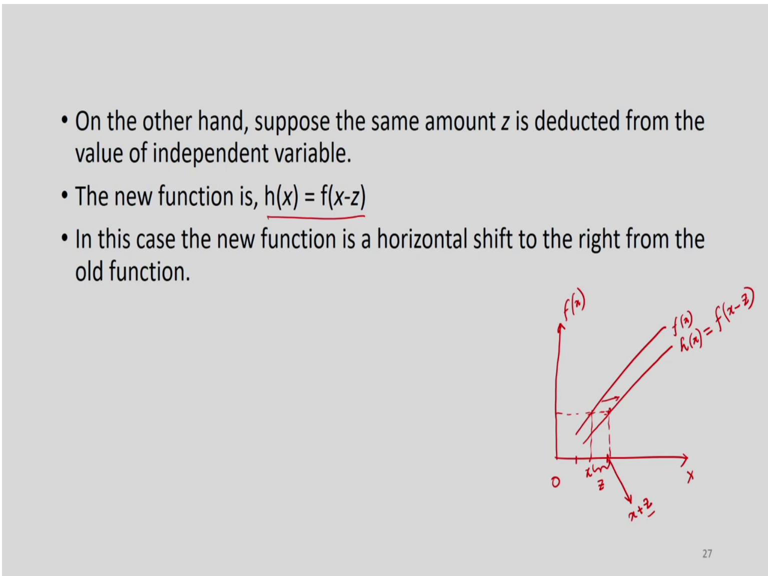

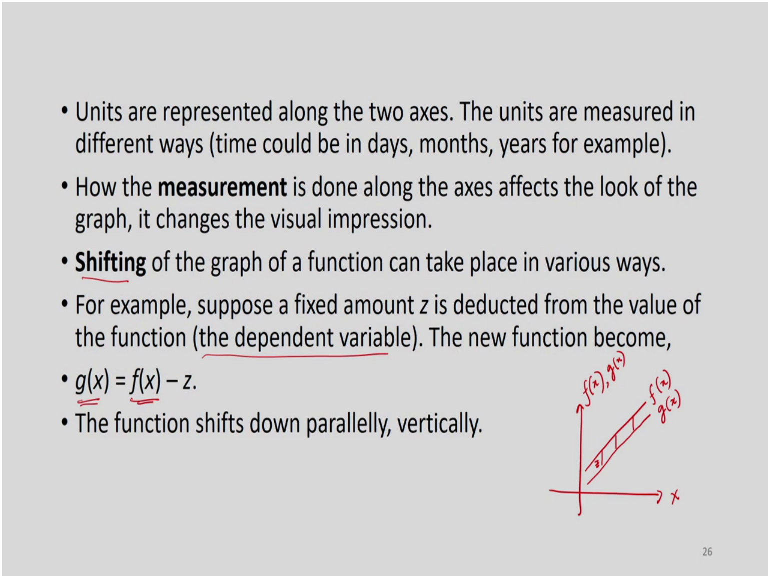

On the other hand suppose the same amount z is deducted from the value of the independent variable. So, the new function here is $h(x) = f(x - z)$. So, the value of the argument is changing and it is declining by a constant amount which is z. Here also the function will shift but it will be a different kind of shift.

So, first you have $f(x)$, suppose it is like this and you have $h(x)$, $h(x)$is where. So, this is $h(x)$, $h(x)$is to the right of the old function $f(x)$, where $h(x) = f(x - z)$ and this gap is z, you can see it from here itself that here if I take $x + z$, I get this value and what is the corresponding value of h, it is this and from this if I just deduct z, I get x and from that if I go to $f(x)$, I get the same value.

So, here also the function is shifting downwards, but it is shifting parallelly, horizontally and the amount of shift horizontal shift is given by z, contrast this with the previous one, where it was shifting downwards, but the vertical shift was z here the horizontal shift is z. So, in either case the shift is taking place, but the nature of shift is different.

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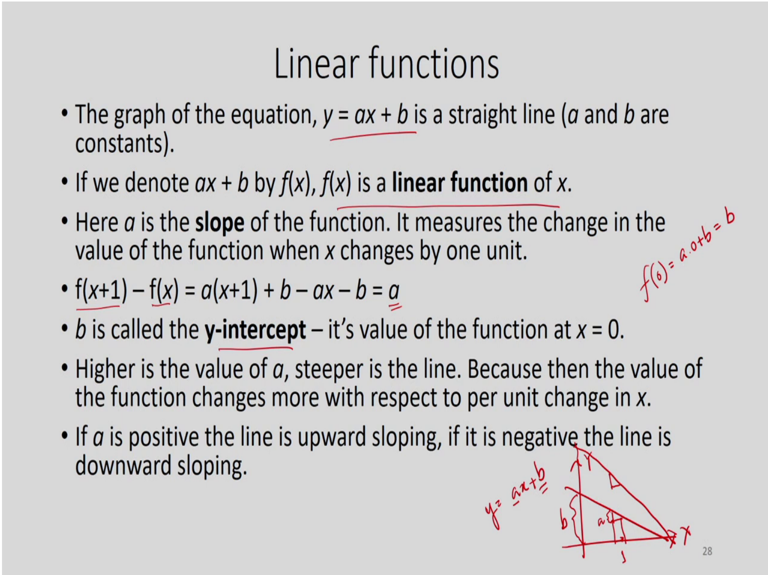

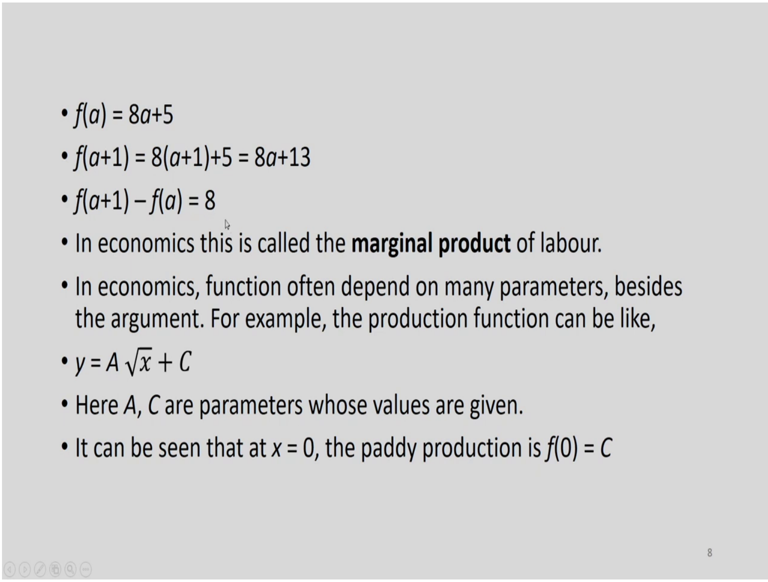

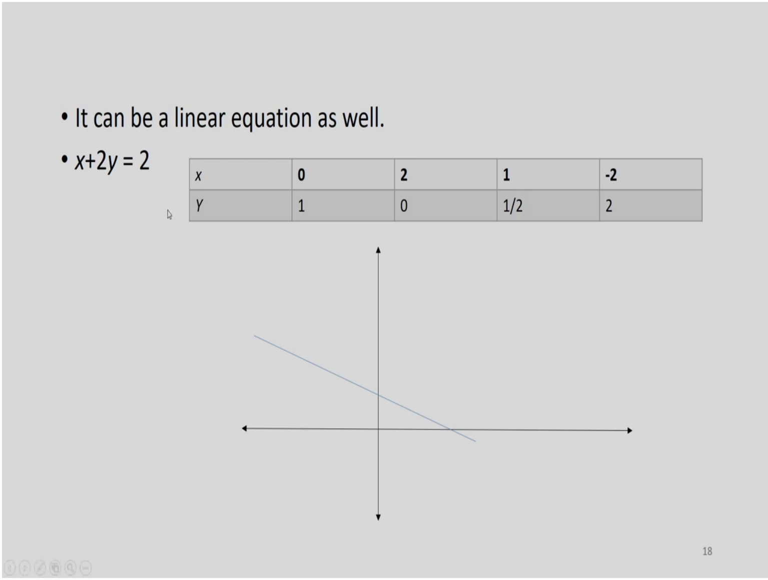

Now, we come to some particular kinds of functions. The, so the easiest function that one can talk about is one of the easiest ways a function can be defined is a linear function. The graph of the equation $y = ax + b$ is a straight line, this we have seen before. So, you take $y = ax + b$.

Now, as we are going to show here, that here a and b will have some specific meanings. Now, $ax + b$, if we denote it by $f(x)$, then $f(x)$ is a linear function of x, here a is the slope of the function, it measures the change in the value of the function, when x changes by 1 unit. So, if I take $f(x + 1)$ and from that I deduct $f(x)$, what do I get, $f(x + 1) = a(x +) + b$, from that I deduct $(ax + b)$.

So, it becomes minus b and so, some terms will cancel out and we are left with a. So, a measures the change in the value of the function when the argument changes by 1 unit. This we have seen before in a previous example, just remember earlier when we talked about marginal product. This was the example. So, here if the value of the function was changing by 1 unit, then we got something which was called the marginal product.

Here is another example of a linear equation. So, right now, we are talking about linear function, but the functional form in both cases in either cases will be similar, a is called the slope of the function if you have $y = ax + b$, this is the equation of the graph, then a which is getting multiplied with x is called the slope of the function. It measures the change in the value of the function that is $f(x)$ when x changes by 1 unit.

So, you can think about a function like this. Here, the value of y is changing with respect to change in the x, value of x and how much is y changing with respect to change of x that is called the slope, here it is going down as x is rising, suppose this is one unit, as x is rising, y is going down.

So, in this case, we say the slope is negative, it is y value is going down. On the other hand, in the expression $ax + b$, this b is a constant term, this is called a y intercept, it is the value of the function at x is equal to 0. So, if you substitute $x = 0$. So, $f(0)$ it will be a multiplied by 0 + b you get only b.

So, that is called the y intercept and visually, this is this value. So, this is b the y intercept, higher is the value of a steeper is the line, because then the value of the function changes more with respect to per unit change in x. So, instead of this particular function, if you take another function like this, more steep. If a is positive, the line is upward sloping if it is negative, the line is downward sloping.

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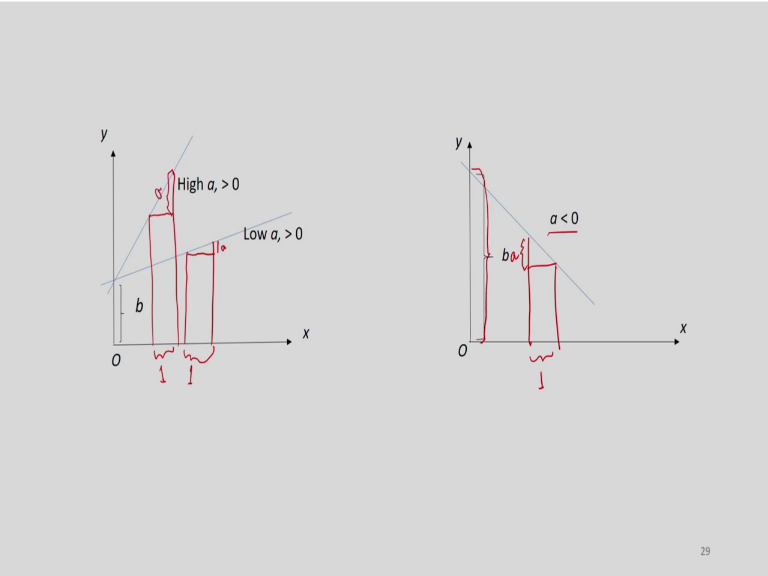

So, here is an example of an upward rising line. So, in the first case, so this case one, a is positive, but it is low. So, this is your a, suppose this is one, one unit of change of x then y is changing by, how much it is changing by a positive amount and it is given why small a.

On the other hand if you take the second line, which is more steeper line there, the value of a will be higher, you check that by drawing perpendiculars here value of a is quite high. So, here also we are assuming that this portion is just 1. So, in either case, I have kept other things same by taking the intercept same which is b is remaining constant.

But what is different between these two lines is that the first line has a low a and the second line has a high a and it is becoming steeper and this is the second case where a is not positive at all. So, a is negative. So, therefore, as x is rising you have a decline of the value of y. So, here a is negative and b here you can check it is positive and b here is given by this portion on the y axis.

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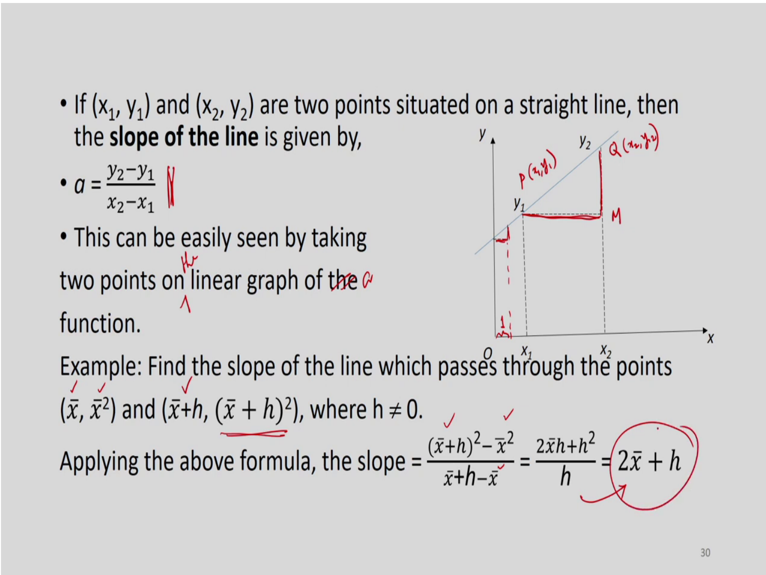

If (x_1, y_1) and (x_2, y_2) are the two points situated on a straight line, then the slope of the line is given by this. Suppose a is the slope, then $a = (y_2 - y_1)/(x_2 - x_1)$, this can be seen by taking two points on linear graph, on the linear graph of a function.

So, here I have taken a particular linear graph, it is a straight line, upward rising straight line, I have taken two points, let us suppose this is P and this is Q. So, P's coordinates are (x_1, y_1), Q's coordinates are (x_2, y_2) and so I have constructed another triangle here with a property that this is equal to 1.

Now, this is equal to 1 and this is parallel to this line, this is parallel to this line. So, these two triangles are similar. So, therefore this divided by this will be this divided by this, but this divided by this is the slope of the straight line. So, the slope of the straight line will be this portion, divided by this portion. But what is this portion Q M divided by P M, what is Q M? Q M is nothing but $(y_2 - y_1)$ and P M is nothing but $(x_2 - x_1)$.

So, the slope of the straight line is $(y_2 - y_1)/(x_2 - x_1)$. So, here is another example, find the slope of the line which passes through the points $(\bar{x}, \bar{x}^2)$ and $(\bar{x} + h,\ (\bar{x} + h)^2)$, where $h \neq 0$. So, we have to find the slope of a straight line, which passes through these two points. So, we apply this formula, this formula is $(y_2 - y_1)/(x_2 - x_1)$.

Let us suppose y_2 is this one. So, $(\bar{x} + h)^2$ is y 2. So, I have written that minus y_1, $y_1$is $\bar{x}^2$ and divided by what is x_2, x_2 is $\bar{x} + h$ minus x_1, x_1 is $\bar{x}$. So, I have put everything here and then I will simplify this and from this expression to this expression, what you are doing is that you are cancelling h from the denominator and the numerator and you are getting to $2\bar{x} + h$. So, this is the slope of the straight line.

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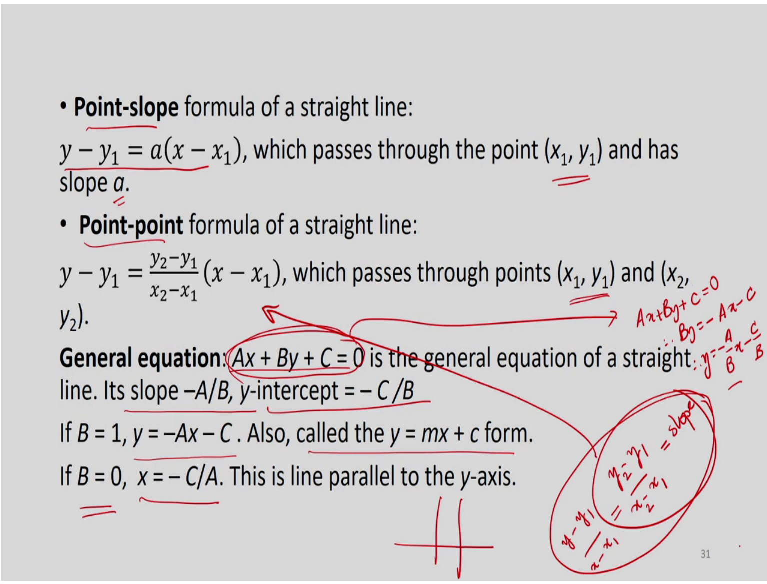

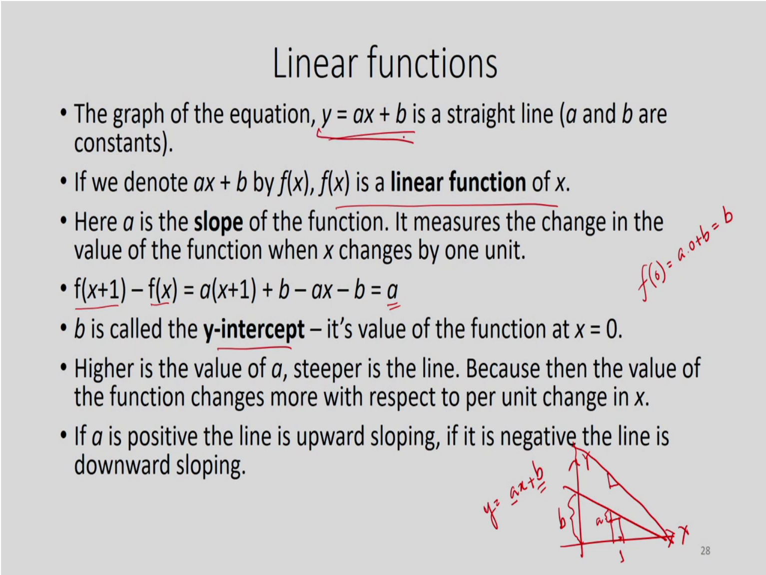

So, there are some formulas of straight line. The first formula we are going to talk about is what is called the point slope, formula of a straight line and this is given by $y - y_1 = a(x - x_1)$. This line passes through the point (x_1, y_1) and has a slope of small a.

So, if we know the slope of a straight line which is given by a and we know that it passes through a particular point (x_1, y_1), then we can find out the equation of that straight line. Here, we are basically using the fact that slope of a straight line is given by $(y_1 - y_2)/(x_1 - x_2)$.

So, suppose (x_1, y_1) is given, that we know it lies on that straight line and we take any other arbitrary point on the straight line whose coordinates are x and y. So, what is the slope according to the formula, it will be $(y - y_1)/(x - x_1)$, this is the slope and we know the slope is given to us, this is given by a.

So we simplify this and we will get this particular form, you can see that it is actually a function because you know y can be represented as the, as dependent on the value of x. This is point slope formula, there is another thing called point formula, where what we know is that there is a line which passes through two points (x_1, y_1) and (x_2, y_2) .

So, that we know. We do not know the slope, unlike the previous case, so that is why it is called point, point formula, not point slope formula, but if we know that it passes through these two points (x_1, y_1) and (x_2, y_2) , then we can use again the fact that the slope of any line is given by this.

So, this is the slope. So, we know that this is the slope because (x_1, y_1) and (x_2, y_2) is given to us. At the same time we take another arbitrary point on this line which is let us say x y. So, this should be equal to the slope formula, if we use x and y. So, $(y - y_1)/(x - x_1)$ should also be equal to this and then from this, this entire thing we get this form that $y - y_1 = [(y_2 - y_1)/(x_2 - x_1](x - x_1)$.

So, these are the two formulas of straight lines. Now, what is the general form of a straight lines function, which is linear. So, this is the general form, $Ax + By + C = 0$, this is the general equation of a straight line and what is the slope of the straight line, it is given by $- A/B$ and what is the y intercept is given by $- C/B$.

So, we take this particular form to verify whether the slope is indeed equal to $- A/B$ and the y intercept is indeed equal to $- C/B$. So, we start with $Ax + By + C = 0$. So, $By = - Ax - c$. So, therefore, $y =- (Ax/B) - C/B$ and this is exactly in the form $y = Ax + B$ and if it is $Ax + B$, then we know A is the slope.

So, therefore, here the slope is $- A/B$ and the intercept is equal to$- C/B$. Now, there are some special cases, suppose B is equal to 1. Now, if B is equal to 1, then what do you get $Y = Ax - C$and this is called the $y = mx + c$ form. So, this is the form we talked about earlier also, we did not say that it is $y = mx + c$, but this is the form $y = Ax + B$, same thing.

And if we assume that B is equal to 0, then I cannot divide both sides by B. So, I just look at this equation and put B is equal to 0, then what do you get is $x = - C/A$, assuming obviously, A is not 0. Now, if $x = - C/A$, then it is basically a straight line, it could be to the left, it could be to the right depending on the sign. So, this is the general equation of a linear function.

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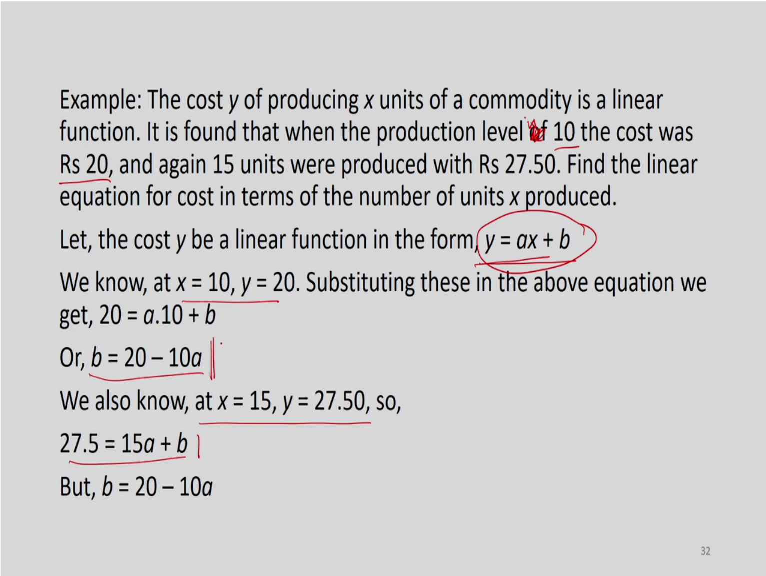

Now, we take an example, the cost y of producing x units of a commodity is a linear function. It is found that when the production level of 10, production level it should be is, is 10 the cost was 20 rupees and again 15 units were produced with the cost of 27.50 rupees, find the linear equation for cost in terms of the number of units x produced.

So, x is the number of units that is being produced and y is the cost. So, we have to find a and b and we are given the information that their relationship is linear. So, what are the information we further know, we know the information that if x is equal to 10, then y is equal to 20, then we can substitute this in this form. If we do that, $20 = 10a + b \; or \; b = 20 - 10a$, we also know that if x is equal to 15, then y rises to 27.50. So, again we can put that here and here we can substitute the value of b from here.

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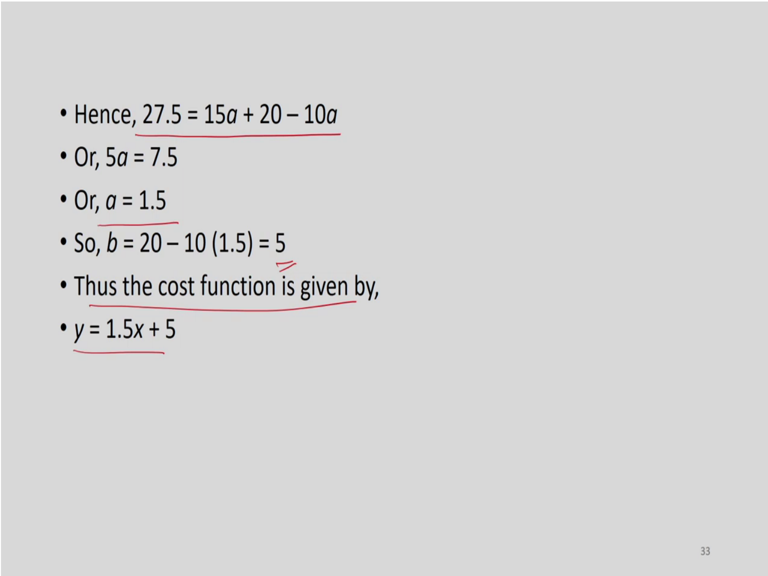

And therefore, we will get $27.5 = 15a + 20 - 10a$ and if we simplify this, then we get $a = 1.5$ and putting this in the equation for b, we get $b = 20 - 10(1.5) = 5$. So, therefore the cost function which we were supposed to find is given by this equation, $y = 1.5x + 5$.

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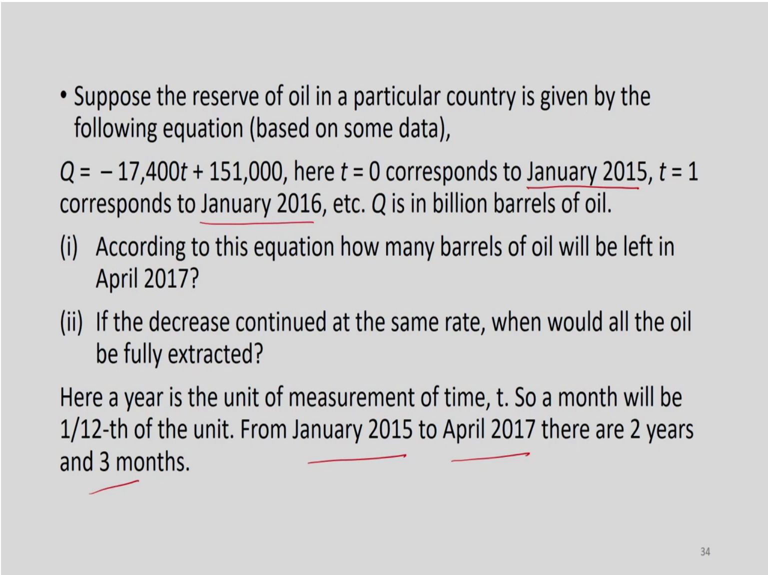

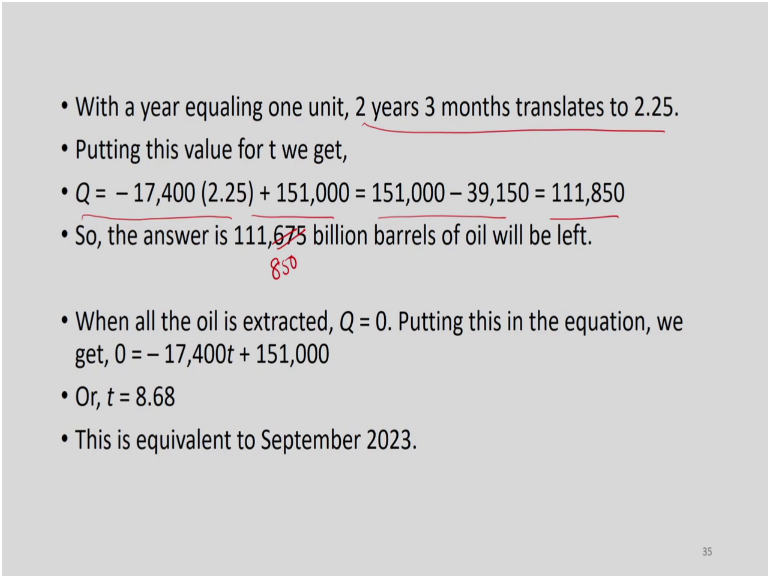

Here is another example, suppose the reserve of oil in a particular country is given by the following equation based on some data. So, Q is the reserve of oil in a particular country it is given by $Q = \ -17400t + 151000$. Here $t = 0$ corresponds to January 2015, $t = 1$ corresponds to January 2016 et cetera.

So, t is time and it is measured in terms of years. So, 1 year is equal to 1 t and the origin is at January 2015. What is Q? Q is in billion barrels of oil. Now, you can see that as t is changing, t always rises, time does not go back. So, as t rises, this Q is going to fall. According to this equation, how many barrels of oil will be left in April 2017?

So, we are starting the 0 point is in January 2015. The question is if we look at the totals reserve, at the point of April 2017 then how many barrels of oil will be left at that point of time. Here one year is the unit of measurement of time. So, which is t. So, a month will be 1/12 of the unit. Now from January 2015 to April 2017 there are two years and three months.

So, two years and three months will be how much it will be equal to 2.25, in terms of the unit that we have here. So, we have to put $t = 2.25$ in the equation that is given to us and if we do so, then we have to after that we have to simplify it, turns out to be this much, which is 111,850. So, 111,850, this should be 850, 111850 billion barrels of oil you will be left.

There is a second part to this, if the decrease continued at the same rate, when would all the oil be fully extracted. Decreased means, you know you can see that Q is going down that means, more and more oil is being extracted. So, the reserve is going down. So, if it continues like this, then at some point obviously, all the reserves will be extracted, then the question is at what time this will happen.

So, for that what do you need to do, you need to put $Q = 0$ and solve the equation. So, I put $Q = 0$ and put this in the equation, I get this much and if I simplify this, this turns out that $t = 8.68$ and this is $t = 8.68$ now, we have to convert this in terms of calendar time and if I convert this in terms of calendar time, it turns out to be September of 2023. So, basically in September 23 all the while will be extracted.

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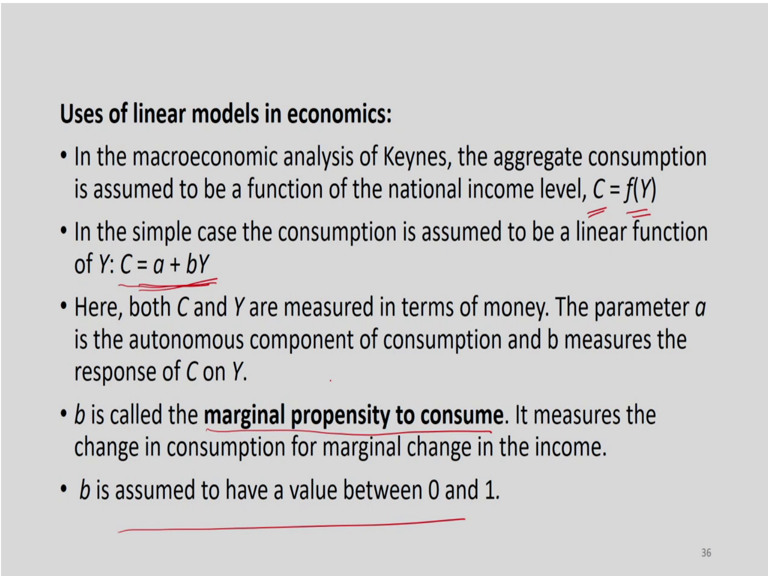

Now, we come to some other applications of linear models in economics, I will just introduce this thing and then maybe we shall call it a day. In the macro economic analysis of Keynes, the aggregate consumption is assumed to be a function of the national income level. So, capital C is the aggregate consumption function of the economy and you can see it is a function of f of capital Y, $C = f(Y)$ Y is the national income and it is supposed to be a linear function, we know that a linear function can be of this form.

So, $C = a + bY$. Here C and Y are measured in terms of money. The parameter a is the autonomous component of consumption and b measures the response of C on Y that we have seen before. So, a here is the intercept term, it does not change with respect to the value of the independent variable, here the independent variable is Y.

So, a is the what is called the Y intercept and b is that slope and slope is there it is b and if it is nonzero, then as y changes, capital C will keep on changing. So, therefore we see that b measures the response of C on Y, Y is changing, therefore capital C will be affected b is measuring that response of C on Y, b is called the marginal propensity to consume, it has a particular name it is called the marginal propensity to consume, it measures the change in consumption for marginal change in the income.

So, this is like the marginal product that we have seen before when we talked about the production function. Here it is called the marginal propensity to consume, change in the consumption level, when the income changes very marginally. b is assumed to have a value between 0 and 1. So, again, I am not going to explain why it should have a value between 0 and 1.

But, if it is having a value between 0 and 1, it means the effect on consumption of change of Y is there it is greater than 0, but it is not very much it is less than 1. We shall stop here. Before we proceed to the next lecture let me just summarise what we have done. We have looked at different linear functions, graphs of linear functions. We have talked about why all shapes are not graphs of functions and we looked at certain applications of linear functions in economics. Thank you.