**Mathematics for Economics 1**
**Functions of one variable, graphs of function**
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**Module: Functions**
**Lecture 5: Definitions**

Welcome to the fifth lecture of the course, Mathematics for Economics part 1. Actually, this lecture, we are going to talk about something called, a module called Functions of One Variable and Graphs of Functions and we introduced this topic in the last lecture itself in the fourth lecture itself. So, we are just going to do a brief recapitulation of what we have seen in the last lecture, and then we shall move ahead.

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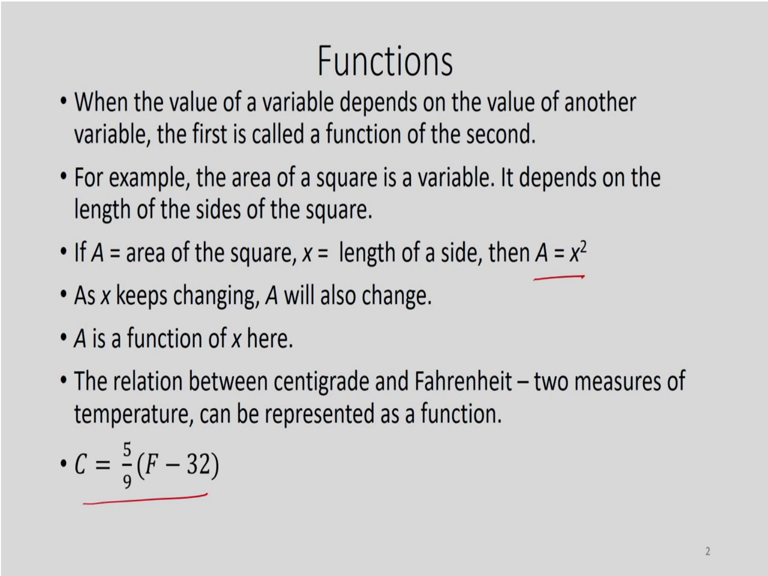


So, what we have done here is that we have given some examples of functions, we say that, when the value of a variable depends on the value of another variable, the first is called a function of the second and we gave some examples, such as you have this function $A = x^2$, where A is the area of a square and x is the length of a side of the square.

So, as you take the square of a side it becomes x square and that is nothing but the area of a square. So, here A depends on x as x goes on changing A that is the area of the square will also go on changing. So, in other words, as the side of a square becomes larger, the area of that square also will become larger and vice versa.

Similarly, there could be another function like this, the relation between centigrade and Fahrenheit. So, we know if C represents centigrade temperature $= \frac{5}{9}(F - 32)$. So, as Fahrenheit goes on changing, that is the temperature of any place for example, goes on changing with respect to Fahrenheit, in centigrade scale also the temperature will go on changing by following this rule.

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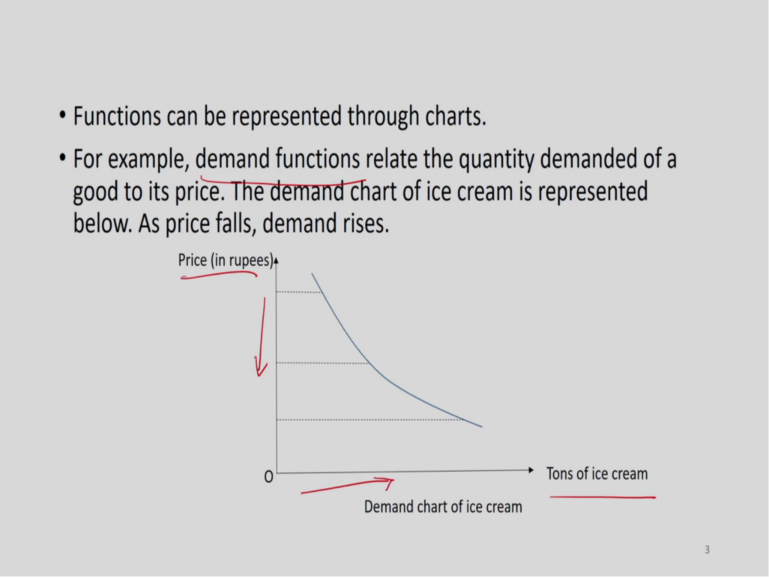


And in economics, we have different kinds of functions; here is an example of what is called a demand function. Here the demand quantity demanded of a good is the variable which is depending on the price of that good, so you have price here and tonnes of ice cream here and as price goes on changing, it falls and you can see that the quantity demanded goes on rising. So, this is an example of a function which is used in economics.

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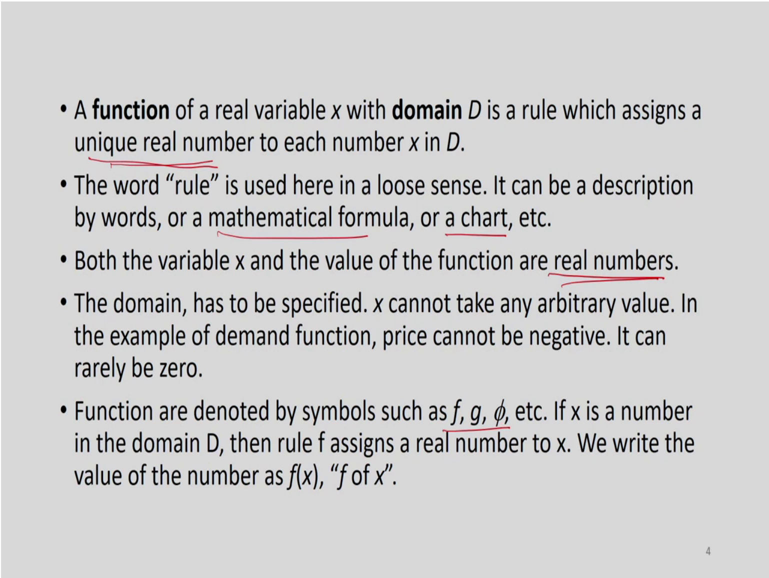


And in function, there is something called the domain. Domain is the set from which the variable x takes its values and what the function does is that it assigns a particular value and unique real number to any x which belongs to the domain D. So, this is important, it assigns a unique real number.

So, it cannot happen that with respect to a particular value of x, you have multiple values of the function, that is not allowed. But a function can be represented in different ways, it can be represented through a mathematical formula, as we have seen $A = x^2$, that is the formula or it could be represented in terms of a chart. For example, we have seen that demand function that was represented through a chart or it could just be described in words.

So, either way you have what is a function and in our case, we are going to take functions where the variable x and the value of the functions are real numbers. So, this is the kind of functions that we are going to restrict our attention to. Functions are represented by some certain symbols. You can use f, you can use g and you can use ϕ.

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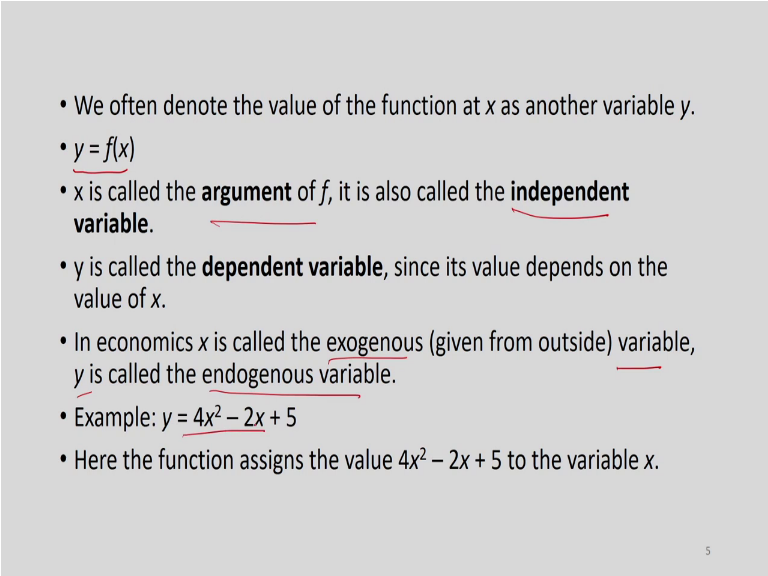


And here are some terms that we have defined, x is called the argument of the function, it is also called the independent variable and y if we write $y = f(x)$, then y is called the dependent variable, because its value is depending on the value of x and in economics, we sometimes use another kind of terms, x is sometimes called the exogenous variable and y is called the endogenous variable. So, y is getting determined, so it is getting determined within the system and that is why it is an endogenous variable whereas x is given from outside. That is why it is called exogenous variable.

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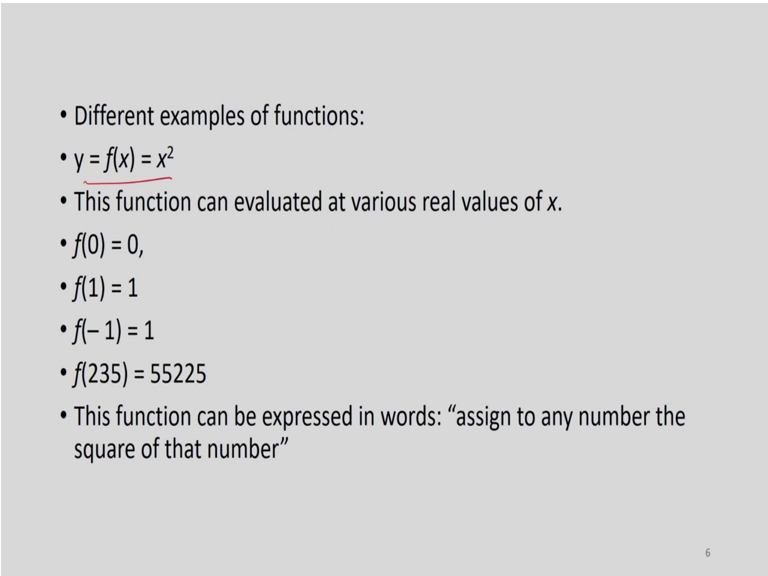

And here are some examples of a function $y = x^2$. Now, it is like the area of a square, where y is equal to area and x is the side of the square. So, $A = x^2$ same as $y = x^2$.

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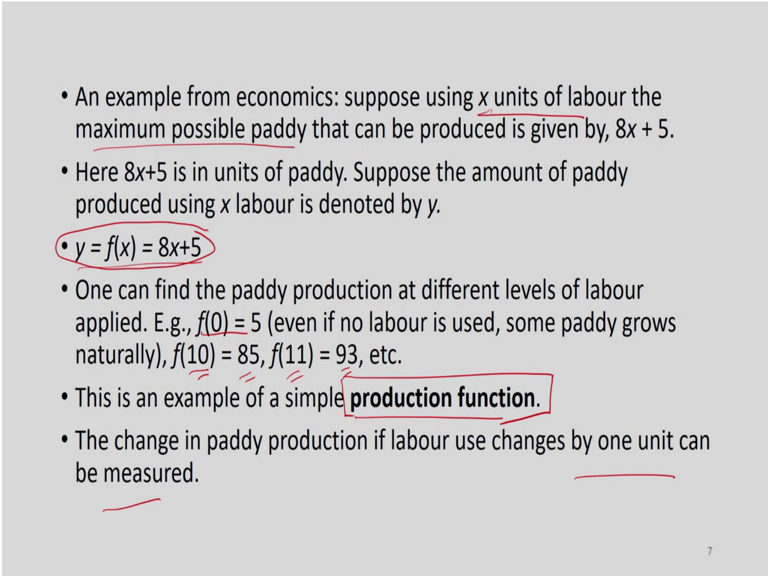


So, this is where we left in the last class, last lecture. So, here is an example of what is called a production function, $y = 8x + 5$ what is x and what is y here? x is the units of labour, how much amount of labour is being put in the process of production?

So, for example, if you have a factory, then how many labourers are you employing in that factory or it could be measured in a different way, x could be measured in terms of hours of labour, instead of human beings, you can more precisely measure it as number of hours that labourers are putting in that factory, in the process of production.

And what is y? y is the amount of production and more precisely it is the maximum possible paddy. So, here paddy is being produced. So, y is the maximum possible paddy that can be produced by putting in x units of labour. Why did I say maximum possible because, it is possible that you are putting so much amount of labour, but some of it is being wasted through inefficiency and other things.

So, that we are ignoring we are assuming that whatever labour is being used, it is used efficiently. So, therefore y represents the maximum possible output in this case paddy that we can produce. Now, this relationship between x and y is also a function and it is called a production function and in economics, we will encounter it a number of times, where we talk about production function, output y is called the output and x is called an input, you know labour is an input.

So, output is a function of the amount of input. You can check how the y value is changing with respect to different values of x. For example, if you take $x = 0$, you get $y = f(0) = 5$. So, what is the interpretation of that, it means that even if no labour is used, even then we can see some production taking place, which is equal to 5 and if x rises to 10, output rises to 85, if it rises further to 11 it rises to 93 etc, etc.

Notice we talked about something called a domain of function. Domain means the possible values of x. Here obviously, x cannot take negative values, you cannot use a negative quantities of labour in production. So, here x system domain is a non-negative set of numbers. Change in paddy production if labour use changes by 1 unit can also be measured. So, suppose output is changing with respect to change in input, but changing input is just by 1 unit, then that can be measured.

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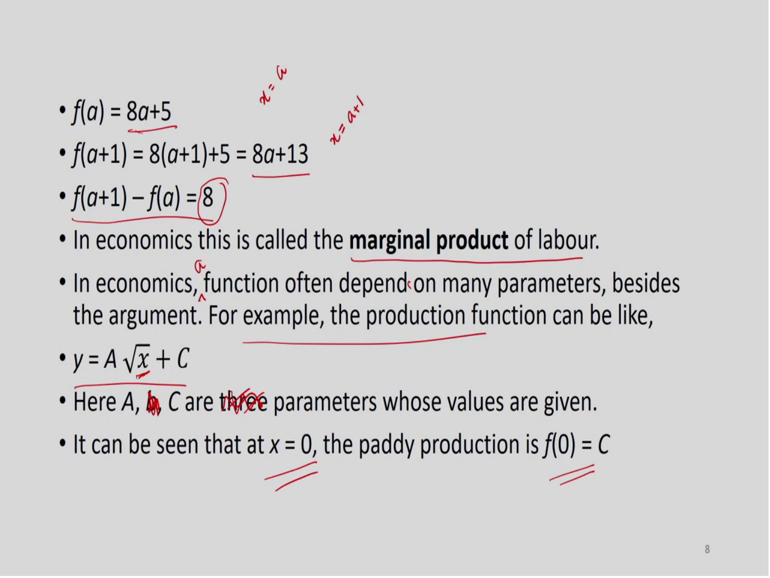


Let us say the amount of $x = a$, so therefore total output will be at $f(a) = 8a + 5$. And here I am taking $x = a + 1$. So, total output will be $8a + 13$ just simplifying this thing. So, what is the change in output or change in the value of the function when input is changing by just 1 unit it is just this minus this and if you do that it will be 13 minus 5, what is 13 minus 5 it is 8.

So, this is a change in the paddy production if labour unit changes by 1 unit. In economics this has a specific term, it is called the marginal product of labour. So, marginal product of labour here the input is labour, if I had taken some other input, if I had taken let us say capital it would have been marginal product of capital and why is it called marginal product? Because it is the change in the total product, if the input in question is changing by a little, it is changing marginally.

So, we are assuming that the smallest possible change that takes place in the inputs is 1 unit. In that case, the marginal product is here in this case is 8. In economics a function often depends on many parameters besides the argument, so a function can depend on the argument, but it depends on other parameters also.

So, here is an example of another production function. Here $y = A\sqrt{x} + C$, ignore this b part. So, here A and C are parameters whose values are given. So, A and C, their values are given from outside that is why they are called parameters and they actually affect y, but since their values are given, so they are not the thing we are going to concentrate too much on, we are going to concentrate more on the endogenous variable the exogenous variable, which is x.

So, we can look at how y changes with respect to x. So, x is a variable, A and C are parameters, variables are the things we concentrate on. Parameters also sometimes talk about them, but they are not really the focus of our attention. Because for example, think about the practical things involved here why we do not focus too much on the parameters is that, suppose this is a production function.

Now, as a producer, a man who is running the factory, he has control over x. So, he can decide how much more labour he can use and that will affect the output which is y, but what about A and C? A and C are let us say the state of technology, the state of technology is depending the values of A and C. Now, since the state of technology at any given point of time is given.

So, this producer has no control over the values of A and C and so, he can change production, but not by changing A and C, he can change the production by changing only x. So, that is why we most of the time concentrate on the value of x rather than the value of the parameters. It can be seen that if $x = 0$ the paddy production at $f(0) = C$. So, just put $x = 0$, you will get the value of $y = C$.

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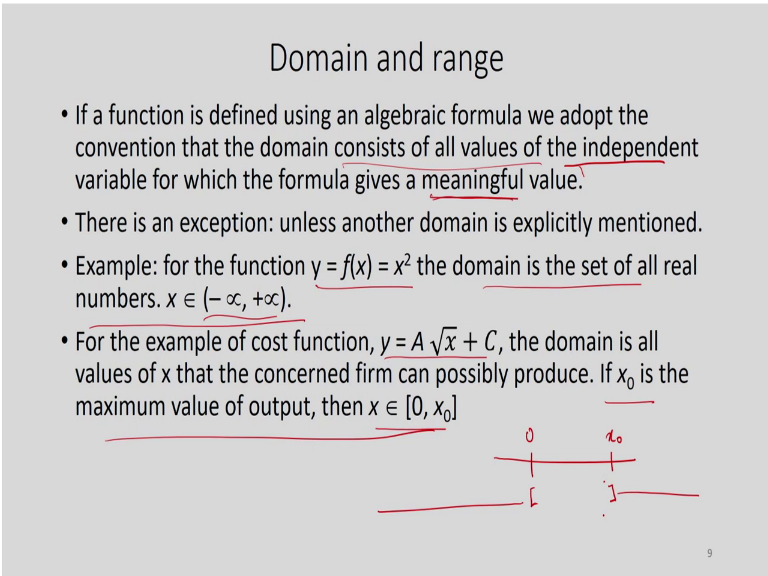


Now, we define certain other things, domain and range. I have already talked about domain. Now, let us look more carefully into the idea of domain. If a function is defined using an algebraic formula, we adopt the convention that the domain consists of all values of the independent variable for which the formula gives a meaningful value. This meaningful value, this phrase is important.

So, what is being said is that domain, it consists of all values of the independent variable that is x for which the function that is the formula, which is a representation of the function will give a meaningful value, meaningful I am going to talk more about this, what is meant by meaningful and when things are not meaningful, but there is an exception unless another domain is explicitly mentioned.

Suppose the domain is explicitly mentioned that for this values of x the function is defined. Then obviously, there is nothing to talk about this, domain is given, but if the domain is not given, then how do you define the domain. Then we have to look at those values of x for which the formula gives us a meaningful value. So, here is an example $y = f(x) = x^2$.

Here the domain is the set of all real numbers; $x \in (-\infty, +\infty)$, because I mean you take any value of x, so, x square which is the value of the function will always be a meaningful number. So, if you take 1, it will give you 1. If you put $x = -1$, it will get plus 1, if you put $x = 0$, it will give you 0, like this as x goes on changing it takes values along the real number line, you get ys which are also along the real number line.

So, here the entire real number line from $-\infty\ to\ +\infty$, it is in the domain. Let us take another example, this example we have seen before, $y = A\sqrt{x} + C$. Here the domain cannot be the entire real number line, here you see where this meaningful word is important.

The domain is all values of x that the concerned firm can possibly produce. So, cost function remember, what is cost function, it gives you the cost of production, where the independent variable is the level of production. So, y here is the cost of production and x here is the amount that is being produced.

If $x_0$ is the maximum value of output, then this is the domain. It starts from 0 it goes up till $x_0$. So, if I draw the real number line, so one is talking about this. So, you are ignoring the entire part to the left of 0 because if you put x is equal to minus something minus something negative, then you are going to get imaginary numbers and that is not allowed.

So, you start at 0, but again you do not go all the way to plus infinity because the maximum possible output a firm can produce is limited. So, that maximum possible output is suppose $x_0$. So, $x_0$ is the maximum value to which the independent variable will go. So, therefore, this is the domain D.

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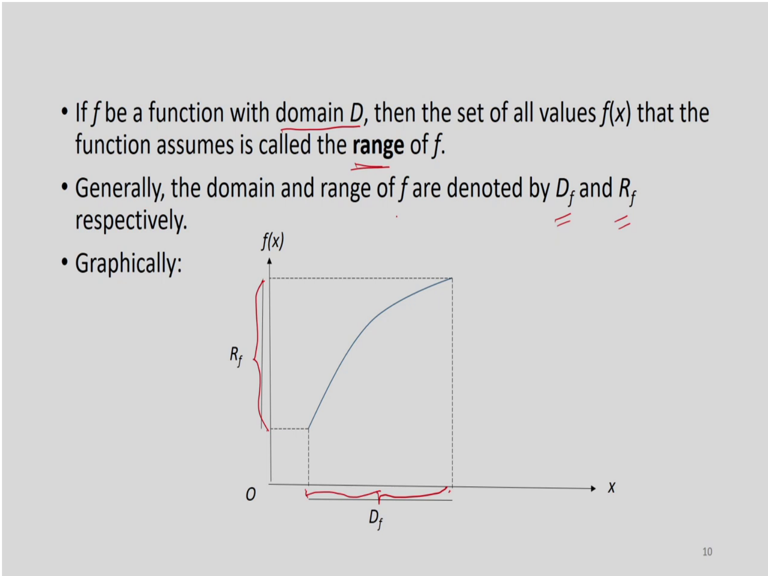


We have talked about domain now, let us talk about the range. If f be a function with domain D, then the set of all values f(x) that function assumes is called the range of f. So, here I have defined what is the range of a function, suppose f is a function, f is defined over the domain which is given by capital D, then you look at all the possible values that f(x) can take and put it under a set and that set will be called the range of f.

Generally the domain and range of f are denoted by $D_f$ and $R_f$ respectively. So, these are the usual notations that we use. So, here is an example, here x is the independent variable f(x) is the dependent variable that is the value of the function. Now, domain here is given, this is the domain $D_f$.

So, this is the $D_f$ from this point to this point and you can look at the function, the function is generally upward rising function it is going up as x is going up and so what are the values of f(x) the minimum value is starting from here, where x is also very low and then as x is rising f(x) is also rising and as x is at its maximum f(x) is also at its maximum. So, here is the maximum value that f(x) can take. So, the range of the function will consist of all values from here to here, so that is how the range is defined.

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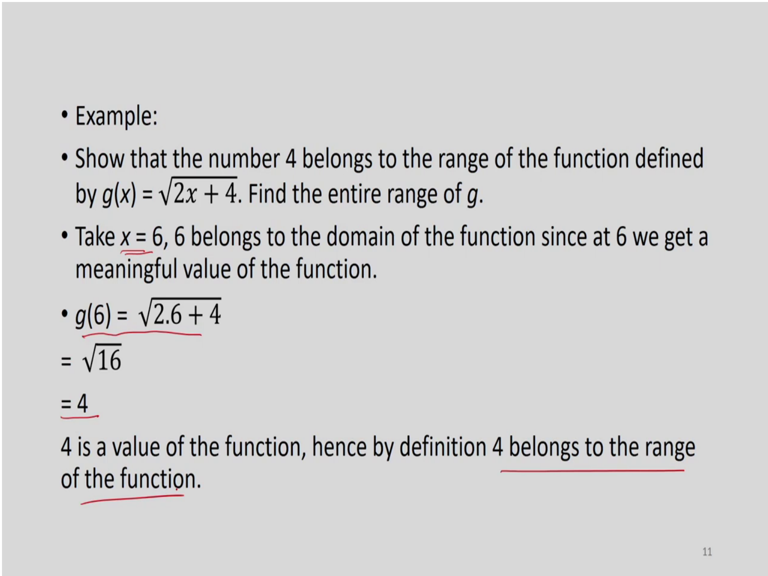


Example show that the number 4 belongs to the range of the function defined by $g(x) = \sqrt{2x + 4}$ and there is a second part to it, find the entire range of g. So, we have to show that 4 belongs to the range of this particular function, this function is called the g function. We are taking a particular value of x, suppose x is equal to 6, does 6 belong to the domain of x.

So, we have to be careful that the particular value of x that we are taking, it has to belong to the domain otherwise it cannot be taken and yes, we can take 6, because if we take 6, then the value of the function will not become something meaningless, because when can this become meaningless, the function can become meaningless.

If 2x+4, it becomes a negative quantity, then it is problematic, it will become root over something minus which is not allowed. But here if I take $x = 6$, that problem does not arise. So, 6 belongs to the domain of the function and so, if I put $x = 6$, what do I get $g(x) = \sqrt{2.6 + 4} = \sqrt{16} = 4$.

Now, that is what we had to prove that 4 is a number which belongs to the range of the function. So, we have put x is equal to 6 and the value comes out to be, the value of the function comes out to be 4 that means, for a particular value of x, we get 4 as the value of the function. Therefore, 4 this number 4 belongs to the range of the function.

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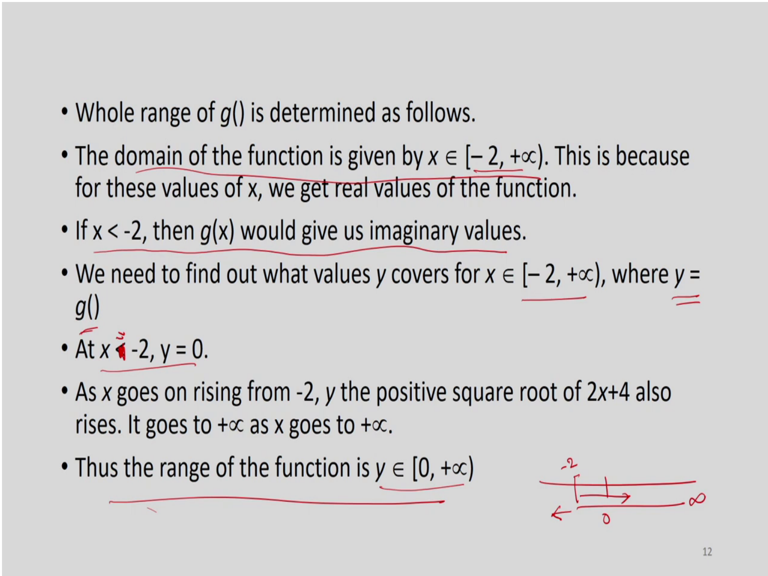


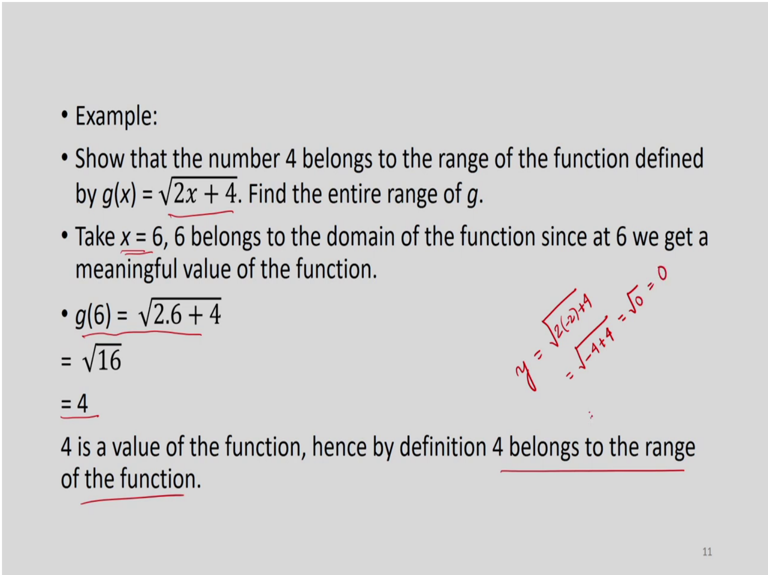


Now, we also have to determine the whole range of g. So, this is a little bit tough, let us remember the domain is what is going to determine the range, because as x is changing along the domain, the value of the function will go on changing and we have to specify the entire set of values that the function takes and that will be the range.

Now, what is the domain of this function, the domain of the function is given by this, $x \in [-2, \infty)$. Why this particular range of values of x. So, you are talking about minus 2 and here is plus infinity. So, you are talking about this, because if you go to the left of minus 2, you get negative values.

So, at $x = -2$, this will give you 0. So, if you reduce x further, then this expression becomes negative and which is not allowed. So, therefore the minimum value of $x = -2$ and it can go all the way up to plus infinity. So, this is the domain. So, if x is less than minus 2, g(x) will give us the imaginary values, we need to find out what values y covers, if x belongs to this particular interval $x \in [-2, \infty)$, where y is equal to g(x).

So, I am just defining another variable here which is y, y is the value of the function. So, here if this should be just equal to, if $x = -2$ we have just seen $y = 0$, because it becomes $\sqrt{2(-2) + 4}$=0. So, f(-2)=0.

That is why I have written at $x = -2$, $y = 0$, as x goes on rising from minus 2. So, here x is going up, y is the positive square root of 2x + 4, it will also go on rising, it goes to plus infinity as x goes to plus infinity. Thus, the range of y is given by this, so this is what we were supposed to find, range of this function is $y \in [0, \infty)$from 0 to plus infinity, where 0 is included.

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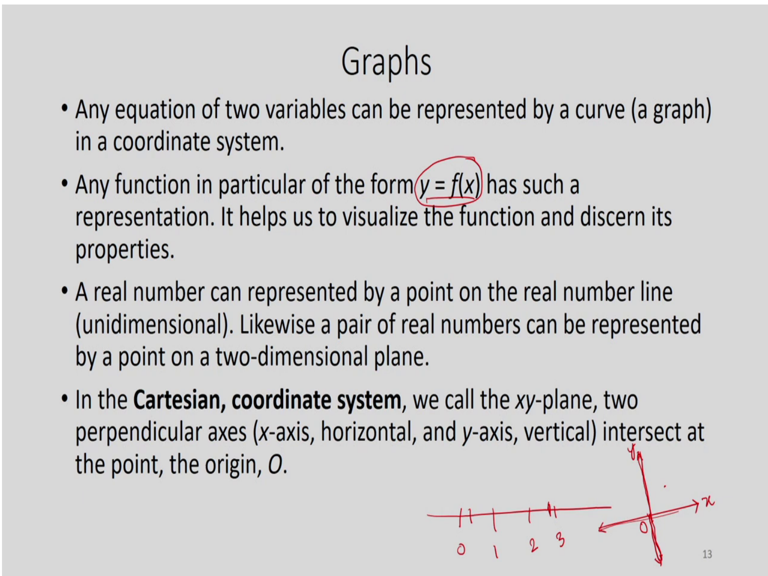


We now come to what are called the graphs of functions. Any equation of two variables can be represented by a curve or a graph in a coordinate system. Any function in particular of the form $y = f(x)$has such a representation; it helps us to visualise the function and discern its properties.

So, the point that is being made is that if you take any equation, I am not talking about function, any equation of two variables, then these two variables, this equation can be represented along a coordinate system and that is called the graph of that particular equation. Now, that same thing is true for a function also of the form $y = f(x)$, what is there in $y = f(x)$, two variables x and y and y is dependent on x.

So, difference between an equation and a function is that in an equation, it is not anywhere given what is a dependent variable, what is an independent variable, those things are not there, just two variables are there. But in a function, you have $y = f(x)$, so x is the independent variable and y depends on that.

But nonetheless, it is a relation between two variables. So, just as an equation can be represented in a coordinate system by a graph, visual representation. Similarly, a function which also consists of two variables can be represented by a graph in a coordinate system. Real numbers now, let me talk a bit about the coordinate system as such, a real number can be represented by a point on the real number line.

So, unidimensional. You take any numbers let us say 2.97, 2.97 will be somewhere here, 0.354 somewhere here, etc, etc. So, any number as long as it is a real number can be located along the real number line, which is an unidimensional representation. Likewise, a pair of real numbers, so we are not talking about one number, but a pair of real numbers can be represented by a point on a two dimensional plane.

So, here you do not have just one dimension, do not have a horizontal line, but what you have is two dimensional plane something like this. Here any point is representation of a pair of numbers. Likewise, a pair of real numbers can be represented by a point on a two dimensional plane.

In the Cartesian coordinate system, we call the x y plane two perpendicular axis. So, these are the two perpendicular axis, x axis is the horizontal axis. So, here you have 0, here you have the x axis and you have the y axis here, x axis horizontal and y axis vertical, they intersect at the point, the origin, which is represented by the symbol O.

So basically, so far, we were talking about a single number, but now we are talking about two numbers and how visually we are representing this pair of numbers and in visually we are representing any pair of numbers in a plane, two dimensional plane and in this two dimensional plane, there are two axis. One is the x axis, which is the horizontal line.

So, this is a real number line actually, you can imagine this itself to be a real number line, but there is another real number line which is the y axis, which is perpendicular to it and so, two axes which are like stretching up to infinity on both sides obviously will intersect they are perpendicular and that point of intersection is called the origin or O.

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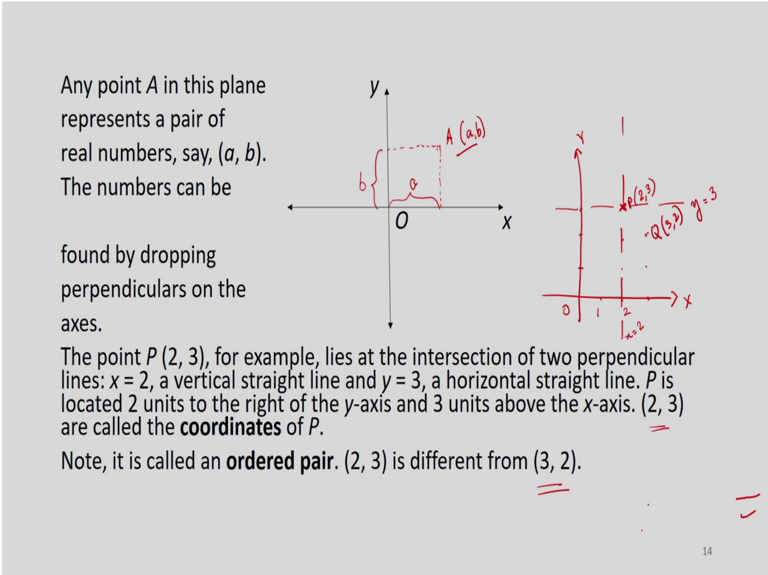


Any point A on this plane represents a pair of real numbers, say (a,b). So, let us suppose, I represent this as this point A, the numbers can be found by dropping perpendiculars on the axis. So, suppose this represents this pair of numbers, (a,b), then this (a,b) have their visual representations.

So, I draw the perpendiculars on the axis. So, when I draw the perpendicular on the x axis, a portion of the x axis I have cut off that is a and similarly, I have drawn perpendicular one the y axis then the portion of the y axis that I have cut off is called b. So, after drawing the perpendiculars I have what is called the x intercept and the y intercept of the perpendiculars and so, those portions of x and y axis that have been cut off from the point of the perpendicular and the origin.

And these values are the values of what is called the coordinates of this particular point. The point P which has a coordinate of (2,3), for example lies at the intersection of two perpendicular lines $x = 2$, let us do it differently. So, 2, 3 here is the point 2, 3 it lies at the intersection of two perpendicular lines $x = 2$, $x = 2$ is this line, it is a vertical straight line as I have drawn it is a vertical straight line and $y = 3$, a horizontal line. So, this is $y = 3$.

So, this point P (2, 3) lies at the intersection of these two lines, 1 is equal to $x = 2$ and $y = 3$. P is located 2 units to the right of the y axis, you can see that here is 1 here is 2 to the right of the y axis, this is the y axis and 3 units above the x axis. So, here is 1, here is 2 and here is 3. That is why it is being said that this point is lying 2 units above the x axis, 2, 3 are called the coordinates of the point P.

Note this is called an ordered pair. Ordered pair means it matters where you are putting this number (2, 3) is different from (3,2). (2, 3) is not same as (3, 2). (3, 2) will be where? 3, 2 will be 3 and 2, this is the value of let us say Q (3, 2). P is not as equal to Q. So, it matters how these numbers are appearing, which is preceding what? If it is 2, which is preceding 3, then you are here P. If it is 3, which is preceding 2, you are here at Q.

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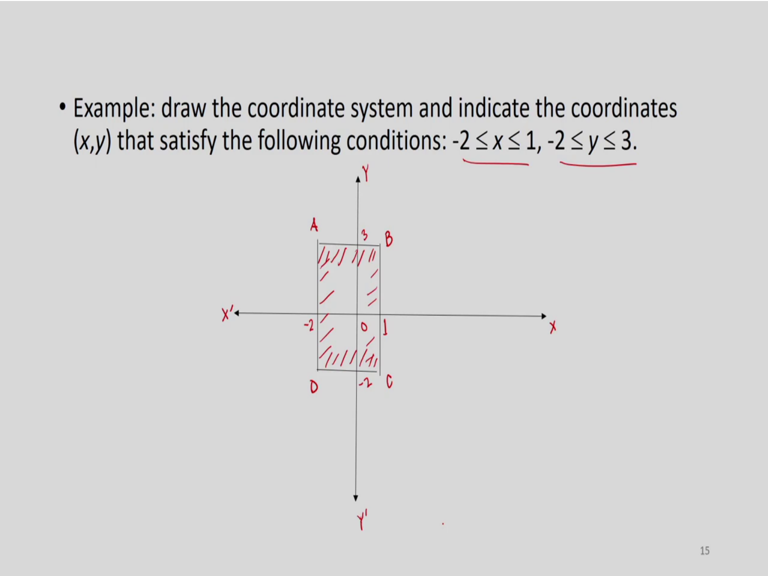


Here is an example, draw the coordinate system and indicate the coordinates x y that satisfy the following conditions $-2 \leq x \leq 1, -2 \leq y \leq 3$. So I have already drawn that, let me just add the name of that axis.

This is the horizontal is the x axis and the vertical is the y axis, in between we have the point of origin 0. Now, how do I write these things, x is lying between 1 and minus 2. Where is 1? Here is 1? Where is minus 2? Here is minus 2 and y here is 3, here is minus 2. So, x has to be less than 1. So, you are talking about this part, but at the same time it is more than minus 2.

So, to the right of minus 2 to the left of 1. So, you have basically a cylindrical kind of shape, vertical cylindrical, but at the same time, vertically also it cannot go up till infinity and minus infinity, vertically also there are limits and that is given by the second condition that y has to be less than 3. So, here is 3. So, y has to be less than that and from below y has to be more than minus 2, more than equal to minus 2. So, you are talking about this.

So, at the end of the day, what you are getting by these two conditions is a rectangular kind of shape. So, we have indicated the coordinates x, y that is satisfying the following conditions. So, the answer is that if you denote this rectangle by A, B, C, D, then all x and y, which lies inside A, B, C, D, as well as on its edges will satisfy these two conditions together.

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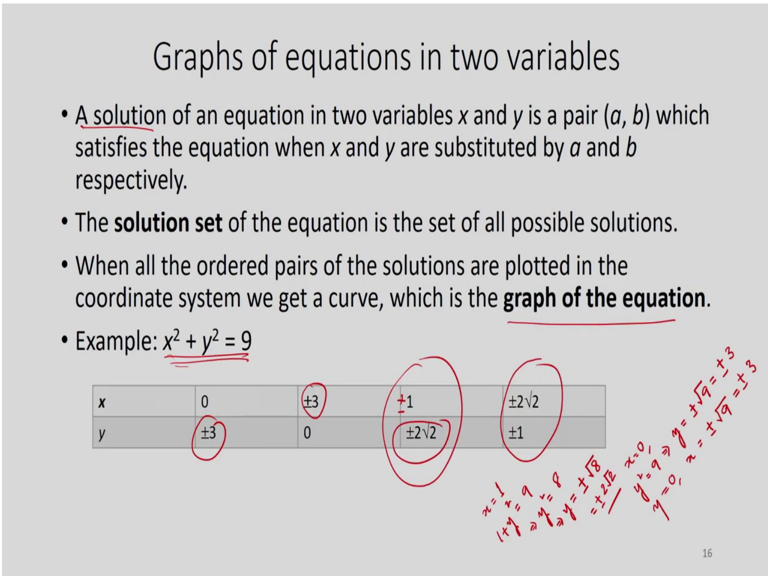


Graphs or equations, what I have done so far, I have just introduced what is a coordinate system. How to work with coordinate systems. Now, I am coming to the graphs, graphs of equations, let us first talk about that, graphs of equations in two variables. A solution of an equation, this is important, what is the solution of an equation? In two variables x and y is a pair (a,b) which satisfies the equation, when x and y are substituted by a and b respectively.

So, this is the definition of a solution of an equation. So, suppose you have any equation and what is the solution of that equation of two variables x and y, what is the solution of that equation, if I substitute x by a and y by b and that equation is satisfied, the left hand side is equal to the right hand side, then that a and b is called a solution.

So, a solution of an equation of two variables will have a pair of numbers like (a,b). The solution set of the equation is the set of all possible solutions. So, there will be many solutions of an equation, because many pair of numbers will satisfy a particular equation. So, if I put all of them together, all the solutions together, then what I get is a set, a set of pairs of numbers, that set is called the solution set.

When all the ordered pairs of the solutions are plotted in the coordinate system, we get a curve, which is the graph of the equation. So, that is the definition of the graph of any equation. First, we talk about solutions and then we talk about set of solutions or solution set, then we talk about plotting the solution set, in a coordinate system.

After we have plotted the solution set in a coordinate system, we get a graph and that is called the graph of the equation. So, here is an example $x^2 + y^2 = 9$. So, this is an equation of two variables x and y, $x^2 + y^2 = 9$. Now let us try to find out what could be the solutions of this equation. So, here are some solutions. So, if you put $x = 0$, then what do you get? $y^2 = 9$ that implies $y = \pm\sqrt{9} = \pm 3$. So, that is what I have written here.

Similarly if you put $y = 0$, you will get $x = \pm\sqrt{9} = \pm 3$, so that is what I have written here. So, here from these two columns, actually you are getting how many solutions? Four solutions. Suppose instead of 0, if you put $x = 1$, then what do you get, $1 + y^2 = 9, so\, y^2 = 8, y = \sqrt{8} = 2\sqrt{2}$. That is what I have written here.

And finally, if you put $y = \pm 1$ also. So, here also if you put $x = \pm 1$, you will get the same solution. Here, I could have written this as this. So, you are getting this solutions, also with respect to plus 1, you are getting two solutions of y with respect to minus 1 value of x, you get $+ 2\sqrt{2}$ and $- 2\sqrt{2}$ as the values of y. So, you are getting 4, from this particular column and here, also you are getting four solutions.

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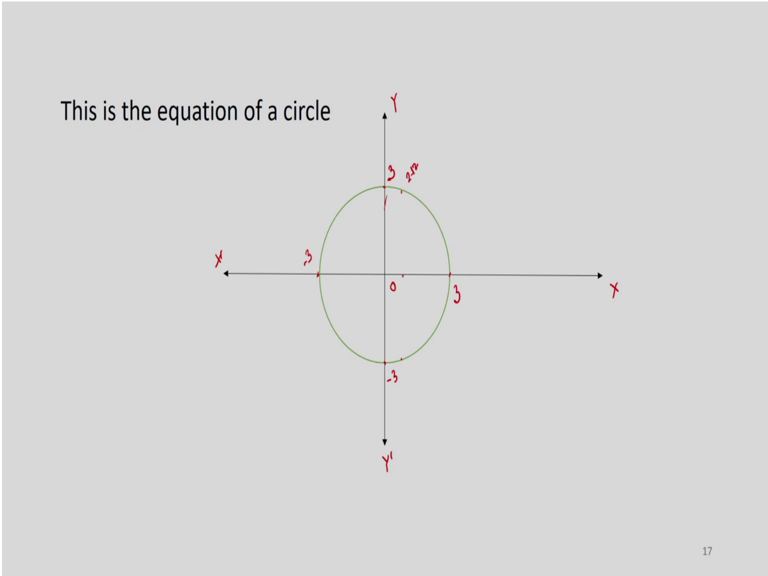


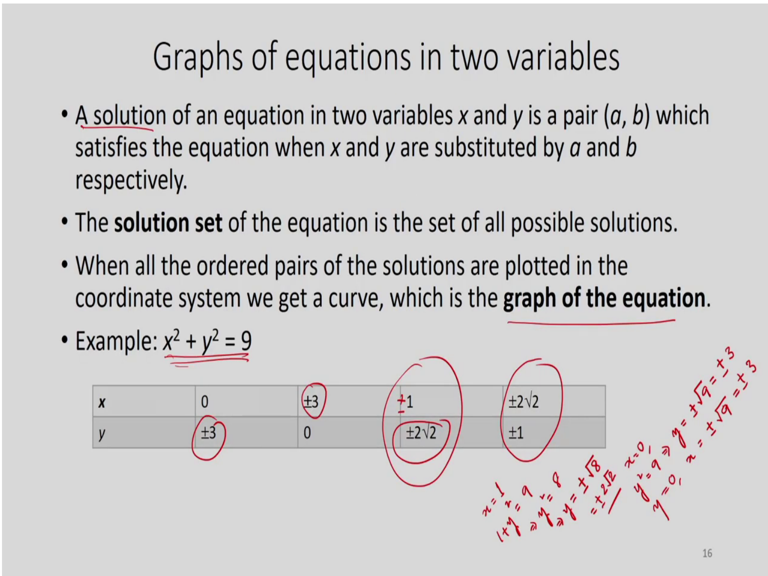


And if you plot all of them, what you are going to get is a circle and this circle has a radius of 1 and let us look at these values. I am sorry, the circle has a radius of 3. How do I know that, just look at this. If we put $x = 0$, y has two solutions, plus 3 and minus 3. So, $x = 0$.

So, along the y axis, $x = 0$, along the y axis, $x = 0$. So, what are the points you are getting on the y axis, you are getting this point, here $y = 3$ and this point here, $y =- 3$. So, these are the two points on the graph. So, these are the two solutions, we know the graph is the solution set, visual representation of the solution set, it is the line that we get after we plot the solution set.

So, that is why 2, 3 0 3 and 0, -3 are points on the graph. Similarly, what you have here is 0, 0, $\pm 3$, if you put $y = 0$ that means, along the x axis, you get either $x =+ 3$ or $x =- 3$. So, these are the 4 points it passes through. Moreover, you also pass through the point 1, 1 is somewhere here, $x = 1$, then y is equal to either this value or this value, these are $2\sqrt{2}$. So likewise, you get all the points and if you pass a line through all the points, then you get the graph of this equation.

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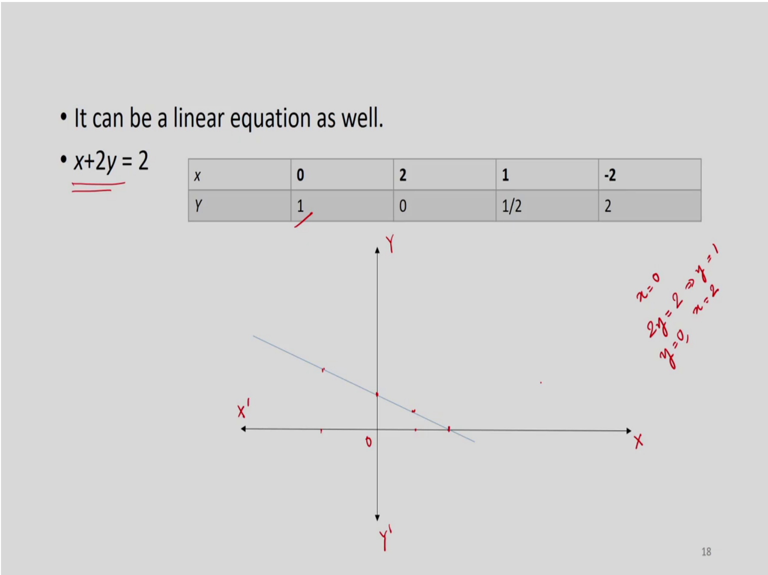


Now a circle, which was the graph of this equation, $x^2 + y^2 = 9$ is a nonlinear equation, you had square terms, you had x square and y square. So, you get basically curve, but it is not necessary that you always get a curve, if you plot the solution set. So, if the equation is like this, this called a linear equation $x + 2y = 2$ and the corresponding graph will not be a curve, then the corresponding graph will be a straight line.

Here, the speciality of this equation is that the powers of x and y, they are just 1. So, it is easy to see how you are getting a straight line. Suppose you put $x = 0$, what do you get? $2y = 2$, so $y = 1$. If you put $y = 0$, then $x = 2$.

So, let us see what these points are x 0 y 1. So, you are talking about this point. y 0 x 2, so, this is the point 2 0. $x = 1, y = 1/2$, so this is the point. $x =- 2, y = 2$. So somewhere here, minus 2, $y = 2$. So, you are getting all these four points and you can see all these four points actually lie on a straight line. It is a declining line. As x is rising, y is declining, so that is why I said it is a declining line. But nonetheless, it says straight line. So, this equation is called a linear equation.

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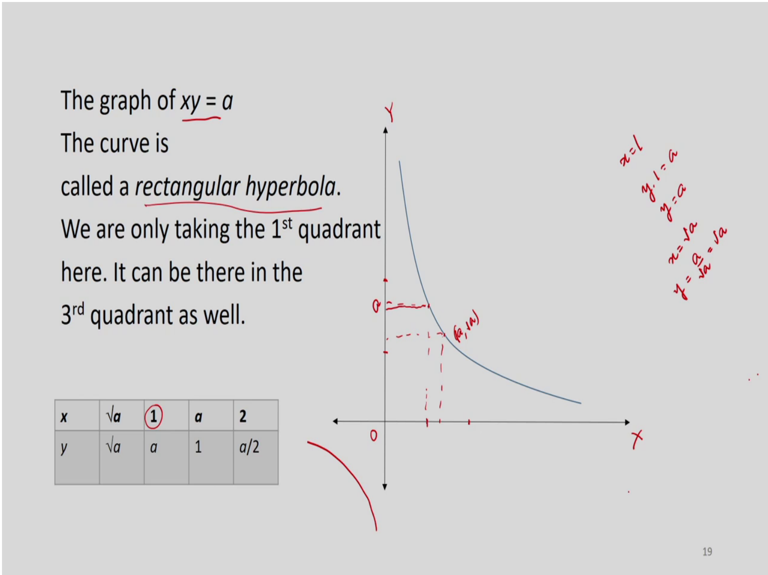


Let us take another example. Suppose I take $xy = a$. Now, the corresponding graph will not be linear in this case. Here x and y are getting multiplied and the graph will not be a straight line. The curve is called a rectangular hyperbola and the curve that I have drawn here is just one example.

I mean, there are two rectangular hyperbola, two graphs of this rectangular hyperbola, one is on the first quadrant, this is called the first quadrant another will be on the third quadrant. Because you take the value of x to be negative and the value of y to be negative. So, negative multiplied by negative will give you another positive number.

So, as long as that positive number is a, $xy = a$, then that x and that y, both of them are negative will be on the graph, but I am ignoring that third quadrant graph, I am concentrating on the first quadrant graph. Here, what are the points on the graph, if I take $x = 1$, then what do you get $1. y = a,\ so\ y = a.$

So, the point 1 a is going to lie on the graph, where is a? What is the value of a here? So, if I put $x = \sqrt{a}$, then you can see that $y = a/\sqrt{a}$= $\sqrt{a}$. So, here are the points values of this two variables are just equal. So, here is that point, particular point of ($\sqrt{a}$,$\sqrt{a}$).

Suppose 1 is somewhere here. These are the values of 1, 2, etc. So if $x = 1$, then $y = a$. So, what is the value of a here, this is the a. In this particular diagram, this is the value of a. So, this is how this graph looks like. It is not a straight line, it is a downward sloping line. That means as x is rising, y is going on declining in value and as I said, there are pairs, a pair of this graph, but I am just concentrating on the first quadrant graph.

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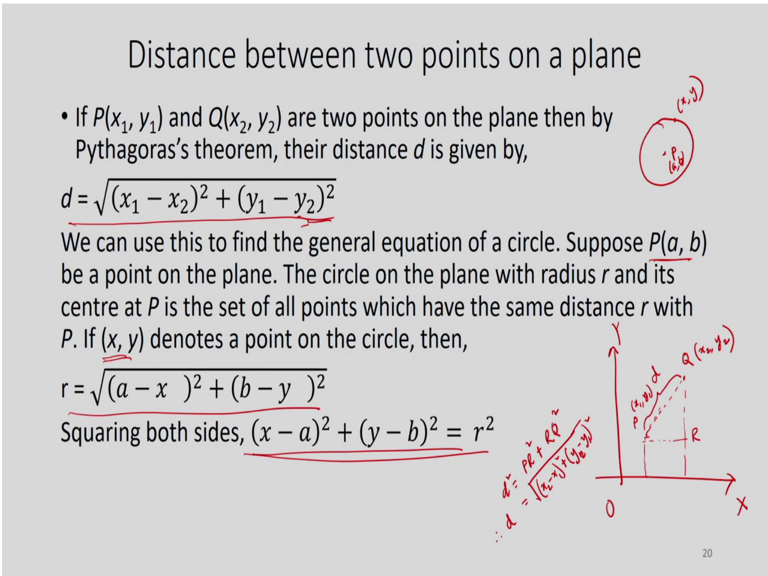


Now, I shall let to derive, what are known as linear functions, linear equations, how they are, what are their properties. I am just going to start with something which is called a distance between two points on a plane. This is again very basic, but let me just review this. Suppose you have two points, whose coordinates are $P(x_1, y_1)$and $Q(x_2, y_2)$, this two points are represented by P and Q and they lie on a plane, then by Pythagoras theorem, they are distance suppose d is given by this.

Let us see why this is so. So I am just talking about the coordinate system P and Q. The coordinates are $(x_2, y_2)$ and this is $(x_1, y_1)$. We want to find out how long is this? This is d. Now, Pythagoras theorem says what? I am just drawing the perpendiculars here, let us suppose this is R.

Now what do we know is that $d^2 = (PR)^2 + (RQ)^2$ and what is $(PR)^2$? $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$. So, that is how we are getting this

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. I wrote it as y 2 minus y 1 here it is written as y 1 minus y 2, but it is really does not matter because I can just flip these two numbers and since it is a square, so the same formula will apply.

So, distance between two points where the coordinates of these two points are given by $(x_1, y_1)$and $(x_2, y_2)$, the distance is given by $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. Now, we can use this to find the general equation of a circle. So, suppose you have a circle, what is the general equation of that circle, suppose P whose coordinates are a and b, P(a,b) be a point on the plane, the circle on the plane with radius r and its centre at P is the set of all points which have the same distance r with P.

So, what is a circle after all? Circle will be a curve on a plane with a radius r and it centre P such that the distance of all points on the circle should have same distance with the centre. So, if I take x y to be any point on the circle that is on the edge of the circle and here you have the centre.

So, here you have (a, b) and here you have (x, y), then the distance between (a, b) and (x, y) should always be equal to the radius. So, that is what I have written. This is the distance formula that I have used, $r = \sqrt{(a - x)^2 + (b - y)^2}$, R is the radius and if I square both sides to remove the square root sign, I get this as the equation of a circle, $(a - x)^2 + (b - y)^2 = r^2$. So, I think I shall end today's lecture here and we will pick up in the next lecture. What are the things that are left after this. Thank you.