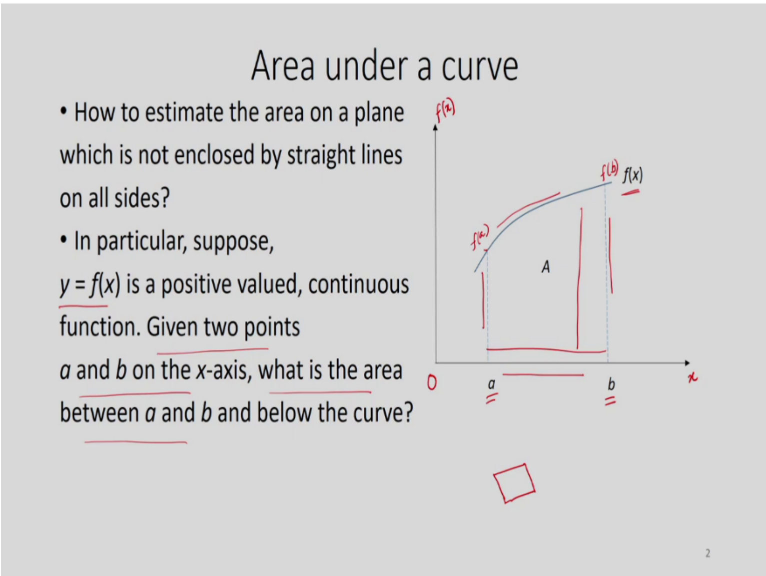
Mathematics for Economics – I
Professor. Debarshi Das
Humanities and Social Sciences Department
Indian Institute of Technology, Guwahati
Lecture No. 25
Area under a curve, indefinite integral

Welcome to another lecture of this course, Mathematics for Economics-Part I. So, in the last lecture, we have covered a topic called optimization with a single variable. We are going to start with a new topic today. It is integration. Now integration as we shall see, it is the opposite of differentiation that we have discussed earlier. And like differentiation, integration also is a very important tool of economists and people associated with financial analysis. So, today we are going to start with this new topic, introduce a few definitions and concepts and then we shall see what we can say about the applications.

(Refer Slide Time: 1:39)



As this is the first slide, you can see it on your screen. And first we are going to interpret integration as area under a curve so that is going to be the interpretation of what we mean by integration area under a curve. So, this is the first question that we are starting with “how to estimate the area on a plane which is not enclosed by straight lines on all sides”.

This is a very old problem, the Greek mathematicians dealt with this problem, so you can understand it is a very ancient problem that has been challenging the human mind. If you have an area which is enclosed by straight lines like this, it is relatively easy to say something about the area enclosed by these straight lines. But if you do not have straight lines on all sides, then it becomes a little bit difficult to estimate the area enclosed by those lines.

And there are practical implications of finding the area, which is enclosed by lines maybe straight, maybe non-straight, think about surveying the land. Different farmers might be having different plots of land. And those boundary lines may be straight lines, may not be straight lines but as a surveyor you might be interested to find out what are the areas that are owned by different farmers not only farmers, every landowner irrespective of being farmer or not a farmer, might be having lands and the administrators might be interested to find out the areas owned by different land owners, why?

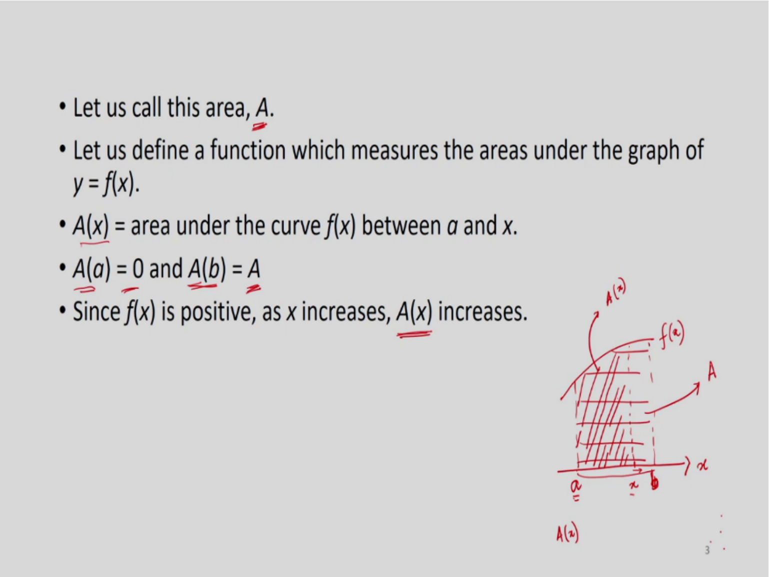
One reason could be the land taxes are imposed on the area of the land under your ownership. So as a surveyor, as an administrator you will be interested to know the area of land owned by different land owners. So, this question of finding the area enclosed by different lines has very important practical implications and that is why the ancient thinkers were grappling with this problem.

Here is the diagram that I have on the screen. You can see that I have a curve $f(x)$ and I have taken 2 points on the x-axis a and b. Now from point a, one can find out what is $f(a)$, this is $f(a)$ and this will be $f(b)$. So, in essence you have one side here, one side here and one side here, all these three are straight lines but on the top you do not have a straight line, you have the curve and this curve can take any shape.

Only thing we need is that this $y = f(x)$ curve or this function, it is positive valued. We are on the first quadrant so $f(x)$ is positively valued. It is continuous. It is a continuous function otherwise there will be gaps and there will not be a continuous line which will be enclosing an area. There will be no enclosure if you do not have a continuous line.

So, the question that we are asking is this, given 2 points, a and b on the x-axis, what is the area between a and b and below the curve? So, this and this below the curve between a and b this entire area, we may be interested to find out this area. So, that is the question that we are starting with and as we shall see integration actually is a tool which will give us an answer to this question. The area between 2 points and below a particular graph.

(Refer Slide Time: 6:52)



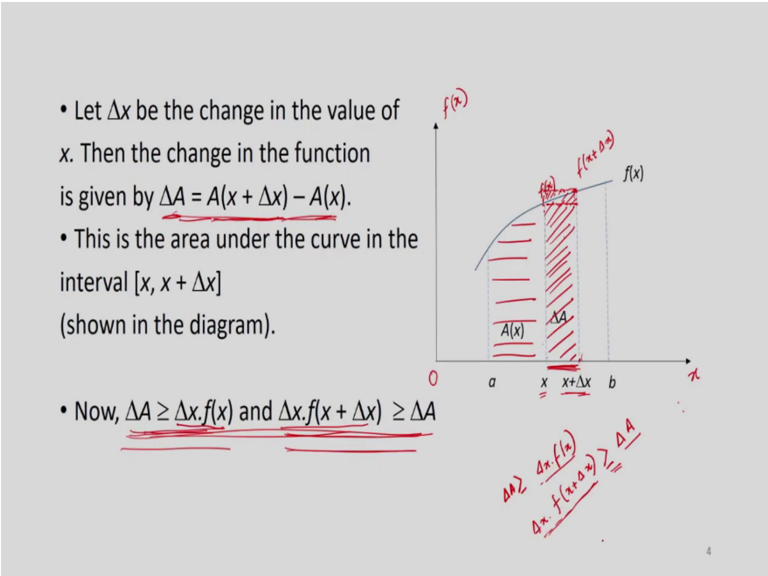
Let us start with an assumption. The assumption is that let us suppose this area is given by A. And let us define a function which measures the areas under the graph of $y = f(x)$ as follows. So, if you have a function $y = f(x)$ and this is the x-axis and let us suppose this is a, this is b and this is x-axis. You are defining something called $A(x)$. What is $A(x)$? Area under $f(x)$ between a, this point, and x. x is an arbitrary point. And this $x < b$ suppose x is here.

So this area, let me shade this area to demarcate it. This shaded area we are calling it as $A(x)$. So, that is what I have written here. $A(x)$ is the area under $f(x)$ between a, a is one point and x, x is another point and obviously $x > a$. Now from this definition of this area $A(x)$ obviously you take the 2 endpoints. If you take $A(a)$, that is you find out what is the area between small a and small a. Between small a and small a there is no area. It is 0. That is one extreme.

The other extreme is A of the endpoint here b, so $A(b) = A$. What is A? A was the area. So, the complete area was defined as A. So, pardon, the symbols that I have used there are repetition of the same letter. So, just to clarify between a and b the area under the curve is given by A and that is why if I take this function A. So, $A(x)$ is a function. If I take $A(b)$ then I get the entire area and that is equal to A. All right so these are the boundary points.

Since $f(x)$is positive, $f(x)$ is positive, that is we are in the first quadrant. It is a positive valued, as x increases this function also increases. The area that we are talking about as you are moving to the right, you are raising the x and since the function is positive valued so the area under the function will rise and therefore $A(x)$ increases so that is one important property. We have defined a function so sum total of this is that we have defined a function, $A(x)$ which basically measures the area under this particular function $f(x)$ between a and x.

(Refer Slide Time: 11:11)



Now we are going to play around with this function, $A(x)$. Let Δx be the change in the value of x. Then the change in the function is given by ΔA. ΔA will be the change in the area and that is given by the new area, which is $A(x + \Delta x) - A(x)$. So, what we are talking about is this extra area ΔA.

Just to clarify we started with this x point and we know $A(x)$ is this area. Let us use a different shade. This is $A(x)$. Now suppose x rises, x rises by how much, by Δx so the new value of x is this much $(x + \Delta x)$. And the new value of the function is how much, function means the A function, it will be $A(x + \Delta x)$, this shaded area plus this shaded area. That will be $A(x + \Delta x)$ because this is the new value of the independent variable.

Now we want to find out what this area is. This ΔA area, that is the increment in the area so that is ΔA and that is given by this expression, $\Delta A = A(x + \Delta x) - A(x)$ that is simple. You are taking the whole and then subtracting the left hand side shaded region. And then you are going to get this slanted shaded region, which is ΔA. This is the area under the curve in the interval x and $(x + \Delta x)$ that we have shown in the diagram. This ΔA is under the curve under $f(x)$ and between 2 values which are x and $(x + \Delta x)$. That is very clear.

Now these are some of the important relationships, so we have to understand that $\Delta A \geq \Delta x. f(x)$ so what is this saying? Remember what is ΔA? ΔA is this entire slanted shaded region, this is ΔA. What is being said here is this area will be greater than $\Delta x. f(x)$. That is what, which is being claimed.

Now what is $\Delta x. f(x)$? What is the meaning of this claim? Well, Δx is the base. This is Δx and what is $f(x)$, $f(x)$ is this height. So, when we are saying $\Delta x. f(x)$ what we have in mind is this rectangle, the area of this rectangle. This rectangle's area is given by $\Delta x. f(x)$. And we have in this example taken an increasing function so we are saying that ΔA which is the area under the curve that should be either greater than or equal to the area of this rectangle.

If the function is not an increasing function, if it is a flat function, think about horizontal line. In that case these 2 parts, the left hand side will be just equal to the right hand side but since we are considering a mildly increasing function, so we are writing this as a weak inequality. This is what this relationship is saying. ΔA which is the area under the curve in this particular interval A is greater than or equal to the area of the rectangle where the height is $f(x)$.

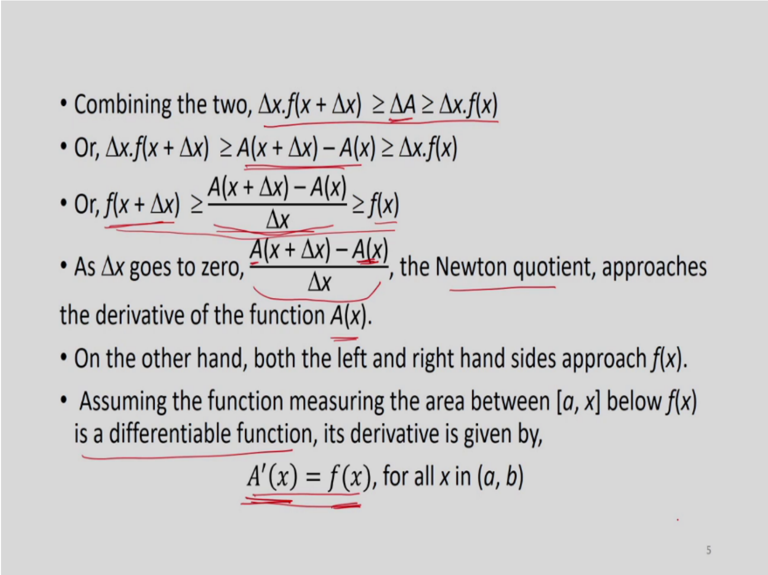
And you have another relationship, similar relationship I should say which is saying that $\Delta x. f(x + \Delta x)$ is greater than or equal to ΔA and this is just similar in the sense that, concentrate on the left hand side expression, Δx. What is Δx? Again the base of this rectangle and what is $f(x + \Delta x)$? It is this, this is $f(x + \Delta x)$ this height.

So, when you are writing this expression $\Delta x. f(x + \Delta x)$, what you have in mind is the area of this bigger rectangle. So, earlier the rectangle was, this rectangle now this one was the smaller rectangle. Now when you are expressing this, you are talking about a small block, which has been kept above the smaller rectangle and you have a longer rectangle.

And what is being said is this is greater than or equal to ΔA and that is obvious, what is ΔA? ΔA is area below the function so that is the right hand side but on the left hand side what you have, you have the area below the function but you have something extra, that is why this should be greater than or equal to ΔA, why equal to, well, greater than one can understand it is obvious, why equal to, again the reason has to do with the fact that this $f(x)$ function could be a horizontal line.

If it is a horizontal line, slope of 0, then you actually have an equality, you do not have any inequality. You have an equality and that is why to make that possibility felt here we have a greater than equal to sign here. So, these two things are correct, both these two things are correct and we can combine these two relationships in a single relationship actually because ΔA is common to both of them.

(Refer Slide Time: 18:50)



Combining these 2 we get this, $\Delta x. f(x + \Delta x) \geq \Delta A \geq \Delta x. f(x)$ just to make sure whether we have done it correctly. $\Delta x. f(x + \Delta x) \geq \Delta A$. Yes that is true here. It is coming from here and $\Delta A \geq \Delta x. f(x)$ that is also coming from here. So, both these things are correct. Therefore I can write this as a chain. This chain is saying that $\Delta x. f(x + \Delta x) \geq \Delta A \geq \Delta x. f(x)$.

Now I am going to use something which I have just found out earlier, that ΔA can be written as this, $A(x + \Delta x) - A(x)$ and this is something we have talked about earlier. I am just going to substitute this in this place. And next we divide all these 3 expressions by Δx. I can do that legitimately because $\Delta x \neq 0$. It is the increment. So, it is not equal to 0, so I can divide all these 3 terms with Δx and then I shall get this expression $f(x + \Delta x) \geq \frac{A(x+\Delta x)-A(x)}{\Delta x} \geq f(x)$.

Now I take the limit of Δx going to 0 in this entire relationship, in this chain. Now if I concentrate on the middle portion the second term, as Δx goes to 0 what happens to this just a notice what is this term, the middle term, this is nothing but the Newton quotient and as Δx goes to 0, I know what happens to Newton quotient, it becomes the derivative of this function, which function A, $A(x)$. So, it becomes $A^{'}(x)$ so $\frac{dA(x)}{dx}$ that is what it becomes the second term. That happens to the middle term.

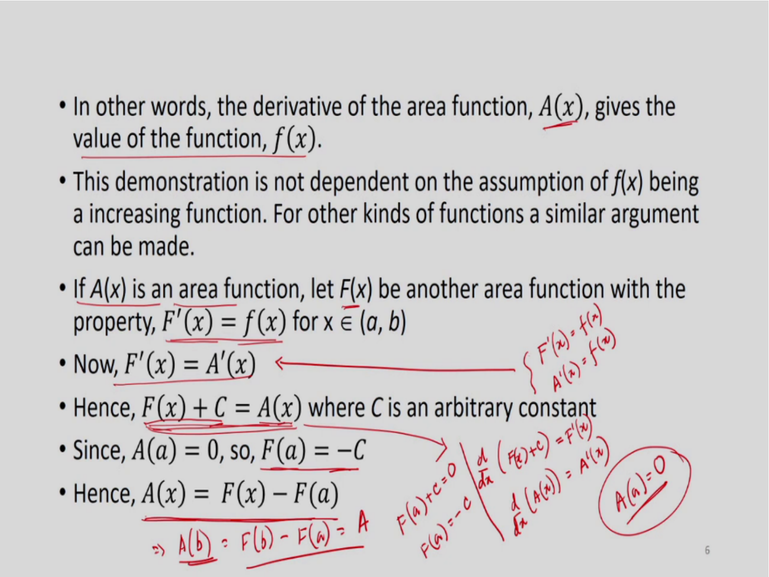
On the other hand, both the left and the right hand sides approach $f(x)$. This is obvious as Δx goes to 0, this term that is $f(x + \Delta x)$ will go to $f(x)$ and this is the first term. Whatever the third term, the third term is independent of Δx so it does not change at all so in a sense both the first term and the third term approach $f(x)$.

Whereas the middle term approaches $A^{'}(x)$. That is what is happening to the 3 terms. The first term and the third term approach the same value which is $f(x)$ and so assuming the function measuring the area between A and x below $f(x)$ is a differentiable function. That is important. So, I have to assume that $A(x)$ is a differentiable function otherwise I cannot take the derivative of it.

Now if it is differentiable, then the derivative of, then this term will become equal to $A^{'}(x)$ that is $A^{'}(x)$and that will be equal to $f(x)$ because $f(x)$ is what the first term and the last term are converging to approaching as Δx goes to 0. And this will be correct for all x in the interval a to b. Remember just to remind you x is always in this interval. It is not greater than b neither is it less than a.

And we have shown, this is a very important result. We have shown that as long as x is between a and b and $f(x)$ is a continuous function then there is this area function which is we are representing by $A(x)$. The derivative of that area with respect to any x between a and b that is $A^{'}(x)$ will be equal to the value of the function which is the $f(x)$, so that is the end result of this exercise. $A^{'}(x) = f(x)$. What is $A^{'}(x)$? The derivative of the area under the function, right under the $f(x)$ function, that derivative of the area is equal to $f(x)$.

(Refer Slide Time: 24:33)



In other words, the derivative of the area function that is $A(x)$ is the area function, derivative of this, gives the value of the function, which is $f(x)$. This demonstration that we have just done is not dependent on the assumption of $f(x)$ being an increasing function. For other kinds of functions a similar argument can be made. So, what is being claimed here is that in this demonstration we have drawn the $f(x)$ to be an increasing function. And then we have shown that $A^{'}(x) = f(x)$.

Now the questions that can be asked is that suppose $f(x)$ is not an increasing function, then can we claim the same thing, can this claim of $A^{'}(x) = f(x)$ being maintained if $f(x)$ is not an increasing function and the answer to that is actually yes even if $f(x)$ is not an increasing function, it can have other kinds of nature in those cases also one can show the same thing. The demonstration will be similar if not the same.

If $A(x)$ is an area function. Now we are talking about something a little bit complicated, suppose $A(x)$ is an area function, let $F(x)$ be another area function with the property that $F^{'}(x) = f(x)$ for all $x \in (a, b)$. Suppose there is another function also $F(x)$ for which the same relationship holds. Same relationship holds means remember for $A(x)$, $A^{'}(x) = f(x)$. But suppose there is another function $F(x)$ for which also $F^{'}(x) = f(x)$ for all $x \in (a, b)$.

Now if this is correct, so what you are essentially saying is that $F^{'}(x) = f(x)$ at the same time $A^{'}(x) = f(x)$. Therefore from these two I can conclude that $F^{'}(x) = A^{'}(x)$. Now

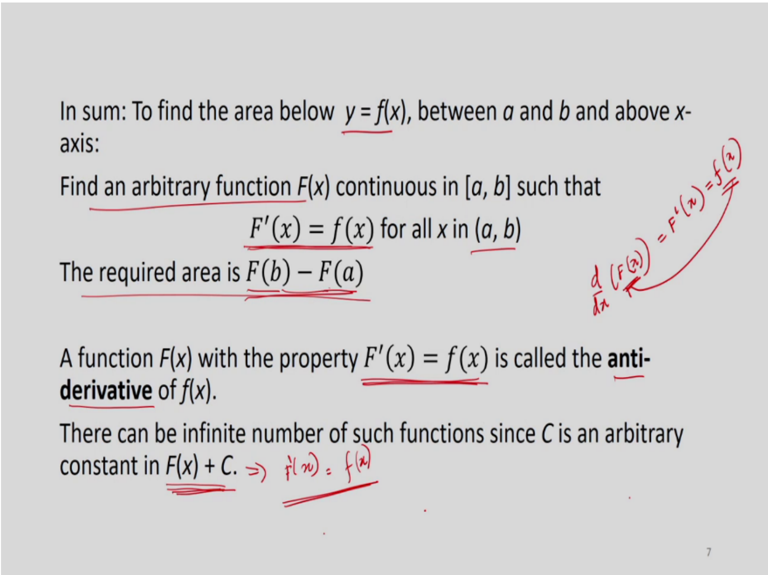
$F^{'}(x) = A^{'}(x)$,that means $F(x) + C = A(x)$ where C is an arbitrary constant. How am I saying this because you take this relationship and you take the derivative of this, what you do you get?

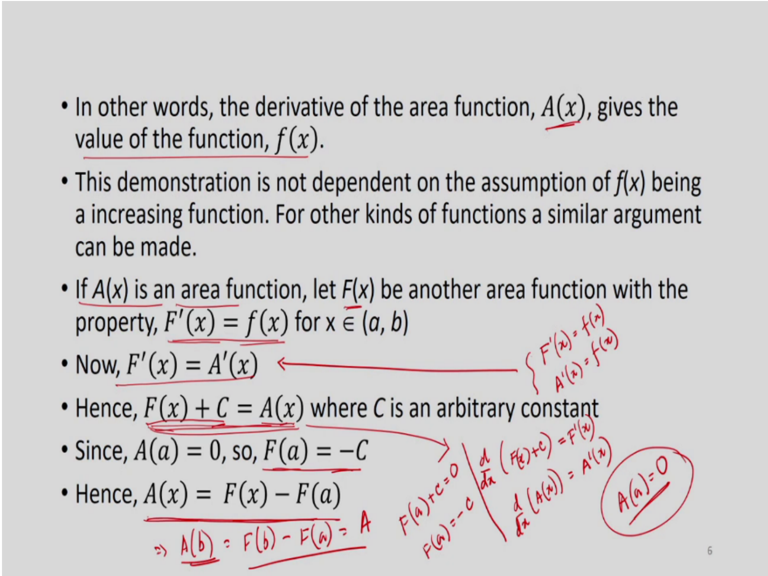
This will just become $F^{'}(x)$ because C will become 0 derivative of C is equal to 0. This is the left hand side. On the right hand side also you take the derivative of $A(x)$ it becomes $A^{'}(x)$. So, $F^{'}(x) = A^{'}(x)$ from that I can actually write $F(x) + C = A(x)$.

Now put $x = a$. If you put small $x = a$ then on the right hand side I know what is $A(a)$, it is equal to 0, $A(a) = 0$ that is how, A this function has been defined. So, therefore $F(a) = \; - C$ because $F(a) + C = 0$therefore, $F(a) =- C$. So, $F(a) = \; - C$.

And therefore I can write $F(x) - F(a) = A(x)$. I have just substituted. In case of C, I have substituted $- F(a)$ in this relationship therefore I get $A(x) = F(x) - F(a)$. Now what is the implication of this relation?$A(x) = F(x) - F(a)$.

(Refer Slide Time: 29:30)





What it means is the following. To find the area below $y = f(x)$ between a and b and above x-axis find an arbitrary function, $F(x)$ which is continuous in [a,b] and such that this relationship is maintained that $F^{'}(x) = f(x)$ for all $x \ in \ (a, b)$. So, this is the first thing that we found.

And there if I can find such a function then the required area will be $F(b) - F(a)$ that I have just seen here because remember $F(x)$ is a function which is such that $F^{'}(x) = f(x)$ and there is another function $A(x)$ which is defined in such a way that $A(a) = 0$ and using this function and using the relationship this, $F(x) + C = A(x)$, I have found out that $A(x) = F(x) - F(a)$ and from this I can get what $A(b) = F(b) - F(a)$ and what is $A(b)$ it is nothing but the area that I wanted to find.

And therefore capital A which is the required area is equal to, $A(b) = F(b) - F(a) = A$. That is what I have written here $F(b) - F(a)$. So, this is the algorithm. This is the important thing that we have to remember that if you want to find out the area below a particular function which is given to us continuous function $y = f(x)$ this function. Between a and b which are given to us, a's value is given, b's value is given.

Then firstly we find another function $F(x)$ such that $F^{'}(x) = f(x)$ for all x in this interval. And if I can find such a function $F(x)$ our task is done. We have to just evaluate this function at b, evaluate this function at a and take the difference that is it.

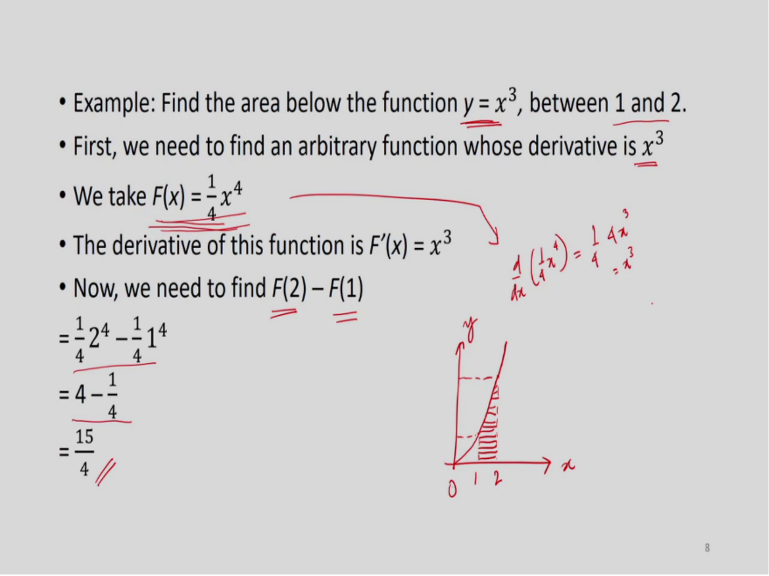
So, that is what this interpretation of an area under the function is telling us. This is called as we shall see this particular F function will be called the integral. The function $F(x)$ with the property $F^{'}(x) = f(x)$ is called the anti-derivative of $f(x)$ Why is it called anti-derivative?

Well if you have this $F^{'}(x) = f(x)$ and suppose how did I get $F^{'}(x)$, I took $\frac{dF(x)}{dx}$. I took $F(x)$ and take the derivative of that then I got $F^{'}(x)$ and that is equal to the function that we are given which is $f(x)$ so to get back to $F(x)$from $f(x)$ I have to do something, which is opposite, to take the derivative. I have to go from here to here so to do that I have to reverse the process of differentiation, that is why it is called the anti-derivative of $f(x)$.

Now interestingly there can be an infinite number of such functions. That is $f(x)$, $F(x)$ which satisfies the property, this property, there can be infinite number of such functions because think about the constant term, $F(x) + C$. C is the constant term. For all these functions, you take different values of C and you will get different functions from here. Now for all these Cs, if you take the derivative, then you will get $F^{'}(x)$ and that $F^{'}(x) = f(x)$.

So, I am not sure which C is the capital C. In fact there are an infinite number of Cs which will satisfy the same condition and all of them are valid. And later on we shall see that we can get around this problem by something called an initial value.

(Refer Slide Time: 34:40)



An example is given here how to find the area below a function between 2 particular values. Find the area below this function, which is given to us $y = x^{3}$. This is given to us, below this function and between the point 1 and 2. So, as we know what the algorithm was, we have to find another function such that the derivative of that function is equal to x^3 and then use that function. That is what we have to do.

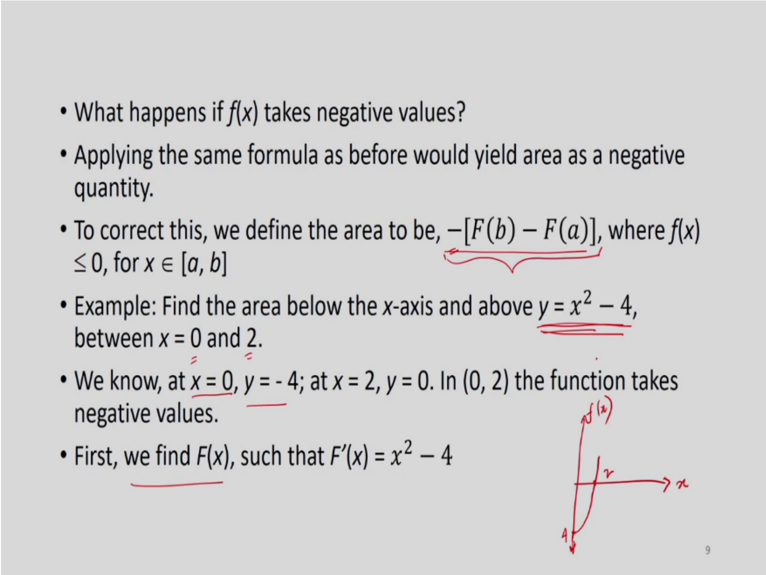
The derivative of the function that we find should be equal to x^3. And you can do some trial and error and you will ultimately find that if you take $F(x) = \frac{1}{4}x^4$ then the derivative of that, that is this function will be equal to x^3. You can just verify that, so you take the derivative of $F(x) = \frac{1}{4}x^4$, so you use the power rule it becomes $\frac{d}{dx}F(x) = \frac{1}{4}4x^3$and that is x^3. So that is the function which is given to us.

So, this is the function we want to find the anti-derivative of x^3 and the last task is quite simple. I have to evaluate this function at 2. This is $F(2)$ and from that I have to subtract the value of the function at 1, $F(1)$ and that is what we are doing here, that is $F(2) - F(1) = \frac{1}{4}2^4 - \frac{1}{4}1^4$. The values are 2 and 1 and there we simplify this a bit and we get $\frac{15}{4}$. So, just to make sure how this looks like.

So, here you have y, think about the shape of this function at $x = 0$. This function will give you 0 value. So, it passes through (0, 0) point. It does pass through the (1,1) point and then it becomes a convex function. It is always a convex function. It becomes steeper and steeper. So, here is your (1,1) and let us suppose this is 2 if you put $x = 0$, then $2^3 = 8$. So, here is 8 and here is 1. So, the area that we are talking about is this area. As you can see it is not an area enclosed by only straight lines.

On the top you have a curve and on the three sides you have straight lines but fear not we can actually find out the area by this method and it comes out to be $\frac{15}{4}$. So, what is the approximate value of that, this is a little bit less than 4 because had it been 16 divided by 4 it would have been 4. So, it is a value little bit less than 4, what obviously is greater than 3.

(Refer Slide Time: 38:12)



Now the important question that may arise here is, what happens if $f(x)$ takes negative values. What does it mean? It means that we are going into the fourth quadrant. So, you have $f(x)$ here and here is the fourth quadrant. Suppose $f(x)$ takes this kind of shape so it is close to the negative territory then does the same formula apply? Can we use the same method to find out the area?

Now what will happen if we apply the same method, it will give the area as a negative quantity. So, if we want to find out, suppose this area between this point and this point, this is the area. Now if you blindly follow the same method as before you are going to get a negative quantity. Now to correct this what we define the area to be is the negative of what we thought would be the area by our previous formula.

So, $-\ [F(b)\ -\ F(a)]$. This is the method that we should apply because if we do not add this negative sign then the area will come out to be negative, which is absurd. So, we apply this thing, this formula where $f(x)$ can be negative between a and b. So, here is an example of how we do it in practical terms. Find the area below the x-axis and above this function $y = x^2 - 4$ and between 0 and 2, $x = 0$ and $x = 2$. So a and b are given, $f(x)$is given. Now we have to just find the area.

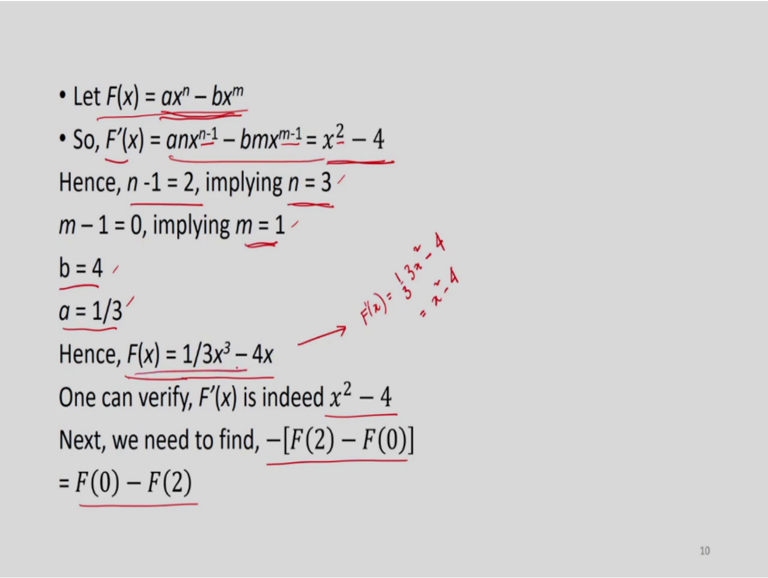
Let us think about this function a bit to see how it looks like. We know at $x = 0$, what is the value of the function if you put $x = 0$ it will give you -4. If you put $x = 2$, it will give you

$y = 2^2 - 4 = 4 - 4 = 0$. So, how does it look like 0, -4 and 2, 0. 2 is somewhere here, let us suppose 2 and so this function is going to go through this point and this point.

How am I sure that is going to be a convex function? I have drawn a convex function. How do I know that it is a convex function? Well you just take the second derivative of this function. Second derivative is 2, so 2 is a positive number. So, it is a convex function. It is a convex function and it passes through these two points, 0, -4 and 2, 0.

So, this basically the main point that I am trying to make is that this basically goes to negative territories so I have to apply this formula rather than the formula that we have talked about before. Now in this new formula or the new method like the old method I have to first find out $F(x)$ such that $F'(x) = x^2 - 4$.

(Refer Slide Time: 41:55)



So, how do I do that? I assume a particular form. $F(x)$ is equal to let us suppose $F(x) = ax^n - bx^m$. You see it is a very general polynomial form. I do not know the value of a, n, b and m. I have to find those values. What I know is that if I take the derivative of this function then I will get $x^2 - 4$ which is the given function to us, $x^2 - 4$, that I know.

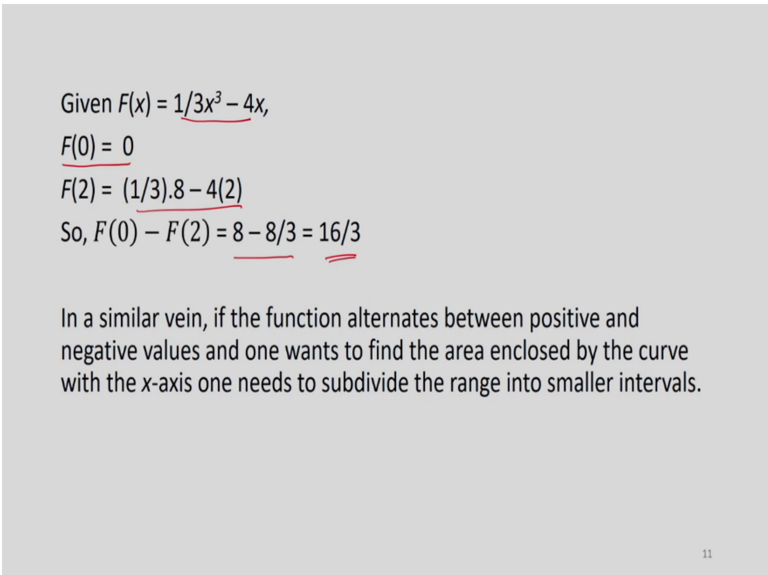
Now from the assumed form which is $F(x) = ax^n - bx^m$. If I take the derivative of that that is $F'(x)$, I will get $F'(x) = anx^{n-1} - bmx^{m-1} = x^2 - 4$. Now the next task is relatively easy, I just have to compare the coefficients and the powers. Now if I compare the coefficients then $n - 1 = 2$. This will straight away give me the value of n, $n = 3$.

And I also know that x^{m-1}, here the power of x is 0, so $m - 1 = 0$. x^0 is because there is no x here. So, $m = 1$ so n and m are found out and using that I can now find out what is b because $-b.1 = -4$, so $b = 4$ and $a.3 = 1$ which means $a = 1/3$.

So, everything is now clear to us. I have found out n, I have found out m, I have found out b, I have found out a. Now I substitute them back to the assumed form, I will get this function. So, it is $F(x) = \frac{1}{3}x^3 - 4x$ and one can actually verify that if I take the derivative of this, what do I get? It becomes $F'(x) = \frac{1}{3}3x^2 - 4 = x^2 - 4$. And that is what is given as $f(x)$ so we are on the right track.

Now there comes the last part which is I have found out $F(x)$ but since the values are negative, value $f(x)$ is negative, then I basically take the $-[F(b) - F(a)] = -[F(2) - F(0)]$. If multiplied by the negative sign, then I get $[F(0) - F(2)]$ and now I use this particular form that I have found out.

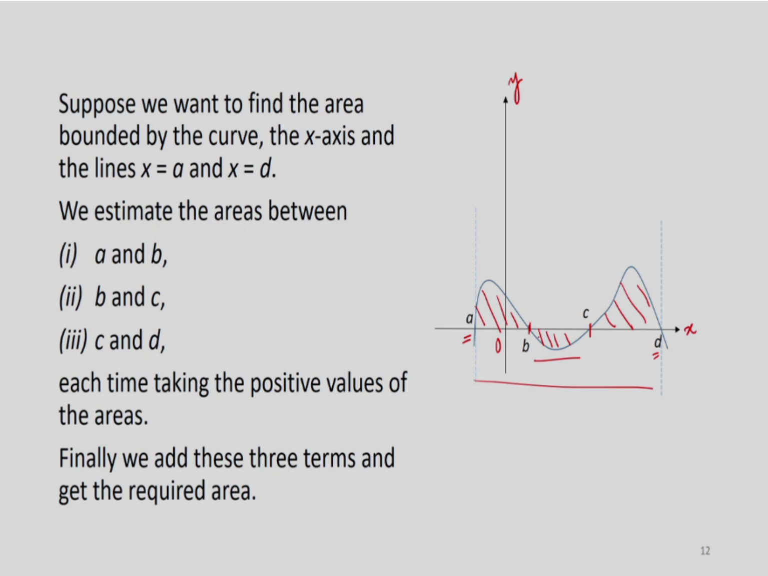
(Refer Slide Time: 45:19)



And if I do so, then I know $F(0) = 0$. This will give me 0. What about $F(2) = (\frac{1}{3}).8 - 4(2) = 8 - \frac{8}{3} = \frac{16}{3}$. So, that is the area. Let us see how, if it makes sense or not, $\frac{16}{3}$, what is the value of $\frac{16}{3}$, it is a little bit over 5.

We are talking about this area. So, what is being said is that the area under this curve is $\frac{16}{3}$. In a similar vein if the function alternates between positive and negative values and one wants to find the area enclosed by the curve with the x-axis one needs to subdivide the range into smaller intervals.

(Refer Slide Time: 46:47)



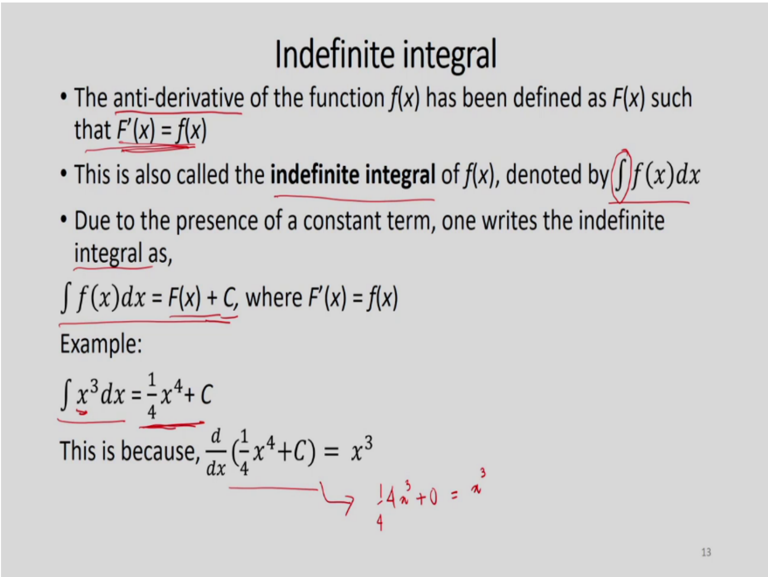
So, suppose the function is such that it snakes around the x-axis like this that is it goes to the first quadrant and it can come back to the fourth quadrant etcetera, etcetera, then suppose you want to find the area this, this summation of all this then what do you do? Suppose we want to find the area bounded by the curve the x-axis and the lines $x = a$ and $x = d$. So, here is $x = a$ and here is $x = d$.

You want to find out what is the area enclosed by the function with the x-axis, that is the area between the function and the x-axis and that is the vertical stretch. Horizontally, the stretch it is from a to d, so the shaded regions that I have demarcated on the diagram those regions, the area of those regions have to be found out

Now here what we do is something quite similar to what we have done before. We subdivide the entire range into smaller portions. So, we are basically first finding out those points where the function is intersecting with the x-axis. Here there are 2 points of intersection b and c. So, between a and b the function is taking a positive value and between c and d also the function is taking a positive value.

So, for those cases we can use the former formula that is $[F(b) - F(a)]$. But between b and c the function is taking a negative value there we have to take that formula which was applied when the function takes negative value which is $-[F(b) - F(a)]$. So, basically what one is saying is that we have to subdivide the entire range according to the points of intersection. And finally, when we have found out this different area separately then we add up these areas and we are going to get the final result. So, this is the method.

(Refer Slide Time: 49:24)



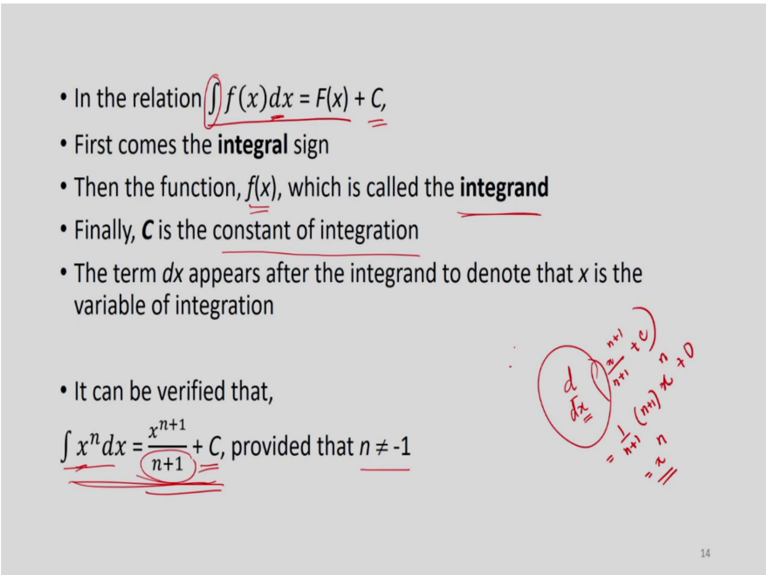
So far what we have done is that we have not used the word integration or integral. We have talked about area under the curve. Right now we are going to start with this idea of integral so we are going to use this term. The anti-derivative we have talked about anti-derivative. This $F(x)$ was the anti-derivative of $f(x)$. The anti-derivative of the function $f(x)$ has been defined as $F(x)$ such that $F'(x) = f(x)$.

Now this anti-derivative is also called the indefinite integral of $f(x)$ and it is denoted by this sign, $\int f(x)dx$. So, this is the integration sign. It looks like the letter S in English but sort of elongated S and then you have $f(x)$ *and* dx. This is called the indefinite integral of $f(x)$. Now remember if this is true if $F'(x) = f(x)$ then there are a host of functions. For all these functions in that group this will be satisfied because you can just add a constant term that arbitrary constant term will give you different functions but for all those functions this property will be satisfied.

So, due to the presence of a constant term, one writes the indefinite integral as follows. This is what one writes: $\int f(x)dx = F(x) + C$. So, $F(x)$ is the anti-derivative of $f(x)$ but $F(x)$ is not the function that one is looking at because there could be a plus constant term. So, here is an example so $\int x^3 dx = \frac{1}{4}x^4 + C$. Why? Because if I take the differentiation of this function on the right hand side, I will get x^3 that can be easily verified.

You take the derivative of $\frac{1}{4}x^4 + C$, what you are going to get is $\frac{1}{4}4x^3 + 0$. So, that is what I was trying to tell that if you have the $\int f(x)dx = F(x) + C$ then that basically means that there is a constant term there and depending on the value of the constant term you can get different anti-derivatives.

(Refer Slide Time: 52:33)



So, this is, in this relationship $\int f(x)dx = F(x) + C$. What is the pattern that one is following when one is writing this integration, the indefinite integral. First comes the integral sign. We have talked about that. It looks like an elongated S. Actually it comes from the letter S, S stands for summation. Then the function $f(x)$ is occurring and this is called the integrand, the function which has to be integrated so it is called the integrand. And finally you have the C term on the right hand side which is called the constant of integration. This is the last term on the right hand side.

The dx term has to appear here. After the integral sign is there, in order to close this matter that dx has to appear. What is the importance of this dx term? It appears after the integrand to denote that x is the variable of integration. So, it is the x variable with which the integration is taking place. It is like the $\frac{d}{dx}$ sign. So in the case of differentiation, I use this $\frac{d}{dx}$sign and x. Why do I write x? Because with respect to x the differentiation is taking place. Here also the integration when we do it, we have to mention with respect to which variable the integration is taking place. So, that is why x is the variable of integration.

Well, now before we close, I just noticed one small formula here which is that if you take any function of the form x^n, sort of power is there on the variable and this power is just an arbitrary power n then it can be verified if you take the integral and then you are going to get this form, $\int x^n dx = \frac{x^n+1}{n+1} + C$and obviously there is this constant of integration provided that $n \neq 1$because if $n = 1$ then the denominator becomes 0 and this term becomes undefined.

So, we have to be careful that $n \neq 1$ and it can be verified easily if you take the $\frac{d}{dx}(\frac{x^n+1}{n+1}) + C$. Let us take the derivative of this. $\frac{d}{dx}(\frac{x^n+1}{n+1}) + C = \frac{1}{n+1}(n + 1)x^n + 0 = x^n$which is there as the integrand. So, this formula is correct. So, this is like the power rule formula of integration. If you take an arbitrary function, x^n, if you want to take the indefinite integral of that, then you get $\int x^n dx = \frac{x^n+1}{n+1} + C$, C is the constant of integration.

Let me stop here. I am going to talk about more such tools of integration in the next lecture. Please join me then. Thank you.