Mathematics for Economics-I

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Lecture No. 02

Logic, Proof

Welcome to the second lecture of this course, called mathematics for economics part 1. My name is Debarshi Das. So, we have already had the first lecture of this course, we are going to start with the second lecture of this course. So, we are going through the topic called the preliminaries right now, in the last lecture we covered real numbers. And we talked about symbols.

And within the numbers, we talked about what are called the rational numbers and irrational numbers, we said that, if we put together all the rational and irrational numbers together, then we get a group of numbers, and the group of numbers has a one to one correspondence with the points on the real number line.

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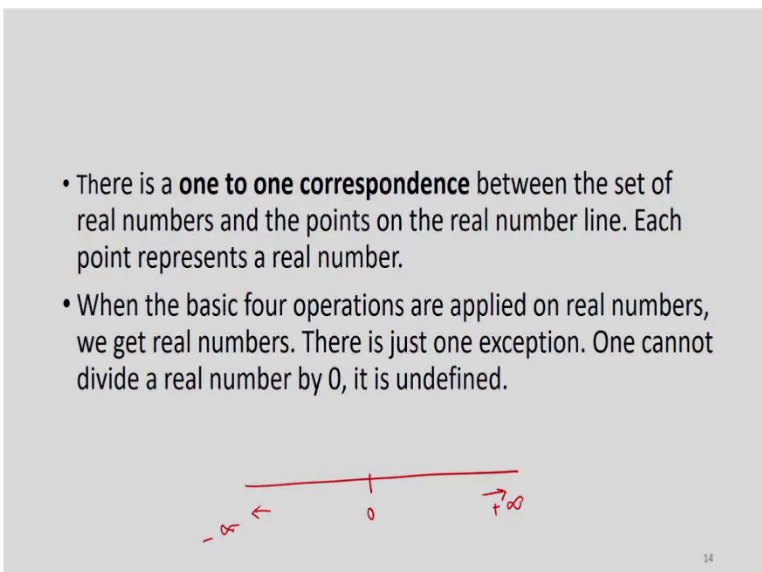

So, just imagine a straight line, horizontal straight line and that line has a point somewhere here, which is called the 0, like the origin. And so, on the left hand and on the right hand, you extend the line towards infinity. So, plus infinity and minus infinity. So, this line is called the real number line.

So, the main point that has to be paid attention to is that, each point on this line corresponds to some real number, that number could be rational, it could be irrational, but it is, has to be a real number. And that real number corresponds to some point on this horizontal line, which is called the real number line.

And we also said that, if we perform the four operations, that is addition, subtraction, multiplication and division on the set of real numbers, then we only get real numbers, we do not get out of the set of real numbers. Next, we talk about some other basic facets of mathematics, we talk about what are known as inequalities.

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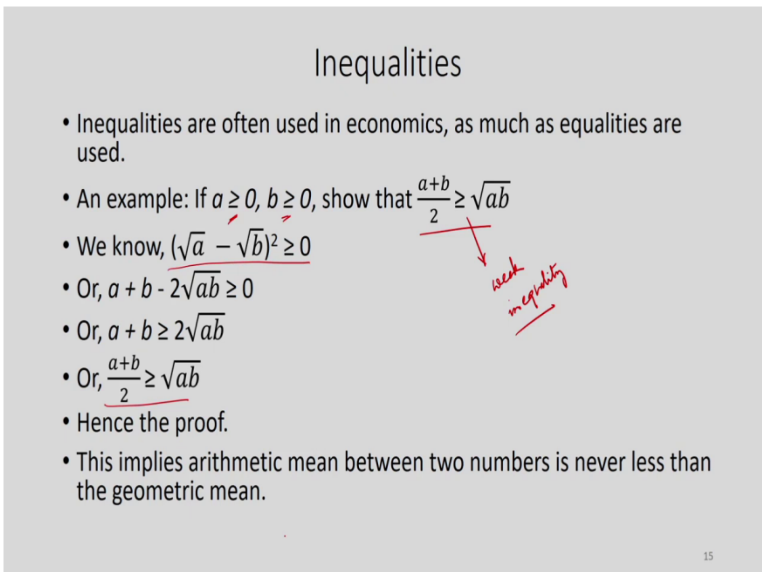

So, inequalities, what are they? They are the negation of equalities in mathematics as well as an, in economics equalities are used, but inequalities are also used. So, here is an example. So, suppose a is any, any number, $a \geq 0$ and $b \geq 0$, this relations are inequality relations. So, they do not represents equality on the left hand side and right hand side, rather they represent some sort of non equality.

Here, the non equality is saying that the left hand side quantity, which is a and b, they can be equal to 0, but they can be greater than 0 also. So, how to show that $\frac{a+b}{2} \geq \sqrt{ab}$. So, this we have to show, this one. So, we start with the basic idea, which is that if you take the square of any number, then the square is a non negative quantity. And so this is greater than or equal to 0. So, here also we are using the idea of inequality.

So, $(\sqrt{a} - \sqrt{b})^2 \geq 0$. Then we use the formula of whole square, if you have two numbers and one minus other, and we take the whole square of that, then we get the first number square, which is $a + b - 2\sqrt{ab} \geq 0$. So, that is the left hand side, left hand side has to be greater than or equal to 0.

And then we simplify this and ultimately we get this result, that we were supposed to derive, that a plus b divided by 2 is greater than or equal to root over ab. So, we started, to go back to the beginning, we started with some inequality greater than or equal to inequality. This is called a weak inequality. So, we started with weak inequality and we got in the result another weak inequality.

And this result is telling us a fundamental property, which is that arithmetic mean of two numbers, which is there on the left hand side $\frac{a+b}{2} \geq 0$, this is the arithmetic mean of a and b, it has to be greater than or equal to the geometric mean. So, $\sqrt{ab}$ is called the geometric mean of these two numbers. So, arithmetic mean is never less than the geometric mean, the minimum value, the AM can take is equal to GM and it can be greater than the GM.

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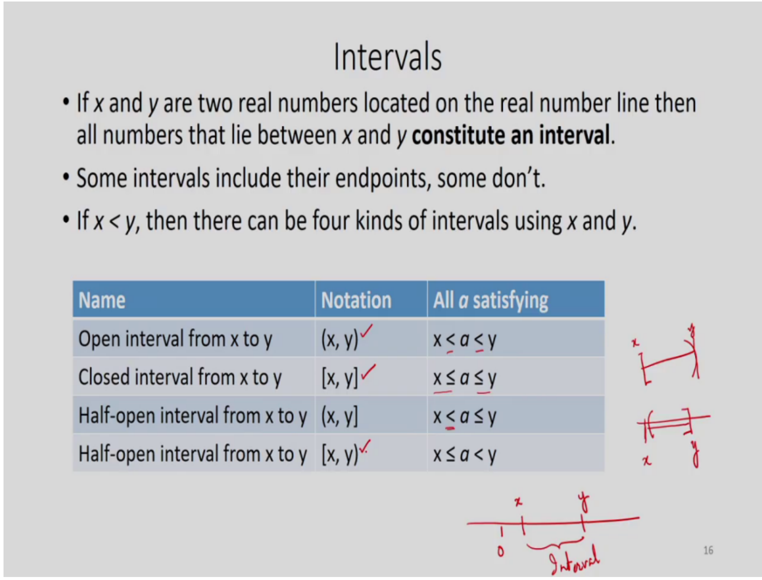

So, next we encounter another basic idea of mathematics, which is the idea of intervals. So, suppose x and y are two real numbers located on the real number line, then all the numbers that lie between x and y constitute an interval. So, think about, just imagine the real number line. Suppose, this is 0 here and here you have x and here you have y. Since, these are two real numbers, as we know by this property that we have seen before, all the real numbers can be plotted on the real number line.

So, suppose x is here, y is here and so, there are some numbers between x and y. So, if we put all these numbers together, then they constitute an interval. So, all the numbers which are between x and y, when we say some numbers between x and y, then we are being a little bit imprecise, because, even if we say they are between two numbers, there could be different ways of being, in between. So, here we are specifying the different ways of being in between, here we are talking about intervals. Some intervals include the endpoints.

So, suppose they include the, the first point that is x and the last point which is y. If they include the endpoints, then they are called closed intervals. So, like this one, closed interval $[x, y]$and this is denoted by this, the third bracket, this bracket. So, you have the start of this third bracket, then x comma the first number then comma then the second number and then end of bracket. And here you can see that, you have a less than equal to sign, equal to is there. So, that means x and y are included in the interval. So, these are called closed intervals. $[x, y]$ $x \leq a \leq y$

And the diametrically opposite case of this, is this one, open interval. (x, y) $x \leq a \leq y$. Here you use the first bracket sign and then the first number comma, the second number and close the brackets. Here the endpoints are not included. So, here you have strictly less than sign. So, you do not have the equality sign. So, these are the two basic kind of intervals, one is closed and the other is open, but there could be a mixture of these two and these are the two other cases, there could be, as a kind of mix of the first two cases. So, these are called half open intervals.

So, here you have the first bracket, here it is starting with the first bracket and is ending with the third bracket. So, first bracket means strict inequality, we know that. First bracket means strict inequality. So, that is where it is starting from. So, here what is happening is, if I have to give you the geometric idea, here is what is happening. Here you have a x and here you have y. So, x is not included, all the points are included. At the same time y is included. So, here you have the third bracket. So, this is called half open interval from x to y.

And this can be reversed also. So, here suppose you are including x and excluding y. So, here is y so, you are just stopping short of y. So, that will be this one, the last one, this will also be a case of half open interval from x to y. So, these are the cases of intervals.

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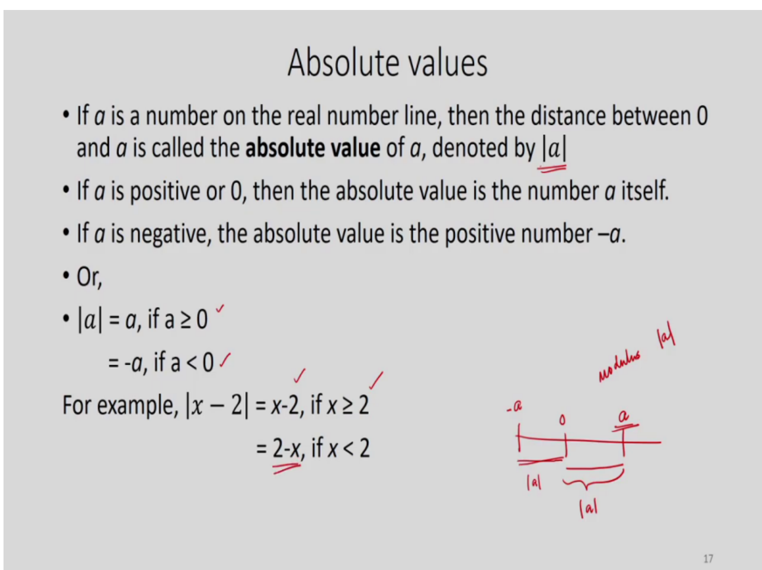

Next we talk about something called absolute values. So, what is an absolute value? Suppose a, small a is a number on the real number line, then the distance between 0 and small a is called the absolute value of a, and this is denoted by this sign, $|a|$,this is called a modulus, modulus, this sign. what I have written here, if I have to represent this geometrically, suppose 0 is here that is the origin, and you have a here.

Suppose a is positive, then this distance is called the modulus, this is modulus of a. So, if a is positive, then modulus of a is just a, because this gap is nothing but a. However, a could be negative also. So, here you have minus a suppose, then modulus of a will be this distance. So, here modulus of a will be this distance, but remember distance is always a positive quantity, distance cannot be negative.

So, if you have a to be negative, then the modulus of a will be negative of that negative which will give us a positive quantity. So, you have this case here, where a is negative and here you have a to be positive, here modulus of a is simply a. And we can generalize this a bit. For example, suppose we are not talking about a, which is particular variable, but we are talking about x minus 2, where x is a variable.

So, you have variable and then you have a logical constant. Then how do you represent this? So, what is modulus of x minus 2? So, it is nothing but the same thing, it is the distance between 0 and x minus 2. But again x minus 2 could be positive, or x minus 2 could be negative. So, x minus 2 is positive or equal to 0 also, that is also included. So, if x minus 2 is positive or 0, then we just write modulus of x minus 2 is equal to x minus 2, this is this case, first case.

On the other hand, if x minus 2 is negative, which basically means that x is less than 2, then I have to add a negative sign in front. And if I do that, so it becomes minus of x minus 2, minus of x minus 2 is 2 minus x. So, that is how we deal with, what are known as modulus or absolute values. And here are some other implications of this modulus.

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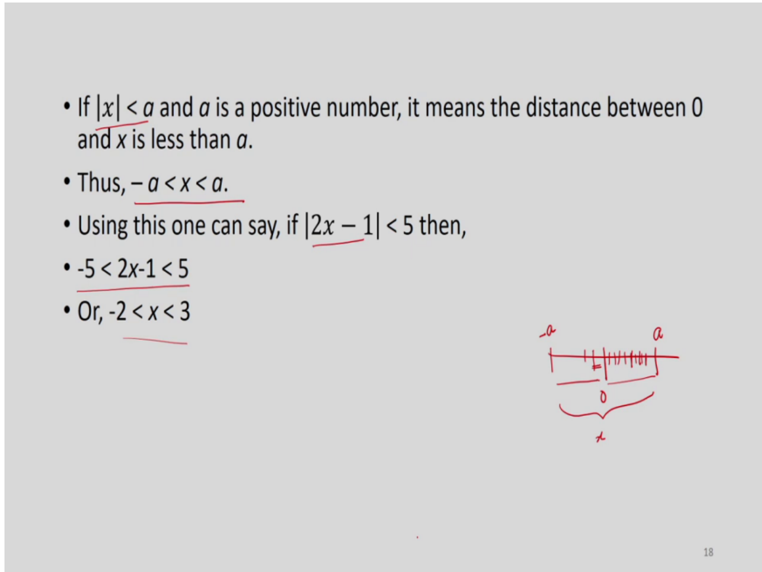

Suppose, you take a variable, which is x and you take the modulus of that, where x is a variable, it can take different values. On the other hand, a is a positive number, it is a logical constant. Suppose modulus of x is strictly less than a, then what does it mean? It means the distance between 0 and x, that is modulus of x is less than a, I am just using the definition.

So, here you have x, here you have a which is a positive number, then what one is saying that whatever be the value of x, if this is correct, if this is correct, then the distance between 0 and x has to be less than a and a is a positive number. So, what does that mean? It means x could be positive, all this values of x are permissible, but x cannot be equal to a or greater than a, because in that case, this condition will be violated.

On the other hand, x could be negative also. So, here also if I take x here, then the modulus of x becomes, what? This distance, and this distance we can see it is less than this big distance. So, x can take negative values also, but, there is a maximum to which, or rather it is a minimum to which x can go. And here is that minimum, it is minus a. So, x can go to negative values, but it cannot go below minus a, it cannot even go to minus a, it has to stop before minus a.

So, this is the range of x. And this is the same thing that is written here. So, x has to lie within this range, it has to be strictly less than a, but it is strictly greater than minus a. The same thing can be generalized a bit, suppose instead of x I take an expression which contains x. So, you have 2x minus 1 and modulus of this expression. And suppose this modulus is, this absolute value, a is less than 5, strictly less than 5.

And what does it mean? I use the same idea as before. So, that basically means that $-5 < 2x - 1 < 5$ and if I simplify this, it becomes this one. It becomes a simpler kind of expression, that $-2 < x < 3$.

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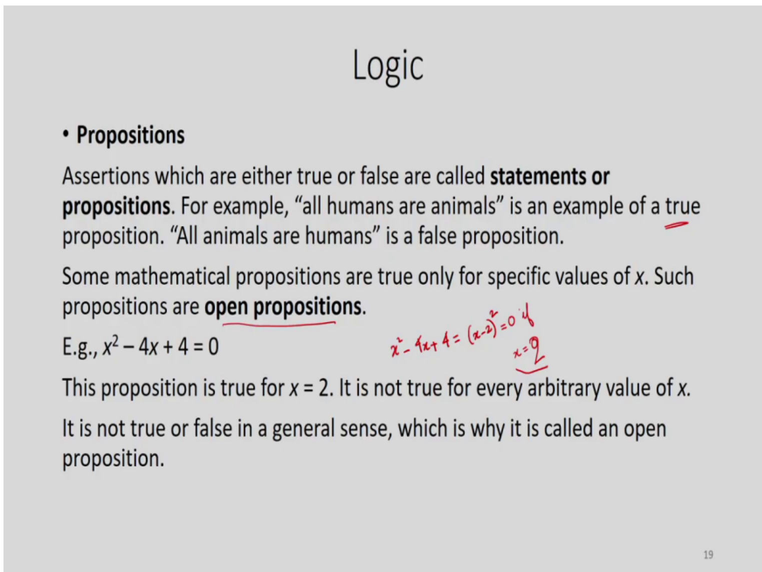

Now, with these things behind us now, we can go ahead and talk about some, a little bit more advanced things. We are going to talk about what is known as logic, few aspects of logic, because these aspects of logic will be used in mathematical proofs that we shall be dealing with. So, the first thing to note in the discussion of logic is the idea of what are known as propositions or statements.

So, assertions which are either true or false are called statements or propositions. So, for example, all humans are animals is an example of a proposition, but this proposition happens to be a true proposition. All humans are animals, it is a true proposition. In order to be human, obviously, the entity has to have life, and this is a true proposition, but if I just reverse this.

So, if I say all animals are humans, then that is also a proposition, it is asserting something but that proposition is not a true proposition, it is a, as we know it is a false proposition. So, there are two kinds of propositions to begin with, these are true and false propositions. Some mathematical propositions are there which are true only for specific values of this variable called x. So, these are also propositions, but they are true in some cases, not true always. These are called open propositions.

So, these are beyond the case of true or false propositions, these are open propositions. An example is given here, $x^2 - 4x + 4 = 0$. This is a proposition and it is an open proposition. Why? Because, if I take the left hand side, $x^2 - 4x + 4$, then if I simplify this, it becomes $(x - 2)^2$. And you can see that this $(x - 2)^2$ becomes 0 when? It becomes 0, if x is equal to 2.

So, this proposition is correct, if x is equal to 2. So, this proposition is correct, for some cases, some values of x, not always. So, it is not true for every arbitrary value of x. It is not a true or false in a general sense, which is why it is called an open proposition.

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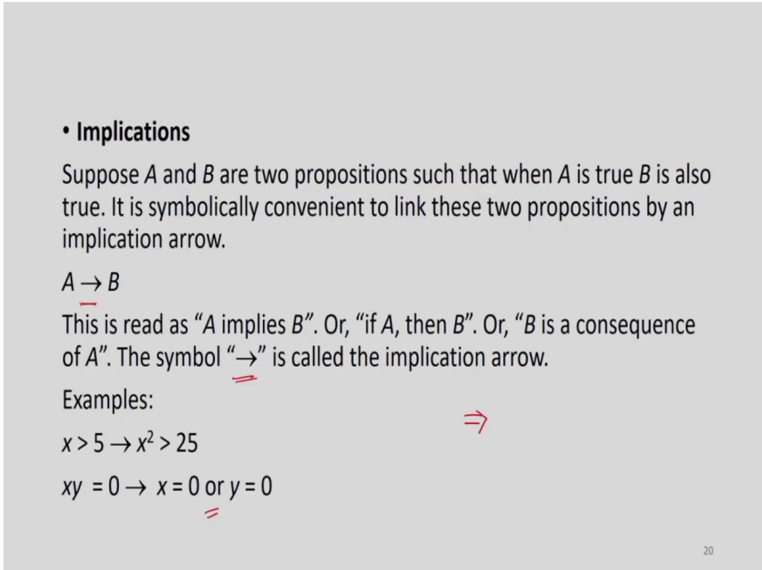

Implications- Suppose A and B are two propositions such that when A is true, B is also true, it is symbolically convenient to link these two propositions by an implication arrow, $A \rightarrow B$. So, this arrow is called an implication arrow. And in words, this symbol is read as A implies B, or it can be read as if A then B, or it can be read as B is a consequence of A. So, there are different ways to say the same thing. This symbol $\rightarrow$ is called the implication arrow.

It is sometimes also written as this symbol, $\Rightarrow$. So you have two parallel lines instead of one parallel line or just one line. Some examples are given here. So, on the left hand side, that is A you have $x > 5$and that implies, that $x^2 > 25$. But here I have to specify that x, that we are talking about, this x has to be a real number, and then another example is given below this. Suppose $xy = 0$, what does it imply?

It can imply that $x = 0$. If $x = 0$, then this will be correct, or $y = 0$. So, $xy = 0$means, either $x = 0$ or $y = 0$, when I say ‘or’, this or term may also includes the possibility that both x and y can be 0.

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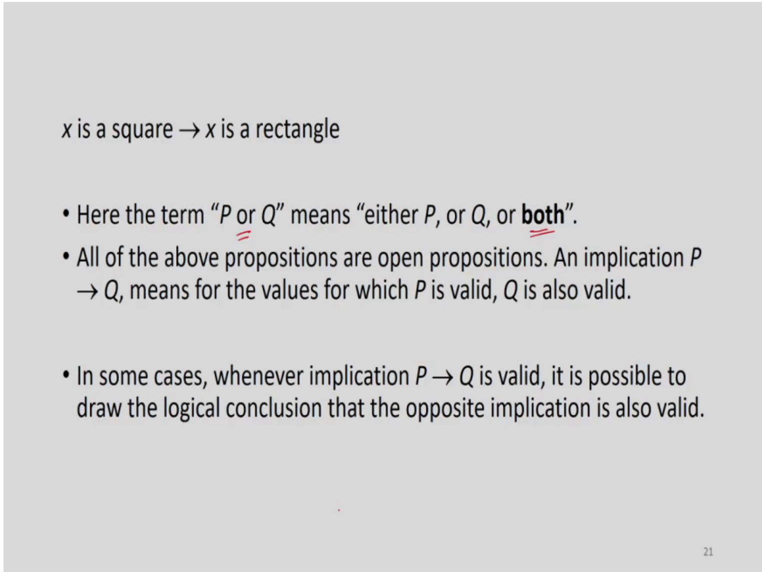

Another example is given here, x, x here is a figure, geometric figure. x is a square → that x is a rectangle. This basically invokes the idea that every square is a rectangle. A square is a rectangle, but it has some other properties, which are not satisfied by rectangles in general. So, this is special kind of rectangles. Here the term P or Q, which I used in the previous example, this x or, $x = 0$ or $y = 0$. P or Q means either P or Q or both.

As I have just said, I mean this, this means that $x = 0$ that is one possibility or $y = 0$, but it also means that x and y both could be 0. All of the above propositions are open propositions. An implication P implies Q means for the values for which P is valid, Q is also valid, remember, what is an open proposition? That an open proposition is true for particular values of the variable x, which is included in that proposition.

So, when I say P →Q, then I mean that for those values of the variable for which P is correct, or P is true, for the same values of x, Q is also true. In some cases, whenever implication P →Q is valid, it is possible to draw the logical conclusion that the opposite of implication is also valid.

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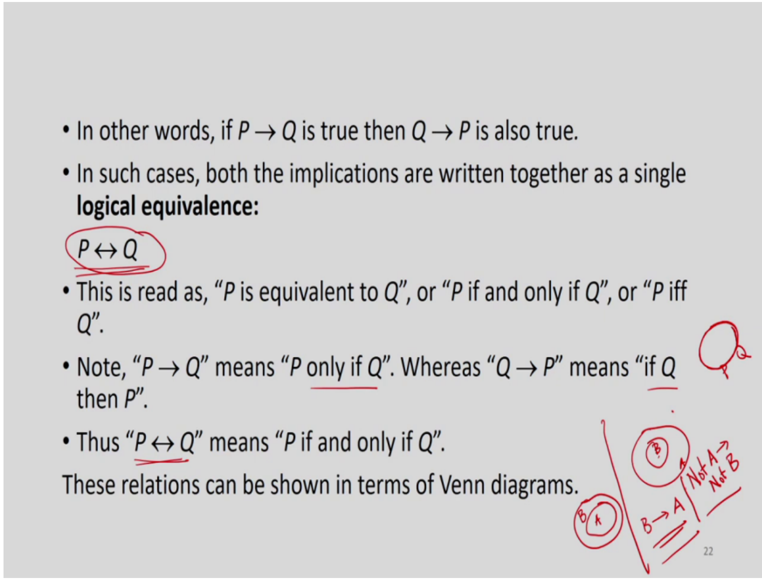

So, here is this. In other words, if P →Q is true, then Q →P is also true. So, this is a special kind of implication where, the arrow sign goes both ways from P it goes to Q and from Q also it comes back to P. In such cases, both the implications are written together as a single logical equivalence P↔Q. So, this is called a logical equivalence. This is read as P is equivalent to Q or P, if and only if Q or P iff Q.

So, here the left hand side implies the right hand side at the same time, the right hand side also implies the left hand side, these are the cases of logical equivalence. Note, P →Q means P only if Q. Whereas, Q→P means, this is the opposite thing, it means, if Q then P. Thus, P logical equivalence Q, P↔Q, that is if and only if, it means P if and only if Q. So, here we are combining these two, only if and if, both are combined together and then you have this logical equivalence thing.

This relations can be shown in terms of Venn diagram. The help of Venn diagram, it becomes very easy to show what is meant by the one sided implication or both sided implication. For example, suppose you have A, so, A is this big circle and B is this small circle. So, these are Venn diagrams as we know. Now here, B implies A because if you are here, right, suppose you pick up a point here within B, B itself is within A. So, that necessarily means that, that point is within A also.

So, here if we have this sort of diagram, then B→A, this sort of diagram. On the other hand, when do you have A implies B, it is just the opposite. So, here it is possible, I am just giving an example, that A is inside and B is outside and outside means the bigger circle, and A is the smaller circle. Here, if A is satisfied then B will also be satisfied, because A is included in B.

Note, when I say B→A, that also means so if we are talking about this example, when B implies A, it also means that if I pick some point which is not in A, suppose here, then that point cannot be in B either. So, this is written in the following sense that not A→not B. So, if A is not true, then B is also not true. So, this thing and this thing, they are same things, this statement and this statement, both are same statements.

When B implies A that also means that not A implies not B and that is correct for the other case also. Now, that raises a question that what happens in terms of Venn diagrams, when you have logical equivalence like this, P↔Q, then how do you show that in terms of Venn diagrams. Actually, when P implies Q, then both these figures which represent P and Q in Venn diagrams, they coincide.

So, if I just draw one, that is a circle, which is representing P and Q is on the top of that same circle, there is no part sticking out. This is the case how we show P and Q imply each other, P→ Q and Q→P. So, both sets are same sets, we are going to talk more rigorously about sets later on, but I guess the idea is clear that two circles in terms of Venn diagrams, they are just coinciding with each other.

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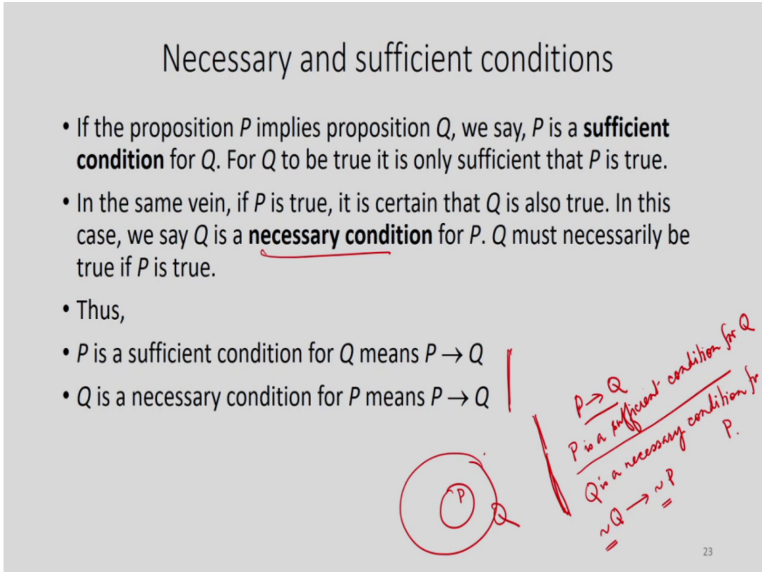

Now, the same ideas, this implications can be put in a different language and these are called necessary and sufficient conditions. And this is the language that we shall be using more in economics, if the proposition P implies Q. So, if we are talking about P implies Q, then we say that P is a sufficient condition for Q. P→Q, P is a sufficient condition for Q. Why you are saying P is a sufficient condition for Q? Because for Q to be true, it is sufficient that P is true.

So, if you want Q to be true, then just make P true, that will be sufficient for Q to be true. In the same way, if P is true, it is certain that Q is also true. In this case, we say Q is a necessary condition for P, Q must necessarily be true if P is true. So, this is just saying the same thing. When I am saying P is sufficient for Q, that also implies that Q is necessary for P. Why this word necessary is coming?

It is coming because if P is true, then Q is necessarily true. Because we know P →Q. So, this is the case where you have, Q is a necessary condition for P. So, these two things and these three things actually, they are saying the same thing, we are putting the same thing in different languages. And this third one, Q is a necessary condition for P, it is also written as this one, in this way.

And actually this is how we are going to write it more often, that ~Q, this sign means not Q implies ~P, ~Q →~P . So, that is a more conventional way of saying that Q is necessary for P.

So, when I say P is sufficient for Q, I also mean that Q is necessary for P and this you can see from the diagram also, as I have done before. So, you have let us suppose, P here smaller circle and Q here, larger circle.

So, if Q is not true, so, suppose you take a point here, so that is not inside Q, then obviously, that point cannot be within P as well. So, not Q means not P. So, this is what we have written here also that P is a sufficient condition for Q means P implies Q and Q is a necessary condition for p means the same thing, P implies Q.

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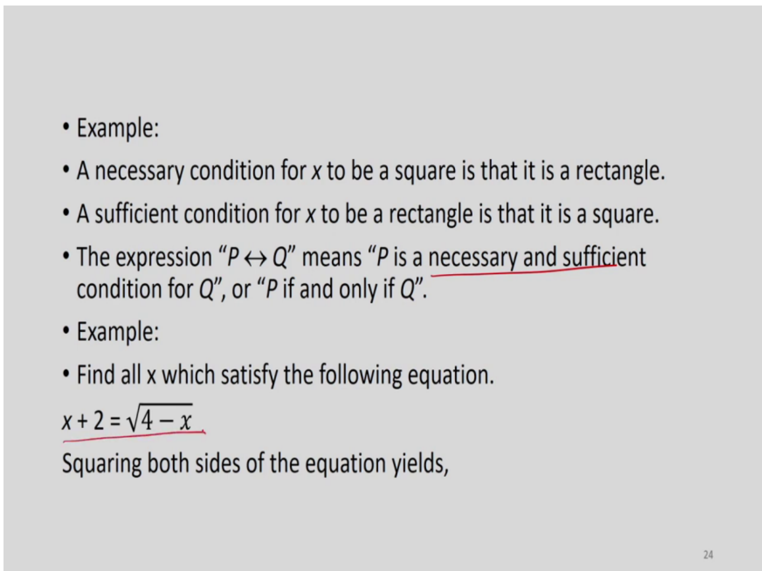

Here I have taken some examples to fix our ideas. A necessary condition for x to be a square is that it is a rectangle. So, without being a rectangle figure cannot be a square. A sufficient condition for x to be a rectangle is that it is a square. So, if a figure is square, then it is necessarily a rectangle. The expression P if and only if Q, means P is necessary and sufficient condition for Q.

So, here you have both the things happening at the same time, necessary and sufficient at the same time. Here I have taken an example to show the difference between necessary and sufficient condition. So, suppose I am given this following problem, find all x, which satisfy the following equation, this $x + 2 = \sqrt{4 - x}$, let us try to solve this.

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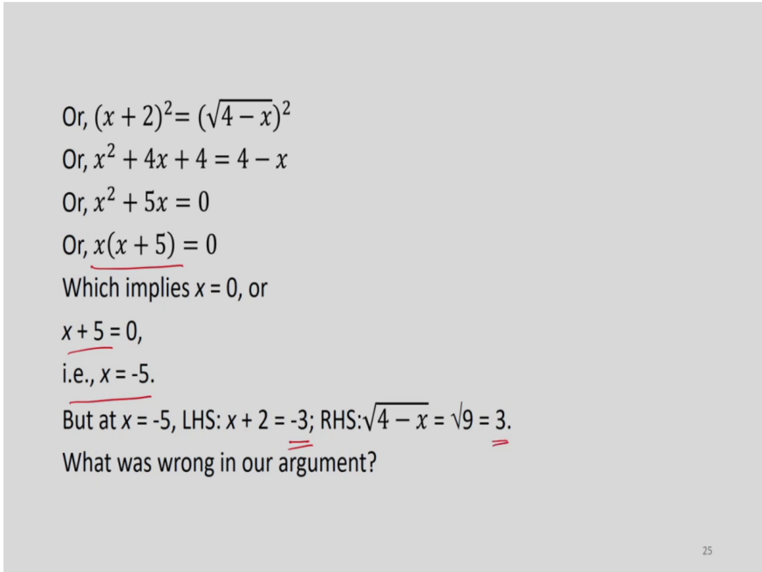

So, what do we do? We first square both sides. And when you square both sides, this is what we get on the left hand side, you have $(x + 2)^2$, on the right hand side you have $(\sqrt{4 - x})^2$. And then this is just simplifying the expressions, $x^2 + 4x + 4 = 4 - x$, and $x^2 + 5x = 0$, because x from the right hand side will come to the left hand side, 4 4 will get cancelled.

And then it turns out to be this one, $x(x + 5) = 0$, which means that $x = 0$ or $x + 5 = 0$. And from this latter one, we get $x = -5$. But at $x = -5$, what about the left hand side? This was the left hand side $x + 2$, if $x = -5$, this becomes -3. And what about the RHS $4 - x$ at $x = -5$ this turns out to be 3. So, left hand side is not equal to right hand side. So, there must have been some mistakes in our argument, what is that mistake?

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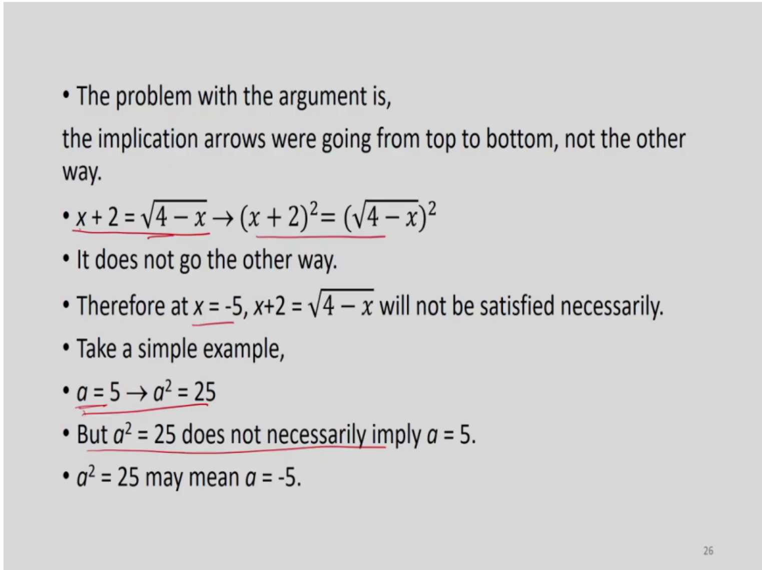

So, the mistake is with the necessary and the sufficient condition. The problem with the argument is that the implication arrows were going from top to bottom, not the other way. So, an example is given here, we started from this one and we came here and the implication arrows were going like this, from the left hand side to the right hand side, it did not mean that the, from the right hand side it can go to the left hand side.

This is sufficient for this, left hand side is sufficient for the right hand side, but the right hand side is not sufficient for the left hand side. So, that was the problem, whereas our argument was that if the right hand side is satisfied, then the left hand side will also be satisfied, which is not correct. Therefore, at $x =- 5$, that is the final solution that we got $x =- 5$, the left hand side will not be satisfied necessarily.

For some values of x, it will be satisfied, but not for all values of x. Actually there were two xs, one was 0, the other was minus 5. So, for x equal to 0 actually the first thing we will get satisfied, and also we have to remember that ,when we take the root of this one on the right hand side, we do not miss the negative root also. So, there could be a negative root, there could be positive root.

Another simple example is that this, suppose $a = 5$. This implies $a^2 = 25$. But $a^2 = 25$ from here, we get two solutions away, we get a is equal to $+ 5$, a is $- 5$, but $a =- 5$ contradicts the first one, first thing that we started which is $a = 5$. So, the implication is going from the left hand side to the right hand side, it does not go from the right hand side to the left hand side.

So, here in this case, from the right hand side, you can get $a =- 5$, that will contradict with the left hand side. So, we have to be careful in which direction the arrows are flowing.

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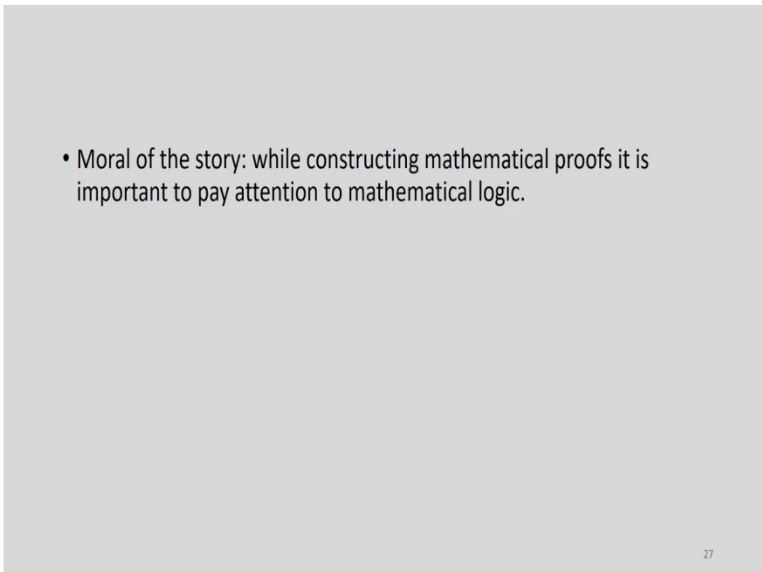

The moral of the story is this, while constructing mathematical proofs, it is important to pay attention to mathematical logic. We have to differentiate between what is a necessary condition, what is a sufficient condition and other things. Finally, we are going to end with what is known as mathematical proof.

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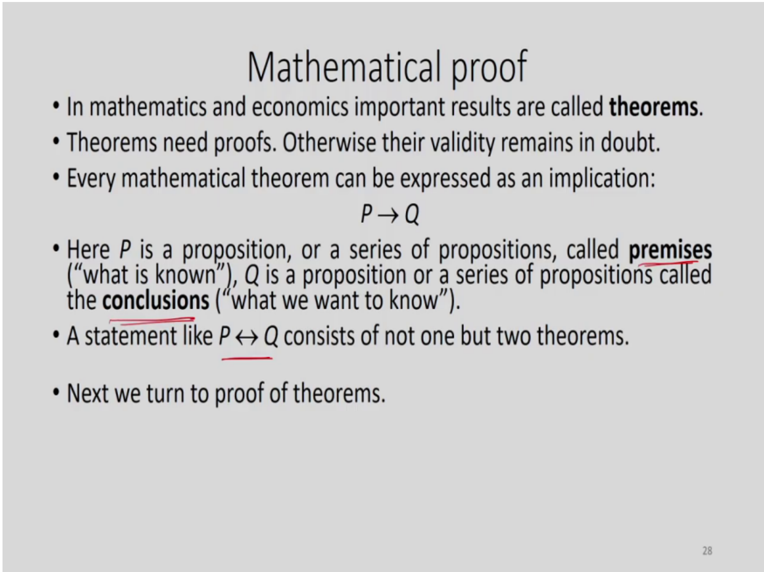

In mathematics and economics, important results are called what are known as theorems. Now theorems need proofs, otherwise their validity remains in doubt. Every mathematical theorem actually can be expressed as an implication, something like P→Q. So, every theorem in mathematics or in economics can be boiled down to this sort of implication, P→Q.

Here P is called a proposition or it can be a series of propositions. And they are generally called the premises. So, these are the premises of the theorem. Sometimes they are also called assumptions or what is known or what is taken as given. And then you have Q, which is a proposition or a series of propositions, they are called the conclusions. And they are also sometimes can be rephrased as what we want to know. So, is it the case that P being true implies Q being true. So, if it does imply that if P→Q, then you have a theorem.

On the other hand, there could be theorems like this also, statements like P↔Q. And here the arrow is flowing both ways. And this kind of statements actually are consisting of two theorems, they have two theorems punched together, because the implication is going both ways.

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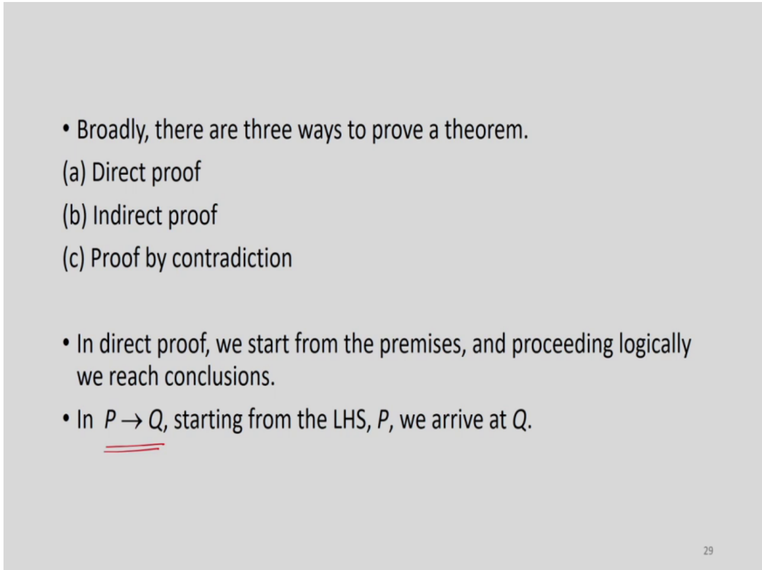

Next, we turn to proofs of theorems. Broadly in mathematics, there could be three ways to prove a theorem. Number 1, a direct proof, number 2 is what is called an indirect proof and thirdly, proof by contradiction. In direct proof, what we do is start from the premises, that is P and proceeding logically, we reach the conclusions. So, in P → Q. Suppose, this is the theorem, we start from the left hand side that is P, and arguing logically we try to arrive at Q, that is how direct proofs are conducted.

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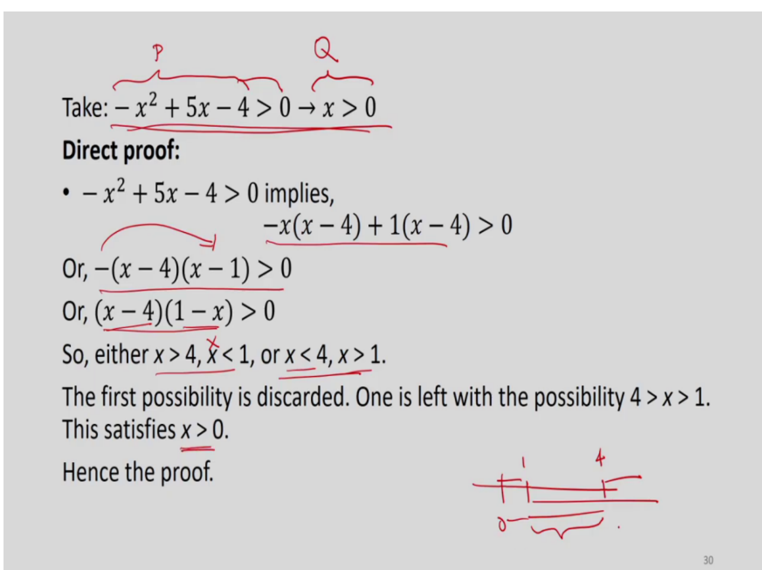

Suppose, we take this example, suppose, we have to show this. So, here you have this as the P and this is the Q, this whole thing and Q is ending here. So, this is not included, P is what? P is saying that $-x^2 + 5x - 4 > 0$, this is P and Q is what? Q is saying that $x > 0$. So, we have to show that P implies Q.

Or in other words, if I have to rephrase this in simple language, for those values of x, for which the left hand side is satisfied, that is P is satisfied. For those values of x, the right hand side will also be satisfied. Now how do we show this by direct proof? Now we start with the left hand side, this is the method of direct proof. Now the left hand side implies, I am just simplifying this by decomposing this 5x, 5x is what?

This $4x + x$. So, I can take x common, and from the first two terms, and from the second time I can take 1 common and this, then I get this simplification, and I can take the minus sign inside, and I get $x - 4$and $1 - x$ and this whole thing should be greater than 0. Now, if this the whole thing has to be greater than 0, then there are two possibilities. One is $x > 4$and $x < 1$. In this case, each of these terms will be positive, the product of two positive terms is positive.

On the other hand, the other possibility is that each of these terms is negative, and that will happen if $x < 4$, at the same time $x > 1$, that is also other possibility. In that case also the left hand side will be positive. Now the first possibility is discarded obviously, because just imagine you have 0 here, you have 1 here, you have 4 here. What this is saying is that $x > 4$, but $x < 1$, that cannot happen, because these two sets are different sets.

So, this is discarded, this is impossible, no x can satisfy this one. But the second thing can happen, what is saying is that $x < 4$. So, you are talking about this entire thing. And $x > 1$. So, you are talking about this entire thing. So, you are basically talking about this range. Now, if this range that we are talking about, then it is obviously true that $x > 0$, because here is 0. So, x is more than 0. So, we have satisfaction of the right hand side as well. So, this is the way in which the direct proof proceeds.

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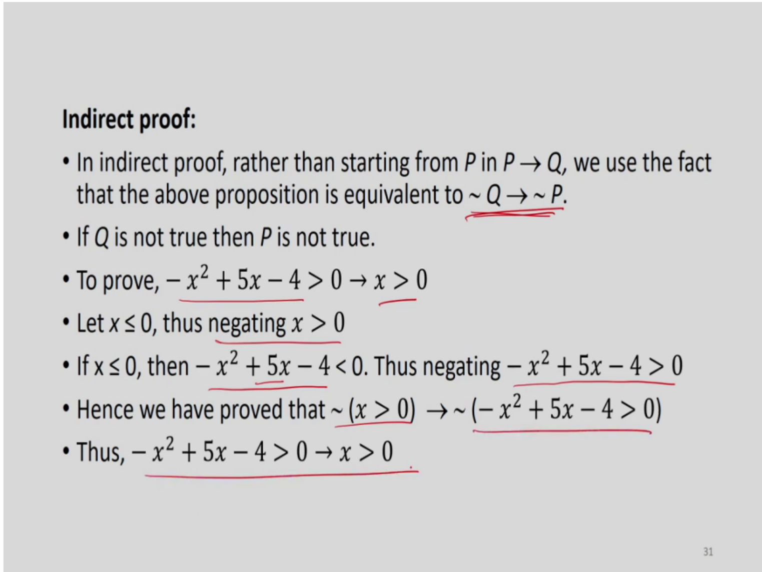

Then we talk about what is known as indirect proof. So, in indirect proof rather than starting from P, we use the fact that the above proposition is equivalent to ∼P →∼Q. So, this is nothing but P is sufficient for Q is equivalent to Q is necessary for P. Now, how do we prove this? We prove this one, the other one. Suppose Q is not true, then what happens? So, what we start with is that Q is not true, if Q is not true, then what happens?

If Q is not true, then $x \le 0$, and if $x \le 0$, then look at the left hand side, this term. If $x \le 0$, this whole term will become negative. Because if x is negative, then obviously minus x square, then you have a negative term here because x is negative and then$-$ 4. So, the whole thing is negative. If x takes the value of 0, then this term drops out, but you have then $-$ 4, which is negative.

So, this then negates that this is true. So, you have basically proved that ∼P =∼Q, and that basically means P = Q. This is the indirect proof.

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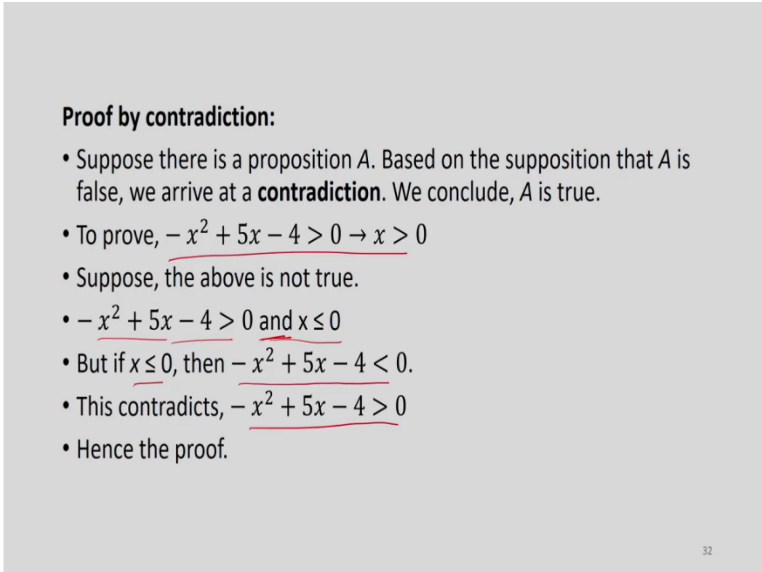

And finally, we come to proof by contradiction. Suppose, there is a proportion A. Now, based on the supposition that A is false, we arrive at a contradiction. And therefore, we conclude that A is correct. So, here we are assuming that this proposition that we are supposed to prove is incorrect, and assuming that it is incorrect, we proceed and we arrive at a contradiction. So, here the example was this one, P →Q.

Now, suppose this above is not correct, that basically will mean that P does not imply Q. So, P is correct but Q is not correct at the same time. And now, at the same time we know that if Q is not satisfied that means $x \leq 0$, then we have just seen that this is also $-x^2 + 5x - 4x < 0$ and which will contradict this one, that P is true. So, Q is not true and P is true, both cannot be happening at the same time. So, you have a contradiction and hence this is the proof.

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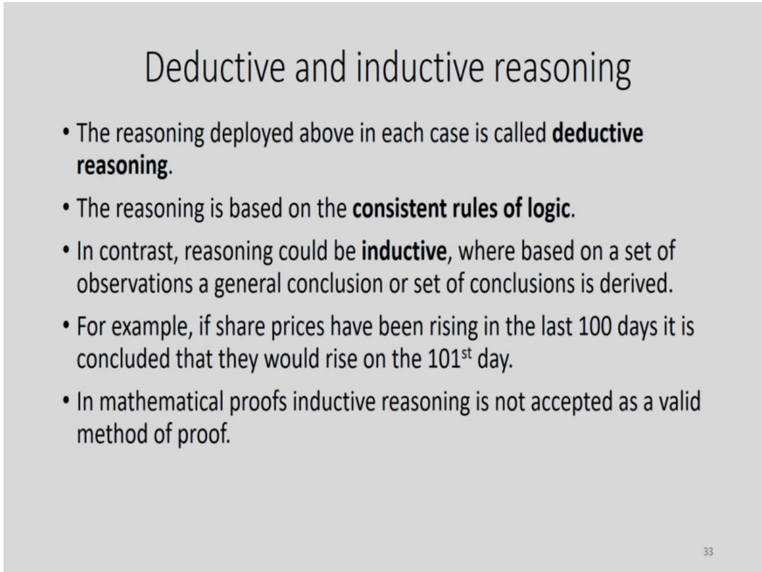

Finally, we talk about what are known as Deductive and inductive reasoning. The reasoning we have deployed above in each case is called deductive reasoning, what is deductive reasoning? This reasoning was based on the consistent rules of logic, we just proceeded logically with some given things and we try to argue our case. In contrast, reasoning could be inductive, where based on a set of observations, a general conclusion or set of conclusions is derived.

An example is given here, if share prices have been rising in the last hundred days, it is concluded that they would rise on the hundred and first day. So, this is called, what is known as inductive reasoning. So, you have seen something happening for a long time and then you deduce, that in the next, next turn of the event also the same thing will happen. But this sort of reasoning of inductive reasoning is used a lot in empirical sciences, in empirics, but these are not considered to be full proof proofs in mathematics.

Okay, we end here in today's lecture, we have covered a lot of ground in today's lecture. We talked about the basic ideas of mathematics such as intervals, absolute values etcetera, etcetera. And then we talked about logic, how different kinds of logical things, propositions can be made, necessary condition, sufficient conditions, logical equivalence, and finally, we talked about mathematical proofs, how critical proofs can be constructed. Okay, I shall see you in the next class. Thank you.