Mathematics for Economics - I
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Lecture 17
Tutorials – 1a

Hello, and welcome to another lecture of this course Mathematics for Economics Part I.

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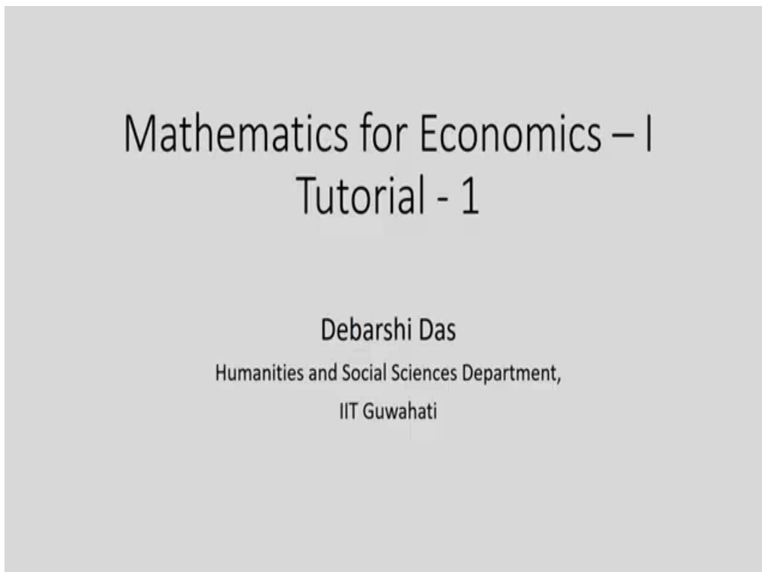

So, as you can see on your screen, what we are doing now is we are doing some tutorials of this course. The purpose of doing the tutorial is to make the students equipped with the skill to handle questions, problem solving skills, so those are the things that they should develop. So, this is the purpose. Also, the tutorials I think will give the students of a flavor of the review, the topics that will be covered in these tutorials, will be sort of summary of the different things, different topics that have been covered. So, without much ado, let us start the tutorial.

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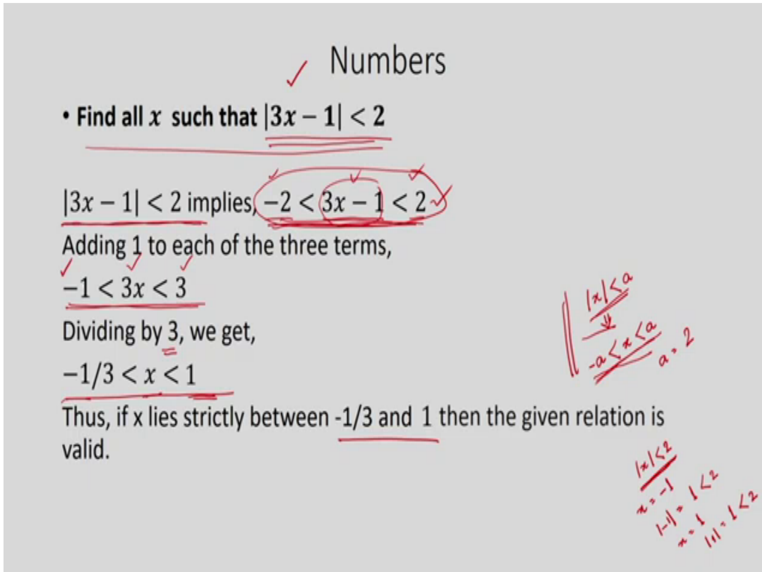

So, we will start with this first topic which is the real number systems. So, I have written numbers on top that is the topic of these questions. So, under each topic, I will be covering a number of questions, number of problems that are relevant for that topic. So, that is the strategy that we are adopting in this tutorial. So, this is the question. The questions are all in boldface and the solutions are not in boldface, as you can see here.

So, the first question is this. Find all x such that $|3x - 1| < 2$. So, let us try to understand what this question is asking. We have to find out all those xs, all those real values of xs, such that this condition holds. So, we start with this condition, what this condition means, in simple terms, and then we shall try to see whether we can find the solution.

Now, this relation that $|3x - 1| < 2$, it implies this, $-2 < 3x - 1 < 2$. So, this is the critical stage of this problem. So, what we are applying here is this property that, suppose $|x| < a$, so this is given to us. What this actually implies is this, that $-a < x < a$. So, that is what it means. a is any constant, a positive constant. So, if $|x| < a$, then that means that whatever be the value of x it lies in this range minus a to plus a.

And you can see the reason for that. The reason is that $|x| < a$, means what, that even if x is negative, then by modulus we take the positive value of that negative number. So, take an example. Suppose $a = 2$. Now, $|x| < 2$,, what it means is that even if $x = -1$, what is modulus of minus 1, it is equal to plus 1. So, this minus 1 will also satisfy this relationship that $|x| < 2$. And of course, if x is plus 1 then also it satisfies that relation because $|1| = 1 < 2$.

So, as you can see that not only $0 \leq x < 2$ will satisfy this relationship $-2 < x < 0$ that will also satisfy this relationship. In other words, x lying in the range from minus 2 to plus 2 will satisfy this relationship. So, that is why I am writing this relation that if $|x| < a$, where a is a positive number, constant, then from that I can straight away write this $-a < x < a$. So, this is the principle that I am applying here in this particular problem.

So, what we are saying is that, since we know that modulus of x minus 1 is strictly less than a, therefore, whatever within the modulus symbol that is $-2 < 3x - 1 < 2$. We are actually following this pattern that is getting replicated here. Now, we have to use this relation to get the different values of x which satisfy the given relation. Now, from here what we do is that adding 1 to each of these three terms. We can add 1 to this term, this term and this term. And if we do so, then this is what we get, minus 2 plus 1 is minus 1, 3x minus 1 plus 1 is 3x and 2 plus 1 is 3. So, this is what we are getting. In the end it is $-1 < 3x < 3$. This is what we have got.

In the next step what we do is that we divide again all these three terms by this number 3. And what is the purpose of doing all these is that we want to get x here in the middle term. So, if we divided 3x by 3 then we are going to get x as the middle term. But this first term and the third term will also get divided by 3, therefore, the first term will become minus 1 divided by 3 and the last term becomes 1. So, this is the final sort of chain that we have got $-1/3 < x < 1$. So, this is the actual solution. If x lies between $-1/3 < x < 1$, then this given relation that is $|3x - 1| < 2$ is valid. So, that is the solution.

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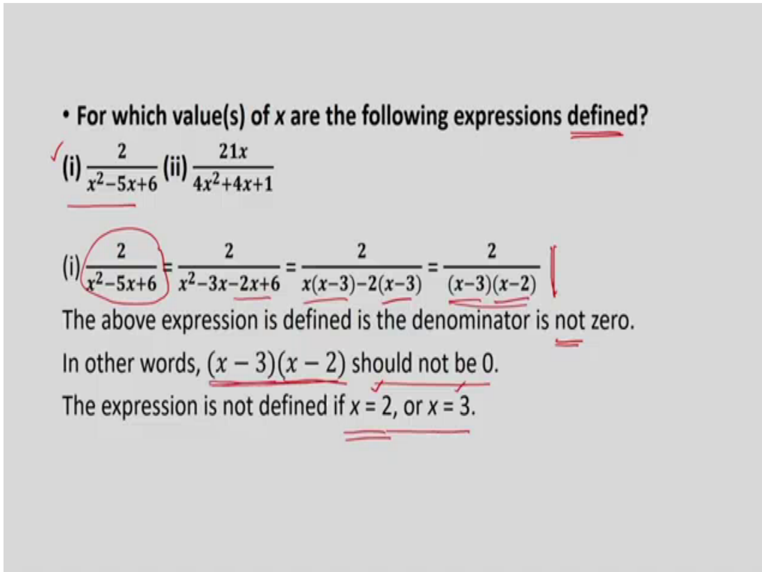

Here is the second sort of question. For which value or values of x are the following expressions defined? So, this term defined is important. And there are two separate problems actually, number one is this $\frac{2}{x^2-5x+6}$ that is the first problem. And the second problem is $\frac{21x}{4x^2+4x+1}$. Let us start with the first problem. And as we can see, $\frac{2}{x^2-5x+6}$, we can write this expression as $\frac{2}{x^2-3x-2x+6}$.

Actually, what we are trying to do is that we are trying to factorize the denominator. So, that is the general idea. So, from this, actually we get 2 divided by, I am taking x common, so it will become x multiplied by x minus 3. And from the last two terms, I can take minus 2 common. So, within the brackets, I am getting x minus 3. And therefore I can take x minus 3 common. So, if I take x minus 3 common, then the other term will be x minus 2. So, in the end, the expression actually becomes $\frac{2}{(x-3)(x-2)}$.

Now, the point that is important in this problem, the point that we are going to use is that this expression is defined that is, $\frac{2}{(x-3)(x-2)}$ is defined if the denominator is not 0. So, that is the idea that we are going to use here. If in a fraction the denominator is equal to 0, then that fraction is not defined. So, that is the basic idea here. Therefore, the denominator cannot be equal to 0. So, that is the property that is being used here. Now, the denominator is $(x - 3)(x - 2)$, that cannot be equal to 0.

Now, when is this expression that is $(x - 3)(x - 2) = 0$? This is equal to 0, if there are two cases when $(x - 3)(x - 2)$ can be equal to 0, either if $x = 3$, in that case this part will become 0, therefore, the product becomes 0. So, that is one case. Or $x = 2$, in that case the second part turns out to be 0, and therefore, the product also turns out to be 0. So, therefore, the expression will become something divided by 0 under two values of x, either $x = 2$ or $x = 3$.

So, that is the answer to the question. For which values of x are the following expressions defined? At least for the first problem, there are two actually values of x for which this expression is not defined. And these two values of x are $x = 2$ and $x = 3$. So, that is the answer. Now, we are going to take up the second question and we are going to use the same tactic. Let us see how we can do that.

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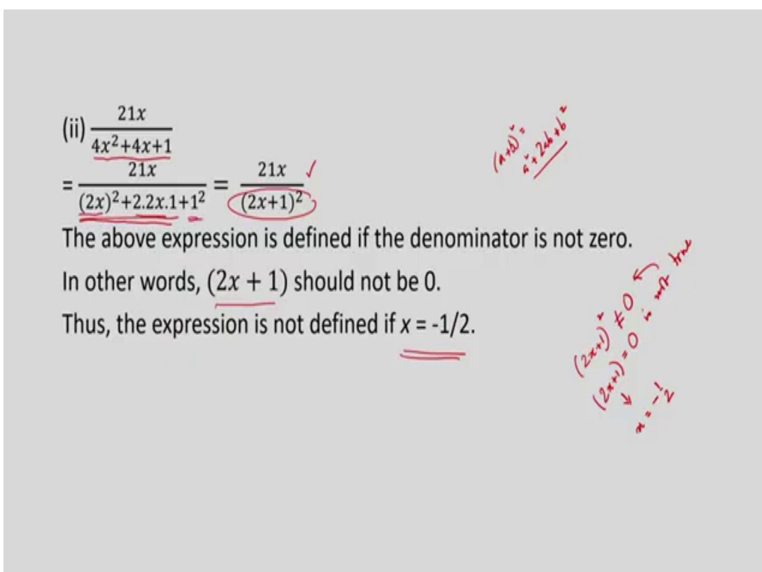

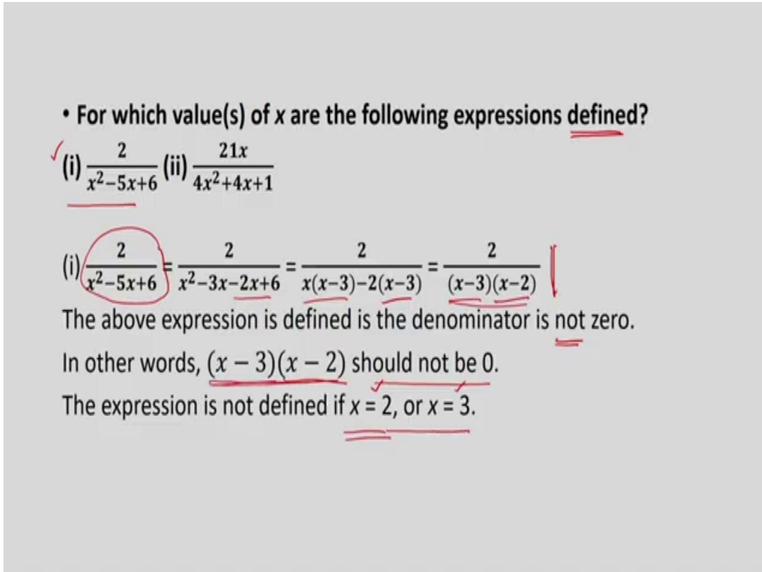

So, this is the second problem, $\frac{21x}{4x^2 + 4x + 1}$. So, again, we concentrate on the denominator. So, denominator is $4x^2 + 4x + 1$. And now we try to play around with the denominator, try to factorize that. And as we can see that this can be actually easily expressed in the $(a + b)^2$ form. The first term is $4x^2$. So, that is nothing but $(2x)^2$, that will give me $4x^2$. The second term is 4x. And 4x can be written as 2 multiplied by 2x multiplied by 1. And the last time is 1. 1 is nothing but 1 square.

So, as we know, $(a + b)^2$is what, it is $a^2 + 2ab + b^2$. Just to make sure that people remember their high school mathematics, $(a + b)^2 = a^2 + 2ab + b^2$, and here, so 2x can be thought of as the first term that is a and 1 can be thought of as the second term that is b, and therefore, this denominator can be written as $(2x + 1)^2$.

Now, again after we have got this term, then we use the previous property that any expression is not defined if the denominator is 0. So, for this expression to be defined, the denominator cannot be equal to 0. What is the denominator? Denominator is this $(2x + 1)^2$. In other words, $(2x + 1)^2$ this should not be equal to 0. Now, this can be equal to 0 if you have $2x + 1 = 0$. So, this is not true. Only then you are going to get this.

In other words, that is why I have written here, in other words, $2x + 1$ should not be equal to 0. So, $2x + 1 = 0$ will land us in problem, and therefore, I can simplify this a bit. It means $x =- 1/2$. So, that is what I have written here that if $x = \ - 1/2$ then the denominator becomes 0 and if the denominator becomes 0, then the expression that is given is not defined. So, this is the answer to the second part.

Now, you can see that interestingly although in both these problems the denominator is a quadratic expression because there are x squares in both the problems. In the first problem, there are two values of x for which the expression is not defined. Whereas, in the second problem there is just one value of x which is$- 1/2$ for which the expression in the second part is not defined. So, that was the answer.

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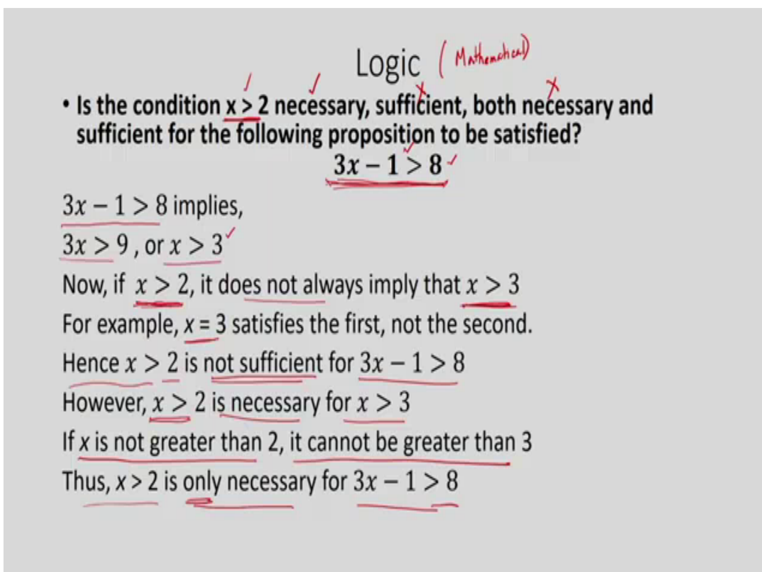

Now, we move off to another topic and take up some problems from that topic. This was **mathematical logic**. To save time I have written it just as logic, but it should have been mathematical logic. We are not talking about any logic here, but the logic that we can handle with mathematics, we are talking about that sort of logic. So, here is the problem. Is the condition $x > 2$ necessary, sufficient, both necessary and sufficient for the following proposition to be satisfied? What is that proposition? This is the proposition, $3x - 1 > 8$.

So, the first thing I think the students should try to do is to whenever a question is asked they should devote some time to understand what the question is trying to enquire, what is the question asking. So, that should be clarified first before jumping into the solution. So, what is being asked in this question is that there is a proposition which is given $3x - 1 > 8$. And in this proposition x is involved. x, as we know, is a variable unknown.

Now, the question that is being asked is this proposition to be valid, for this proposition to be valid is this condition $x > 2$ a necessary or a sufficient or both necessary, sufficient. Again to repeat, x is an unknown here. So, x can take any value. So, it is possible that for multiple values of x this given proposition is valid. But if we are given this particular condition then this condition is it a necessary or sufficient or both necessary and sufficient for this proposition to be valid.

Now, how do we attack this problem? We start with this condition, try to make it a little bit simpler and try to compare this proposition with this condition that will give us a clearer idea as to what is the relationship between this condition and this proposition. We have to simplify the proposition. So, that is the line of attack. We start with $3x - 1 > 8$ and we add 1 to both sides. So, we shall get $3x > 9$. And in the next stage we divide both sides by 3. So, we should get $x > 3$. So, this is what this proposition boils down to.

Now, what is the condition that we have been given? The condition is $x > 2$. Let us now try to understand the relationship between the condition and the proposition and that will give us the answer to the question. Now, if the condition is sufficient for this proposition, then if the condition is satisfied, then the proposition will also be satisfied. So, let us try to see if this condition $x > 2$ is sufficient. So, if $x > 2$, it does not always imply that $x > 3$.

And here is an example. Suppose $x = 3$, if $x = 3$, then this condition is satisfied because 3 is greater than 2, but $x = 3$ will not satisfy this proposition. The proposition is saying $x > 3$. 3 is not greater than 3. So, here this is an example, where the condition is satisfied, but the proposition is not. And there are obviously an infinite number of such examples where the x is such that the condition is satisfied, but the proposition is not being satisfied.

So, what we can say is the following, that$x > 2$ is not sufficient, it is not a sufficient condition for $x > 3$. And $x > 3$, as we know, it is equivalent to saying $3x - 1 > 8$. So, that is the conclusion that we are getting that the condition is not a sufficient condition for this proposition to be valid.

Now, the next question is, is it necessary. If $x > 2$ is a necessary condition for $3x - 1 > 8$. And here actually the answer is correct that $x > 2$ is necessary for $x > 3$. And the reason is the following that remember the definition of a necessary condition that if this condition is not satisfied, then the proposition is also not correct. Now, that is the idea that we are going to apply here. So, if this condition is not satisfied, that means $x > 2$.

Now, if x is not greater than 2, then x cannot be greater than 3 either. So, x is not greater than 2 means what, x could be 1, x could be 0, x could be something negative. Those are the cases where x is not greater than 2. But in all these cases, x is not greater than 3 also. So, that is what I have written here. If x is not greater than 2, it cannot be greater than 3 and that is the idea of a necessary condition. Therefore, we conclude the following that $x > 2$ is necessary for $3x - 1 > 8$. But remember what I have written here is only necessary, which means it is not sufficient and it is not both necessary and sufficient. It is only necessary. So, this is the way this sort of problem can be tackled in logic.

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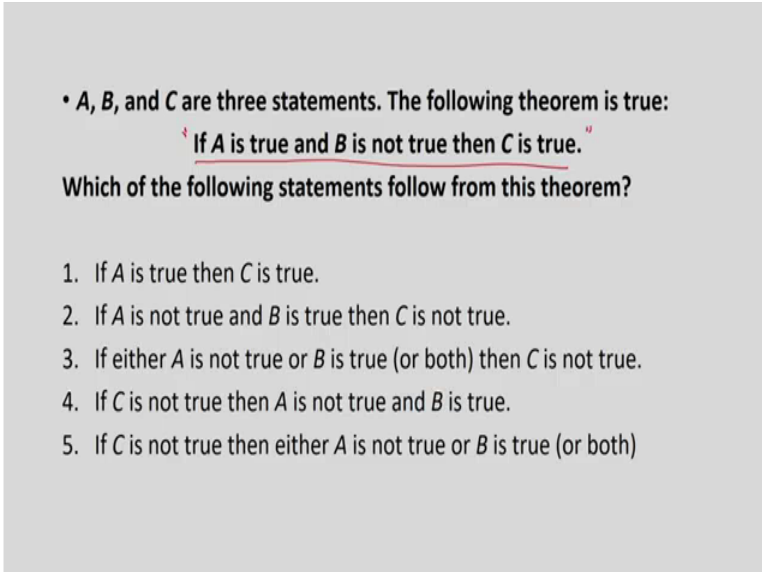

Here is another problem from mathematical logic. So, here is the question A, B, C are three statements and the following theorem is true. What is this theorem? Theorem is this. If A is true and B is not true, then C is true. So, this is a theorem which is given to us. This theorem is correct. It is a valid theorem. Which of the following statements follow from this theorem? So, that is the question. And there are five options which are given to us. We have to find out which of them actually follow from this theorem.

Number one, if A is true, then C is true. Number two, if A is not true and B is true, then C is not true. Number three, if either A is not true or B is true (or both), then C is not true. So, this was the third sort of statement. The fourth statement is if C is not true, then A is not true and B is true. And fifthly, if C is not true, then either A is not true or B is true (or both). So, these are the five options. We have to find out which of them is correct, which of them actually follow from this theorem. given theorem. Let us see how we can again solve this problem.

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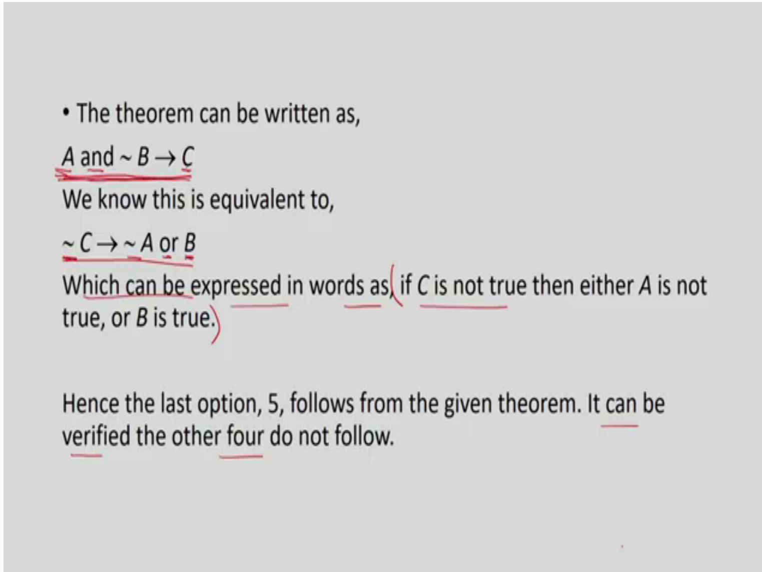

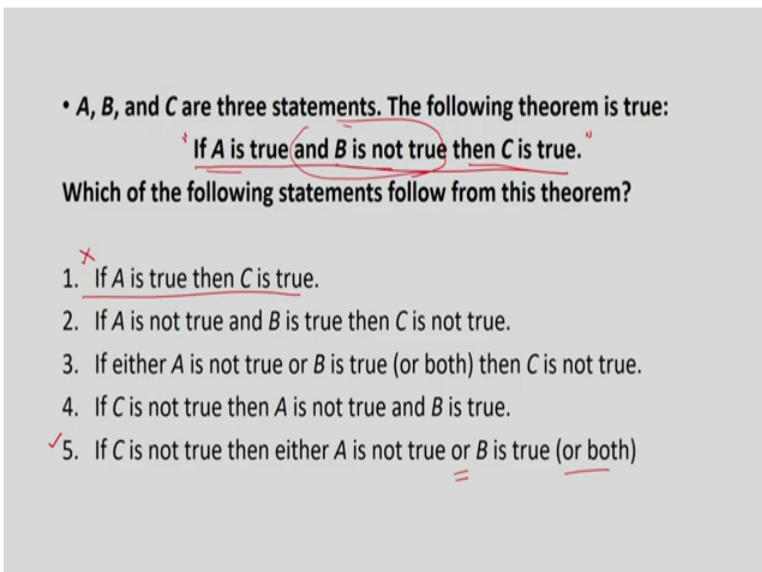

Now, the theorem can be written in short form as this, $A\ and\ \sim B \rightarrow C$. If A is true and B is not true, then C is true. So, that is why I have written it in short form as $A\ and\ \sim B \rightarrow C$. Now, as we know this sort of statement is equivalent to this statement that $\sim C \rightarrow \sim A\ or\ B$. So, try to understand how we are doing it. We are flipping it around and not the negative of this statement is not C implies negative of the first one that is A was there that becomes minus that is not A and here there was an operative that becomes or and not B becomes B. So, that is the pattern. And so, in the end we are getting $\sim C \rightarrow \sim A\ or\ B$.

Now, this is in terms of symbols. What does it mean in terms of words? This can be expressed in words as, if A is not true. Remember, there is a wiggle expression here that means A is not true. If C is not true then either A is not true or B is true. Now, let us see if this is there in any of the five options. What is there? If C is not true, then either A is not true or B is true. So, that is the thing that we have to see whether it is there.

And actually if you inspect all these five options, then it is there in number 5. If C is not true, then either A is not true or B is true. Here it is also given as or both, but or both is implied. Whenever or is written, it also means that both of them can be true. So, the fifth option is the correct option. If C is not true, then either A is not true or B is true. That is the correct option. But is any of the other four correct. It can be verified that the other four do not follow. Let us take one or two to understand how they are not following from the theorem.

For example, suppose you take the first one, if A is true, then C is true, does it follow? What is this theorem saying, if A is true and B is not true, then C is true. So, this part has been ignored when you are writing this one. So, the first one is not correct. Similarly, you can see the other three are also not correct. For example, if A is not true, and B is true then C is not true. This does not follow, because if A is not true and B is true. So, you are taking actually negative of this and you are taking negative of this and you are saying then that implies the negative of this. That obviously, that sort of statement obviously does not follow from this theorem.

So, the last option, that is, the fifth option is the correct one. The other four, the first four are not following from this theorem.

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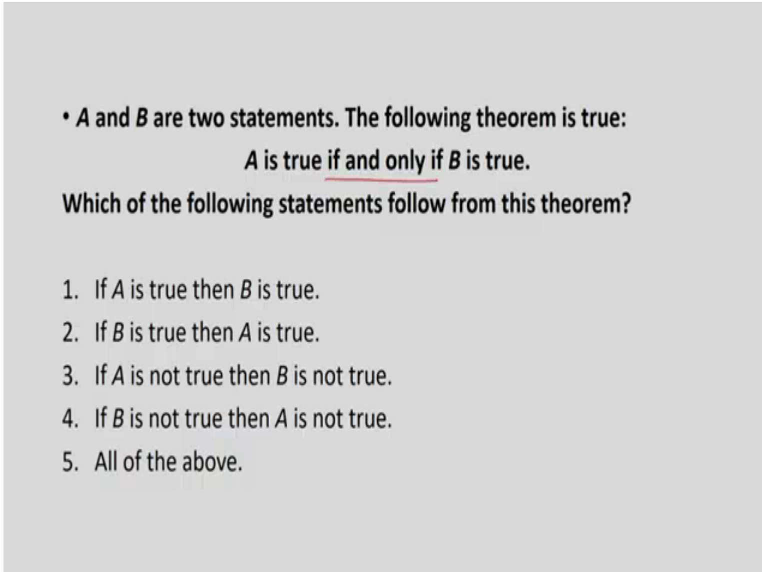

Here is another problem from mathematical logic. A and B are two statements. The following theorem is true. A is true if and only if B is true. Which of the following statements follow from this theorem? The first option is; if A is true, then B is true. Second option is if B is true, then A is true. Third, if A is not true, then B is not true. Fourth, if B is not true, then A is not true. And fifthly, all of the above. These are the five statements. We have to find out which of them is correct.

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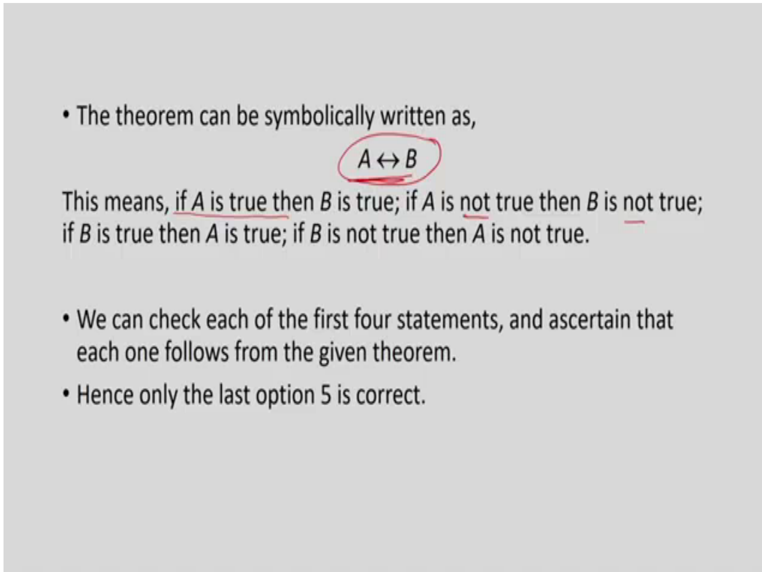

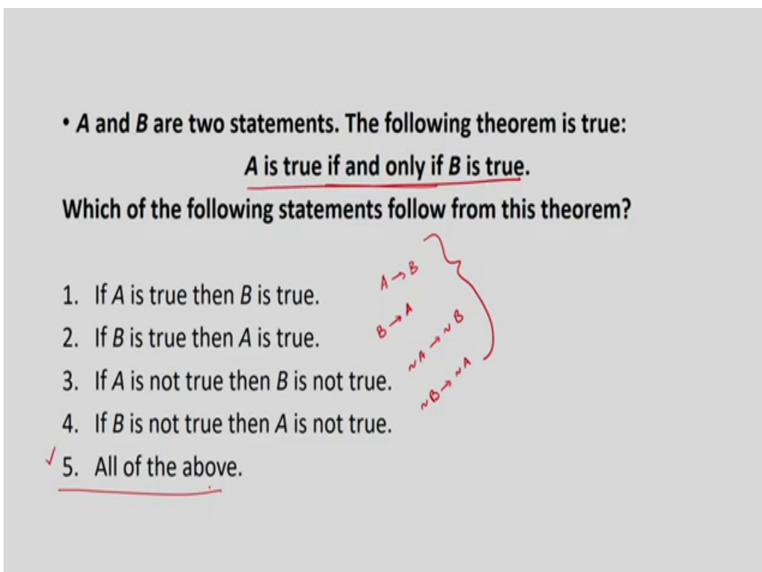

Now, the theorem can be symbolically written as $A \leftrightarrow B$. Remember, this is what we have discussed in the lectures that if there are two statements A and B and the relationship between them is this that if A then B and if B then A. Both of them are correct, then we can write it like this, $A \leftrightarrow B$. And if we have something like this, $A \leftrightarrow B$ then actually it means the following; if A is true, then B is true. That is A sufficient condition for B. If A is not true, B is not true. That is, A is a necessary condition for B. At the same time, if B is true, A is true. That is B is a sufficient condition for A. And lastly, if B is not true, then A is not true and this basically means B is necessary for A.

So, all these four statements actually follow from this, $A \leftrightarrow B$. We can check each of the first four statements and ascertain that each one follows from the given theorem. Hence, only the last option five is correct. So, you can check, if A is true then B is true that is A implies B. If B is true then A is true that is B implies A. If A is not true, then B is not true. That is A is necessary for B. If B is not true then A is not true, this. And all of these actually are following from this statement, from this theorem. Therefore, I have to choose the last option, because if I choose one then that is just partially correct because 2, 3, 4 are also true. So, fifth is the correct option that all of the above.

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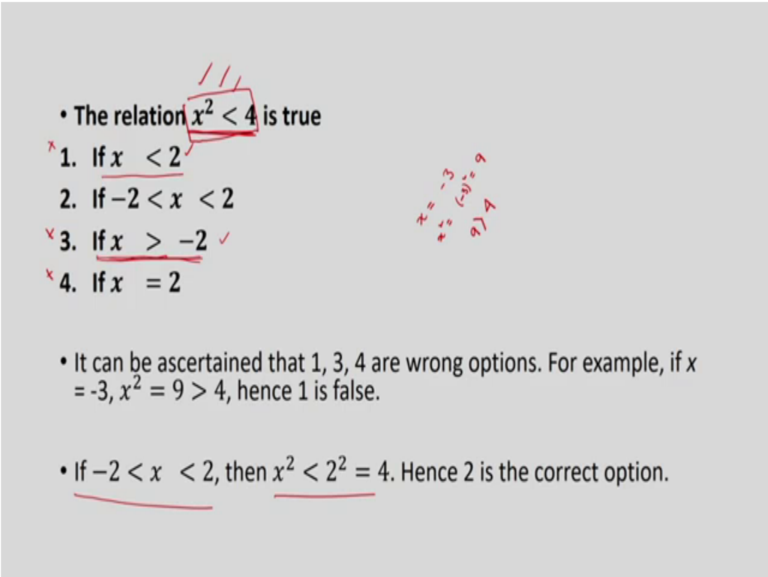

Here is another problem from mathematical logic. The relation $x^2 < 4$ is true; number one, if $x < 2$; number two, $-2 < x < 2$; number 3, $x > -2$; and number 4, $x = 2$. How to again tackle this problem? It can be ascertained that 1, 3 and 4 are wrong options. 1, 3 and 4 means this one, this one, this one, they are not correct. Why they are not correct? Let us take one example.

The first one let us see, if $x < 2$, will it satisfy this? For example, if you take $x = -3$, then what is x^2, $x^2 = (-3)^2 = 9$. But $9 > 4$. So, this is not being satisfied. We needed $x^2 < 4$.

What we are getting is $x^2 > 4$. So, x being less than 2, it does not mean that $x^2 < 4$. So, that is why the first one is not correct.

The third one is also not correct, $x > -2$. Well, if you say that $x > -2$, x could be, let us suppose, 3, 3 is greater than minus 2. Again, if you take the square of 3, you will get 9 and 9 is greater than 4. Again, that will violate this one. If $x = 2$, then obviously x^2 will give you 4. 4 is not less than 4. So, that is why 4 is also, the fourth option is not correct.

Whatever the second option, if $-2 < x < 2$, then if you take the square of x, then that is less than 2^2. And what is 2^2? $2^2 = 4$. So, that is, $x^2 < 4$ and that is what is given here. $x^2 < 4$. Therefore, the second option is the correct option.

What is important here in this problem to note is that suppose you take $x >- 2$. Now, if $x >- 2$, it might mean that $x = 0$. And if $x = 0$, then this relation will be satisfied. But that is for a particular value of x. If $x >- 2$, then for some values of x, the solution will be satisfied, but not for all x, which satisfy $x >- 2$. And that is what is important. We have to check for all values of x which satisfy this condition. And if at least for one value of x which satisfies this, but does not satisfy this, then we cannot say that this is a correct option. So, the second option is the correct answer here.

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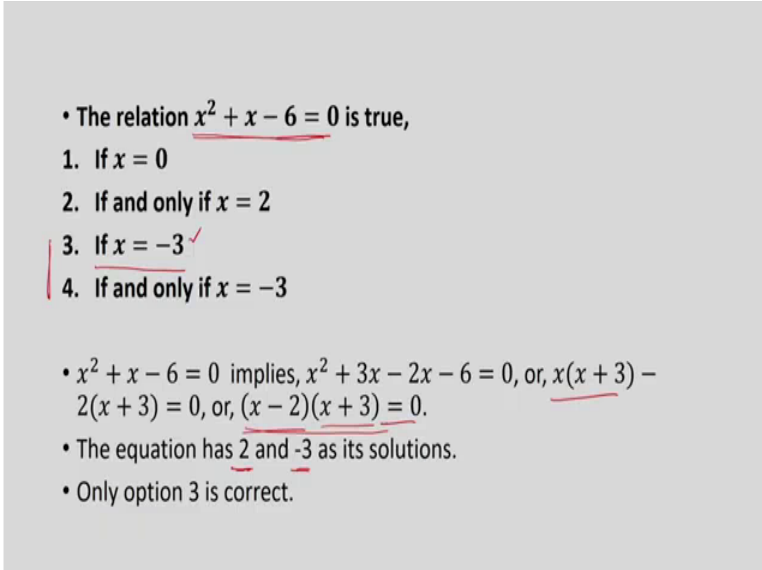

Here is a related problem again from mathematical logic. The relation $x^2 + x - 6 = 0$ is true, if $x = 0$, if and only if $x = 2$, if $x =- 3$, and lastly, if and only if $x =- 3$. Notice, there is a difference between the third option and the fourth option. The third option is saying, if $x =- 3$, and the fourth option is saying, if and only if, if $x =- 3$.

Now, how to again tackle this? We start with this relation $x^2 + x - 6 = 0$ and we factorize. So, it becomes $x^2 + 3x - 2x - 6 = 0$, take x common, $(x + 3)$, take 2 common, so that will give you $(x + 3)$, you take $(x + 3)$ common, so $(x - 2)(x + 3) = 0$ is equal to 0. So, this is an equation, a quadratic equation with two roots of 2 and - 3. That basically means that this relation is valid for two values of x, one is 2 and the other is - 3.

Now, check if any of these options is correct. For $x = 0$, obviously, this is not going to be satisfied. If and only if $x = 2$, no this is also not correct, because it is true that for $x = 2$ it is correct, but not only for $x = 2$, it is also true for $x =- 3$. So, that is why the second option is not correct. The third option is correct. If $x =- 3$ then obviously we know - 3 is the root of the quadratic equation. So, therefore, this is correct. And the problem with the fourth option is that, yes, for $x =- 3$, the relation is correct, but it is wrong to say that it is correct only for $x =- 3$. So, therefore, only the third option is the correct option.

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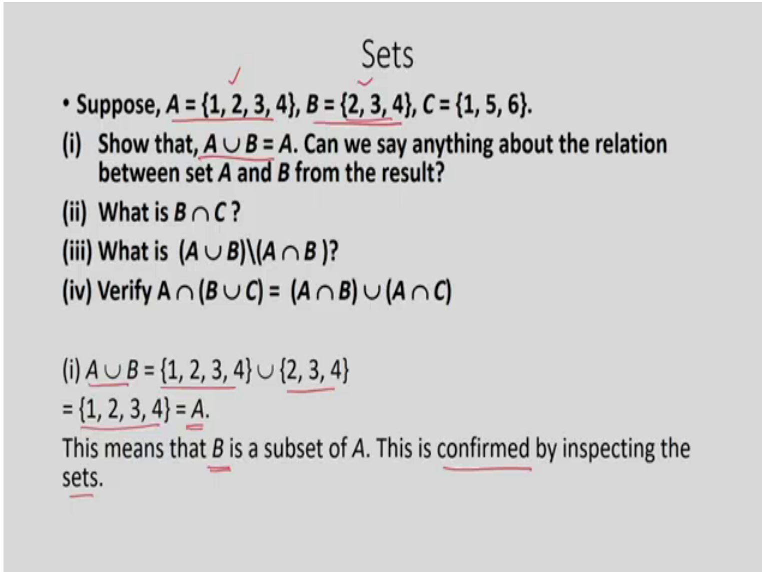

Now, we **move to another topic which is sets and set operations**. So, we will start with the first problem here. Support, three sets are given. First set is A. A has four elements {1, 2, 3 and 4}. And then we have set B. Set B has three elements, {2, 3 and 4}. And then you have the third set C, which has three elements {1, 5 and 6}. So, these three sets are given to us. We have to answer four questions.

Show that $A \cup B = A$. Can we say anything about the relation between set A and B from the result? What result, the result that we have given here that is $A \cup B = A$. The second question is what is $B \cap C$? The third is what is $(A \cup B)\backslash(A \cap B)$? And the fourth one is to verify that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

So, let us start with the first one. A is known to us. B is known to us. So, we can find out what is $A \cup B$. So, $A \cup B$ is equal to this set that is A= {1, 2, 3, 4} and B= {2, 3, 4}. The union of these two sets will consist of those elements which are there either set A or set B. So, it will be {1, 2, 3 and 4} and this is nothing but set A. So, that is what we were supposed to prove that $A \cup B = A$. Now, what does it mean? Can we say anything about the relation between set A and B from this result?

This actually means that B is a subset of A. When you are taking the union of two sets, if the union turns out to be one of those sets, that means that the other set does not have any element which is not there in the other set. So, that basically means that the first set is a subset. Let me repeat that once again. If we are taking two sets A and B and we are taking the union of them and the union turns out to be set A, then that means that in set B there is no element which is not there in set A also and that basically means that B is a subset of A.

It is not necessary that it is a proper subset, but in general in that case B is a subset of A. And actually this is confirmed by inspecting the sets also if you look at set A and set B, then each element of B is also an element of A and that basically makes B a subset of A. The second problem is what is $B \cap C$?.

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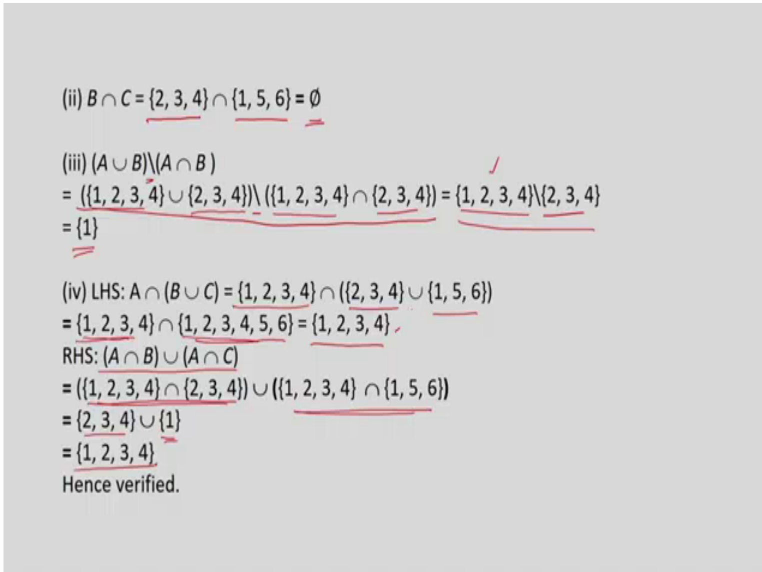

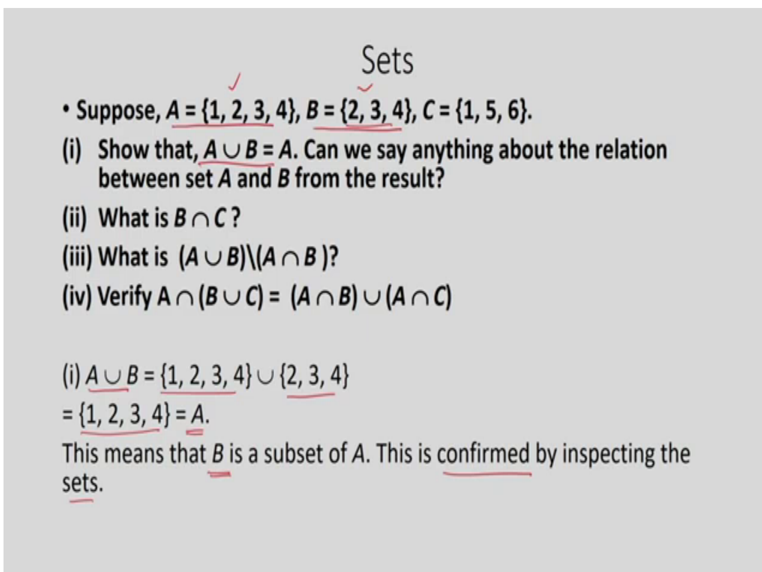

What is B? B is {2, 3, 4} and intersection C {1, 5, 6}. Now, in intersection sets we take elements which are common to the sets. Now, as we can see that between B and C there are no common elements. So, therefore, the intersection set is a null set. The third is this, that we have to find out $(A \cup B)$ that is within the brackets. And this backward slash, this symbol basically, denotes minus. So, backward slash within the brackets $(A \cap B)$. So, we expand this and that is what we are going to get.

So, this is A {1, 2, 3, 4} union B, what is B, {2, 3, 4} and then you have this symbol and then within the brackets again {1, 2, 3, 4} the first set and intersection {2, 3, 4} the second set B. Now, if we take the union of {1, 2, 3, 4} and {2, 3, 4} you get {1, 2, 3, 4}. And if you take the intersection {1, 2, 3, 4} and {2, 3, 4} what are the common elements, they are just {2, 3 and 4}. So, this is what we are getting {1, 2, 3, 4} \ {2, 3, 4}. Minus means we have to take out those elements in the first set which are there in the second set as well. Here {2, 3, 4} are there in the second set. So, we take them away and we get only {1}. So, this is the answer.

And last problem, the fourth problem was verify $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. So, we start with the LHS, that is the left hand side, $A \cap (B \cup C)$. A is known to us it is {1, 2, 3, 4} and then intersection B {2, 3, 4} and union C that is {1, 5, 6}. And then in the next step A remains as it is. We take the union of {2, 3, 4} and {1, 5, 6} and that becomes {1, 2, 3, 4, 5, 6}. And if we take the intersection of these two sets, that is we are taking the common elements from this set and this set and this is nothing but {1, 2, 3, 4}. This was the LHS, the left hand side.

What about the right hand side, right hand side was this within the brackets $(A \cap B) \cup (A \cap C)$. So, $(A \cap B)$ means this {1, 2, 3, 4} ∩ {2, 3, 4} and ∪ $(A \cap C)$ that is {1, 2, 3, 4} ∩ {1, 5, 6}. So, from the first term, we are going to get {2, 3, 4}. So, {2, 3, 4} are common. And from the second, we are going to get only {1}, because there is only one element which is common to both these sets. So, {2, 3, 4} ∪ {1} and that will give us {1, 2, 3, 4}. And you can see this is the set that we got from the left hand side as well. Hence, we have verified.

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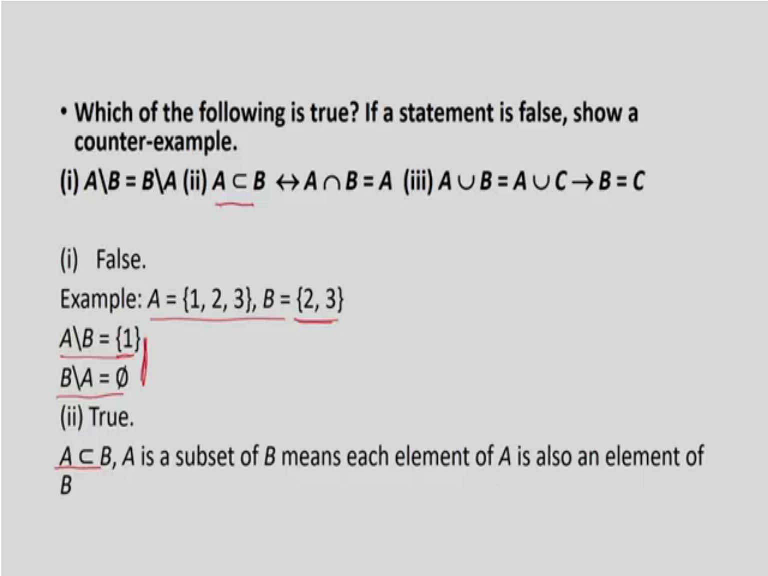

This is again a problem from the sets. Which of the following is true? If a statement is false, show a counter example. $A \backslash B = B \backslash A$, whether it is false or true that we have to say. If we think it is a false statement, then we have to show a counter example where this statement is not valid. $A \backslash B = B \backslash A$, let us see how it is false. We take A to be $\{1, 2, 3\}$ and B to be $\{2, 3\}$. So, what is $A \backslash B$? $A \backslash B$ means we take out those elements of A which are there in B also. So, this will only leave us with one element that is $\{1\}$.

And $B \backslash A$, if we take B which is $\{2, 3\}$ from that if we take elements of A that is $\{1, 2, 3\}$ we can see that all the elements of B are there in A also. So, there is no element in $B \backslash A$. So, it is a null set. And as you can see these two are not equal, $A \backslash B \neq B \backslash A$. So, that is why the statement was not correct.

The second problem is this. $A \subset B \leftrightarrow A \cap B = A$ whether this is correct or incorrect. Our answer is it is correct. It is true. What is the reason why it is true? It is not being asked that we have to show the reason. Anyway, but we are trying to clarify why we think that this is correct. Now, when we say that $A \subset B$ it means that each element of A is also an element of B that is the definition of a subset.

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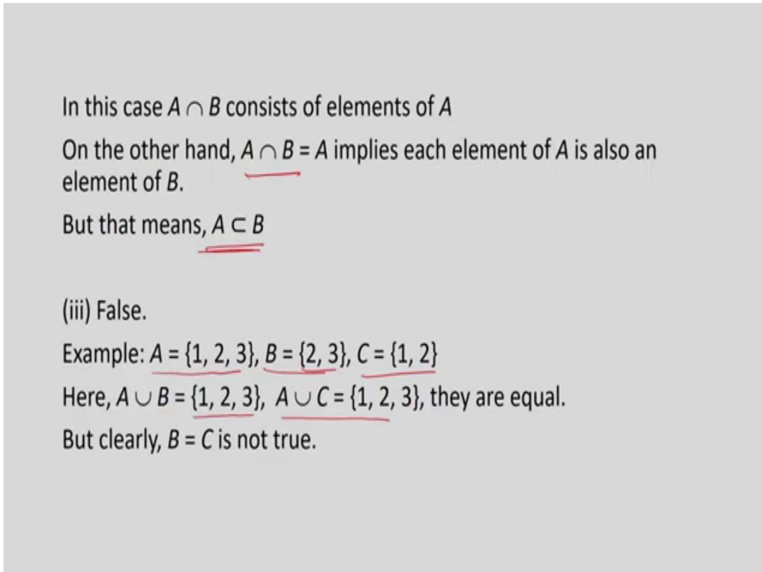

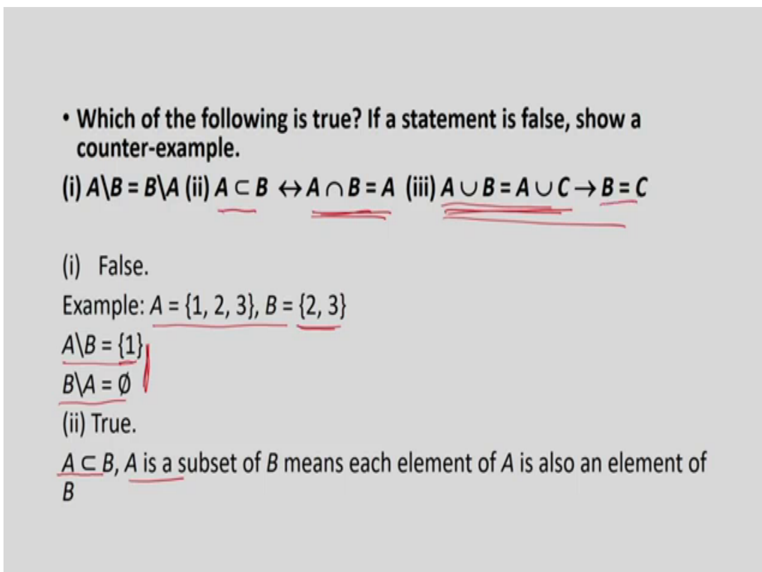

In this case $A \cap B$ consists of elements of A. So, if we take one to be a subset of the other and if we take the intersection, the intersection set becomes the subset. On the other hand, $A \cap B = A$ implies that each element of A is also an element of B. So, that was the right hand side $A \cap B = A$. So, if this is true then what does it mean? It means each element of A is also an element of B. So, that basically means that $A \subset B$. So, that is why from this, this is implied and from this, this is implied. So, that is why we are saying that the second statement is correct.

The third, what is this third saying, $A \cup B = A \cup C \rightarrow B = C$. And our answer to this is that it is a false statement. And if it is a false statement as per our claim, then we have to give an example. And here is the example. So, we have taken A to be this set {1, 2, 3} B to be {2, 3} and C to be {1 and 2}. Now, let us suppose these are the three sets then if we can show that this is not correct. In this case, what is $A \cup B$? $A \cup B$ will be {1, 2, 3}. And what is $A \cup C$? It is also {1, 2, 3} both are equal.

So, the left hand side is satisfied in this case. But obviously, B is not equal to C in this case. B is the set {2, 3} and C is the set {1, 2} obviously, they are not equal. So, we have constructed one example where even if this is correct, the implied part, the putative implied part is not following. So, that is why this third statement is not correct.

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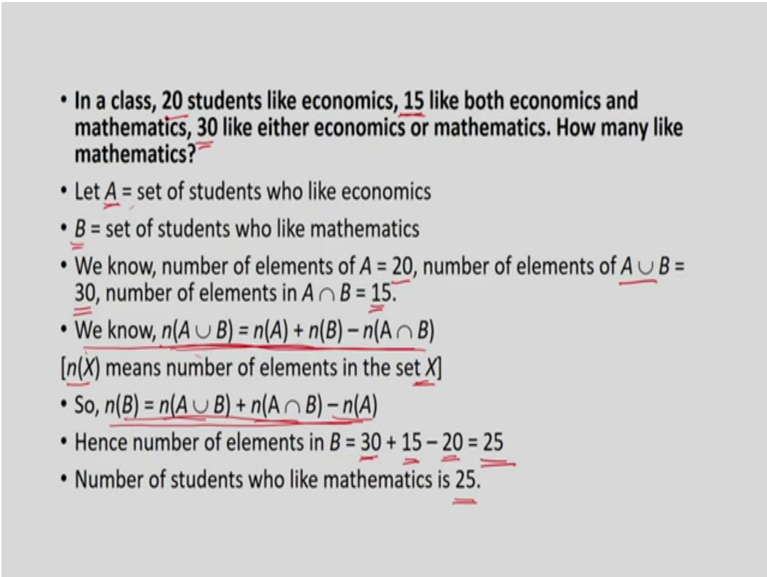

Here is another problem from set theory. In a class, 20 students like economics, 15 like both economics and mathematics, 30 like either economics or mathematics. How many like mathematics? So, that is the question. Here let us suppose that A is the set of students who like economics and B is the set of students who like mathematics. What do we know? We know the number of elements of A is 20. A is the set of students who like economics that is 20. Number of elements in A union B, that is the students who like either economics or mathematics, that is given by 30. And the number of elements in $A \cap C$ that is the students who like both economics and mathematics. That number is 15. And this is what we know.

This is a theorem that we are going to use here that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. What is n? n(X) means the number of elements in this set of X. So, this is a sort of new notation that is being introduced here. If X is any set, then n(X) denotes a number, that number is the number of elements in that set X. So, if that is how n is defined then this theorem follows that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

And from here we can actually manipulate it a bit and we shall get this relation that $n(B) = n(A \cup B) + n(A \cap B) - n(A)$. That will actually solve our problem if we use this relation, because we know $(A \cap B)$ is 30, students who like either economics or mathematics, plus 15 students who like both economics and mathematics, minus 20, the students who like economics. So, 30 + 15 - 20, that will give me 25. So, that is the answer. The number of students who like mathematics is 25.

That is more or less about set theory. We talked about actually three topics so far. We started with numbers and real number system, then we talked about mathematical logic, and finally, we talked about set, set operations. Those are the three topics that we have covered.

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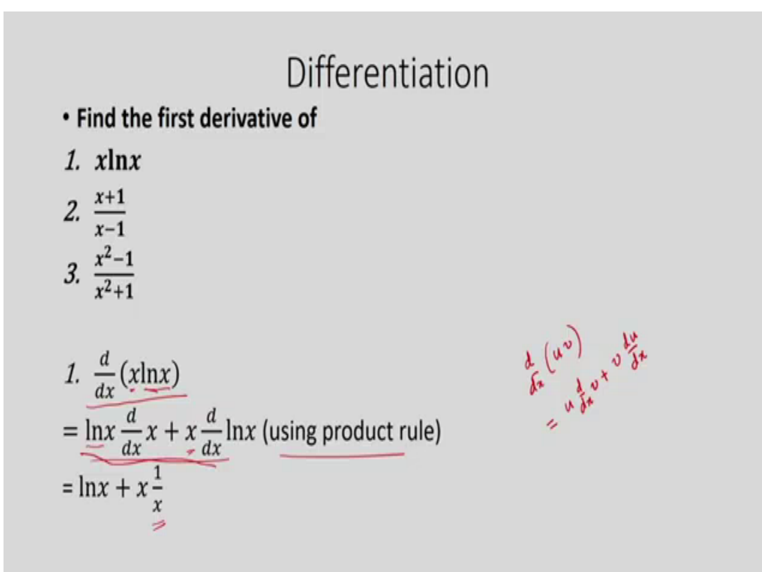

Now, let us move ahead a bit and start talking about what is known as differentiation. This was I think somewhere down the line in our course topics. So, differentiation, derivatives that is what this topic is about. So, here are some practical problems. First, these are the three problems that we are given. Find the first derivative of; number one, $xlnx$, log natural x in this case; secondly, $\frac{x+1}{x-1}$; and third, $\frac{x^2-1}{x^2+1}$. So, we have to find this in the first problem, d dx of $\frac{d}{dx}(xlnx)$.

Now, what we do is that we take the help of product rule. Remember, what was the product rule. So, you have $\frac{d}{dx}(u.v)$. u and v both are functions of x. Then what do we do is that we take the first function, let us say, u write and then we take the derivative of the second function plus you take the second function multiply that with the derivative of the first function. So, that was the product rule, just in case you remember. And that is the thing that we have used here.

We take one of these functions, suppose log x, we take it out and take the derivative of the other function which is x and we take then x out and take the derivative of the first function which is log of x. So, this was the product rule. We are applying it here. So, in the next step, what do we get is $lnx\frac{d}{dx}x + x\frac{d}{dx}lnx$, d d of x of x is by the power rule it will just become 1. So, I am not writing that 1 here, plus $x\frac{d}{dx}lnx$. What is $\frac{d}{dx}lnx$ is it is just $1/x$. And so, it can be simplified a bit more.

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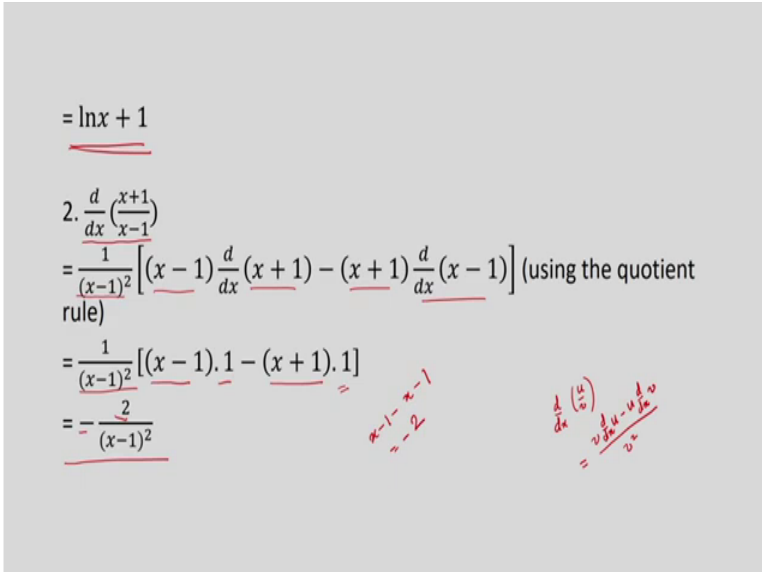

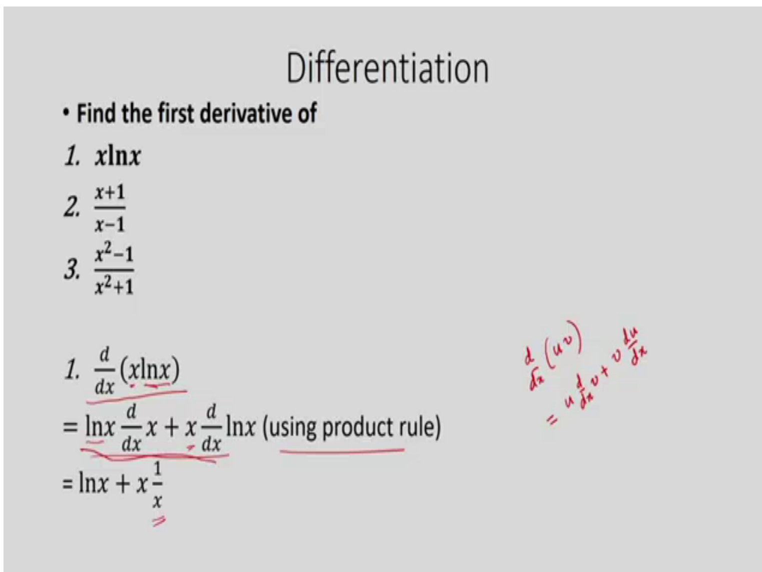

It becomes $lnx + 1$. So, this is the answer. Second question was $\frac{x+1}{x-1}$. We have to take the first derivative of this function. So, $\frac{d}{dx}(\frac{x+1}{x-1})$. So, the rule that we are using here is the quotient rule. Again to recall, so what we are doing here is using this rule. You have two functions u and v, both functions of x suppose, and you want to take the derivative of $\frac{u}{v}$. And the quotient rule tells us that this is $\frac{v.\frac{d}{dx}u - u\frac{d}{dx}v}{v^2}$. And that is what we are applying here.

In this case, $v\, is\, x - 1\, and\, u\, is\, x + 1$. v is the denominator. So, the denominator square multiplied by $\frac{d}{dx}$ of the numerator, the numerator is $x + 1$, minus the numerator $x + 1$ multiplied by the derivative of the denominator so $\frac{d}{dx}(x - 1)$. And in the next stage, it will become $\frac{1}{(x-1)^2}\left[(x - 1)\frac{d}{dx}(x + 1)\right]$. So, again, $\frac{d}{dx}(x)$ is 1 and $\frac{d}{dx}(1) = 0$. So, it becomes just 1, minus $x + 1$ and $\frac{d}{dx}(x)$minus 1 again $\frac{d}{dx}(x)$ is 1 and minus 0, so it becomes 1.

And here I am jumping some steps here. So, in the numerator, it will be what, it will be $x - 1 - x - 1$ and this is - 2. That is what I have written here, - 2. Minus sign is coming in the front. And the denominator is $(x - 1)^2$. So, this is the derivative, first derivative of $\frac{x+1}{x-1}$.

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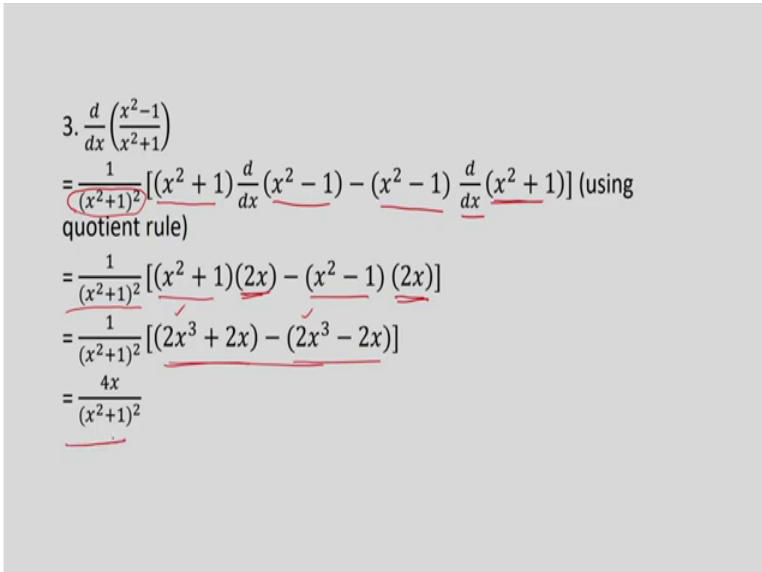

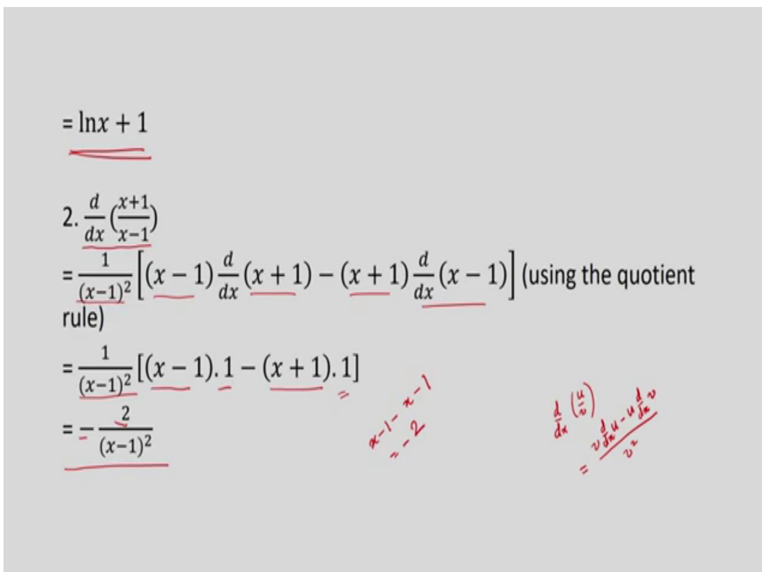

The third problem is a little bit complicated as you can see as compared to the second problem. In the second problem, there was no x square, but now we have x square. So, $\frac{d}{dx}(\frac{x^2-1}{x^2+1})$. Now, since it is a quotient, that we have to take the derivative of, we apply the quotient rule once again. So, the denominator square is coming here in the denominator multiplied by the denominator $x^2 + 1$ multiplied by the $\frac{d}{dx}$of the numerator, that is $\frac{d}{dx}(x^2 - 1)$, minus the numerator that is $(x^2 - 1)$multiplied by $\frac{d}{dx}(x^2 + 1)$. The denominator is $x^2 + 1$.

The rest of the problem is just to labor through the expressions. So, $\frac{1}{(x^2+1)^2}$ and on the numerator we are going to get $(x^2 + 1)$, $\frac{d}{dx}(x^2 - 1)$, we use the power rule and that will give me 2x, because $\frac{d}{dx}1$ is 0 and minus $(x^2 - 1)$ and $\frac{d}{dx}(x^2 + 1)$ again we are going to get 2x using the power rule.

In the next step, we simplify it further. So, we are multiplying 2x with $(x^2 + 1)$. So, $(2x^3 + 2x) - (2x^3 - 2x)$. And if I simplify the numerator, I am going to get 4x, because this and this will get cancelled. So, the answer is $\frac{4x}{(x^2+1)^2}$. This is the answer.

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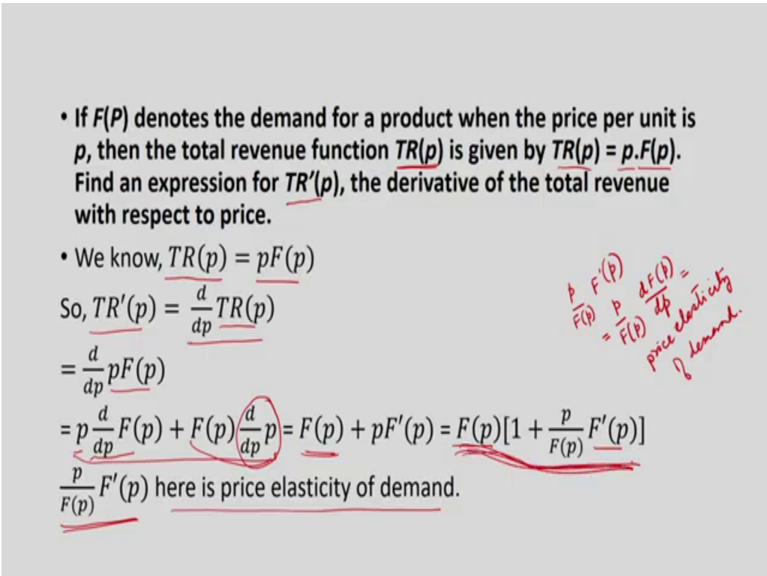

We are going to attempt a lot of questions from differentiation and optimization after differentiation. So, here is another problem about differentiation. If $F(P)$ denotes the demand for a product when price per unit is p, then the total revenue function $TR(p)$ is given by

$TR(p) = p.F(p)$. Find an expression for $TR^{'}(p)$, the derivative of the total revenue with respect to price. That is what we are supposed to do. Find $TR^{'}(p)$.

Now, $TR(p) = p.F(p)$. Just try to understand the economics of it. Total revenue is equal to price multiplied by the quantity demanded. How much people are buying, the quantity people are buying, multiplied by the price per unit. So, that will give me the total amount of money that they are selling out and that money is going to the producers as revenue. So, we have to take the first derivative of total revenue with respect to price. So, that is what we are doing here. And instead of TR, that is total revenue function, we write it as $p.F(p)$, demand multiplied by price. And then, we are using simply the product rule in the next step.

We take out p here and differentiate $F(p)$with respect to p and plus $F(p).\frac{d}{dp}p$, that is that derivative of price with respect to price and this will give me $F(p)$, from the second term we are going to get $F(p)$, because this is 1, plus $pF^{'}(p)$, $F^{'}(p)$ is the derivative of the demand function with respect to price. And in the next step, I take $F(p)$ common. If I do so, then I am going to get $F(p)[1 + \frac{p}{F(p)}F^{'}(p)]$. So, this is the expression therefore. This is the end result that the derivative of the total revenue with respect to price is $F(p)[1 + \frac{p}{F(p)}F^{'}(p)]$.

Now, interestingly this expression that $\frac{p}{F(p)}F^{'}(p)$ is nothing but the price elasticity of demand, because just think about it, $\frac{p}{F(p)}F^{'}(p)$ is equal to, so this is nothing but price elasticity of demand. So, this was the answer. And let me call it a day for the time being and I shall see you in the next class. Thank you for joining.