Mathematics for Economics-I
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Lecture No. 01
Aim of the Course, Real Numbers

Good afternoon everyone. So, we are going to start a new course for the undergraduate students, of engineering and science, and also for other disciplines. The name of the course is mathematics for economics part 1. My name is Debarshi Das, I teach in the department of humanities and social sciences, IIT, Guwahati. So, we are going to just introduce this course today, and maybe cover a couple of topics today itself so as to make you ready for the main content of the course.

The course is called mathematics for economics. And it is part 1, which means that there could be more advanced level courses of the same kind. This course is meant for whom?

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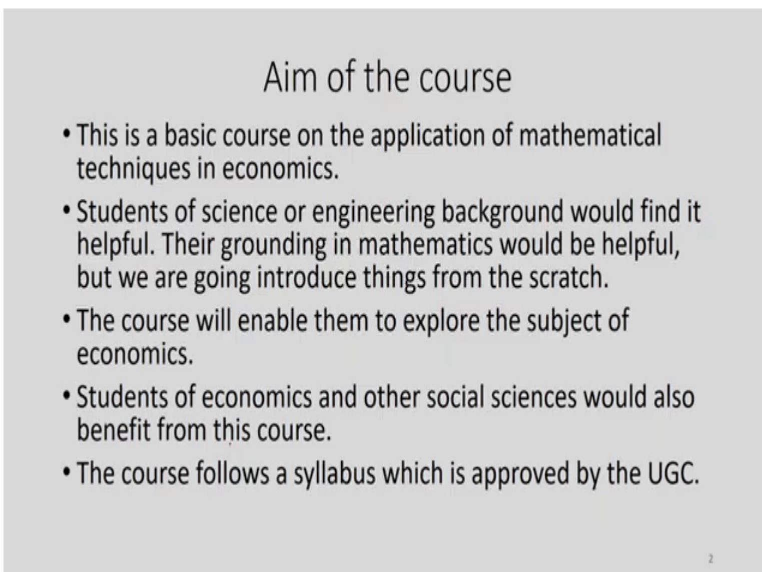

This course is, you can see on the screen, this is the aim of this course. This is a basic course, on the application of mathematical tools and techniques in economics. So, the point to note is that, we are not going to cover each and every topic of mathematics, only those topics of mathematics will be covered, which are applied in the subject of economics. So, that is why I said that it is mathematics for economics, not for some other subjects.

Now, who are the students who are going to be benefited from this course? Students of science and engineering background would find this course helpful, because they are students of engineering and science, they do not actually know which mathematical tools are used in economics. At the same time, they are trained in the tools of mathematics, because they had mathematics in their higher secondary level at least.

So they are, they have a basic grounding of mathematics, they know calculus, they know for example, advanced algebra matrix, linear algebra, but they do not know how these things are applied in economics, and which tools are applied in economics. So, this course, is going to be beneficial for them, because they will get to know, firstly which tools are used in economics, which mathematical tools are used in economics.

And secondly, how and in what context, these tools are used in economics, they have their grounding in mathematics, which is helpful. But what we are going to do is that we are going to start these tools. So we are going to introduce the tools from the very scratch, which means we are not going to assume that they know a lot about these tools, we are going to start from the very scratch.

This course will also help them to explore the subject of economics, because I am assuming that they did not have any grounding in economics in their plus two level or in the undergraduate level. So, they do not know anything about the subject of economics. But through this course, they will get to know at least some of the topics which we cover in economics, at least in the undergraduate level.

Because, when we are going to use the mathematical tools, we are going to use those mathematical tools in economic problems. So, they will get a glimpse of those economic problems as well. Apart from this batch, apart from this group of science and engineering students, this course is going to be useful for students of economics as such, students of economics means the students who have taken honors in economics, or maybe not honors, but they have credited a pass course in economics.

Those students will find this course helpful. And of course, as we know, economics is a social science. So, the students who are not economics honor students, but they have some interest in economics, they will also find this course helpful. Finally, what about the syllabus? When framing the syllabus for this course, we have been very careful so as to make this course a standard course, not a kind of exceptional or out of the ordinary course.

It becomes a standard course, it becomes acceptable to the maximum number of students. Those things have been on our mind, when we framed the syllabus. So, what we did is that, we looked up the syllabus of the UGC, UGC means University Grants Commission, and UGC has a model syllabus for the undergraduate students for many subjects, including economics. So, we have taken the model UGC syllabus for economics honors students.

And we know that economics honors students have to undergo courses on mathematical economics. So, we have taken that as our sort of benchmark. And we have made some changes here and there, maybe added a few topics here and there. And that is how we framed the syllabus of this course. So, that the final syllabus that we have framed, is in line with most of the university syllabus of India, because Indian universities and colleges follow the UGC pattern.

So, our syllabus is consistent with them, maybe a bit advanced here and there, but at least that level has been maintained, what you find on the screen is a list of topics that we are going to cover in this course.

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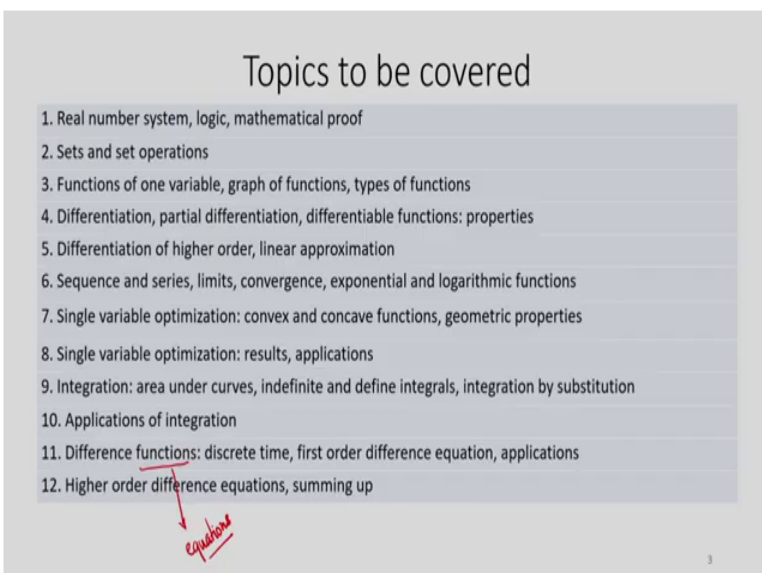

As you can see, we have listed as many as 12 topics, these 12 topics will be covered in this course. Now, as I told you, this is mathematics for economics part 1. So, I am not claiming that all these topics that have been listed here, that will be sufficient for the undergraduate students of economics. Because there has to be a little bit more advanced course, apart from this particular course. So, these are the topics, let me just read them out, we are going to cover real number system, logic, mathematical proof.

This is the first topic we are going to cover, these are the basic things that any student of mathematics has to learn, because without understanding how logic works, or mathematical proof works, you cannot move one step forward. And as we know, the number system is the basic of, founding stone of mathematics. Then we are going to talk about sets and set operations. And the third topic is functions of one variable.

We are going to explain what is a function and when there is a single variable, then how the functions can be defined, and how they look like, what are the different kinds of functions and graphs of functions. And functions can look different ways and how the graphs are derived for the functions. Types of functions, we are going to talk about different kinds of function, for example, in a polynomials and logarithm function, exponential function. So, we will talk about them.

The next topic is the topic on differentiation. So, if you have a function, then how do you differentiate that function? What is the definition? At the same time, we are going to talk about what is known as partial differentiation. So, when you have a function, which has more than one variable, as its arguments, then one can talk about partial differentiation. And this kind of differentiation is used a lot in economics.

Because in economics, the functions that we deal with, they are multivariable functions, there are more than one variable in your set of arguments. And differentiable functions, which functions can be differentiated, not all functions can be differentiated. So, there is something called differentiability. So, one has to be careful which functions are differentiable and which functions are not differentiable. That has to be there.

And we are going to talk about these things, I mean, properties of partial differentiation, differentiation, differentiable functions. Then we are going to talk about differentiation of higher order. Suppose, you have differentiated a function. But you want to find out what happens if you differentiate it further. So, those are called higher order differentiations. And so we are going to talk about how to do that and what are the interpretations of higher order differentiation.

And in the same way, we are going to talk about linear approximation. So, if you have a function, the function may not be a linear function, it could be a nonlinear function, but can you linearize it? So, that is called linear approximation. Can you see a nonlinear function as an approximation to a linear function? So, that is the fifth topic. Sixth topic is sequence and series. So, you have series of numbers, so those numbers have an order in which they are coming.

So, those things could be either sequences or series we are going to talk about that. And connected to this topic is the topic of limits, we are going to precisely define what is a limit. Actually, without the idea of limits, differentiation cannot be defined. So, here we are going to precisely define what is a limit. Earlier when we talked about differentiation, the idea of limits will be used, but in a more common sensical way, rather than a rigorous way.

So, in topic 6, we are going to talk about the limits in a rigorous manner, we are going to talk about convergence, divergence when a series or a sequence diverges, when does it converge, etcetera, etcetera. And very importantly, we are going to talk about these two special kinds of functions, why these are special? These is special, because they are used a lot in economics, these are called exponential functions and logarithmic functions.

So, whenever we talk about growth rate, for example, India's growth rate is 3 percent or 4 percent, or price is rising at a certain person, then at the back of our mind, we have this growth rate and the idea of exponential and logarithmic functions are connected to this idea of growth. So, over time, how the things are changing. So, those things will be dealt with in this functions, which are called exponential or logarithmic, or for example, you have kept money in your bank, and it is giving you some rate of interest.

So, how do you find out what is the value of the money will be after let us say, 5 years, or 6 years, at 6 percent rate of interest. So, those things can be dealt with very easily if we know how to use exponential and logarithmic functions. Then, in chapter 7, we come to what is known as single variable optimization. So, you have suppose just a function of one variable, that is why I have written a single variable.

And you want to optimize it, you want to find out where it is getting its maximum or minimum, then that exercise is called optimization. So, we are going to talk about that, how to do that. We are going to talk about what are known as convex and concave functions, these things related to the idea of optimization, as we shall see. Geometric properties of concave and convex functions, that also we shall deal with.

Number 8 is single variable optimization, the same topic but now we are going to talk about the results, and the applications. So, application is very important. We are going to talk a lot about the applications in economics of these various topics. Some of the topics you may have encountered as a student of mathematics, who had mathematics in your class two, some of these things might be familiar, but you may not have encountered how these things, these techniques are used in economics. So, that is how the applications are very important.

Then the integration, again integration is something which is covered in plus two level mathematics. So, area under curves, indefinite and definite integrals, these are introduced to students, integration by substitution, but what is important is that application, which will be covered in topic number 10 So, in economics, how integration is used? So, that again will be important here.

And then final topic that we are going to cover is what is known as, this should be not functions equations, difference equations, discrete time. So, this should be difference equation. So, what is meant by difference equations, so here time is discrete. So, it is not changing continuously, but changing one after another, you can count time as it were. And first order difference equation, we are going to talk about that. And how it is applied in economics.

And finally, higher order difference equations, and those things will also be very useful as we shall see, if we have something like a cycle, for example, people talk about, if you read newspapers or watch television, you will find economists talking about, recessions and booms, recovery and inflation. So, as if the economy is going through different kinds of cycles, going up and coming down, all those things are happening. So, these are called trade cycles or business cycles. And to explain them, we shall see that the higher order difference equations will be quite useful.

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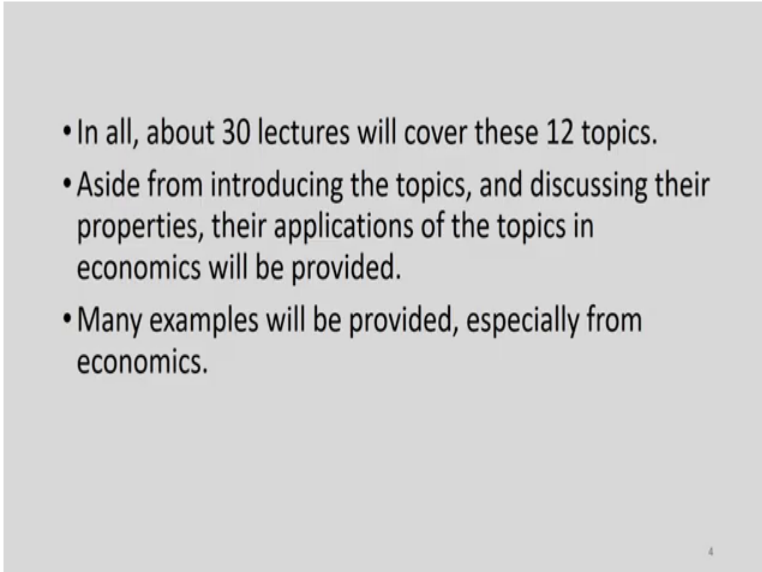

So, after this brief roadmap that I have just discussed. So, these 12 topics are the different, signposts in this road that we are going to undertake. There will be around 30 lectures that will cover these 12 topics. What we shall do in this discussion of 30 lectures, we shall aside from introducing the topics, we shall discuss their properties of these topics, and their applications of the topics in economics will be provided. Many examples will be provided, especially from economics, I have talked about that, so, I do not want to repeat this.

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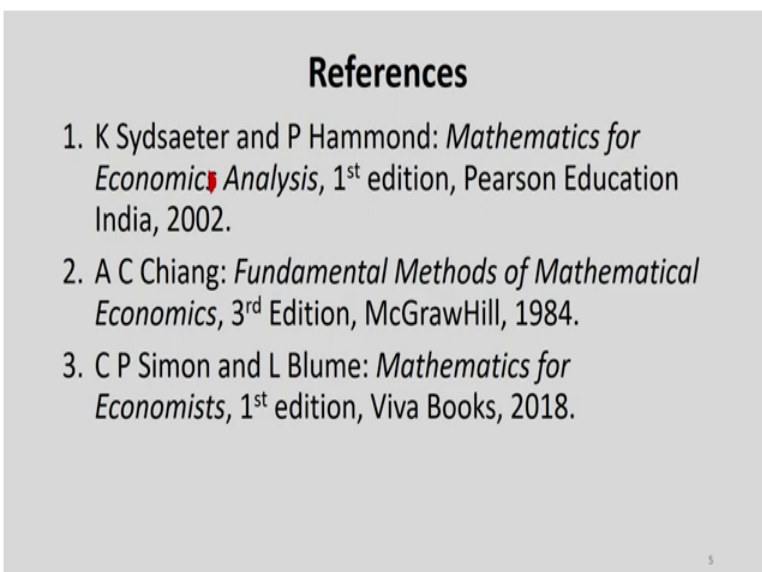

So, these are the references, three books I have, in my mind. The first one is mathematics for economic analysis, I guess this is not economics analysis, mathematics for economic analysis, first edition, Pearson Education, India 2002. So, this is going to be the standard sort of textbook that I shall go back to. But apart from that, you have Alpha C Chiang, fundamental methods of mathematical economics 1984, this is the third edition 1984.

But this is the edition that, that is most familiar to me, and it is a very good book, it is a very classic kind of book, which generations of economics students have read and benefited from. And then, number 3 is mathematics for economists, first edition, Viva books 2018, by Simon and Blume. And this is going to be a little bit advanced compared to the first and the second book, but very good book, all three of them are quite insightful and interesting books.

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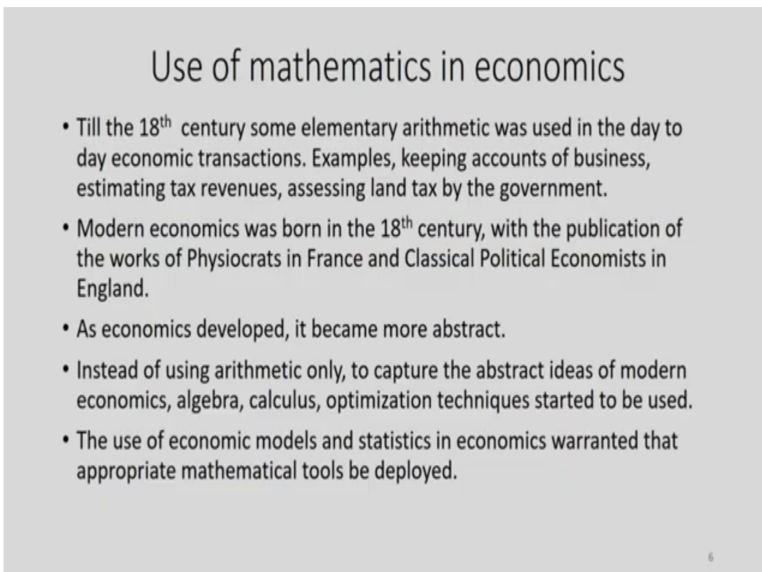

Okay, Now without wasting time, we are going to start with some of the topics. Because we do not have a lot of time, we have to cover a lot of ground in our 30 lectures, so we do not want to spend a lot of time talking about things which are not important. So, we start with the use of mathematics in economics. So, why do you want to use mathematics in economics in the first place? So, if we look at the history, then we find that, that till the eighteenth century, some elementary arithmetic was used in the day to day economic transactions.

So, by this what we mean is that, for example, there were businessmen, what would the businessmen do? They will do business, they will sell goods, they will get the money, keep a track of the money, also, they will buy different goods from different other people, and they will keep track of them. So, a lot of monetary transactions were done by the trades people, business people. Also, the manufacturers were there, rudimentary manufacturers, they will also have to keep track of the things.

So, keeping accounts of business through that, people were using mathematics, but this is like arithmetic, simple arithmetic. Not only business people, also the government, if you have a running government, the government has to collect tax revenue. And so it has to keep track of the income of the people, for example, if you want to collect income tax. So, that also involves some amount of arithmetic.

And also, the government used to collect tax, a lot of tax on land. So, on that also keeping track of the land amount people owned, and how the tax will be collected from them. That was important. So, these are the examples of how mathematics was used in not economics, I will not say that they were used in the subject of economics, but they were used in economic transactions. So, during that time, that is before the eighteenth century, the modern science of economics was not even born.

So, modern economics was born in the eighteenth century, to be more precise, late eighteenth century. So, this is like 1760s, 1770s, 1780s. This is the time I am talking about. So, just before the French Revolution, and you have the start of the American Civil War and American Revolution, that is also the time when the industrial revolution took place in England. And this is the time when the modern economics was also taking its shape.

And it took its shape with the publication of the works by a group of economists called the physiocrats, in France, this is 1760s and 50s just before the French Revolution. And also by the, what are known as the classical political economists in England. So, any economist will tell you that the first ground shattering book of economics was by Adam Smith, the Wealth of Nations.

So, that was written in the 1770s, it came out in 1776. And then after him of course, Malthus has written, Ricardo has written. So, all these people belong to what is known as the classical political economy school. So, that is the birth of modern economics as we know it. Now, as economics developed in this manner, the ideas became more abstract. So, rather than just keeping track of business transactions, what you now have people are talking about let us say the Wealth of Nations. So, what they meant by that, it is not something that the economists actually saw physically, it is not something tangible, the Wealth of Nations it is more abstract.

So, you have a country like England. And England has so much millions and billions pound sterling of wealth. So, Adam Smith obviously, has not seen that with his own eyes. But he is imagining how that amount of money is being generated, how it can go up or down and what does it have to do with other things such as, trade of England with other countries, or let us say division of labour, whether division of labour is good for the Wealth of Nations. So, those things are more abstract, rather than just keeping track of how much money you are earning in your business.

So, instead of arithmetic only, to capture the abstract ideas of modern economics, algebra, calculus, optimization techniques started to be used. So, that these are the new things that came to be used in economics, such as algebra, obviously, algebra is more abstract than arithmetic, because now you have some symbols rather than numbers. And calculus, calculus obviously, the subject was known before the eighteenth century, but, by the nineteenth century, the calculus started to be used in economics.

And optimization techniques were also started to be used in the twentieth century. The use of economic models and statistics in economics warranted that appropriate mathematical tools be deployed. So, as we shall see in our course of lectures, that economic models are very important part of the subject of economics.

We are going to talk about what our economic models, but what is important to understand at this point of time, that if you have economic models, it uses mathematics and therefore, mathematical tools became relevant and also statistics. So, this is again, very new thing in economics. So, economics not only talks about economic models, which are quite abstract, but it also deals with data. Many people think that economics is an empirical science. So, it deals with data. So, if you are dealing with data again, mathematical tools become important.

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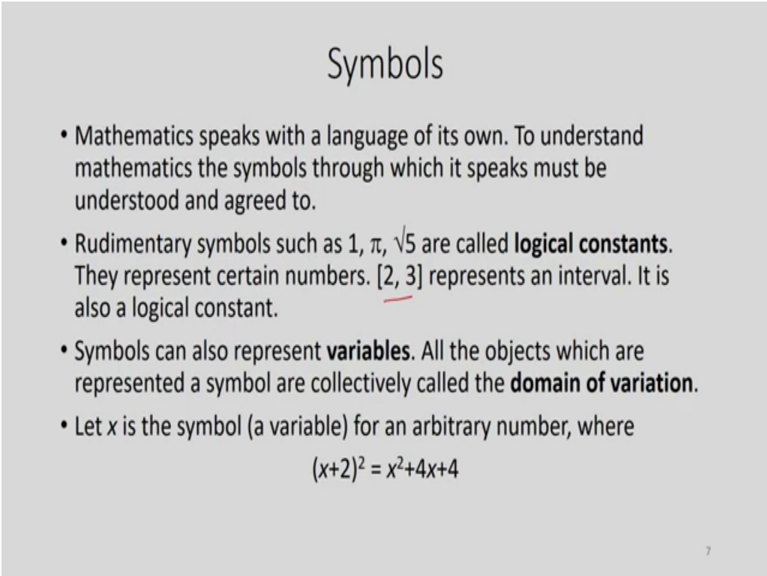

Okay, we are going to start with something called symbols, symbols are the starting points of mathematics. Mathematics speaks with a language of its own, to understand mathematics, the symbols through which it speaks must be understood and agreed to. So, mathematics it has a language on its own means it does not speak in English in words, it speaks in terms of symbols. And those symbols are agreed to all the mathematicians know what a particular symbol represents.

For example d dx, differentiation d dx. So, all the mathematicians around the world agree that this is representing differentiation, with respect to x. So, they have this language. Now rudimentary symbols are like what? Like a number, like 1 is a rudimentary symbol, π or $\sqrt{5}$. These are logical constants, they represent certain numbers, logical constants means they do not change in their value. So, they are constants.

Also take this particular this thing. So, you have here [2, 3], what it represents? It is an interval, but it is also a logical constant. So, it is like a number, because it is representing a set of numbers only, nothing more than that. So, these things are logical constants. But symbols can be more than logical constants, they can also represent what are known as variables, ok.

All the objects which are represented by a symbol are collectively called the domain of variation, okay. So, here the number represented by a symbol can vary actually. Here is an example, let x is the symbol of arbitrary number, where $(x + 2)^2 = x^2 + 4x + 4$. So, x is a symbol here, but it is not a logical constant, it is a variable. And look at this particular relation $(x + 2)^2$, which basically means $(x + 2)(x + 2)$is equal to on the right hand side you have $x^2 + 4x + 4$. So, this is in terms of symbols, I have written this.

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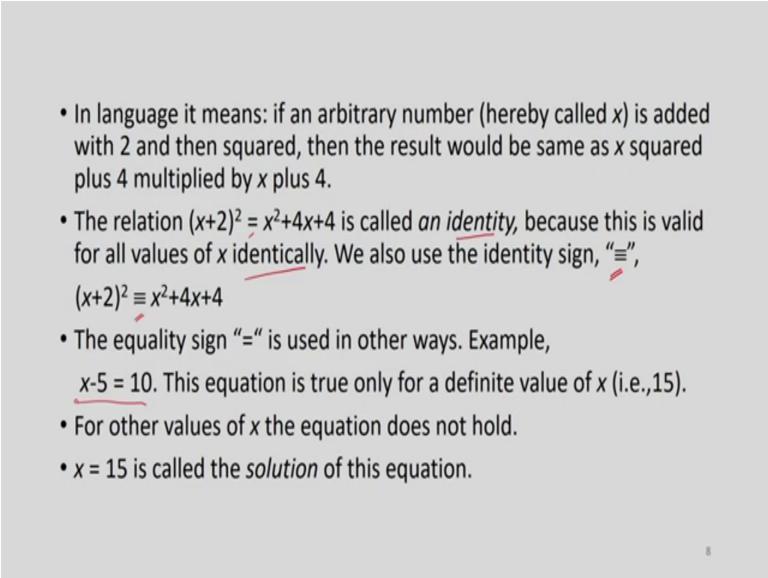

But if I want to say the same thing in terms of ordinary English, then what does it mean? If an arbitrary number hereby called x is added to 2 and then squared, then the result would be same as $x^2 + 4x + 4$, okay. So, this is a whole lot of long sentence, which I have written in English, which has been sort of condensed in terms of symbols in this short form, $(x + 2)^2 = x^2 + 4x + 4$. The same thing can be written in language, but it becomes quite long.

Now, here in this relationship, x could be any value, there is no bound on x, it can be any value and this thing will be true. The relation $(x + 2)^2 = x^2 + 4x + 4$ is called, also called an identity. Because this is valid for all values of x, identically. So, I have marked this as a as italics, because it is called an identity. We use this sign, this, three parallel lines, this is the identity sign.

So, instead of equality sign I could also write it as this, $(x + 2)^2 \equiv x^2 + 4x + 4$. The equality sign just two parallel lines is the equality sign, is used in other ways also. For example,

$x - 5 = 10$here I have used not the identity sign, but the equality sign. This equation is true only for a definite value of x, that is 15.

So, this relationship$x - 5 = 10$ is correct only for a particular value of x, which is 15. For other values of x, this will not be correct. For example, if you put $x = 5$ for example, then $5 - 5 = 0$. So, left hand side will not be equal to right hand side. So, LHS is equal to RHS only if x has a particular value, and that is why this is not an identity, it is an equation. For other values of x, the equation does not hold, $x = 15$ is called the solution of this equation. So, when you have an equation, it may have a solution. So, here in this case, the solution is 15.

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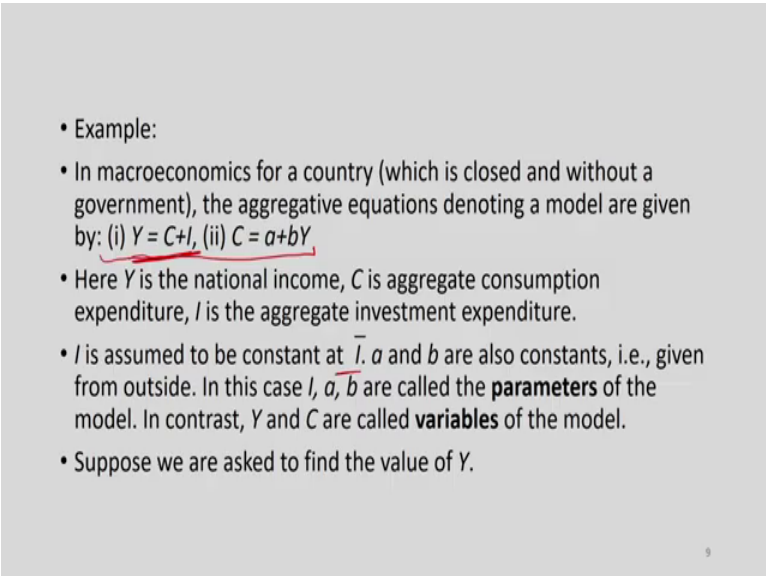

Another example I give here, which is from economics, in macroeconomics for a country which is close country and which is without a government, the aggregative equations denoting a model are given by (i) $Y = C + I$ (ii) $C = a + bY$. I talked about models. So, this is an example of a model.

So, in this case, this model is denoting the aggregate income of the country, Y is the national income or the aggregate income. And this is denoted by the first equation, $Y = C + I$, what is C? C is aggregate consumption expenditure, and what is I? I is the aggregate investment expenditure. And the second equation here, in the model is giving me the function of C,

$C = a + bY,$ small a plus small b multiplied by Y. Y is the same Y that we are talking about, capital Y, which is the national income.

Now, here I is assumed to be constant at $\bar{I}$. Small a and small b are also constants, that is given from outside, constant means, they are not determined by the model, they are given from outside. So, these are also called exogenous. So, they are not determined by the model. So, they are called exogenous. In this case, I, a and b are called parameters of the model also.

In contrast, Y and C are called variables of the model. So, there are some parameters which are given from outside, these are exogenous. And there are some variables which are determined by the model. Here Y and C are determined by the model. These are called variables. Suppose, we are asked to find the value of Y, suppose we want to find out what is the value of Y. So, how should we do that?

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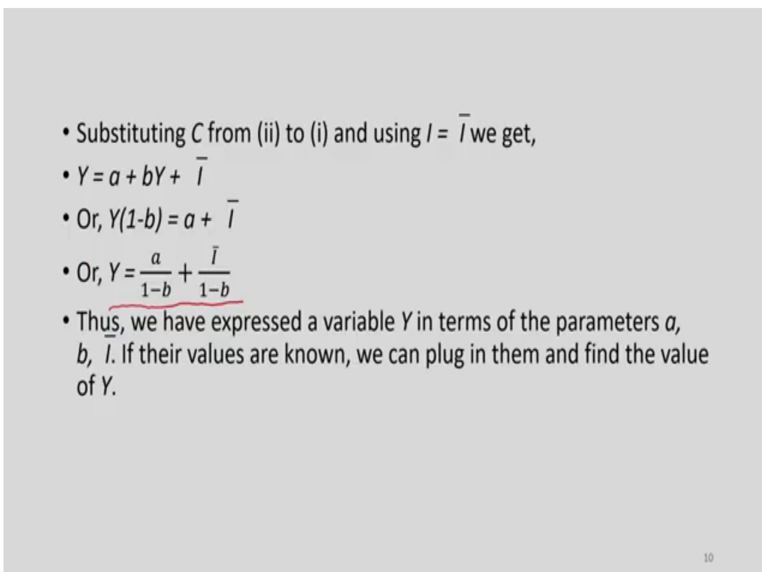

So, it is very simple, we substitute C from (ii) to (i). And we use the fact that $I = \bar{I}$. So, it is given there. So, we use these two things, and we use then, the equation number one, we put everything here. So, we are substituting C, $C = a + bY$, and $I = \bar{I}$, that is what we are doing. So, we are getting $Y = a + bY + \bar{I}$. And then the next step, we are taking all the Y terms to the left hand side.

And then we are just dividing both sides by $1 - b$, assuming $b \neq 1$. If $b = 1$, then we cannot divide both sides by $1 - b$. So, we get this thing, the last line. So, what this line is telling me is that, national income, $Y = \frac{a}{1-b} + \frac{\bar{I}}{1-b}$. Thus, we have expressed a variable Y in terms of the parameters, small a, small b and $\bar{I}$.

If these values are known, we can plug in them and find the value of Y. If the values of the parameters are known to us, then we can just put them on the right hand side here. And that will give me the value of the variable, which is Y. So, that is how you can see, that the parameters and variables are related. If we have this, if this equation is derived that $Y = \frac{a}{1-b} + \frac{\bar{I}}{1-b}$.

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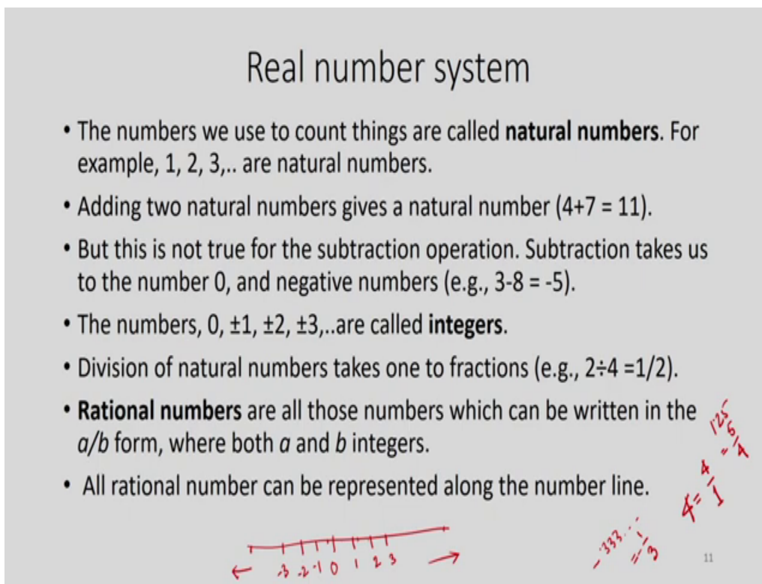

Okay, so, we have talked about symbols. Now, we come to what is known as the real number system. So, we are going to cover a part of this, and then we shall call it a day And pick up the topics in the next lecture. Real number system, so, let me try to go step by step. The numbers we use to count things are called natural numbers. So, that is the starting point of the real number system, natural numbers, the primitive ban, the first time they encountered with number is when they started to count things.

So, how many stars are there in the sky, they will have to count that, or how many cows they have. And that is very important. Unlike counting the stars in the sky, counting your cows is more practical. Because suppose you have 4 cows, and only 3 came back. And you could not count them, then you will be making a lot of loss. So, counting precisely is very important. But how do you count, you count with natural numbers, you count by 1, 2, 3, 4. These are called natural numbers.

Since they are coming natural to you, because you want those numbers to count things, though. So, that is why these are natural numbers. So, 1, 2, 3, 4 like that you can go on to infinity. All these are natural numbers. Now, if you add two natural numbers, then you get another natural number. So, $4 + 7 = 11$, 11 is also a natural number.

So, by adding you are staying inside the domain of natural numbers, but this is not true for subtraction operation. So, if you subtract one natural number from another natural number, do you get another natural number? Not necessarily, subtraction takes us to the numbers 0 and to negative numbers. So, if you take 4 and you do $4 - 4 = 0$, 0 is not considered to be a natural number.

Similarly, if you take $3 - 8 = -5$, -5 is not a natural number. So, subtraction, if you do that, then you go outside the group of natural numbers. And then the numbers that you get, if you include all of them, then what? They are called the integers. So, the numbers 0, ± 1, ± 2, ± 3 like that, you can go on, they are called integers.

And if you take integers and divide one by another, then you go outside the domain of integers. Where do you go? You go to the domain of fractions. So, for example, if you 2 ➗4= 1/2 , 1/2 is not an integer. So, it is a fraction. So, then we come to what is known as rational numbers. Rational numbers will now include, the fractions. Rational numbers are all those numbers which can be written in the form of $\frac{a}{b}$, where a and b both are integers.

Now, you can ask if I take 4, is 4 an integer? The answer is yes, 4 can be written in the form of $\frac{a}{b}$, because 4 is equal to what? 4 is equal to, $\frac{4}{1}$, both are integers, 1 is an integer, 4 is an integer. So, it is in fact in the $\frac{a}{b}$form. So, by that definition, 4 is a rational number. And so is let us say, 1.25, 1.25 can be written in what form? It is 5/4, 5 is an integer, 4 is an integer. So, 1.25 is a rational number.

Now, all rational numbers one can think of, for example, you can think about let us say, 0.33333. So, what is this number equal to? It is 1/3. So, again, you have one integer divided by another integer. And if you take the minus of this, it becomes the -1/3. So, all these numbers can be represented along, what is known as the number line.

If you have forgotten what is the number line, let me just draw it for your benefit. So, here is a 0 number and here you have 1, 2. So, this is going to plus infinity, and you have numbers decreasing here. And here, you go to minus infinity. So, this is called a number line. Now, all these numbers that I have talked about, for example, minus 5 will be somewhere here, 11 will be somewhere here, minus 0.333 will be somewhere here, 5 by 4 will be somewhere here, 1.25. So, every number that I have talked about, so far, will have a place here in this number line.

So, all rational numbers, these are all this group of numbers which are called the rational numbers, can be represented along the number line. So, so far, so good. Now, the natural question to ask here, is that if I put all the rational numbers along the number line, will they cover the entire number line or there will be some gaps? And the answer is there will be some gaps. So, if you think about all kinds of rational numbers, and if you plot them on the number line, still there will be infinite number of other gaps on that same number line.

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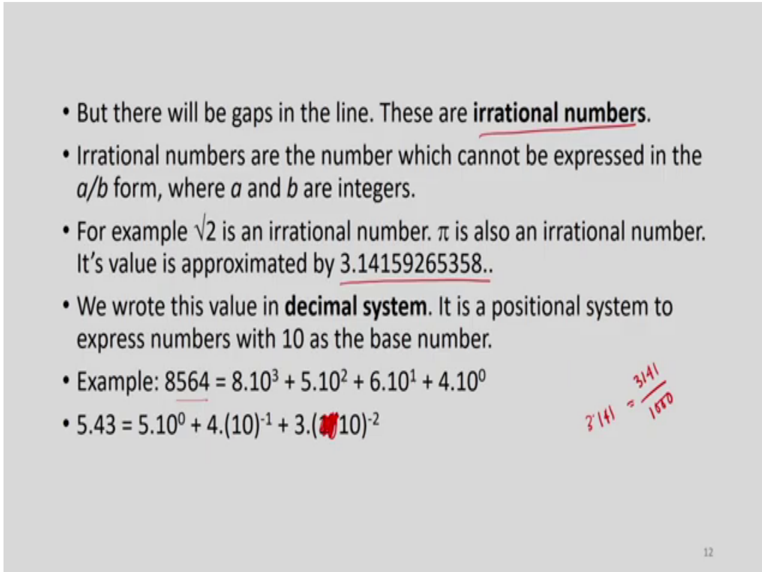

And which are these gaps? These gaps are called irrational numbers. So, irrational numbers are numbers which are on the number line, but they are not rational numbers. So, irrational numbers are the numbers which cannot be expressed in the $\frac{a}{b}$ form, where a and b are integers. Remember, that was the definition of rational numbers, that they can be expressed in the $\frac{a}{b}$ form. So, irrational numbers are those numbers which cannot be expressed in this form.

For example, this $\sqrt{2}$ is an irrational numbers, this is a famous irrational number, π is also an irrational numbers and its value is approximated, this is the value of π, it is approximated by 3.14159265358 so it goes on. Now see, if it goes on, then you cannot put it as something divided by something. If it had been 3.141, suppose it stopped there, then I could have written this as 3141 ÷ 1000. And here 3141 is an integer, 1000 integer. So, it is a rational number, end of story.

But here it is not stopping anywhere, it is going on like that. So, if it is going on, I cannot write it in this form, $\frac{a}{b}$ form. We wrote this value in the decimal system, it is a positional system to express numbers with 10 as the base number. So, what is a decimal system? This is just a small sort of digression. For example, 8564, this has been written in decimal system. So, what does it mean?

It $8564 = 8.10^{3} + 5.10^{2} + 6.\ 10^{1} + 4.10^{0}$. So, this is how decimal system works, you have the base as 10, and each number in the decimal system represents the power of 10, and that gives its this number is position.

Similarly,$5.43 = 5.10^{0} + 4.10^{-1} + 3(10)^{-2}$.

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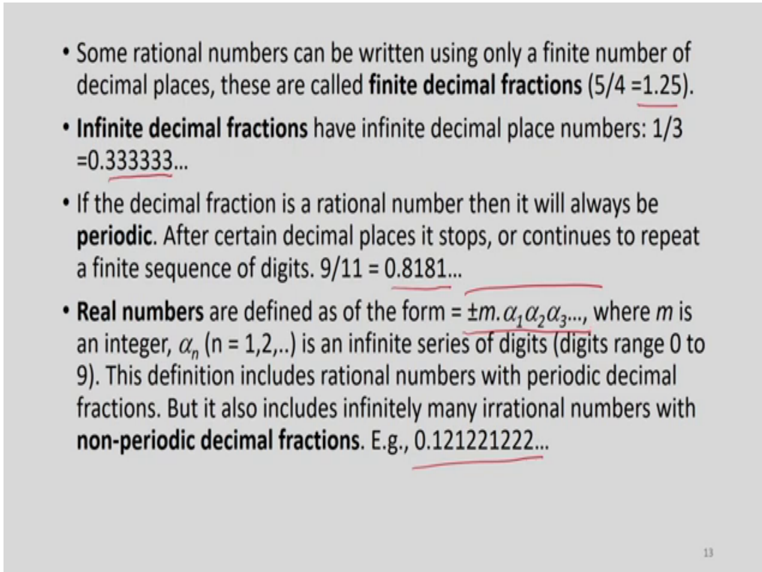

We have talked about rational numbers, we have talked about irrational numbers. Now, one important thing is that some rational numbers can be written using only a finite number of decimal places, these are called finite decimal fractions. So, for example 5/4 = 1.25. So, only a finite decimal places, 0.25. But some rational numbers are there which are called infinite decimal fractions. For example, 1/3 = 0.333333 it will go on like that.

If the decimal fraction is a rational number, then it can always be periodic, after certain decimal places it stops or continues to repeat a finite sequence of digits. So, these are all rational numbers and if you have rational numbers, then they are periodic, which means you have repetition of the same number like here, 9/11 = 818181, here also it is repeating 818181 like that, or they are stopping. They are stopping here, I mean stopping means after that 0 is getting repeated. So, if you have a rational number, it will always be like that, either it will stop or it will go on repeating the same number, same pattern of numbers.

And finally, we define what is known as real number, which is like the summation of rational and irrational numbers. Real numbers are defined as of the form = $\pm m.\alpha_1.\alpha_2.\alpha_3.\alpha_4$......, it will go on like that, alpha 4, alpha 5. Where m is an integer, m is any integer and alpha n, where n goes from 1, 2, 3, 4 like that to infinity is an infinite series of digits, so digits range from 0 to 9.

This definition includes rational numbers with periodic decimal fractions, but it also includes infinitely many irrational numbers with non periodic decimal fractions, like this one, E.g., 0.121221222….. So, it goes on like that till infinity, you can see that it is not repeating, there is a pattern, but it is not repeating. So, 0.12, 0.122, 0.1222 after that there will be 4 twos, 5 twos, but you cannot claim that it is repeating some number or some pattern of numbers.

So, this is an irrational numbers. So, any number which can be written in this form $= \pm m.\alpha_1.\alpha_2.\alpha_3.\alpha_4......$, it includes the rational numbers, it includes the irrational numbers as well. And therefore, it is called what is called the set of real numbers.

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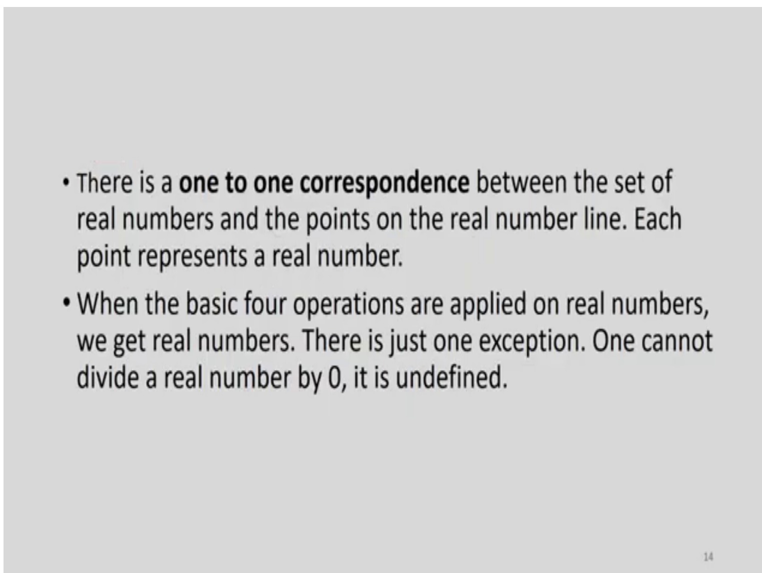

There is a one to one correspondence between the set of real numbers, and the points on the real number line. Which means, if you take any real number, and you plot that on a real number line, and you plot all the real numbers, then this real number line will look completely full, there will not be any gaps in that. And that means that every point on the real number line represents a real number.

When the basic 4 operations are applied on real numbers, we get real numbers. So, basic 4 operations means, addition, subtraction, multiplication and division. So, if you add two real numbers, you get a real number. If you subtract one real number from another real number, you get another real number. If you multiply one real number with another real number, you get a third real number, like that.

But, there is a one exception, which is that, though 0 is a real number. You cannot divide any real number with 0, because that is undefined. It is then going beyond the realm of real numbers. I guess we shall stop here. We are going to continue with where we left in the next lecture. So, here in this lecture, to sum it up, what we have done is that we have introduced what are known as symbols, we have talked about different kinds of symbols, variables, logical constants.

And we have introduced the idea of real numbers, they all fall on the number line. So, in the next lecture, we are going to talk about more, about the basic, these are the basics notion of mathematics, we have not entered the realm of economics as such. So, these are the basic tools of mathematics we are going to introduce and then we are going to introduce economic concepts when the time comes. Thank you.