Introduction to Market Structures
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Lecture - 8
Tutorial on Production and Cost Curves

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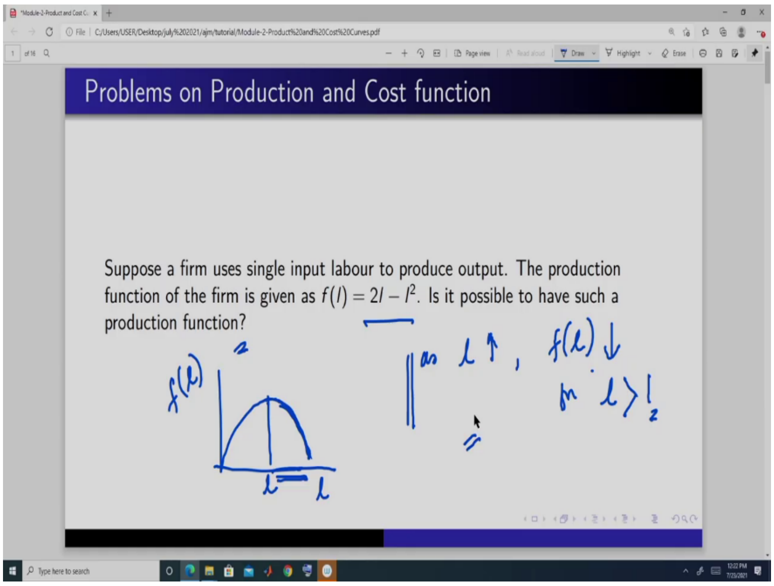

Okay, let us solve some problem for the topic that we have done in module two, that is production and cost. So, here suppose a firm uses a single input that is labor to produce output and the production function of the firm is given by this function- $f(l) = 2l - l^2$, okay. Now, is it possible to have such a production function? Now, if we plot this, if we look at labor here and if we look at this, this function is something like this output here.

Now, if we look at this, what is happening? This portion, it is decreasing but labor is increasing. So, what we are getting? As labor increases, it falls after for l greater than 1 this point is actually because if you solve this the maximum point is at point 1, okay. So, it cannot be a production function, why? Because we know that if we increase input the output should always increase, but we are not getting this here. If we increase labor further more than 1 then output is decreasing. So, that is why this is not a production function, okay.

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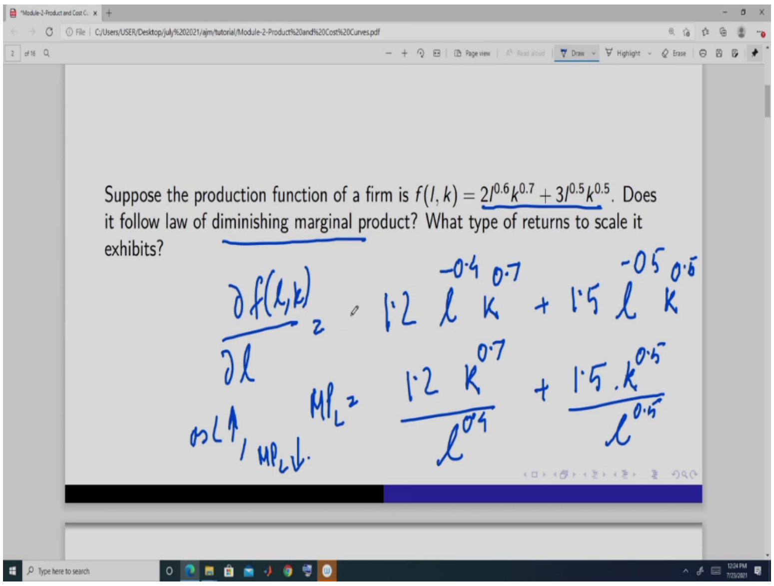

Now, let us take another example and suppose this is the production function of the firm- $f(l,k) = 2l^{0.6}k^{0.7} + 3\,l^{0.5}k^{0.5}$. Does it follow law of diminishing marginal product? So, what do, do we know that this is a differentiable production function and it has two inputs labor l and capital k. So, if we take the derivative of this with respect to l, we are going to get 1.2 $l^{-0.4}k^{0.7} + 1.5\,l^{-0.5}k^{0.5}$ or you can write this, this- $1.2\frac{k^{0.7}}{l^{0.4}} + 1.5\frac{k^{0.5}}{l^{0.5}}$. Now here, you will see this expression as we increase labor this is going to go down, as we increase labor this is going to go down. So, that is why marginal as l increases marginal product of labor it falls. So, that is why it follows law of diminishing marginal product.

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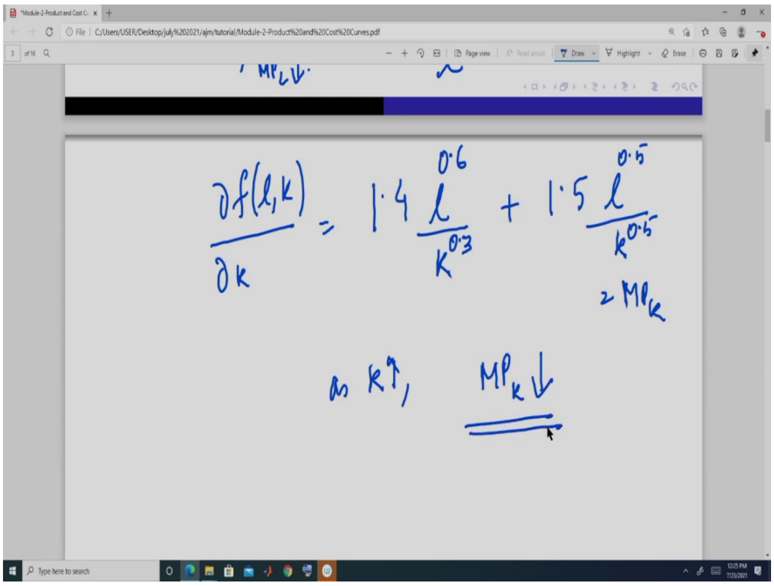

Similarly, if you do the, take the derivative with respect to k, you are going to get this is 0.7. So, it is 1.4. This- $\frac{\delta f(k,l)}{\delta k} = 1.4\frac{l^{0.6}}{k^{0.3}} + 1.5\frac{l^{0.5}}{k^{0.5}}$, now here this is the is equal to marginal product of capital. And if you look at this, what do we get? As k increases, this portion goes down and this portion also goes down because ks are in the denominator. So, that is why marginal product of capital goes down. We get this, so that is why law of diminishing marginal product is satisfied. Now, what type of returns to scale it exhibits. So, let us increase both the labor and capital by a factor theta, okay.

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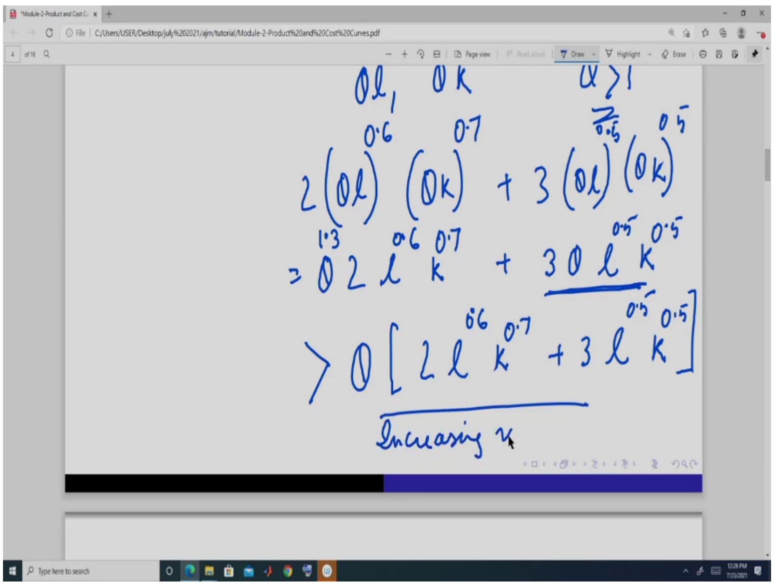

And theta is greater than 1. So, we will get this- $2(\Theta l)^{0.6}(\Theta k)^{0.7}$ plus this-$3(\Theta l)^{0.5}(\Theta k)^{0.5}$. So, this is equal to this-$(\Theta)^{1.3}2l^{0.6}k^{0.7} + 3\Theta l^{0.5}k^{0.5}$, now if you look at this here this portion $3\Theta l^{0.5}k^{0.5}$is theta times this. But here it is theta to the power 1.3. So, it is more. So, this term you can write it is more than theta terms, this-> $\Theta\ [2l^{0.6}k^{0.7} + 3l^{0.5}k^{0.5}]$. So, that is why we see that there is increasing returns to scale, okay, we get this.

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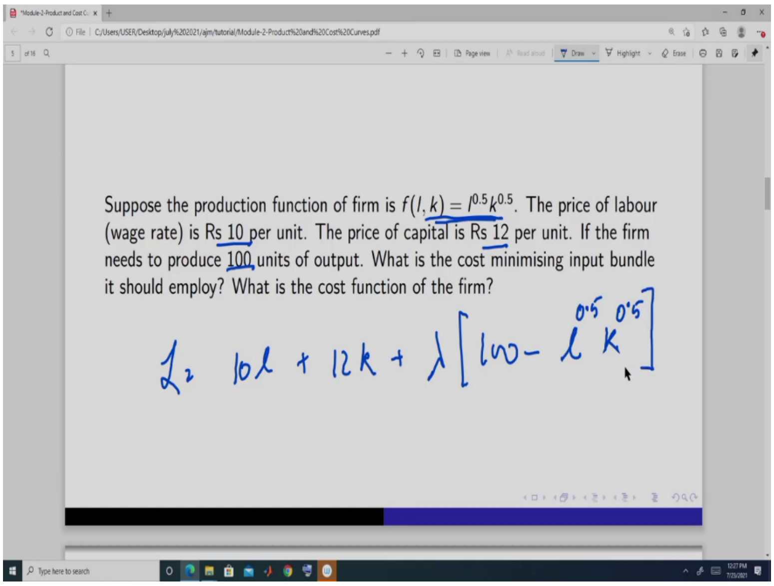

Now, let us do another problem and this is cost minimizing problem. So, here is suppose the production function is this- $f(l, k) = l^{0.5}k^{0.5}$ and the price of capital is 12, price of labor is 10. And the firm wants to produce 100 units of output. So, we see that the production function is a differentiable production function. So, we set the Lagrange on in this form this- $L = 10l + 12k + \lambda[100 - l^{0.5}k^{0.5}]$, this and what do we get?

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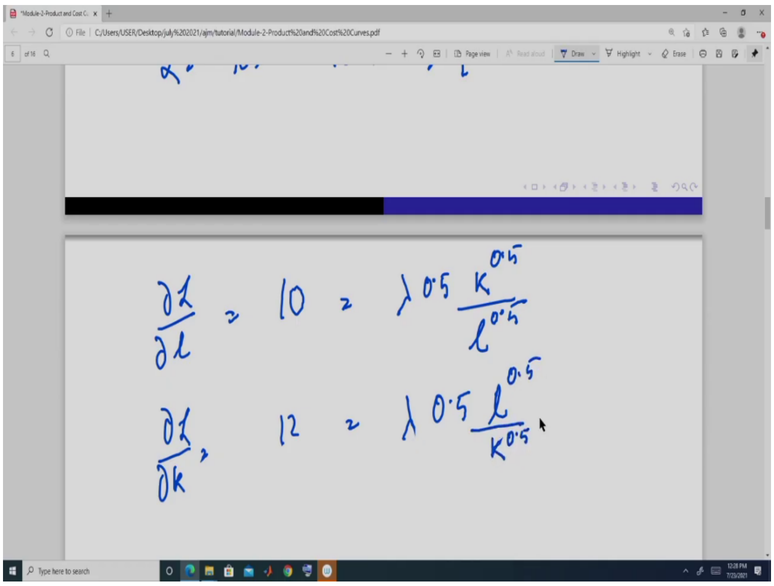

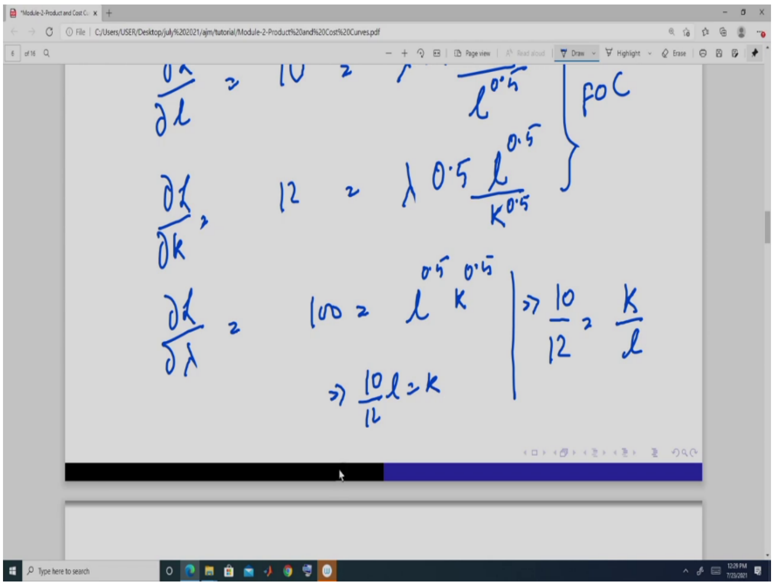

We get the first order condition. I am not, I am simply writing the first order conditions and we are going to get this as- $\frac{\delta L}{\delta l} = 10 = \lambda^{0.5} \frac{k^{0.5}}{l^{0.5}}$, $\frac{\delta L}{\delta k} = 12 = \lambda^{0.5} \frac{l^{0.5}}{k^{0.5}}$, $\frac{\delta L}{\delta \lambda} = 100 = l^{0.5} k^{0.5}$, okay. These are the first order condition. So, from these two, first two equations, we get 10 by 12 is equal to, if I take this divided by this, then I will get this is equal to k by l, okay, sorry, this- $\frac{10}{12} = \frac{k}{l}$. Now, from here, what do we get that 10 is equal to k, i.e$\frac{10}{12} l = k$. So, you plug in here, you will get- $100 = l^{0.5}\left(\frac{10}{12}\right) l^{0.5} \Rightarrow 100 = l\left[\frac{10}{12}\right]^{0.5}$, this.

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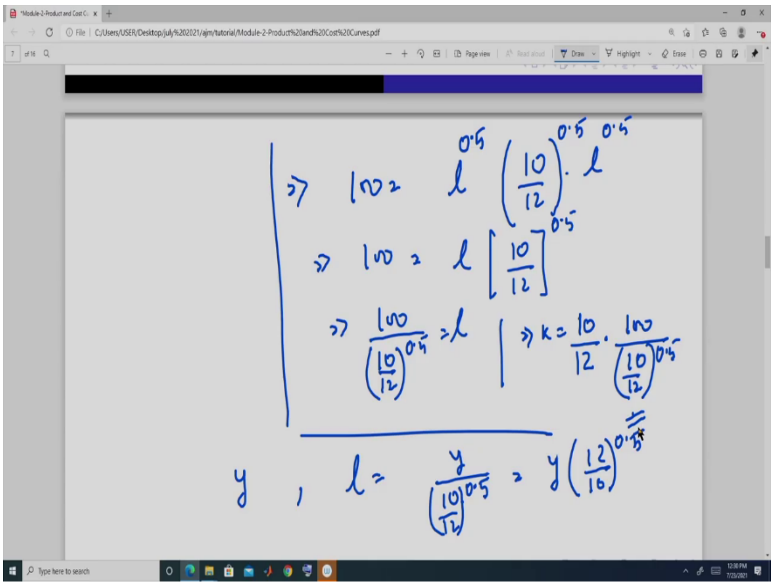

So, this is l. So, k is equal to 10 by 12 into l- $k = \frac{10}{12} \cdot \frac{100}{(\frac{10}{12})^{0.5}}$ you can simplify this. Now, we are again asked, what is the cost function of this? Cost function is a function of output. So, instead of taking 100 if we take some output a suppose y is, output is suppose y, then here what do we get? Labor is equal to y by this- $l = \frac{y}{\frac{10}{12}^{0.5}}$, or you can write this.

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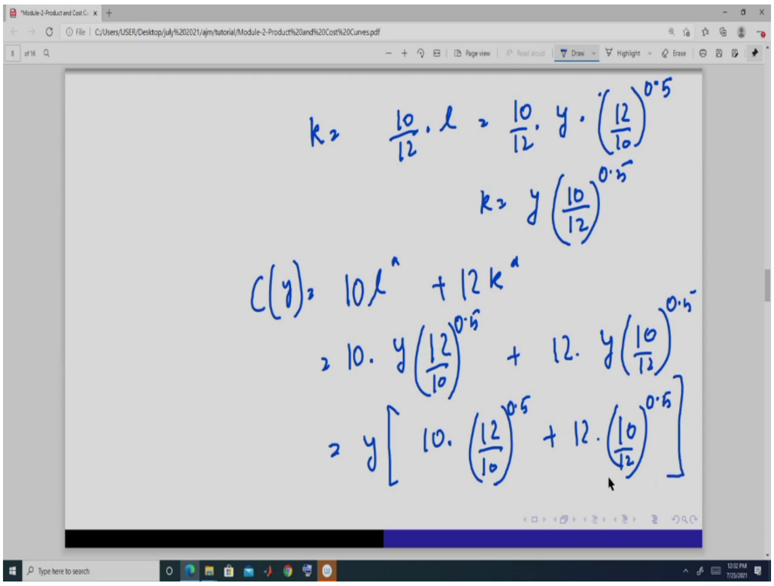

And capital k is equal to y, this 10 by 12 into l, which is 10 by 12 into y, this- $k = \frac{10}{12}. l = \frac{10}{12}. y. \left(\frac{12}{10}\right)^{0.5}$. So, k is equal to y this- $k = y\left(\frac{10}{12}\right)^{0.5}$. So, the cost function is 10 l star plus 12 k star, so 10 into this plus 12 into this. So, this is what? y so this is the cost function in this case, i.e $c(y) = y[10. \left(\frac{12}{10}\right)^{0.5} + 12\left(\frac{10}{12}\right)^{0.5}$

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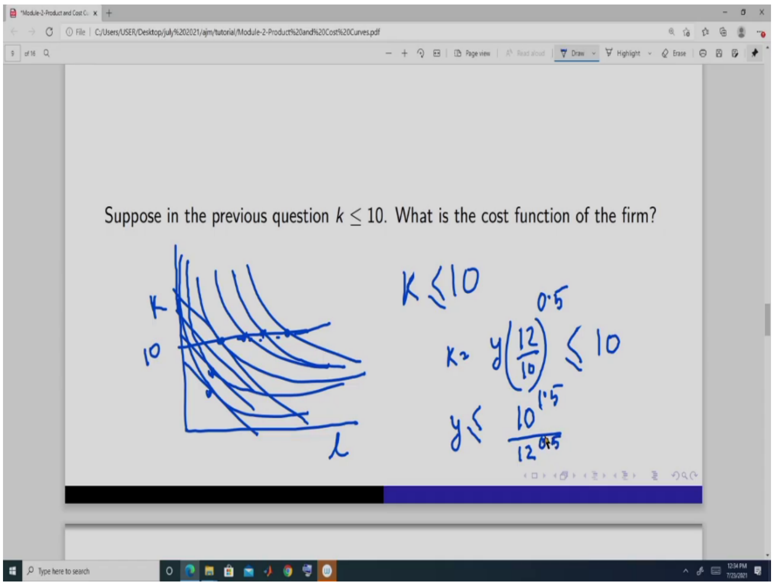

Now, suppose we are fixing the k, k cannot be greater than this 10. Now, what is going to be the cost function? Now, so, this means that if we take labor and capital given the isoquant it is something like this. And we are fixing this k 10 price are such that we will, it is 10, 12. So, it is going to be somewhere here like this. Now, we may get like this, we may get like this. But once we are here we our outputs are going to be at this point, optimal points are this.

We have done this while doing this in the class, okay. Why these are the optimal points? Okay. So, what do we find? Now, k is should be less than or equal to 10, what is the demand for a k? K is always equal to y 12 this- $k = y\left(\frac{10}{12}\right)^{0.5}$. So, this should be less than equal to 10. So, y should be equal to 10 to the power 1.5 and 12 to the power 0.5, 0.5 this- $y \leq \frac{10^{1.5}}{12^{0.5}}$. Whenever, y is less than this we will get the cost function to be of this nature.

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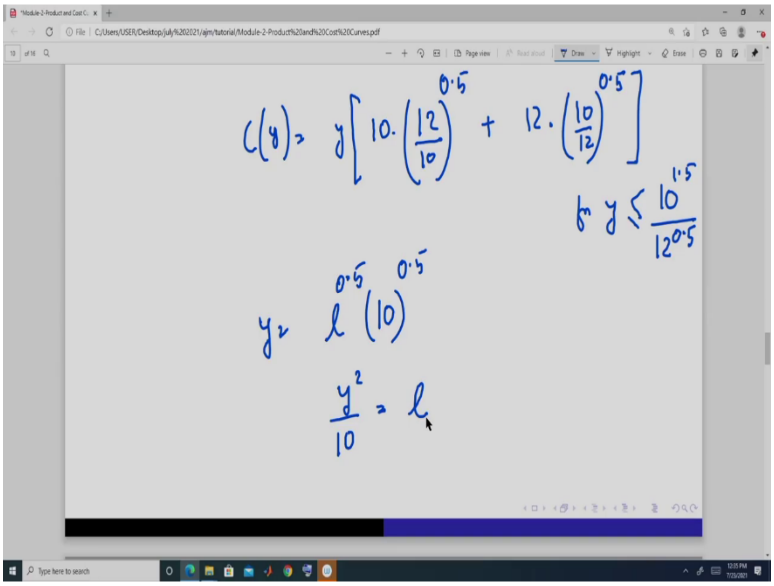

So, we get the cost function is this- $c(y) = y[10.\left(\frac{12}{10}\right)^{0.5} + 12\left(\frac{10}{12}\right)^{0.5}$ for y less than equal to this- $y \leq \frac{10^{1.5}}{12^{0.5}}$. And whenever y is greater than this, then we know how the output is produced. This from the production function it is this, and capital is fixed so it is 10. So, the demand for labor is, it is this, right. So, what is going to be the cost function?

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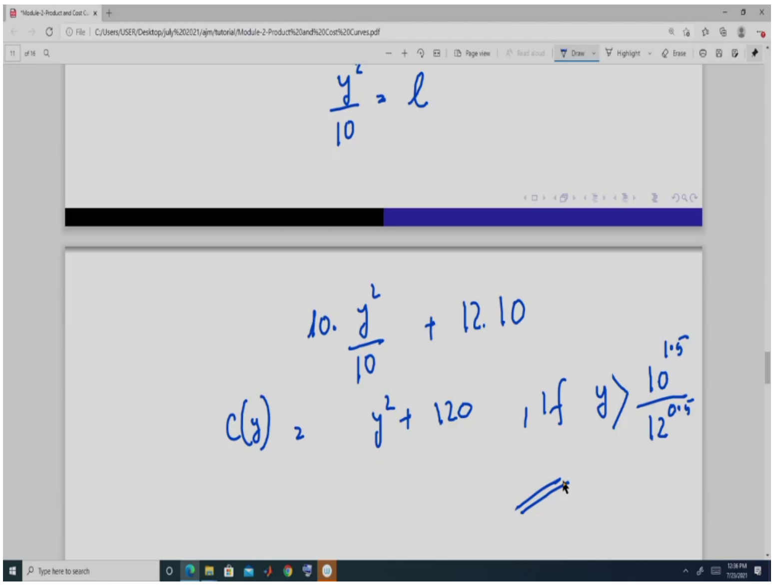

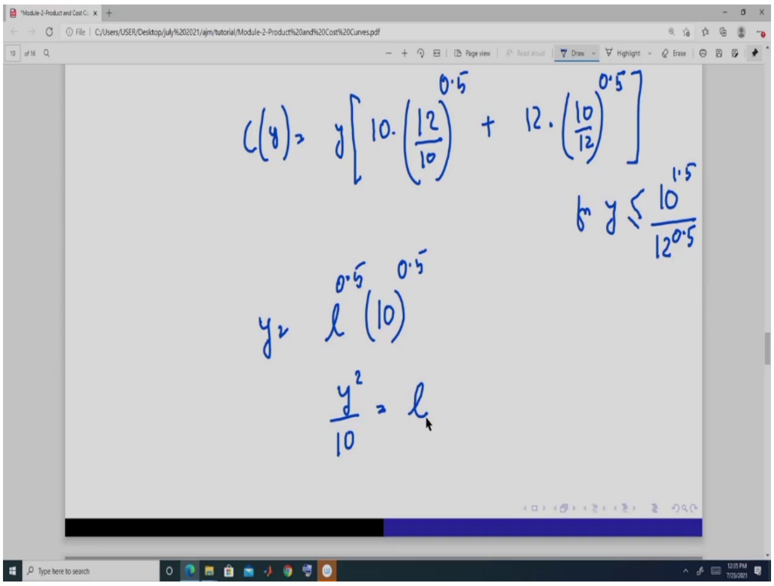

Cost function, now is 10 into this y to the power plus 12 into 10- $c(y) = y^2 + 120$. Because 10 is the a, so, this is square plus 120, if y is greater than 10 to the power 1.5 12 to the power this. So, this is the cost function, okay. And for this, it is this, okay. Thank you.