

**Introduction to Market Structures**  
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**Lecture 7**  
**Derivation of Cost Curves**

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The screenshot shows a PDF slide with the following content:

- We have derived the cost functions. The cost function gives the minimum amount of cost incurred in production of a given amount of output. We get it from the cost minimization subject to a given level of output.

Handwritten notes on the slide include:

$$\min \quad wL + rK$$
$$\text{s.t.} \quad y_0 = f(l, k)$$
$$l^*, k^*$$

A vertical line separates the objective function from the constraint. To the right of the line, the expression  $wL^* + rK^*$  is written above a horizontal line, with the label "cost fn" written below it.

Welcome to my course, Introduction to Market Structures. So, today we are going to do cost curves. Now, we have already derived the cost function, how we have done the cost function? Cost function, we have got in the following way. We have minimize this-  $wl+rk$ , this expenditure on labor, expenditure on capital. This is wage into labor plus price capital that is interest rate into the amount of capital subject to some production function this-  $y_0 = f(l, k)$ .

And we have specified that suppose, we want to produce  $y$  naught amount of output. Then, when we solve this, we get a cost minimum amount of labor and cost minimum amount of capital. And when we plug in this, here in the objective function- $wl^\alpha + rk^\alpha$  we got this with a cost function. We have done that in the last class. So, cost function gives me the minimum amount of cost incurred to produce a fixed amount of output, okay, or given level of output.

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The screenshot shows a presentation slide with the following content:

- In the example done in the last class, we get the following cost function when both labour and capital can be varied  
 $c(w, r, y) = y^{\frac{1}{\beta+\alpha}} \Lambda$ ,  
where  $\Lambda = w \cdot \left(\frac{r\alpha}{w\beta}\right)^{\frac{\beta}{\beta+\alpha}} + r \cdot \left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\beta+\alpha}}$
- Suppose capital is fixed at some level  $\bar{k}$ . This is the minimum amount the producer has to install in the firm. In that case, the firm is always incurring a fixed cost of the amount  $r\bar{k}$ .

Handwritten blue annotations on the slide include:

- A blue arrow pointing to the  $\Lambda$  term in the cost function.
- A blue arrow pointing to the  $\bar{k}$  in the fixed capital constraint.
- Handwritten blue circles around  $\bar{k}$  and  $r\bar{k}$ .
- A blue arrow pointing from the  $r\bar{k}$  circle to the text "fixed cost of the amount  $r\bar{k}$ ".

Below the slide, the text "In this case the cost function is" is partially visible.

Now, if we take this as a Cobb Douglas production function, then we get a cost function of this nature-  $c(w, r, y) = y^{\frac{1}{\beta+\alpha}} \Lambda$ , where this capital lambda is actually given by this expression-  $\Lambda = w \cdot \left(\frac{r\alpha}{w\beta}\right)^{\frac{\beta}{\beta+\alpha}} + r \cdot \left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\beta+\alpha}}$ , we have derived this in the last class. So, what do we get? We get that our cost function is actually is a function of the output that we want to produce and the price of labor and the price of capital. But since this firm always takes the price of capital and labor as given, this firm cannot determine these prices.

So, we take this cost function to be only a function of output, okay. Now, so, we get a cost function of this firm. When both these inputs, labor and capital are divisible and you can employ any amount you want to, want to, to produce that output, right? So, none of these labor input is fixed. But in short run, we know that capital can be a fixed input. We cannot use more than a given amount of capital. So, in that case, what happens this suppose capital is fixed at  $\bar{k}$ , then price of capital is  $R$ .

So, this become a fixed cost, whatever amount of output you produce, you have to bear this cost. Then this component, because you need some amount of capital and you know that capital is only available in suppose chunk, capital is not divisible continuously. So, in that case, suppose

the minimum capital you want to have is this, you need to have is  $k$  bar. So, in that case, you will always incur this much amount of costs, whatever amount of output you want to produce.

Now, suppose you want to employ more capital. So, you will further increase your amount of capital, but that will take time. So, in the short run, only this amount is available. So, now, if you want to produce more and more amount of output, what you can do? You have to employ more and more labor. So, in this case, we have seen that we get a cost function of this nature-

$$c(y) = w \cdot \left( \frac{y_0}{k^\beta} \right)^{\frac{1}{\alpha}} + rk$$

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The screenshot shows a presentation slide with the following content:

- In this case the cost function is  $c(y) = w \cdot \left( \frac{y_0}{k^\beta} \right)^{\frac{1}{\alpha}} + rk$ . A handwritten note  $y = l^\alpha k^\beta$  is written in blue ink above the equation.
- $c(y) = c \cdot y^\alpha + F$ . Here  $F$  is the fixed cost.
- Types of cost: variable cost, fixed cost, sunk cost.
- Variable cost: varies with the level of output. If the level of output increases, the variable cost always increases.
- Fixed cost: A fixed amount of cost independent of the level of output. Even if the level of output is zero, the producer bears fixed cost.
- Sunk cost: cost which cannot be recouped. Like amount of money spent on market research before entering a market. While refurbishing the firm: one example can be the amount spent on painting, but furnishes can be resold and some amount of expenditure can be recouped.

If our production function is a Cobb Douglas production function that is of this nature-  $y = l^\alpha k^\beta$ . Right? We have seen that, now here you can say that this portion is a fixed cost, and this is a variable cost, okay. So, our cost function may have two component, one is the variable cost, another is the fixed cost. And further we may have another type of cost and that is a sunk cost okay. So, first what is a so, what is a variable cost?

Variable cost is that cost, when you want to change your output that cost also changes. That means, that if you want to increase your output, you will require more input, at least one of the inputs needs to be employed in more quantity. So, the cost in that related to that input is going to

go up. So, that component is giving you the variable cost. So, here it is this part-  $\gamma y^{\frac{1}{\alpha}}$ , okay. And if you want to reduce your output, then you are not going to employ that much amount of input.

So, in that case you will reduce your employment of that input. So, your cost, you are going to incur in that factor or input is going to go down. So, this portion is going to change. So, it can change when you increase your output. So, the cost will go up, when you decrease your output, the cost is going to go down and that is going to happen in this portion. Fixed cost is this given-F, you cannot, the moment you keep on increasing output, this cost is going to stay same, it is going to be fixed.

If you reduce your output, even if you are producing zero units of output then also you will incur that cost and that is this f fixed costs. Sunk cost is another type of, it is a fixed cost only it is a cost which you cannot recoup. Like in fixed cost if you have seen, that we have this capital these machines, now you can resell this machine if you are just going out of the business, or if you think that you are not going to produce this much amount of output, you are going to reduce your capacity.

So, you do not require that much amount of machine, then you can sell off some of your machines and you will get some price. So, you will get some money from that. But suppose, you have entered this market and before that you have done some market research. And that has cost you some amount of money. Then that cost you cannot resell that thing, anything. So, that is a sunk cost or suppose, you have refurbished your plant.

So, you have put new paints in this say. So, this paint you cannot sell to anyone, because you have painted your firm. So, this is going to be a kind of sunk cost, you cannot get any return on it. If you have incurred it, it is gone. So, you have incurred that cost. But if you think of furnitures or if you think of machines, then you can resell them. And in that case, you while reselling you will get some money from them, while reselling.

So, you will be selling them at some price. So, at least there is some way of generating some amount of money from by reselling. But if you incur the cost of this nature like painting your firm or market research, those kind of thing, then you cannot resell those things. So, that is why,

these are kind of sunk cost, okay. But we will not explicitly require sunk cost in this chapter. We may introduce sunk cost when we do entry deterrence later in this course, okay. So, we have seen this different type of cost.

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$c(y) = c_v(y) + F$

- Most common form of cost function is  $c(y) = c_v(y) + F$ . Here  $c_v(y)$  is the variable cost component and  $F$  is the fixed cost.
- Average cost: cost per unit of output.  
$$\frac{c(y)}{y} = \frac{c_v(y) + F}{y} = AC$$
- $AC = \frac{c_v(y)}{y} + \frac{F}{y}$ . Here  $\frac{c_v(y)}{y}$  is average variable cost (AVC) and  $\frac{F}{y}$  average fixed cost (AFC).
- Nature of average variable cost curves and average fixed cost curve.

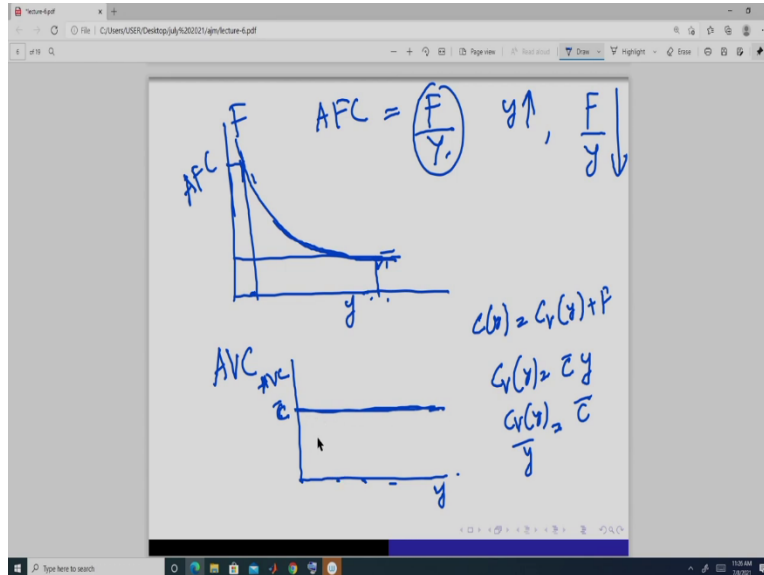
And most commonly, we are going to use this type of cost function-  $c(y) = c_v(y) + F$ , where this part is the variable cost and this part is the fixed cost, okay. So, since this is variable, so we have put a subscript v and this fixed, so, we have put it v, this F. Now, average cost is so, this function  $c(y) = c_v(y) + F$  is giving me the total cost. If I want to produce y unit of output. So, total cost is given by this function. And it is this-  $c(y) = c_v(y) + F$ , it has a variable component and a fixed component.

Now, average cost is the cost per unit of output. So, it is total cost divided by the amount of output I am producing, this-  $\frac{c(y)}{y} = \frac{c_v(y) + F}{y} = AC$ . So, this when you open it up this total cost is variable cost plus fixed cost divided by the total unit of output. This is called the average cost and we write it as AC, denoted it as AC that is the average cost. Now, AC you can see, it is this -  $\frac{c_v(y)}{y}$  plus this-  $\frac{F}{y}$ . So, this part is called the average variable cost, and this part is called the average fixed cost, okay.

So, average cost is sum of two type of costs, that is average variable cost plus average fixed cost. Average variable cost gives you the average cost coming from the variable component. And this

is coming from the fixed cost component, okay. Now, we look at the nature of these two functions okay.

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So, fixed cost is fixed, whatever be the amount of output, it is fixed. So, average fixed cost, AFC is this divided by units of output. Now, if we plot output in this axis, and average fixed cost in this axis. Then you will see that this curve is going to be something like this, it will go down asymptotically hit  $y$  at infinity. And at output equal to 0 this will take a very high value. So, average fixed cost is always going to go down.

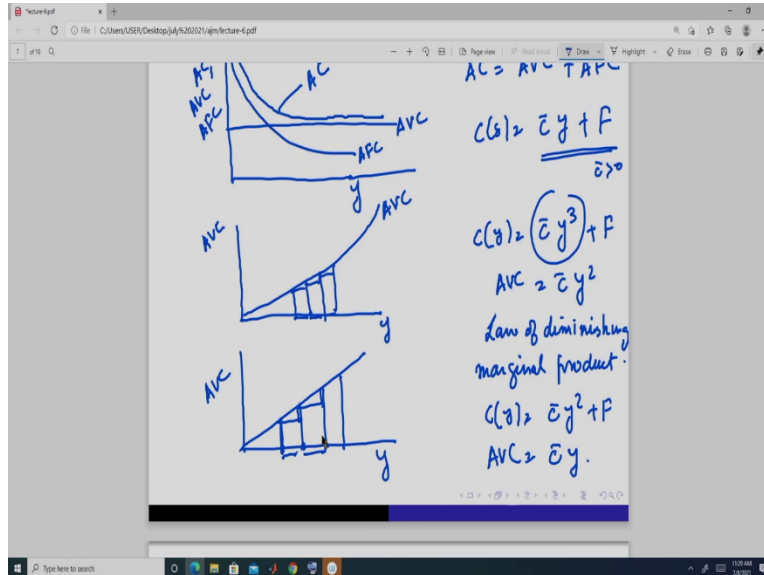
Because as output increases, because as  $y$  increases this  $f$  is fixed. So, this portion is going to go up so, this is going to go down. So, it is going to be something like this, right. So, this is the average fixed cost. So, fixed cost goes on decreasing. So, it means that more amount of output gives you less amount of average fixed cost, less amount of output gives you higher unit of average fixed cost, okay. Why? Because the fixed cost is same, so, the more you are producing, so less per unit it is costing, okay.

Now, we looked at the average variable cost, okay. Suppose, the average variable is of this nature. When we can have this kind of average variable cost? This is the variable component-  $c_v(y)$  and there is a fixed cost- $F$ . And suppose, the variable component is of this nature-  $c_v(y) = cy$ . So, it is a constant. Then this is the average variable cost and this is equal to this, i.e  $\frac{c_v(y)}{y} = c$ . So, it is constant. So, if this is independent of the output of the firm, whatever may be



the output it is producing, its average variable cost, that is cost per unit of output on the variable input it is remaining constant.

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So, here if we have that now, what do we have? Our average fixed cost is this, average variable cost is this. So, this is our average cost, because average cost is sum of average variable cost plus average fixed cost in this axis, we have taken output and all the cost, average cost, average variable cost, average fixed costs are in this axis. So, we get a curve and this average cost is going to be asymptotic to this here.

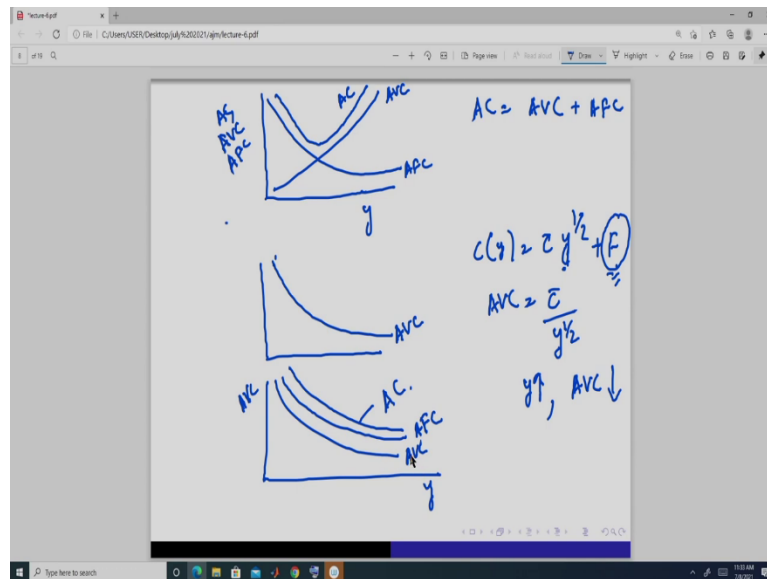
So, we get this kind of average cost curve, average variable cost curve. For example, if we have the cost function is of this nature-  $c(y) = \bar{c}y + F$ , where  $\bar{c}$  is some positive number. Now, we may have a different set. Now, average fixed cost is always going to be of this nature. Our average variable component of this cost coming from the variable component, that can be of this nature also, this is can also average variable cost.

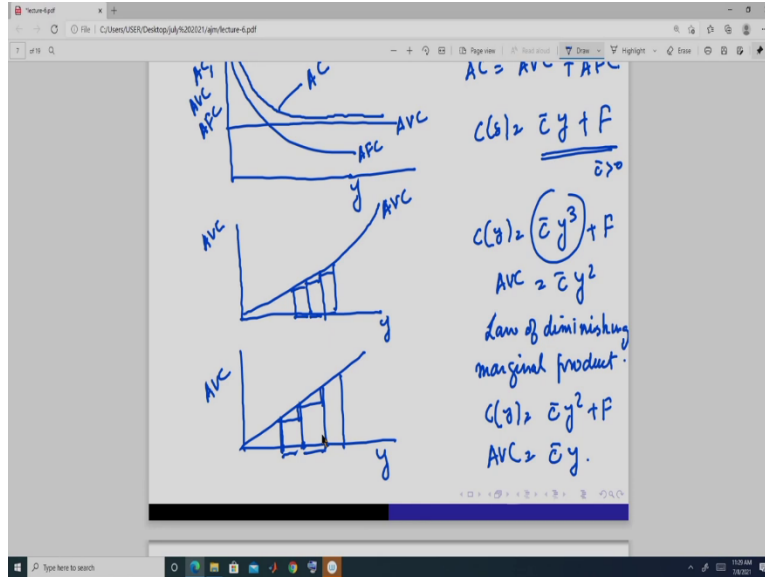
When we can have suppose for example, our is this-  $c(y) = \bar{c}y^3 + F$ . So, this is the variable component. So, the average variable cost here is a  $\bar{c}y^2$ . So, it is this, it is increasing. And when do we get such kind of one, when one factor is already fixed. So, due to law of diminishing marginal product. So, what is happening? As we are increasing the output, per unit cost is increasing, right? here. So, this means, what? This means, we are actually employing more of this a, to get this get same increase in output.

And this is why because the marginal product is decreasing, so we need to employ more and more of input to get the same additional amount of output. So, that is why our average variable cost is increasing here. One another example of this kind of a can be this-  $c(y) = cy^2 + F$ , where AVC if you see, it is this and it is a straight line, upward sloping straight line.

Here also you see that as we go on increasing the output, the average cost or average variable cost per unit of that is here, average variable cost is higher, here it is higher than this, this is higher than this. So, actually what is happening? Because the marginal products are decreasing, so we are employing more and more input. So, that is why the cost is increasing.

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Now, if we have average variable cost of this nature which are always upward sloping, then the average cost is going to be of this nature, average variable cost is suppose this, we know average fixed cost is always this. So, the average cost is of this nature. Because average cost is sum of average variable cost plus average fixed cost. So, we get a u shaped average cost when, when do we get a u shaped average cost?

One possibility is, when the average variable cost is always increasing and since average fixed cost is always decreasing. So, this sum of these two will give us a u shaped average cost, okay. Now, another form of this is going to be of this nature. Suppose, the average variable cost is of this nature. When do we get this? One example, can be of this-  $c(y) = cy^{\frac{1}{2}} + F$ . Here average variable cost is this-  $\frac{c}{y^{\frac{1}{2}}}$ . So, this means what? That as  $y$  increases  $AVC$  goes down.

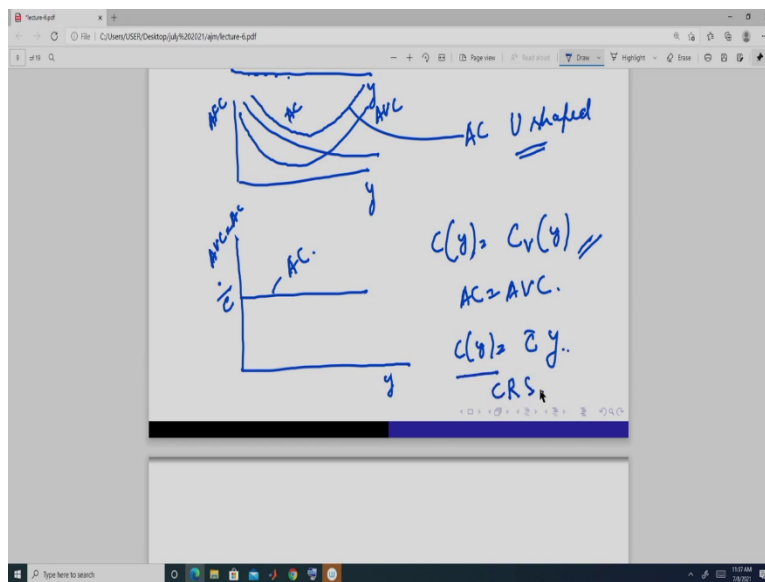
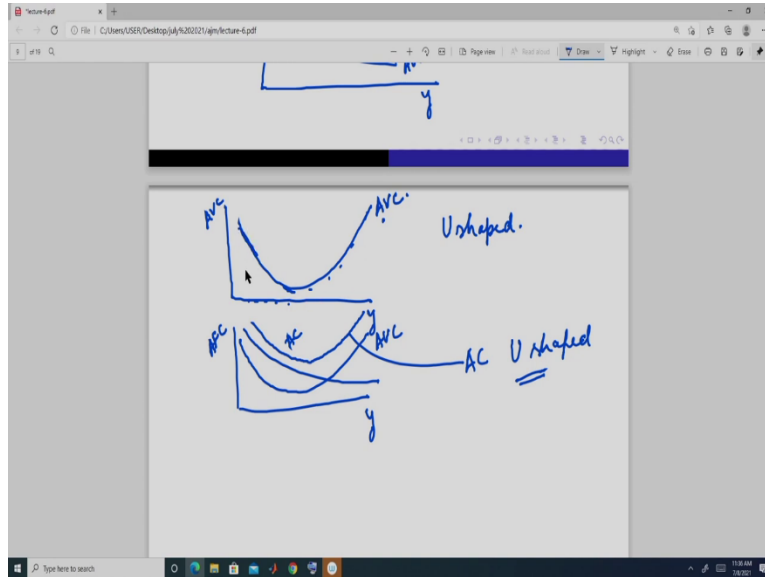
Now, these when do we get such? Now so, the problem here we have a fixed component also, fixed cost. Now, here if we, we can obviously say that if we have increasing returns to scale, then we may have a this kind of variable cost component. But since we have fixed cost, so, here we can say that in few, suppose, this is the plant size, the fixed cost is coming from the plant size and it is fixed given, and or you can say think of like the building, the building is fixed.

But you can vary the labor and the machine. So, the cost of the rent that has incurred in the hiring that building or that plot of land it is fixed, but, you can get different combination or technology or different technique by combining different amount of labor and machine. And that is coming from here. And you are having suppose increasing returns to scale in this return. So, this can be one possibility, when we have a situation like this.

And since average variable cost is of this nature, and we know so, in this situation, our average variable cost is this. And suppose, the average fixed cost is this, then the average cost is this. So, average cost is always downward sloping, when the average variable cost is also always downward sloping.

Now, we have got this are the nature of average variable cost, an average fixed cost and, and some of these two giving us the average cost of this nature, when we have only one form of average variable, means either it is strictly it is constant like this or it is strictly increasing like this or it is strictly decreasing like this.

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But we may have a situation where, our average variable cost is u shaped, it is like this. So, in this kind of situation again, what we do? suppose this is our, our building is fixed, our plot is fixed. And we are paying the same fixed amount of rent for that. Now, initially when we are combining the labor and capital in some combinations, we the production function is such that it is giving us in some form of an increasing returns to scale.

And so, that is why it is, it is so, here when we are seeing increasing returns to scale we are fixing the land or the building and only we are wearing two inputs, that is machines, that is

capital and labor, labor, okay. And then it is giving me. So, what is happening? Average variable cost is going down, as we go on increasing output the average cost. So, the cost per unit of output that is going down.

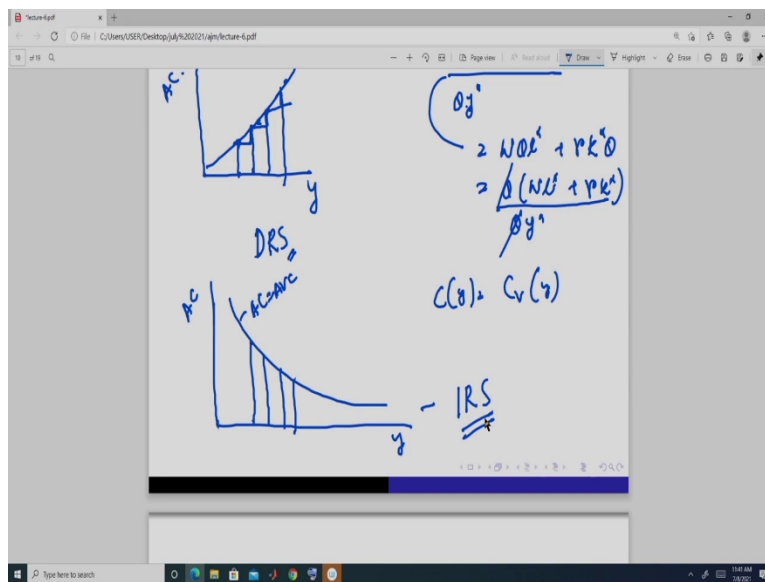
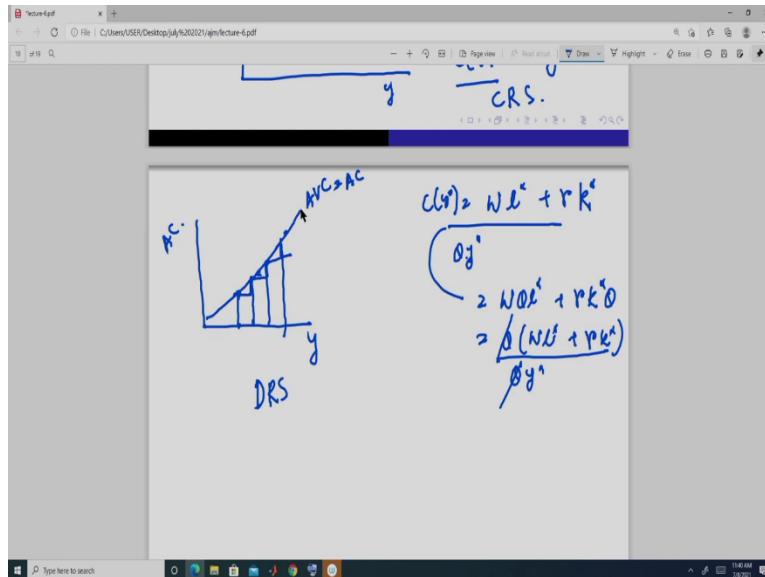
Because we are employing less and less amount of labor and capital to get the same additional amount of output, okay. And that is because we are having some form of an economies of scale. And then, after a point it starts increasing and it is of this nature. And it is here mainly, because now the land size is or the building is fixed. And if we keep on increasing the machines and the labor, then it becomes congested.

So, it is, we are going to get some form of a diseconomies of scale. And that is why it is average variable cost is increasing. So, if we have a average variable cost of this nature, that is u shaped, it is this nature. And since, the average fixed cost is always like this. And suppose our average variable cost is this, then the average cost is going to be of this nature. So, this is, so again average cost is u shaped.

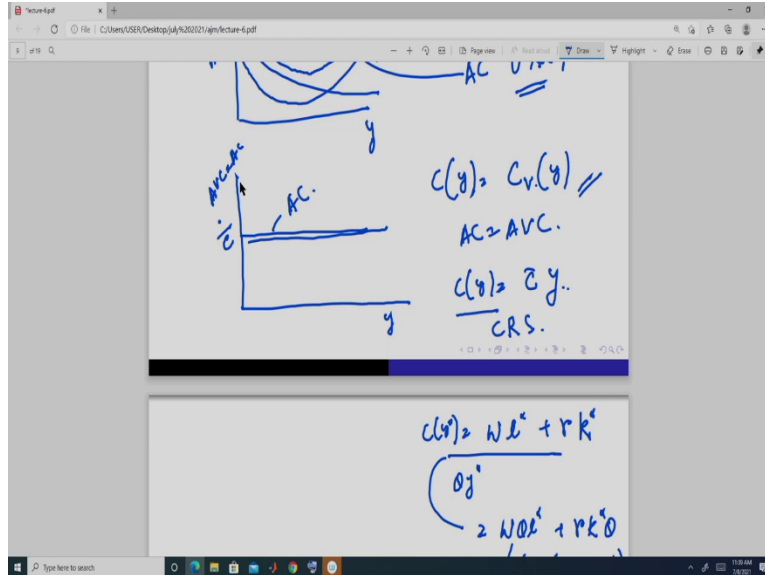
So, we get u shaped average cost curve, because of two reasons. One, when the average variable cost is always increasing, and when the average variable cost is u shaped, that is it has, it exhibits some form of economies of scale and then it has some form of diseconomies of scale. So, we get this. Now, suppose we can have another thing like, suppose our cost function is simply this-  $c(y) = c_v(y)$ . We do not have any fixed component.

That means none of the factors are now fixed, you can vary all of them. Then we may have a situation like this, this is the average cost. Now, here if you look at this average cost is same as average variable cost, because there is no fixed component. And if we have this, then this is something like, and this is  $\bar{c}$ . When do we have a situation like this? So, this is when we have CRS, constant returns to scale.

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So, that means that as we are increasing, so, your cost function, this is actually what?  $L$  star, this-  $c(y) = wl^\alpha + rk^\alpha$ . Now, for a given fixed level. Now, you increase the  $a$ . Suppose, we want to now increase it to  $\theta$  this-  $\theta y^\alpha$ . So, this will be what? From CRS we know, it is going to be because that is why it is CRS. So, it is this-  $c(y) = \theta(wl^\alpha + rk^\alpha)$ . and then this-  $\theta(wl^\alpha + rk^\alpha)$  divided by this-  $\theta y^\alpha$ , so this cancel so it remains same. So, that is why it is a CRS.

So, this if we have a cost function of this nature only, then the, and when average variable cost is same as average cost. So, it is a situation where there is no fixed cost. Then this shows that there is a constant returns to scale. Another thing is, suppose we have a situation like this. And our average variable cost is suppose equal to average cost and it is like this. So, here what you see, as you are increasing the output per unit cost, average cost is increasing.

It is here, now it is this much. So, this much increase in addition per unit cost, this much increase in per unit additional increase. So, that is why it is increasing, average variable cost or average cost it is increasing. So, we get this, when suppose we want to increase the output by  $\theta$  unit. Now, if we have decreasing returns to scale, what is going to happen? We, to get suppose you want to increase output 2 by 2 times.

Then you have to employ the labor and capital more than two times. So, that means, what? Your cost is going to be more, if you want to increase your output by 3 times then you have to employ

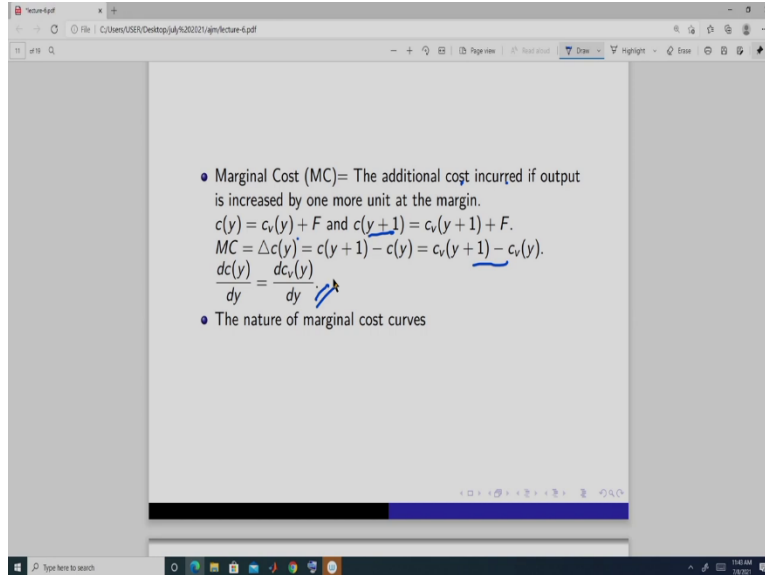
labor and capital by more than 3 times. So, if you look at this situation, so, this denominator, numerator-  $\theta(wl^\alpha + rk^\alpha)$  is increasing by more than the theta times, then the in a. So, that is why, you will get a increasing average cost curve.

So, when our production exhibits decreasing returns to scale, okay. Another is this, AC is this, when we have only this-  $c(y) = c_v(y)$ , we do not have any fixed cost. In this situation, what is happening? As the output is increasing, our average cost that is cost per unit of output it is going down. So, when do we have this situation? When we have IRS, increasing returns to scale. When we have increasing returns to scale, it means that if we increase the output suppose now two times, then we do not need to employ the labor and capital two times.

We can get two times output, twice the output with less than two times labor and two times capital, or when you want to increase the output by some theta times, we do not require to employ labor and capital as theta times of labor and theta times of capital. We can get theta times of output less than the theta times of labor and theta times of capital. So, that is why the cost is going down.

So, because we are getting increasing returns to scale so that is why, our average cost is going down. So, we get a situation like this, when we have increasing returns to scale. So, these are the different forms of average cost curve and average variable cost and a average fixed cost.

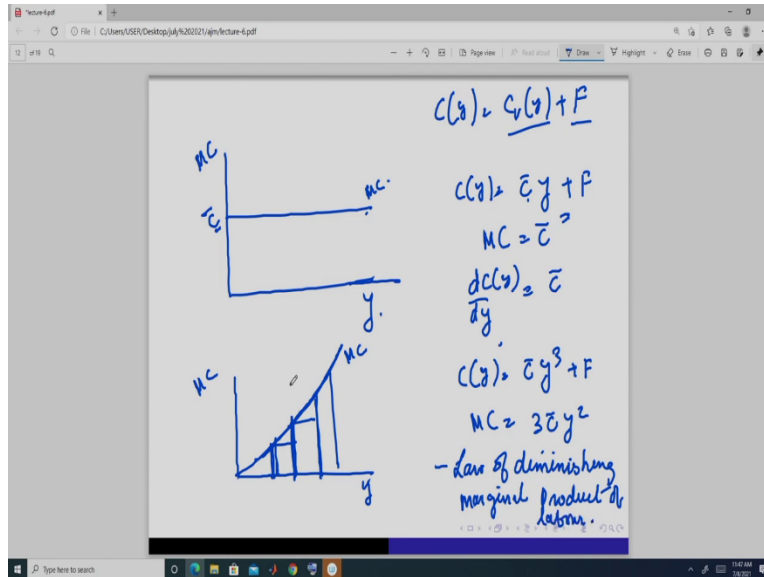
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Now, we move to another concept and that is the marginal cost, what is the marginal cost? Marginal cost is the additional cost incurred if output is increased by one more unit at the margin. Average cost is, cost per unit of output. Marginal cost is, if we increase one more unit of output at the margin suppose, we are already producing 10 units of output. Now, you want to produce the 11th unit, then how much additional cost we are going to incur?

So, it is supposed like this-  $c(y) = c_v(y) + F$ , and then we increase-  
 $c(y + 1) = c_v(y + 1) + F$ . So, the difference between these two is going to be this-  
 $c_v(y + 1) - c_v(y)$ . So, actually this comes from the variable component, fixed cost is to fixed.  
 If we have a cost curve function of this nature, or if we simply take the derivative of this-  
 $\frac{dc(y)}{dy} = \frac{dc_v(y)}{dy}$  we get the cost at the margin, okay. So, at the margin, we simply taking the derivative and plugging in that output, we will get the.

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Now we will look at the nature of marginal cost. Now suppose, our cost function is of this nature-  $c(y) = c_v(y) + F$ . So, we have a variable component and we have a fixed component, okay. So, this is coming from this suppose fixed plant size, and this from differing different combination of labor and capital, or suppose the capital is fixed and you are simply this is the cost you are incurring in the fixed capital and you can only vary the labor and that is coming from this.

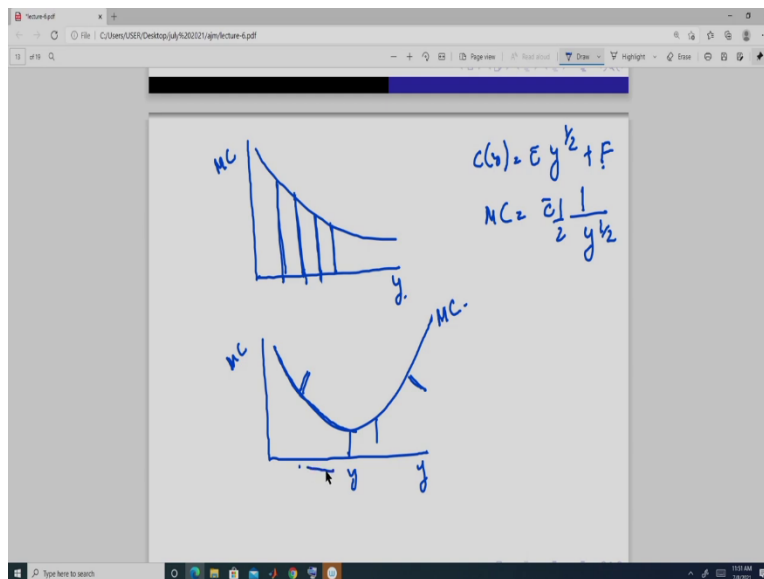
Now, if our, is this nature-  $c(y) = c_v(y) + F$  then marginal cost is simply this-  $MC=c$ . Because you take the derivative of this, we get this-  $\frac{dc(y)}{dy} = c$ . Now, this is when we have a marginal cost is constant, fixed. Whatever may be the input you are employing, you are getting the same cost, you are incurring the same cost at the margin, it is not changing, okay. This can be one form. Another form here, can be of, if we take this.

Because, suppose this is this-  $c(y) = c y^3 + F$ , then marginal cost is how much? So, it is upward sloping. This is the marginal cost-  $MC = 3cy^2$ , . So, as we go on increasing the output, our additional cost that we are incurring at the margin that is increasing, right. From here to here, we are increasing this much. From here to here, we are, output increase. So, and we get this

when we have a cost function of this nature, it is mainly because of law of diminishing marginal product.

Easiest way to explain, because we have capital is fixed and we require more and more labor to produce the same amount of output, one more additional unit of output. Because marginal product is decreasing. That means, if I increase one more unit of labor, the additional output we are getting, that is going down. So, I have to employ more labor to produce one unit, as we go on increasing more and more output. So, that is why, we get a this kind of thing, and it is mainly because of law of diminishing marginal product of labour, you can say of labor.

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Another form of marginal cost, can be this nature, when suppose our is this-  $c(y) = cy^{\frac{1}{2}} + F$ . So, marginal cost here is-  $MC = \frac{c1}{2} 1/y^{\frac{1}{2}}$ . So, what is happening? As we are increasing the output, at the margin the additional cost we are going to bear, that is going down. This, as we have explained earlier, when we take this to be a component, fixed component coming from the plant or from the building or from the rent that we pay for the land or for the building.

And suppose we can vary both labor and capital and our production is and so that exhibit suppose some form of economies of scale here. So, what do we get here? As we go on increasing

the output so additional cost we are going to incur it is going down, okay. But since we have this fixed component, so we do not say exactly that this is mainly because of increasing returns to scale because we cannot vary all the factors.

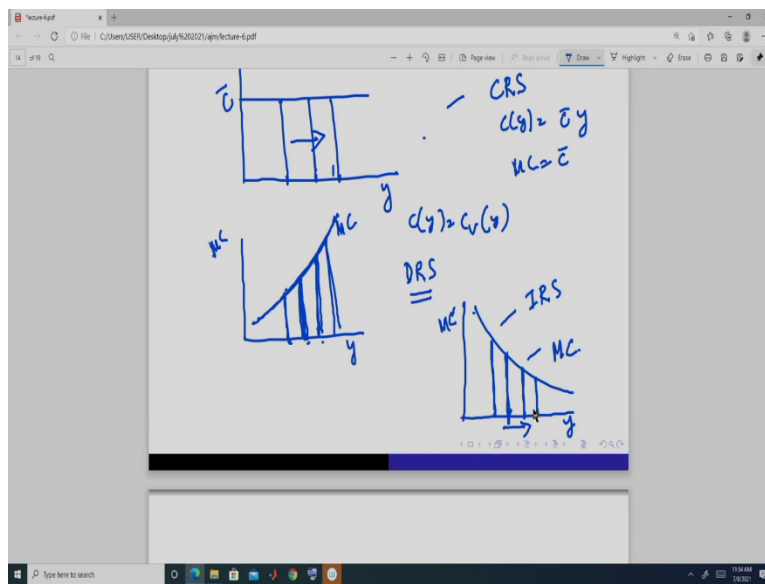
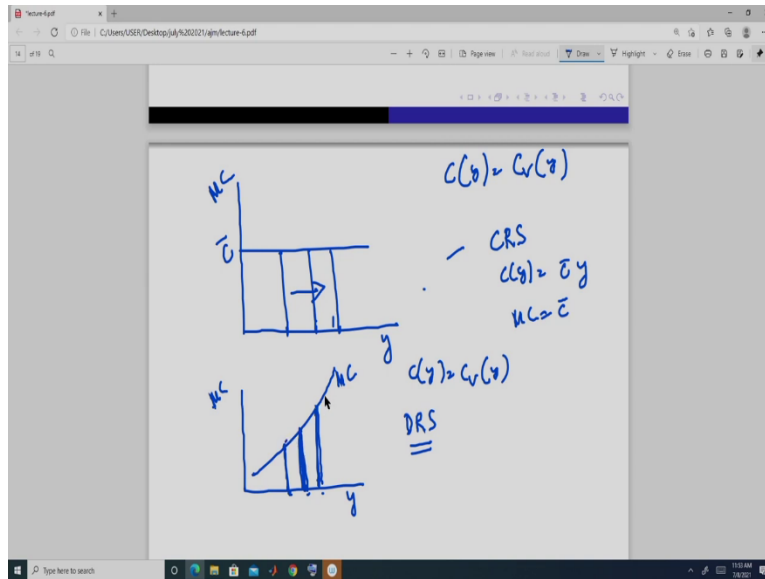
But we get a idea, that suppose the plant size is fixed or the land is fixed and all other inputs are variable like labor and machines or labor and capital, these are variable. Then, we get, we may get a situation like this, okay. So, this is simply take the derivative of this you will get this. So, we get a marginal cost of this nature. Another form of marginal cost, that we may get is of u shaped. So, the argument is same.

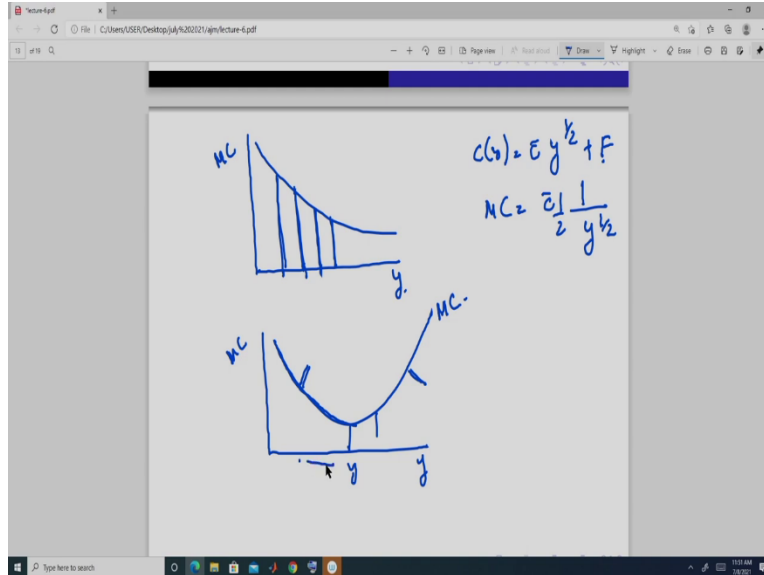
So, till this level of output, our additional cost incurred to produce one more unit of output at the margin, that is going down. So, here we are getting some form of economies of scale and then here we are getting some form of diseconomies. So, the marginal cost is starting, why? Because suppose your plant size is fixed, so as you go on employing more and more labor and capital, more machines, then what is happening?

It is becoming more congested and then your law of diminishing marginal product may operate. But, when I say law of diminishing marginal product, it has a specific meaning. It means, keeping all the other factor fixed if we vary one input. So, we cannot directly use that argument. So, what we are saying, the idea is something like this that since, it is more congested means, subsequently if we can increase the plant size also or if we are looking at suppose agriculture cultivation, we are employing more machine, more labor, but if our fixed, land size is fixed, then after a point we do not get any additional benefit.

And our additional cost is go on increasing. But, if we can vary the land size also, then we may not have that situation, but as of now, we are keeping that land size is fixed. So, we are getting a u shaped. So, for this reason our costs are going down, marginal costs are going down as we go on increasing the output. And after this our marginal costs so that is the additional cost is going up. This is one.

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Now, when we have a cost function of this nature-  $c(y) = c_v(y)$ . So, we do not have any fixed cost. So, all the factors are variable. Then, if we get a situation like this, then this is CRS. We can directly say it is CRS. And we get a situation, CRS, this. And you can do, if you get it from the simply cost minimization problem and you will get that it is a CRS kind of thing. So, what is happening? Whatever may be the output you go on increasing the output, your marginal cost is same.

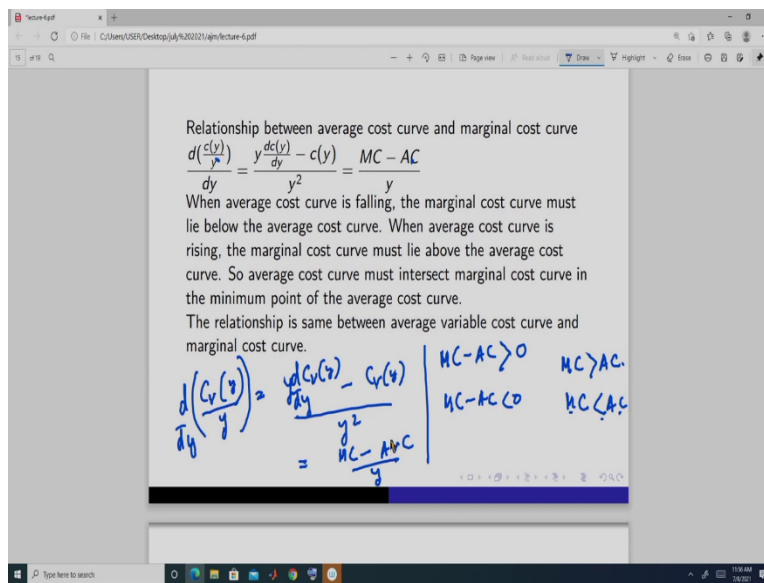
So, the idea is same, that is you employ labor and capital in the fixed ratio, and you go on doing that. If you want to increase your output two times, you have to increase your labor and capital also two times, okay, whatever be the level of output. So, that is why the marginal cost remains same. So, this is CRS things. Now, if our marginal cost is this, and our cost function is of this nature-  $c(y) = c_v(y)$ . , so we do not have any fixed cost.

So, then we have a decreasing returns to scale and that is DRS. Why? Because as we go on increasing the output, the additional cost that we are getting, that is higher this. So, because as we go on increasing, the output, what do we need? We need to employ more labor and more capital, if we want to double the output, we need to employ labor and capital more than double. So, that is why, marginal cost also goes on increasing, okay. So, we get a curve like this.



And another form that we may get, it is of this nature, it is this marginal cost, this is the marginal cost curve, the output. This we get, when we have increasing returns to scale. Because as we go on increasing the output, here our additional cost is going down. Because, as we are increasing more and more output, suppose we are increasing output two times, three times now, we are employing less than three times labor and capital. So, that is why the marginal cost is going down, okay. So, these are the nature of marginal cost. And another marginal cost is of this nature u shaped.

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Now, we look at the relationship between marginal cost and the average cost. Now, if you look at this-  $\frac{d\left(\frac{c(y)}{y}\right)}{dy} = \frac{y\left(\frac{dc(y)}{dy} - c(y)\right)}{y^2} = \frac{MC - AC}{y}$ , it is what? This is the derivative of the average cost. So, derivative of average cost with respect to output is giving me this-  $\frac{MC - AC}{y}$ . So, this means what? So, this portion, this is positive when marginal cost is greater than average cost. So, that means whenever average cost, this is what? Slope of the average cost, average cost is decreasing out, this has to be increasing.

Then marginal cost is greater than average cost. And when we have this, this means marginal cost is less than average cost. So, whenever slope of average cost is negative, then is marginal cost should be less than average cost. Now, here if you look at this situation-

$\frac{d\left(\frac{c(y)}{y}\right)}{dy} = \frac{y\left(\frac{dc(y)}{dy} - c(y)\right)}{y^2} = \frac{MC-AC}{y}$ , instead of this if you take this  $-\frac{d}{dy}\left(\frac{c_v(y)}{y}\right)$  you will get the

same thing. So, you will, this  $-\frac{ydc_v(y) - c_v(y)}{y^2}$  which is equal to marginal cost minus average cost

divided by  $y$ . So, it is same, right, here it is AC because we have taken this the average cost, this we have taken average variable cost so that is why it is AVC, average variable cost, okay.

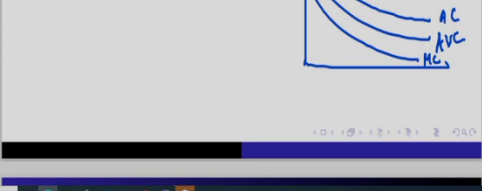
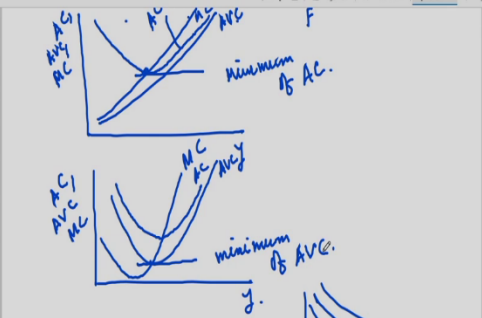
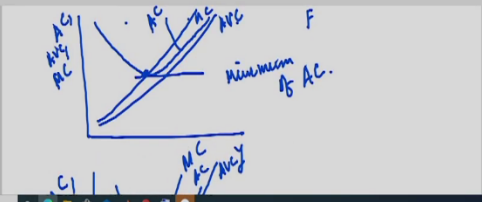
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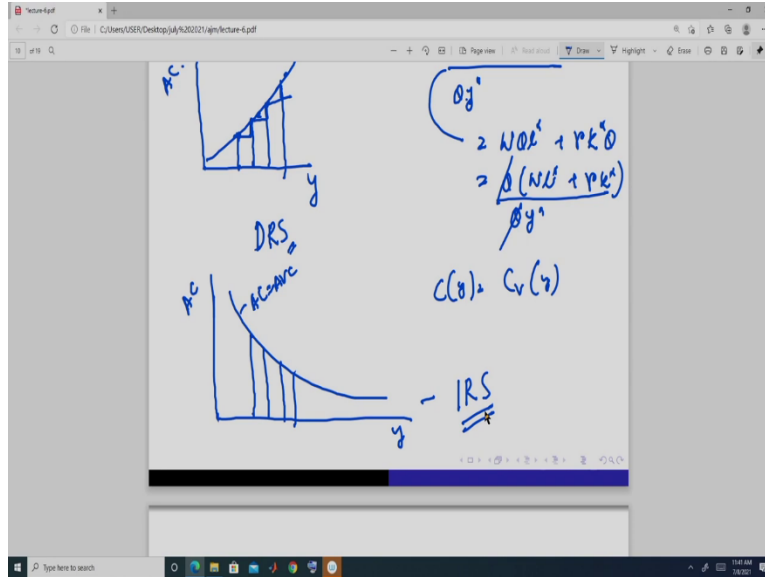
curve. So average cost curve must intersect marginal cost curve in the minimum point of the average cost curve.

The relationship is same between average variable cost curve and marginal cost curve.

$$\frac{d\left(\frac{C_v(y)}{y}\right)}{dy} = \frac{y \frac{dC_v(y)}{dy} - C_v(y)}{y^2} = \frac{MC - AVC}{y}$$

$MC - AC > 0$	$MC > AC$
$MC - AC < 0$	$MC < AC$





So, from this we get that, so all the cost AC, AVC, marginal cost like this. Now, suppose our average variable cost is of this nature. It is always increasing, then if this is the AVC, then marginal cost will always lie above it, right. And the average cost in this situation, average cost is of this nature right. If we have a fixed component, if suppose average cost is not same as average variable cost, then this will hit at the minimum of AC.

Because, average variable cost is this and then we will get a u shaped, this curve is going to be the AC because of the fixed component part. So, this will intersect at the minimum. So, when we have increasing portion of AC, AC will be less than the marginal cost. When we have decreasing portion of AC, AC will be higher than the marginal cost, like this, okay. Now, if we have a situation like this suppose, the average variable cost is of this nature.

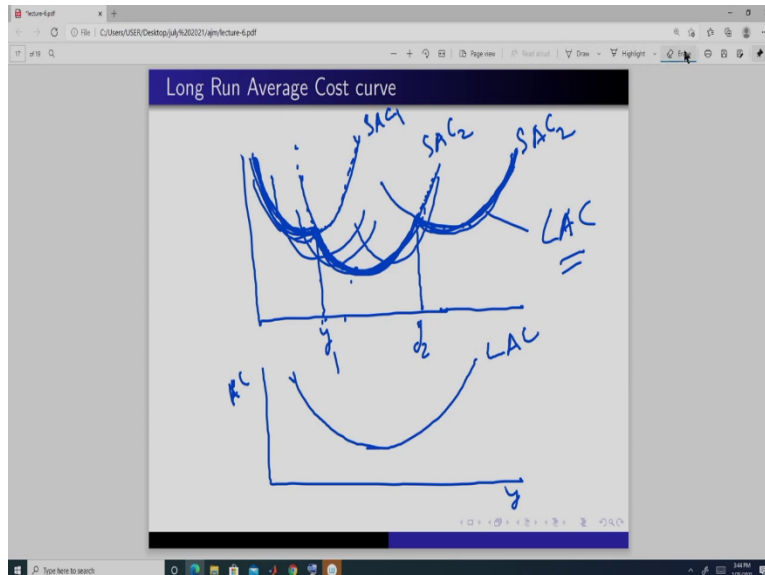
Then the marginal cost is going to be like this, again this is going to be the minimum of AVC. So, this is clear from actually this condition. Now, here this is equal to 0. So, that is why at this point they should intersect minimum, slope is 0. Here, slope is 0 when these two are equal. So, when they intersect, it should intersect at minimum of AC. So, that is why, and in this situation, average cost curve is also going to be something like this, right.

So, here in this axis, we have taken all the cost, and in this axis we have taken all the output. So we get, so mainly this is the relationship between the average cost and the marginal cost, okay.

And if they are decreasing, suppose continuously. Average variable cost is also of this nature, average fixed cost to we know, so average cost is of also this nature, then the marginal cost is going to be always below this, okay, from this  $-MC-AC < 0$  and this condition-  $\frac{MC-AVC}{y} = 0$ , okay.

So, next, so all these things are for the, we have done it for the short run. So, at least we have our plant size was fixed or when we talk about increasing or decreasing returns to scale like in this situation or in these situations, we have ignored that the plant sizes can also vary. But, so, this was mainly the analysis was for short run.

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Next, we move to long run average cost curve. Now, in the long run what may happen, we may have a situation like this, output and suppose this is one average, short run average cost curve. This is suppose for one plant. So, when we say long run, here we can change the plant size or you can think that we may have one size of plot where we have one plant and we can keep on increasing the land plot and we can set up a bigger and bigger industry.

So, this is suppose another short run average cost curve and we have further another short run average cost curve of this nature, okay. So, the plant size are increasing, as we move in this. So, this is like this, this is like this and this is like this. Till this level of output, we should always

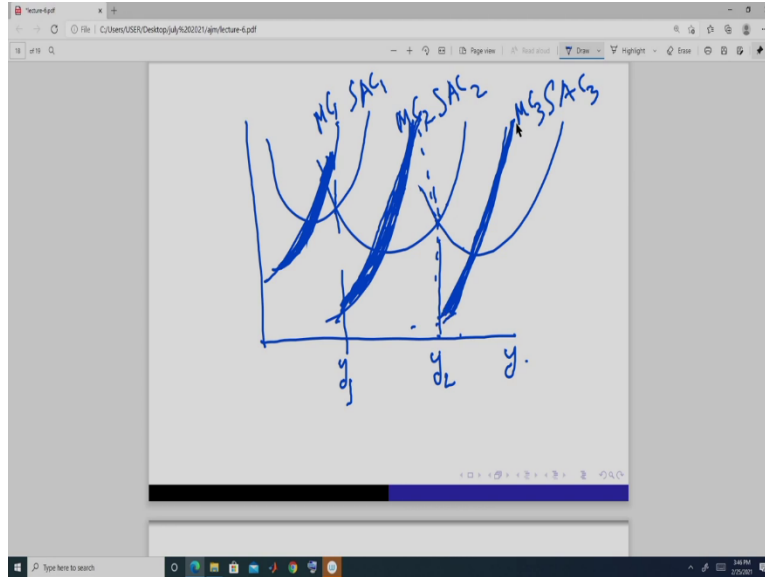
choose this plant size. Because if we choose this plant size, the average cost is higher, right? So, till this output, we will choose the plant one.

But if we move to produce more than this, and if we continue with this plant our cost, average cost is this per unit cost, but average per unit cost is can be lower, if we shift to this plant two. So, we will do this, we will continue this in plant two. But now, suppose we want to produce further and if we continue with this plant, our average cost is here, right. But if we shift to plant three, our cost is going to be like this. So, we can go on.

So, this outer envelope that is the lower envelope of all the short run average cost gives me the long run average cost curve. So, in the long run, the average cost will always be the lower envelope of the average short run average cost curve, okay, it is something like this. Because, when we are producing this much, we can use the plant one and since it is long run we can change the plants, plant sizes are also variable.

So, here now, if we can continuously change the plant size, then we will get a this kind of long run average cost curve, from here like instead of this we may have another here like this, we may have another like this, we may have like this. So, if we do go on like this, then we can get a smooth curve like this. And it is, it will be of this nature, okay. So, this is the long run average cost. Now, what is long run marginal cost?

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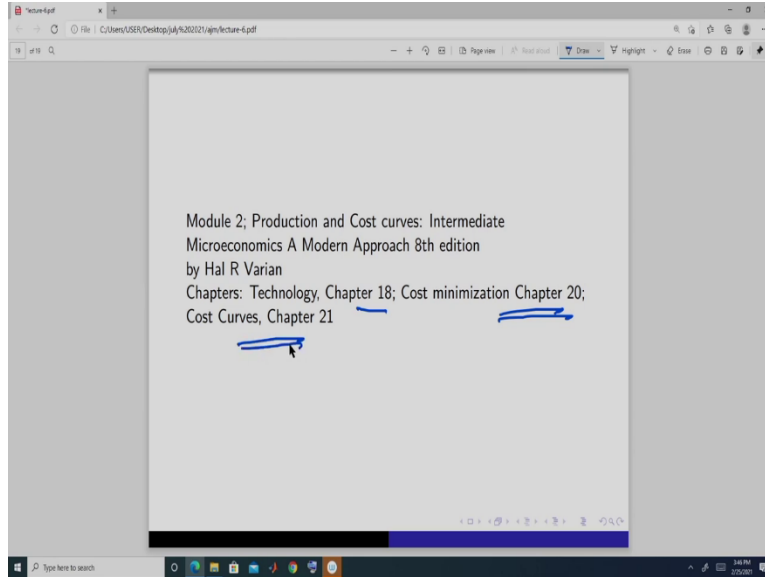


See, so in the long run, we have seen that the plant size are variable. So, we have done, taken three plants. This is short run average cost of plant one, this is short run average cost of plant two, this is short run average cost of plant three. This plant, we suppose, it has a marginal cost curve of this nature, this has a marginal cost of this nature and this has a marginal cost of this nature. Now, we know we will produce till this output in plant one.

So, our marginal cost is going to be of this marginal cost. Then we are going to shift to this plant and here the marginal cost are this. And we are going to continue to produce till this level of output. So, this is the marginal cost, right, plant two. And then plant three we will shift after this much level of output. Because, average cost is less here then continue using plant two, if you continue using plant two.

So, long run marginal cost is like this is, so we will get a discontinuous marginal cost curve in the long run, why? Because we are changing the plants, okay. And so, our cost curves are also changing, okay. So, this is what we require in this module. And if you want to read it, because these notes are sufficient, this class a.

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Or if you want to read it, so you can read chapter 18 from Hal Varian, Intermediate Microeconomics, A Modern Approach, chapter 18 for technology part that is production function for cost minimization, you can read chapter 20 and for cost curves, this kind of curve you can read chapter 21, okay. Thank you.