Introduction to Market Structures

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Module 2: Production and Cost Curves

Lecture – 6 Cost Minimization Problem

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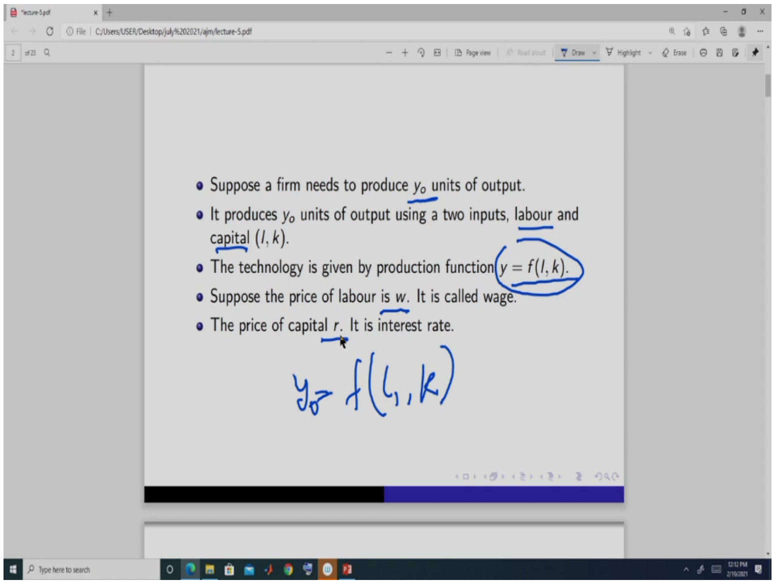

Hello, welcome to my course Introduction to Market Structures and today we are going to do cost minimization. So, now, suppose we have a firm and firm needs to produce this much y naught units of output. Now, we know to produce output it requires inputs and in our simple case we assume that it requires two inputs that is labor and capital, okay. And we have a technology which is given by the, or represented through a function this is production function.

So, we plug in the value of labor and capital and we get output here- y=f(l,k). So, to get y naught amount of output, we will have to plug in some specific amount of L and some specific amount of K to get this okay. Now, to get this input the firm needs to hire this from the market like labor market, from labor market it will hire labor, from the capital market it will hire capital. So, it will have to pay some price, it will pay wages that is ‘w’ for hiring labor and it will pay ‘r’ which is interest for hiring capital.

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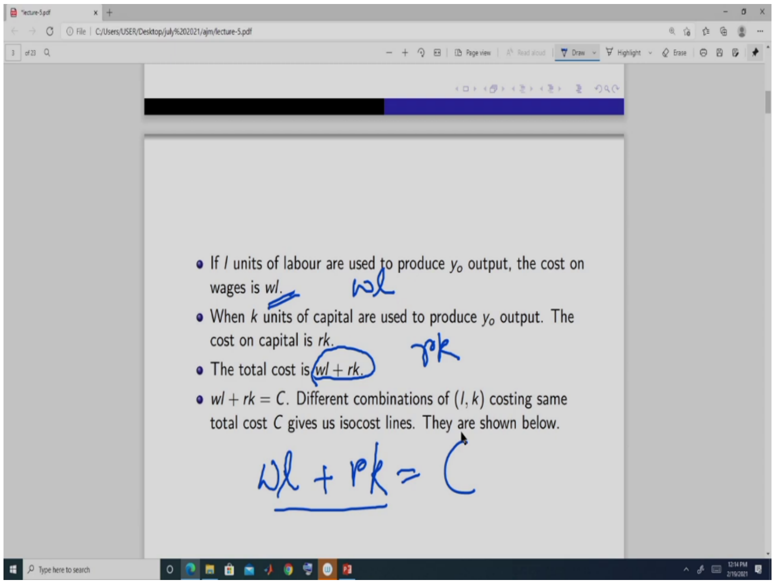

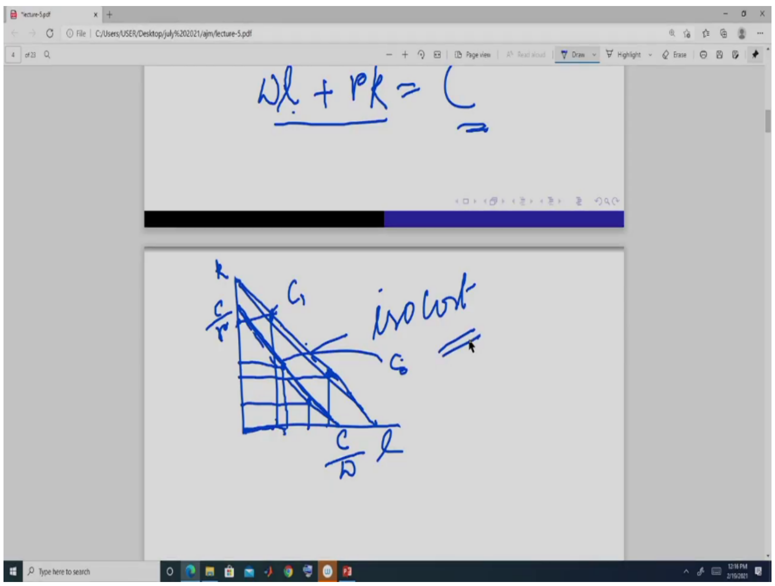

Now, suppose it so, the expenditure that it is going to incur on labor is this much- wl, w that is the wage rate price of labor into total units of labor it has bought that is wl and the expenditure it is going to incur on the cost it is going to incur on the capital is r into k. So, this is r is the rate of interest or the price of capital and k is the amount of capital. So, together this- wl+rk is the total cost that a firm is going to incur to produce output.

Now, what we call this we get a relation like this wl and this you can think something similar as a budget constraint and this is suppose equal to some number that is C, i.e wl+rk=C. This if we look at different combinations of l and k such that the C is same. These lines are called isocost lines and they look something like this if we plot l here and K here capital, this is you can say an isocost line where this is C divided by r and this is C divided by w and this is C divided by r okay.

Now, if we change the amount of total cost, if we increase it, this curve is going to shift like this. So, these are called isocost lines or you can say these lines if we are at any point in this line, then it gives the total expenditure that a firm going to incur is this much for this different combination. So, at this point, we are having this much amount of capital and this much amount of capital and this much amount of labor, but the total cost is this C suppose 1.

Here the total cost remains same, but our combination of capital and labor is different, here we have less capital and more labor here in this a, total cost is less than C one and suppose this is C naught here the combination of capital is this and labor is this much or here we have increased the amount of labor and we have decreased the amount of capital but our cost is same as C naught like this. So, these lines are, gives me the total expenditure that we are going to incur. So, different combinations of labor and capital that their total cost is same okay.

So, this is these are the isocost lines and if we are moving along an isocost like this from this point to this point what we are doing, we are decreasing capital and we are increasing labor but total cost is same and like this we can get what is called the slope of this a, isocost line and that we derive in this way- $dC = \frac{\delta(wl+rk)}{\delta l}Dl + \frac{\delta(wl+rk)}{\delta k}dk.$

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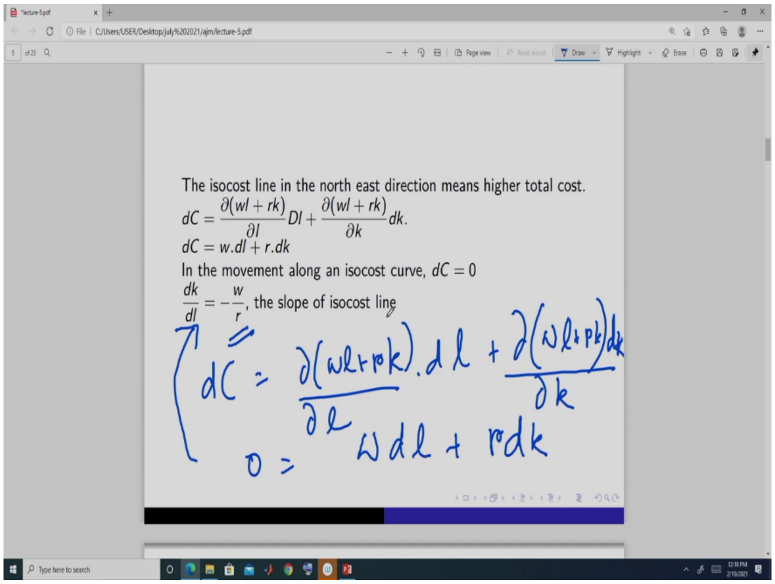

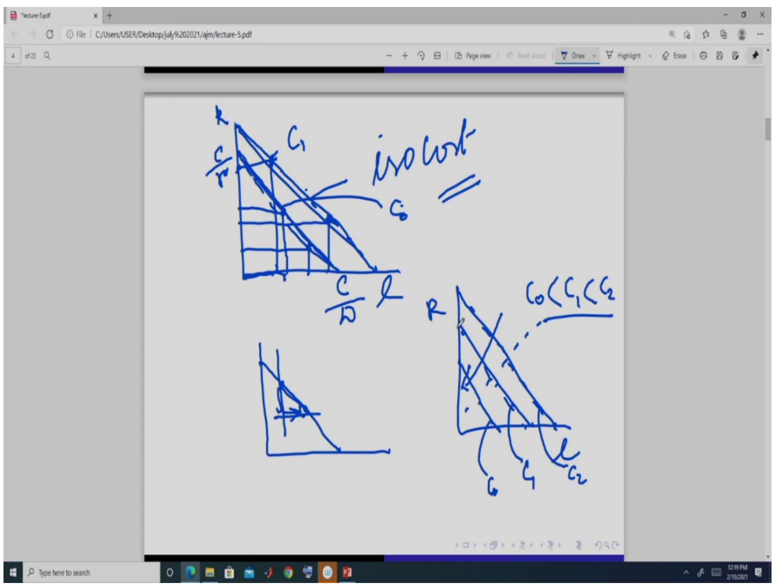

So, if we take the total differentiation of the isocost curve, so, it is what, it is this $\frac{\delta(wl+rk)}{\delta l}dl$ plus $\frac{\delta(wl+rk)}{\delta k}dk$. So, this is simply what? This- dC= wdl+ rdk. And now the changes along an isocost curve is 0. So, this is **a** and so, we get the slope as this. So, this means that if we want to increase one more unit of labor, then if we want to use one more unit of labor, then how much unit of capital the market is allowing us to substitute so, that our cost remains same. So, it is given by this ratio- $\frac{dk}{dl} =- \frac{w}{r}$. So, if I increase one unit of labor, so, I have to decrease some amount of capital.

So, how much the market is allowing us to do so, that our total cost remains the same. So, this is the idea of the slope of the isocost line, okay. So, what we have got that if we look at this capital here and suppose this is the isocost line, now, if we look at this isocost line here, all the total cost is higher in for all these combinations of labor and capital and compared to this and if we take this, this even lower.

So, here if this is C naught, this is C1 and this is C2 then total cost, the rank of this total costs are like this or the position order, okay. So, in this direction in the northeast direction, total cost is increasing okay, and if we have to reduce the total cost we have to move in this direction okay. And we will use this in solving the cost minimization problem okay.

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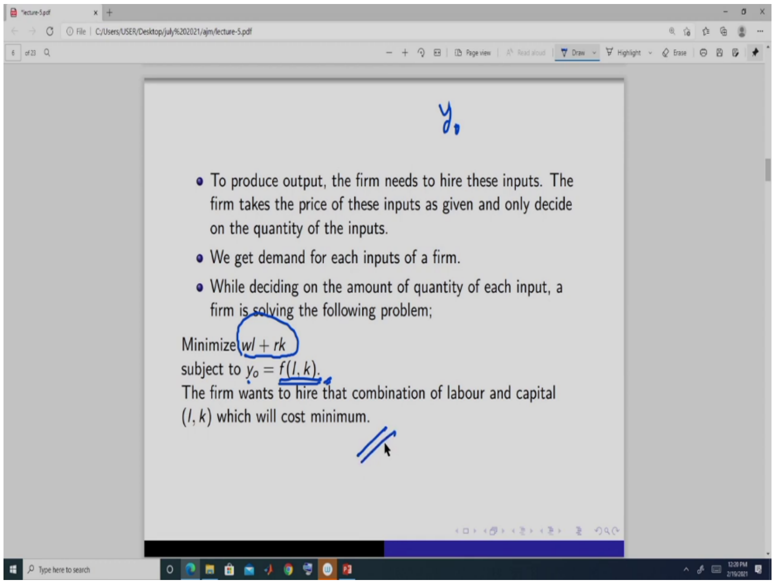

Now, what happens, so the firm wants to produce an output of suppose, y naught unit so, for that it needs to hire labor and capital. Now, while deciding the a, the amount of labor it is going to hire and the amount of capital it is going to hire, it takes the price of labor that is w and the price of capital that is r as given, the firm cannot determine the price of labor and capital, it is given.

So, now, the firm only decides how much amount of this each of these labor, they are going to hire how many units okay. So, the firm is actually going to solve this problem that is its wants to minimize this cost it is w into l plus r into k such that it is subject to it wants to produce this given the production function or technology in this way.

So, now, suppose firm wants to produce 100 units of output then to produce 100 units of output, it can use several combinations of labor and capital. Now, it will choose that combination of labor and capital such that the cost is minimum. So, this is the idea of this problem okay.

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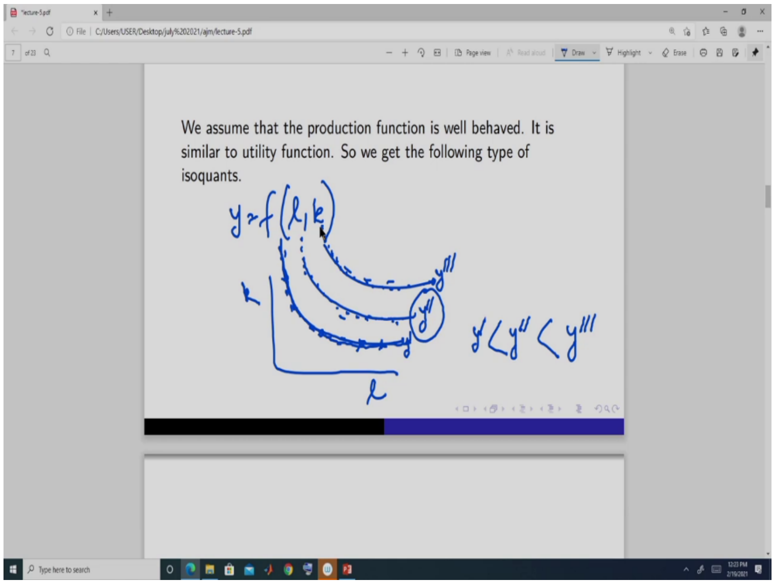

Now, so here to solve this problem, we will be given a production function of some nature this- y=f(l,k). We assume that the production functions are well behaved. So, we get following types of iso-quants. Iso-quants are level curves like this. So, this is for suppose I have no output y dash this is for output y double dash and this is for output y triple dash and here output in y dash is less than output in double dash and here even, i.e $y^{'} < y^{''} < y^{'''}$.

So, outputs are increasing in this way so, we will get this now, suppose we want to produce y dash units of output then we this is fixed so, we can choose from these combinations of capital and labor either we can choose this, we will choose this, we can choose this any one of this, okay. Now, out of these points or these combinations, we want to choose that one which is costing us minimum or suppose now, we want to produce **y** double dash amount of output.

Then all these combinations of labor and capital are allowing us to produce y double dash units of output out of these combinations we want to choose that one which is going to give us or which is going to cost us minimum. Similarly, when we want to produce suppose y tripled dash units of output then these combinations of labor and capital is actually allowing us to produce y triple dash units of output and out of these combinations of labor and capital, we want to choose that combination which is giving us or which will cost us minimum, okay. So, now, how to proceed?

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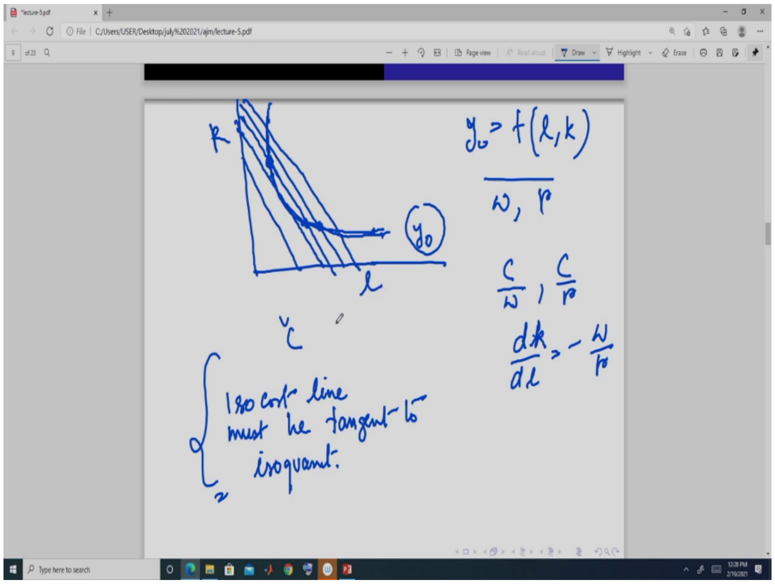

So, first we will solve this graphically and then we will solve these through Lagrange multiplier, okay. So, suppose labor is measured in this axis, capital is measured in this axis and we want to produce y naught unit of output and our production function is suppose something like this- $y_o = f(l, k)$, now we have fixed. So, we will get isoquant and suppose this is the isoquant and this is y naught, okay. Now, we have assumed that this is well behaved so it is convex to the origin and it is also continuous all these properties are eligible.

Now, suppose the price of labor is w and price of capital is some r, okay. So, we have isocost line like this and isocost lines are going to be parallel, why? Because isocost lines are given by joining these two points C and R since w and R are constant, it is taken as fixed or given by this firm. So, these are going to be parallel and the or you can say this since the slope this is equal to w by R and it is going to be same. So, the angles are going to be same. Now, here we can have all these different isocost lines and this is our isoquant, right? we want to produce y naught units of output.

So, we can produce using this combination of labor and capital, this, this, this, this, like this, now if we choose this combination suppose, then, we are in this isoquant, instead, if we choose a point here it we will be at a lower level of isocost line. So, it means our cost can be reduced. So, it is better to choose a combination here rather than this point. Now, suppose we are choosing a combination suppose this if we choose this combination of labor and capital, then we are at this level of isocost line.

But if we switch from this point to this we increase some amount of capital and reduce some amount of labor. Then we are at a lower level isocost curve. So, our total cost is less now, so this is costing us less than this. So, we move in this a, and finally, we reach that isocost curve which is tangent to the isoquant. So, isocost line must be tangent to the isoquant, okay. So, this ensures that the cost is minimum.

Because if we are at any other point compared to this the point of tangency then there is always a possibility that the cost can be reduced provided our production functions are well behaved so, isocost, isoquants are like this, okay.

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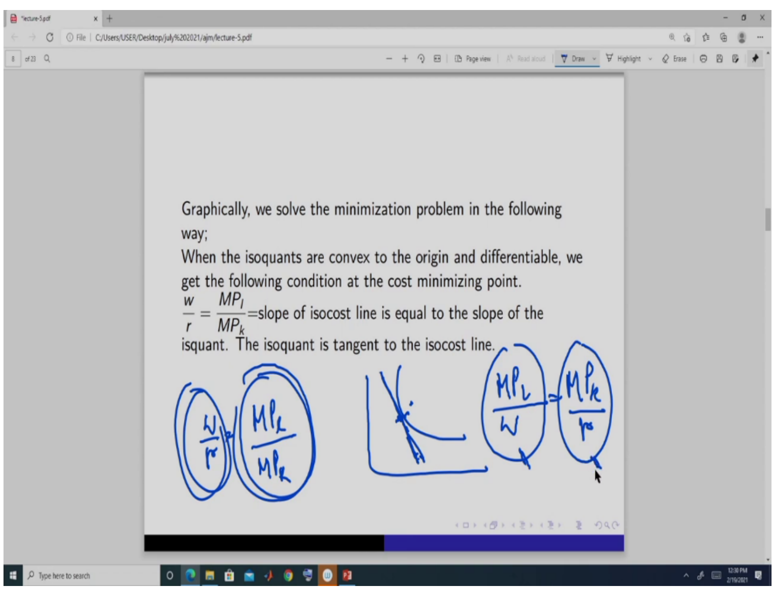

So, this condition you can say is something like this- $\frac{w}{r} = \frac{MP_l}{MP_k}$. So, when they are tangent when the isocost line is tangent to the isoquant it means what? that the slope of isocost and the slope of isoquant are same, right. At that point slope of isoquant at that point, so, it is something like this. So, suppose this is the slope of at this point, right? is given by this, so, again this is a tangent to this, so, the slope must be same of this line and the isoquant at this point, so we get this condition-$\frac{w}{r} = \frac{MP_l}{MP_k}$.

So, this we have got from the slope of isoquant and this we have got from the slope of isocost. So, this should match that means, how if we want to increase one unit of labor, how much unit of capital we should decrease, how much it is possible given the technology that is given by this isoquant and how much the market is allowing us to do given the prices, it is given by this-$\frac{w}{r}$. So, when we our cost is minimum these two things should match or from here you can say that we can also write it in this way-$\frac{MP_l}{w} = \frac{MP_k}{r}$., this condition is similar to what we have done in the consumer behavior.

So, this much is to increase one additional unit of output, the amount of expenditure we have to do on labor it is this-$\frac{MP_l}{w}$ and to increase one unit of output the amount of expenditure we have to do in the capital it is this-$\frac{MP_k}{r}$. So, these two should match. So, if we want to increase one unit of output, then the amount of expenditure done in the labor should be equal to amount of expenditure done on capital when we are at a optimal point and here optimal point is the point at which cost is minimum, okay. So, this is the important condition that the isocost line should be tangent to the isoquant at the optimal point okay.

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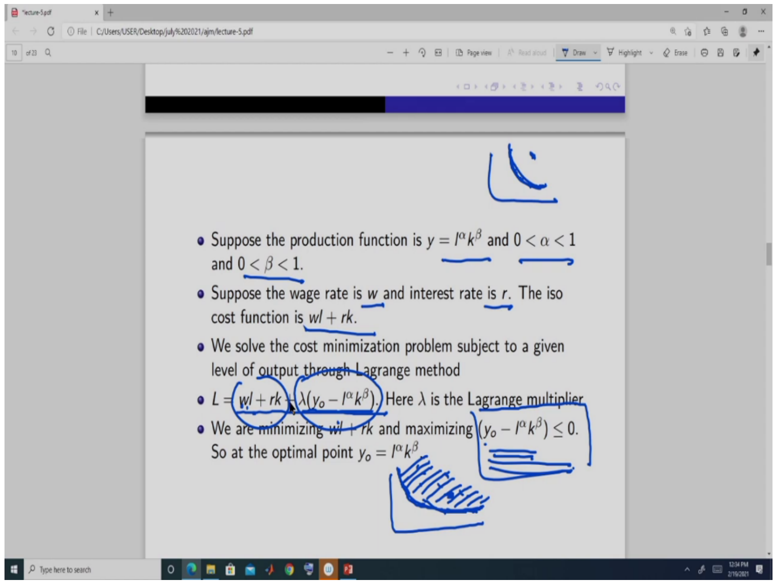

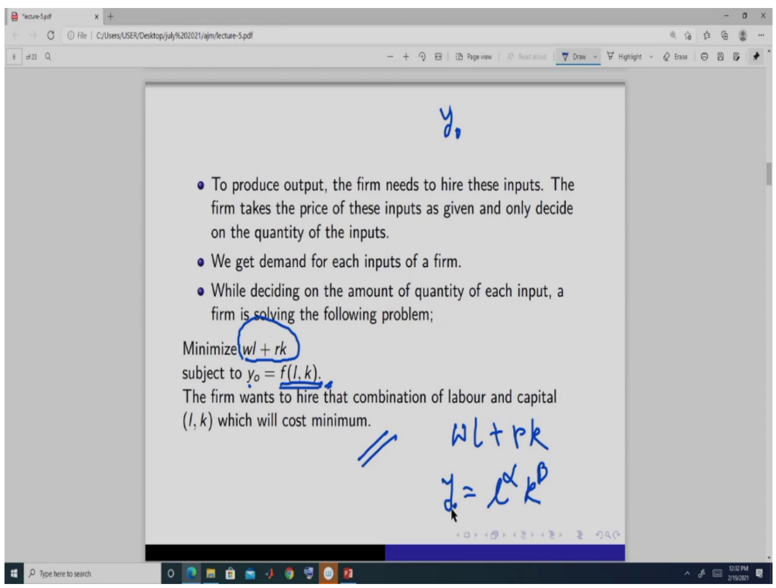

Now, let us solve this problem algebraically, how do we do? So, for this we assumed a very specific form of production function it is this- $y = l^{\alpha}k^{\beta}$ l to the power alpha and k to the power beta, l labor, k capital and alpha takes a value any value between 0 and 1 and beta takes a value between 0 and 1, okay. So, here again we assume that the wage rate is w and the interest rate is r some positive numbers and so, the isocost line is given like this w into r, w into l plus r into k.

Now, we solve this problem this, we want to minimize this- wl+rk, w subject to our production function like this- $y = l^{\alpha}k^{\beta}$, suppose we specify some amount of output and that is suppose, y naught, okay. So, we write the Lagrange in this form- $L = wl + rk + \lambda(y_o - l^{\alpha}k^{\beta})$. So, this Lagrange it is saying it is a function and you can think it to be something like it is a saddle point also you can say we want to find the saddle point of this a. So, what we are doing, we want to minimize this component, this portion- wl+rk and we want to maximize this portion- $\lambda(y_o - l^{\alpha}k^{\beta})$. and this portion is written in this form- $(y_o - l^{\alpha}k^{\beta}) \leq 0$.

So, this is maximized whenever it is equal to 0 because otherwise it is because if we fix the isoquant, right, what we are doing this whole set we take any combination of this when we can produce. Now this is the most efficient way in the sense employing minimum labor and capital we can produce the output. Now here you bring this combination also we can produce this much, but here we are employing more labor and capital, right.

So, in this sense, we want to be always at the boundary. So, that is what that while minimizing the cost, we always want to be at the boundary or we always want to be at that isoquant which specifies that level of output, okay, this. So, since we want to maximize so, when we want to find the optimal combination of labor and capital, so we will always be at this point we will not use any combination of like this we will always use like this.

So, this is now here this lambda is the Lagrange multiplier. So, this thing- $L = wl + rk + \lambda(y_o - l^{\alpha}k^{\beta})$. is what? this thing is in terms of output and this is in terms of value because it is price into quantity, price of labor into quantity of labor, so it is in value terms. So, this is actually converting this output into this. So, this you can say as some price of this good, of this output or you can say like shadow price, I will not discuss that or you can think it is simply the price of this, okay.

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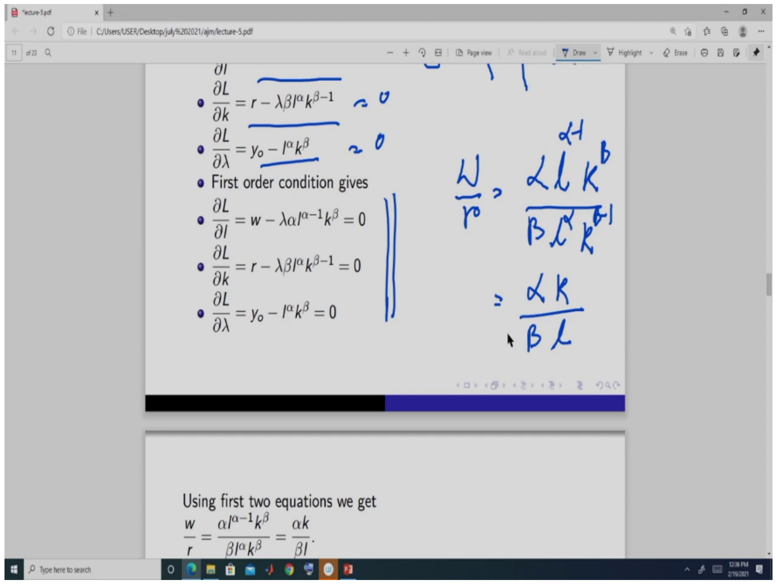
- Suppose the wage rate is w and interest rate is r. The iso cost function is $wl + rk$.
- We solve the cost minimization problem subject to a given level of output through Lagrange method
- $L = wl + rk + \lambda(y_o - l^\alpha k^\beta)$. Here λ is the Lagrange multiplier
- We are minimizing $wl + rk$ and maximizing $(y_o - l^\alpha k^\beta) \leq 0$. So at the optimal point $y_o = l^\alpha k^\beta$.

- Since the production function is differentiable in (l, k). So we take the following derivatives;
- $\frac{\partial L}{\partial l} = w - \lambda \alpha l^{\alpha-1} k^\beta$
- $\frac{\partial L}{\partial k} = r - \lambda \beta l^\alpha k^{\beta-1}$
- $\frac{\partial L}{\partial \lambda} = y_o - l^\alpha k^\beta$
- First order condition gives
- $\frac{\partial L}{\partial l} = w - \lambda \alpha l^{\alpha-1} k^\beta = 0$
- $\frac{\partial L}{\partial k} = r - \lambda \beta l^\alpha k^{\beta-1} = 0$
- $\frac{\partial L}{\partial \lambda} = y_o - l^\alpha k^\beta = 0$



$\frac{\partial L}{\partial k} = r - \lambda\beta l^{\alpha}k^{\beta-1}$

$\frac{\partial L}{\partial \lambda} = y_0 - l^{\alpha}k^{\beta}$

- First order condition gives
- $\frac{\partial L}{\partial l} = w - \lambda\alpha l^{\alpha-1}k^{\beta} = 0$
- $\frac{\partial L}{\partial k} = r - \lambda\beta l^{\alpha}k^{\beta-1} = 0$
- $\frac{\partial L}{\partial \lambda} = y_0 - l^{\alpha}k^{\beta} = 0$

Using first two equations we get

$\frac{w}{r} = \frac{\alpha l^{\alpha-1}k^{\beta}}{\beta l^{\alpha}k^{\beta}} = \frac{\alpha k}{\beta l}$.

Now, what we are going to do we have set this Lagrange and Lagrange in our case is this L is equal to w L, w into L plus r into k plus lambda which is Lagrange multiplier y naught minus L to the power alpha k to the power beta like this- $L = wl + rk + \lambda(y_o - l^{\alpha}k^{\beta})$. Now, we take the first derivative of this with respect to three variables that is L labor, k capital and also lambda. Lambda is also a variable in this case, okay.

Now, and the first order condition gives us this or you can say the derivative this- $\frac{\delta L}{\delta l} = w - \lambda\alpha l^{\alpha-1}k^{\beta}$ is w minus this portion, okay and this is-$\frac{\delta L}{\delta k} = r - \lambda\beta l^{\alpha}k^{\beta-1}$ r minus this first derivative of this and if take the derivative of this expression l Lagrange with respect to lambda we will get this- $\frac{\delta L}{\delta\lambda} = y_o - l^{\alpha}k^{\beta}$because, this portion is going to . Now, the first order condition says that we always we should equate this to 0, this to 0 and this to 0.

So, these- $\frac{\delta L}{\delta l} = w - \lambda\alpha l^{\alpha-1}k^{\beta} = 0$, $\frac{\delta L}{\delta k} = r - \lambda\beta l^{\alpha}k^{\beta-1} = 0$, $\frac{\delta L}{\delta\lambda} = y_o - l^{\alpha}k^{\beta} = 0$, are the first order conditions. Now, from here what we get if we use these two conditions, first two equation we can write it in this form- $\frac{w}{p} = \frac{\alpha l^{\alpha-1}k^{\beta}}{\beta l^{\alpha}k^{\beta-1}}$this, right. And this expression we can reduce it into alpha into k divided by beta into l this- $\frac{\alpha k}{\beta l}$.

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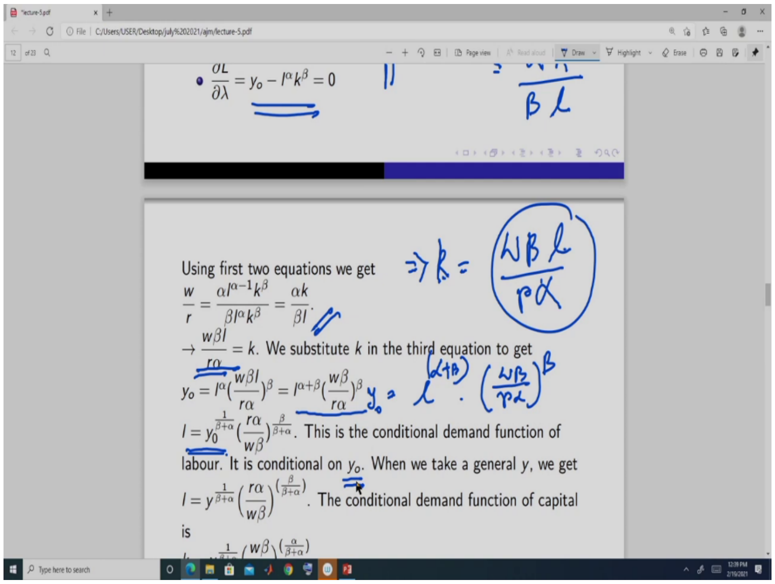

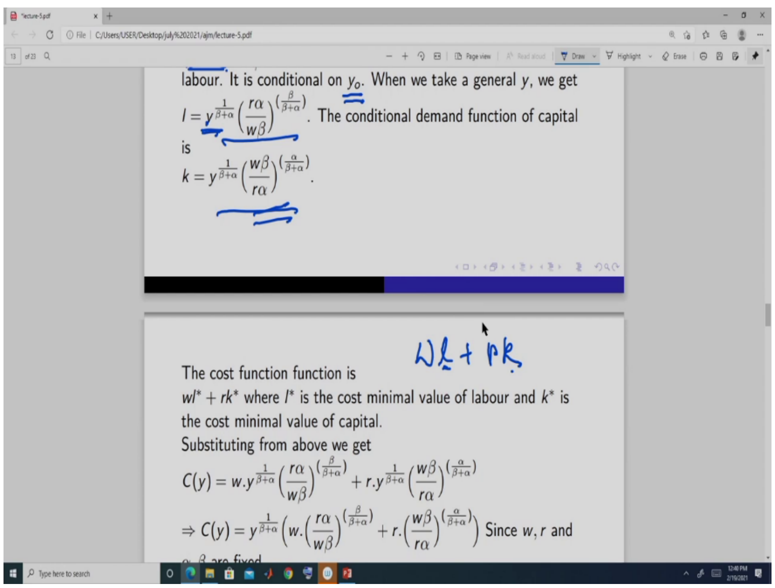

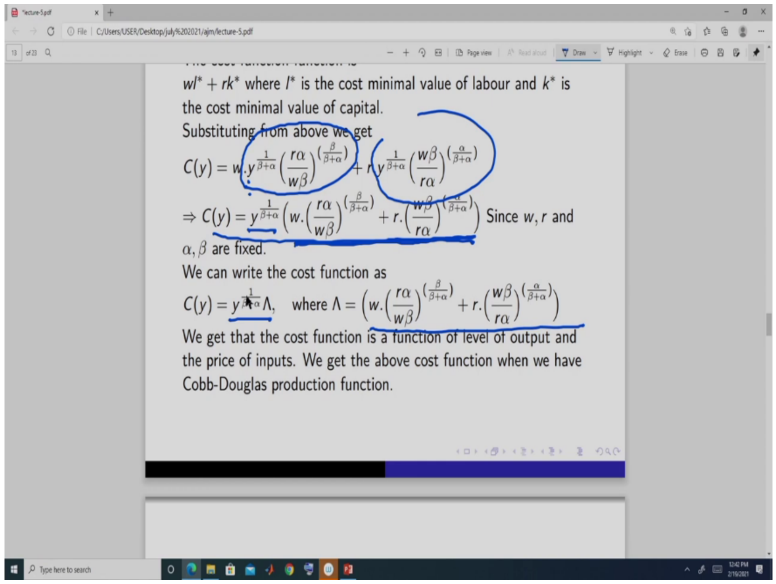

Now, from here we can write labor in terms of capital or capital in terms of labor. So, we what we do we write now capital is equal to w beta L divided by r alpha, i.e $k = w\beta l/r\alpha$, okay.So, this and now, so we know that if we specify any value of l we know how much amount of k to we should buy this, right. Now, again what we do, we substitute this in the production function here because this is also another first order condition.

So, if we do that what do we get this L to the power alpha where k to the power beta. So, in this place of K we plug in this and we get this whole expression this- $y_o = l^{\alpha}\left(\frac{w\beta l}{r\alpha}\right)^{\beta} = l^{\alpha+\beta}\left(\frac{w\beta}{r\alpha}\right)^{\beta}$ where l is to the power alpha plus beta and then we have an expression where this is, to the power beta, okay. So, this now, from here, we can find out the demand for labor and that is if we take this portion to the, because y naught is equal to this we know how much amount of output we want to produce.

So, when we want to produce y naught? So, if we want to produce y naught then what is going to be the demand for l? So, y naught now, if we take this to this side and if you take the power to be 1 by alpha plus beta then we know the demand of labor. So, this is the demand for labor, i.e $l = y_o^{\frac{1}{\beta+\alpha}}\left(\frac{r\alpha}{w\beta}\right)^{\frac{\beta}{\beta+\alpha}}$. So, now, plug in the amount of output you want to produce you will start getting the demand for labor it is this now, from here what you do you plug in this, here you will get the demand for capital.

Now, this is something called a conditional demand why? because it is conditional on y naught it is like conditional on the amount of output we want to produce if we know the amount of output we want to produce we will know how much amount of labor we want it is this given the price of the labor and capital. Similarly, we can find this demand for capital and it is this again you can see it is conditional on y, i.e. $k = y_o^{\frac{1}{\beta+\alpha}}\left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\beta+\alpha}}$

Now, from y naught we make it a general y because you plug in any y you will get the this function only, okay. So, that is why we take instead of y naught we make it y the same here okay. So, this gives us what this gives us that when we are want to choose a combination of labor and capital subject to a production function and the prices of the labor and capital we choose, we get a demand for labor and demand for capital okay.

Now, when we plug in these optimal values or this demand curve in this so, our total cost is this- wl +rk, right?. And if we want to produce some specific unit of output, our demand for capital is this- $k = y_o^{\frac{1}{\beta+\alpha}}\left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\beta+\alpha}}$ and demand for labor is this- e $l = y_o^{\frac{1}{\beta+\alpha}}\left(\frac{r\alpha}{w\beta}\right)^{\frac{\beta}{\beta+\alpha}}$ now plug in this demand for labor and demand for capital here optimal demand less and that gives us something called the total cost. So, total cost, this total cost at the optimal point is called the cost function.

Because cost function is actually the minimum cost of producing some huge amount of output given the price of labor and capital. So, when we plug in this, this here- $C(y) = w.y^{\frac{1}{\beta+\alpha}}\left(\frac{r\alpha}{w\beta}\right)^{\frac{\beta}{\beta+\alpha}} + r.\ y^{\frac{1}{\beta+\alpha}}\left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\beta+\alpha}}$, so this is the demand for labor and this is the demand for capital. So, we plug in here, we get y to the power 1 by beta plus alpha and this term and here we get r into y to the power 1, 1 by beta plus alpha into this term. Now, from here you can take this- $y^{\frac{1}{\beta+\alpha}}$ common this and what is left this term- $w.\left(\frac{r\alpha}{w\beta}\right)^{\frac{\beta}{\beta+\alpha}} + r.\ \left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\beta+\alpha}}$.

So, here if you look at this term- $C(y) = w.y^{\frac{1}{\beta+\alpha}}\left(\frac{r\alpha}{w\beta}\right)^{\frac{\beta}{\beta+\alpha}} + r.\ y^{\frac{1}{\beta+\alpha}}\left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\beta+\alpha}}$ a firm takes the price of labor that is w as given it cannot determine w. It takes the price of labor that is a price of capital that is r as given it cannot determine it, its production function is given. So, this beta and alpha is given. So, this whole portion, this portion, this is already given to this firm. So, this is fixed. Now, whenever a firm has to decide, that it is going to produce some amount of output that is y then its cost function is given like this in this form- $C(y) = w.y^{\frac{1}{\beta+\alpha}}\left(\frac{r\alpha}{w\beta}\right)^{\frac{\beta}{\beta+\alpha}} + r.\ y^{\frac{1}{\beta+\alpha}}\left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\beta+\alpha}}$ provided that the production function is a Cobb Douglas production function of this nature.

So, we can write it in this one in a more compact way - - $C(y) = y^{\frac{1}{\beta+\alpha}}\wedge$ where this big lambda $\wedge$ is this- $\wedge = w.\left(\frac{r\alpha}{w\beta}\right)^{\frac{\beta}{\beta+\alpha}} + r.\left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\beta+\alpha}}$ which is always given as fixed a firm so it cannot determine this, right. So, this is the cost function of a firm and this is the total cost function, okay. So, we see that the total cost function is a function of output, okay. Now, let us do some another example.

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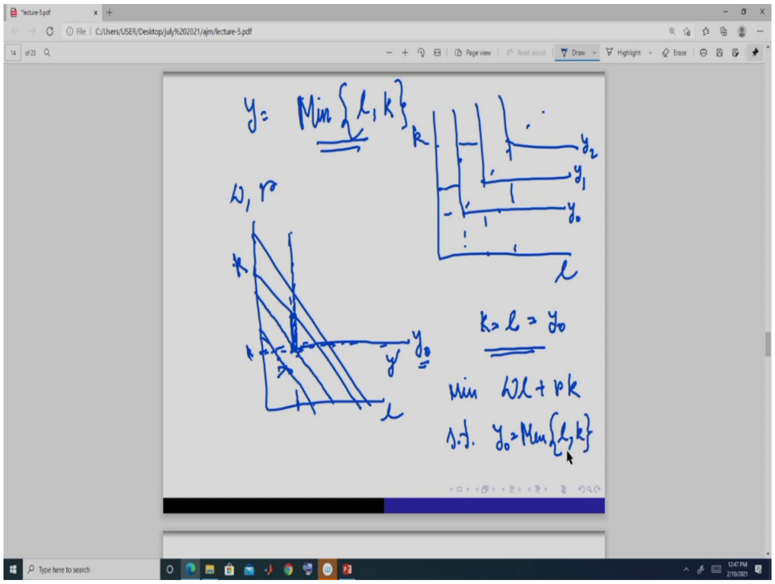

Suppose we take the example of fixed proportion, production function is suppose like this- y=Min{l,k} okay. So, the firm what it does if the production function is this then the isoquants are of this nature. This is suppose y naught, this is y 1, this is y 2 and this is the point of equality, equality of labor and capital like this. So, this production function is not differentiable okay. But it is, you can see convex to the origin if you take any combination of this and this it will at least you can produce this much amount of output, right.

Now, you see what we are, suppose the wage rates are also and the price of the capital is fixed. So, isocost lines are like this and suppose, we want to produce y naught units of output and this is here. Now, this point is going to be the optimal point, why? See, now, because we can produce by combining any point here this y naught unit, but here we have already employed this much k amount of capital. Now, if we go on increasing labor more than this, then what is happening we are not getting any additional output, output is remaining same.

So, there is no point in it. So, we should employ this much only if we have employed this much amount of labor and if we keep on increasing capital we are not getting any extra output, right. So, it is better to say or you can think in this term isocost lines are something like this and like this. Now, this isocost line is lying below this right. So, any point here it is not going to give us y naught unit of output. So, this is not possible. So, isocost line must cross this isoquant, right. if we use this combination, then compare this and this, this isocost line lies below this isocost line.

So, the cost is less here. So, this is better, because this much extra unit of capital is not giving us any extra output, right and we only want to produce y naught units of output. So, we should produce this. So, the optimal point is given in this way, where k is equal to l in this case and that is equal to y naught. Now here if we know the output, we should employ that much amount of labor and that much amount of capital. So, in this problem minimizing min wl+rk this subject to in this- y= Min{l,k}what do we get?

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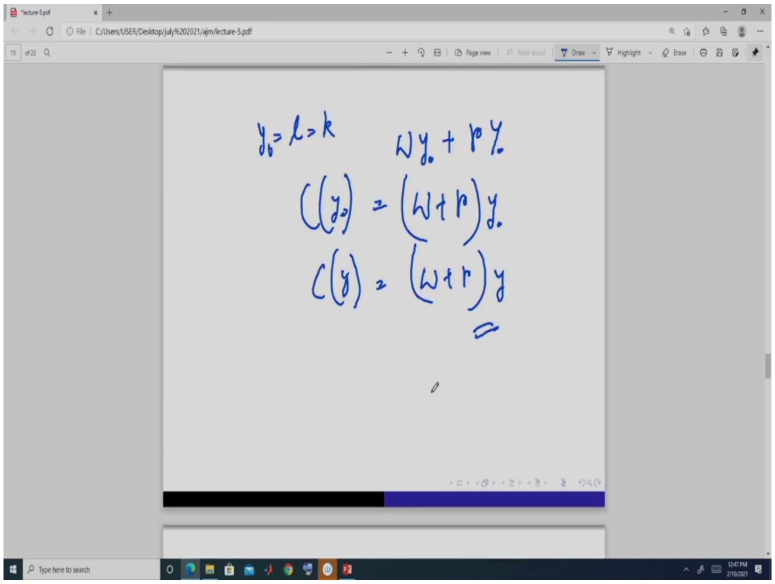

We get that the optimal point is when y is equal to l is equal to k. So, our cost is going to be w. So, this- $Wy_o + ry_o$ is this$C(y) = (W + r)y_o$, now here you plug in different values of output, you will get the cost function of this nature. So, the general cost function is this- $C(y) = (W + r)y$,right. Now, we will see that, how it looks, how this cost function looks when we take different when the production function different exhibits different types of returns to scale, okay.

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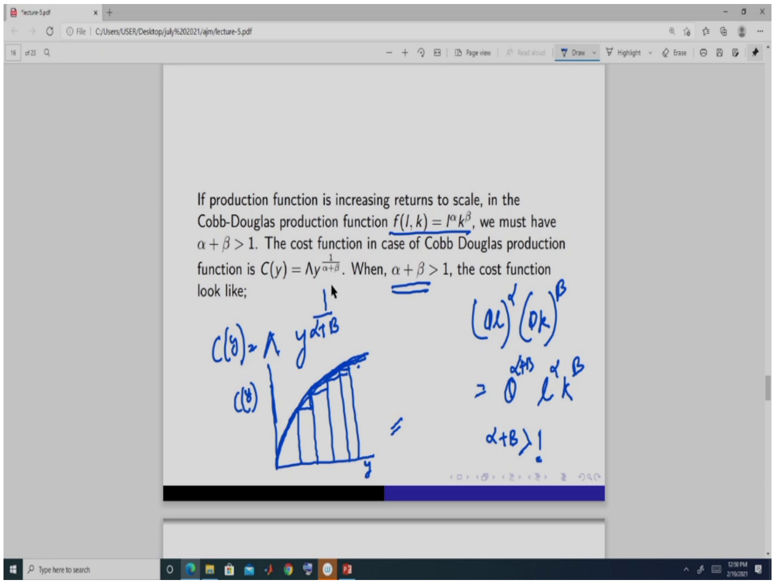

Now, suppose we have increasing returns to scale and the production function is Cobb Douglas. If it increasing returns to scale and Cobb Douglas we have seen in the last class what happens so, this if we take here, this a, what do we get? that if we take a multiple of, theta multiple of both the inputs then this we get theta to the power- $\Theta^{\alpha+\beta} l^{\alpha}k^{\beta}$. And if it is increasing returns to scale then this output should be more than theta times this and this is possible when alpha plus beta takes a value greater than 1, right.

So, whenever we are using a Cobb Douglas production function and we want it to exhibit increasing returns to scale, then we always we must have this- $\alpha+\beta>1$, right. Now, we must have this condition that alpha plus beta should be greater than 1. Now, our cost function here when we have a Cobb Douglas production function is this-$C(y) = \wedge .\ y^{\frac{1}{\alpha+\beta}}$. So, alpha plus beta is taking a value greater than 1 then and this is you can take this big lambda to be some constant, some positive number.

So, if we try to plot the cost function here you take output and here you take the this. Now, plug in 0 amount of output cost is 0 here, but this alpha plus beta is greater than 1. So, the cost function is going to look something like this nature. So, what is happening here as we go on increasing output the additional cost is going down, okay. So, we will do this in detail in the next class, okay. So, this is what that total cost curve looks like when we have increasing returns to scale and the production function is Cobb Douglas okay.

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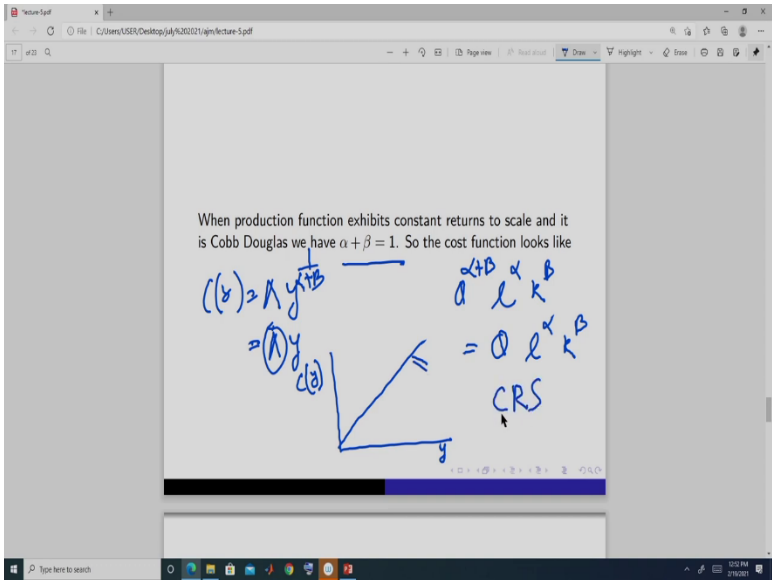

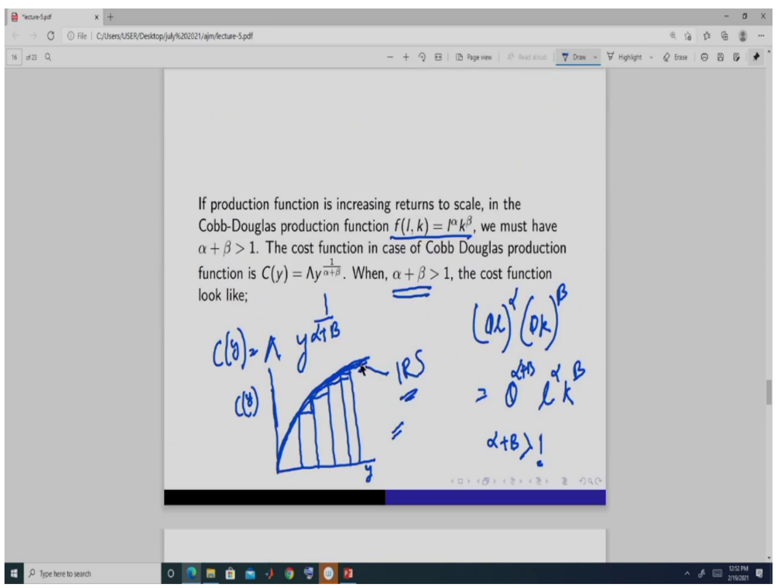

Now, suppose we have constant returns to scale. So, in case of constant returns to scale what we will have, we will have this -$\Theta^{\alpha+\beta} l^{\alpha} k^{\beta}$should be equal to this-$\Theta\, l^{\alpha} k^{\beta}$. So, this means what? Alpha plus beta should always be equal to 1, right because if we take a theta multiple of labor and capital so output should also increase by the same multiple right. Now, our cost function for the Cobb Douglas is big capital lambda y to the power alpha plus beta this- $C(y) = \wedge .\ y^{\frac{1}{\alpha+\beta}}$.

Now, here this is equal to 1. So, alpha is big lambda. So, our total cost function if we take plot y here and cost here, it is going to be a straight line where the slope is given by this big lambda, okay, capital lambda. So, this is our total cost function, when we have CRS that is Constant Returns to Scale and when we have IRS Increasing Returns to Scale, our cost function is like this.

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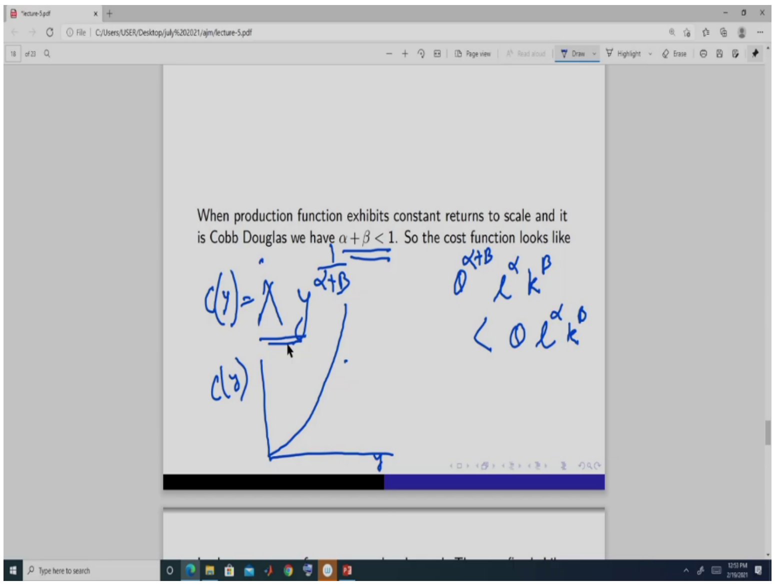

Now, suppose we have decreasing returns to scale. Decreasing returns to scale means, when we have Cobb Douglas production function if we take theta multiple of both the inputs, then this- $\theta^{\alpha+\beta} l^{\alpha} k^{\beta}$should be less than theta where theta takes a value greater than 1, okay. So, this means that alpha plus beta should always be less than 1 if it is less than 1 then the our cost function in the case of Cobb Douglas, it is this- $C(y) = \wedge .\ y^{\frac{1}{\alpha+\beta}}$, right. So, this alpha plus beta is less than 1.

So, it will be of this nature this function, right is it I hope you are following. So, this is 1 because Cobb Douglas is mostly used in the literature. So, we have concerned, we have only, we have stick to Cobb Douglas, you can try any other form also but technique is going to be same. And next we come to a very important thing like we have discussed at the beginning that in the short run, not today in the last lecture, some of these factors can be fixed. And that is why we see something called Law of Diminishing Marginal products.

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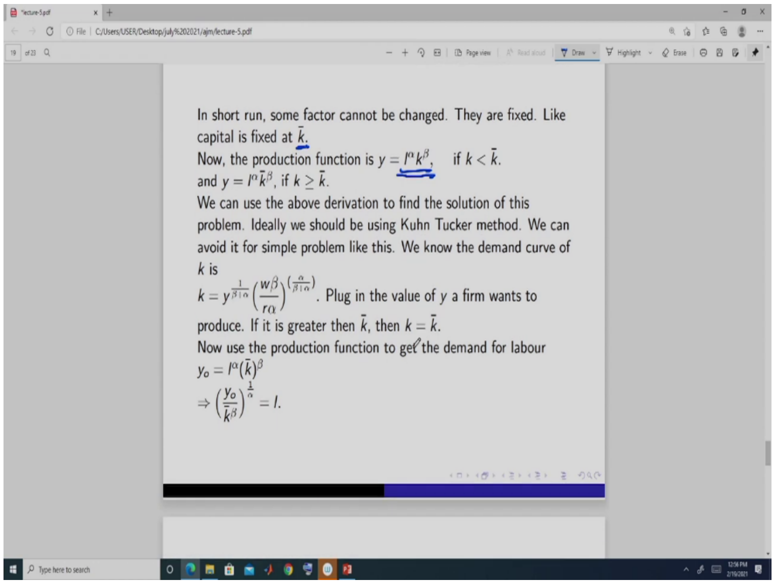

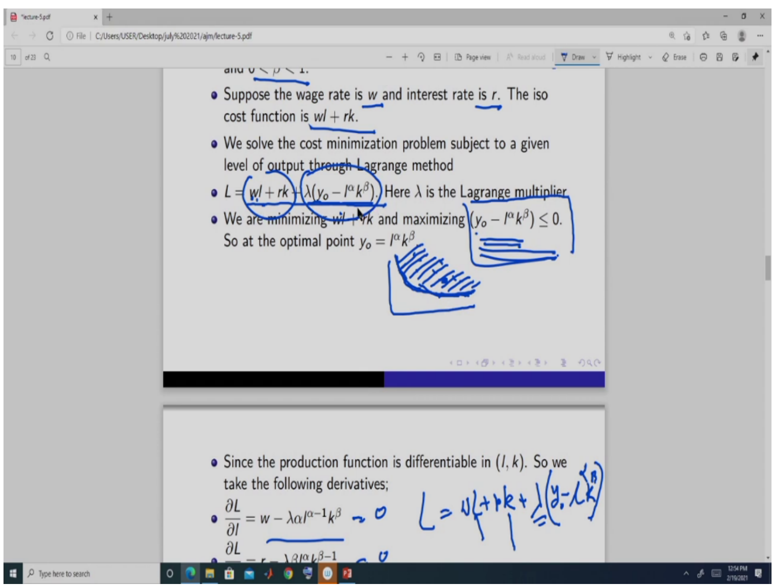

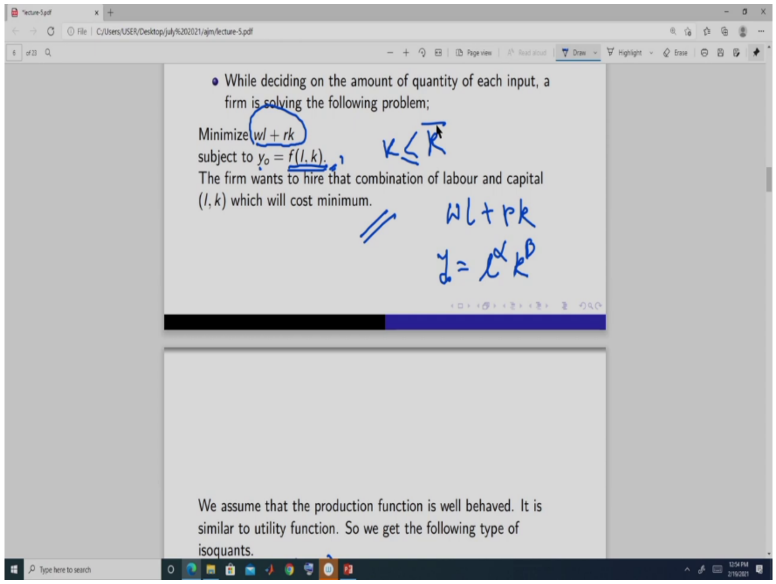

And we assume that suppose k is fixed, okay. Now let us do this problem this what we have done here we have solved this Lagrange thing- $L = wl + rk + \lambda(y_o - l^{\alpha}k^{\beta})$, we have solve this problem using a Lagrange method and this problem was minimize this- wl+rk subject to this- $y_o = l^{\alpha}k^{\beta}$. Now we add one more thing that the k has to be less than equal to k bar, okay. We cannot use more than k bar.

So, here if we are provided some kind of constrain like this then we should actually use a method that is Kuhn-Tucker method, okay, but we will not do that method, what we will do we will use a more simpler method in this case because this problem is a very simple problem because we have only two inputs labor and capital, but if we have multiple than Kuhn-Tucker would have been a better option. Because here we can do it graphically so that is why it is easier to do without using Kuhn-Tucker.

Now, suppose we keep the production function same as the Cobb Douglas. Now here the production function is like this if k is less than a then we will get production function is of this nature- $y = l^{\alpha}k^{\beta}$ l to the power alpha, k to the power beta. Now, if we want to have more k then k bar it is not possible. So, our production function is something like this l to the power alpha into k bar to the power beta. Now, if we want to increase more output, we can only change labor, capital is fixed we cannot change.

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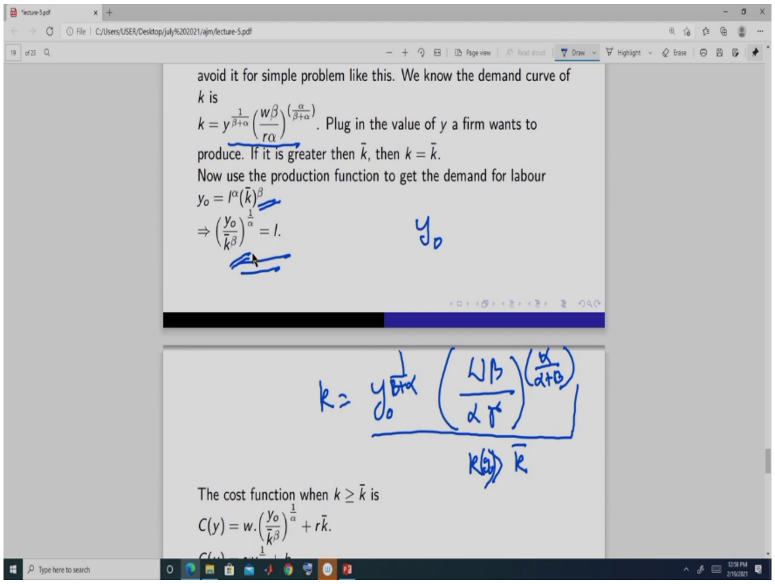

So, what we will do? So, we know from the Cobb Douglas thing the demand curve of labor is a capital is this- $k = y_o^{\frac{1}{\beta+\alpha}}\left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\beta+\alpha}}$. Now, if we want to find out how much is going to be the demand if we have some fixed amount of output that is y naught we know it is going to be given like this where k is equal to 1, sorry k is equal to output alpha into this divided by this is power alpha divided by alpha plus beta okay. given this,

Now, we know the output y naught now plugin y naught here. Now, if so we get a specific value of k if this k is suppose greater than k naught when we have y naught output then we cannot use it, right. So, here what we can do the maximum we can use is k bar. So, plug in k bar, right. So, k is fixed now, once k bar is, k is fixed it is at k bar then we know how much amount of labor we need to employ to produce y naught amount of output it is given by the production function it is this- $y_o = l^{\alpha}\left(k^{-}\right)^{\beta}$ you plug in k equal to k bar and then you will get what? this is the demand for labor.

Now, the question is whether this is an optimal point or not, optimal point in the sense whether this is a cost minimizing point or not it may. Now, from the, since the wages and the price of the capital is fixed, and the production function is also given in this way. So, we know from the

Cobb Douglas production function that the demand for capital is going to be like this- $k = y_o^{\frac{1}{\beta+\alpha}}\left(\frac{w\beta}{r\alpha}\right)^{\frac{\alpha}{\beta+\alpha}}, right.$ Now, since we cannot employ the optimal a, because this optimal capital because this is more than what is available to us. So, the minimum that is we can employ is this- $k = \frac{y_o^{\frac{1}{\beta+\alpha}}\left(\frac{w\beta}{\alpha r}\right)^{\frac{\alpha}{\alpha+\beta}}}{k}$.

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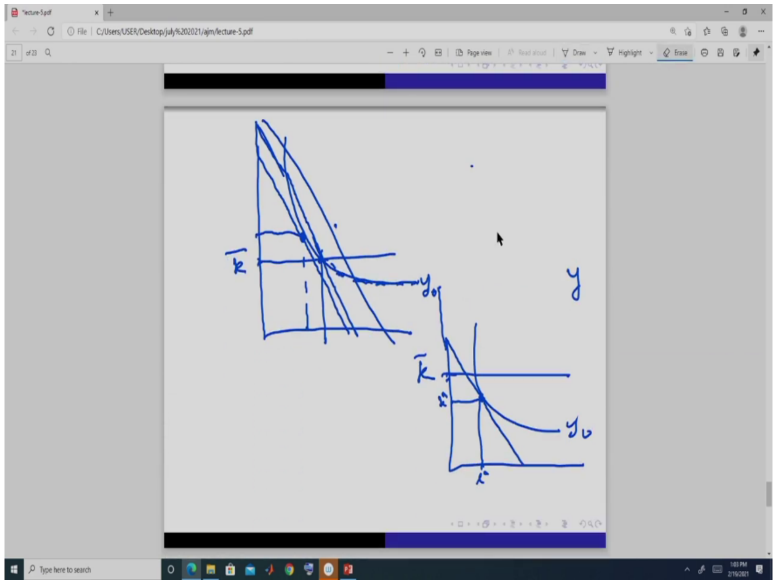

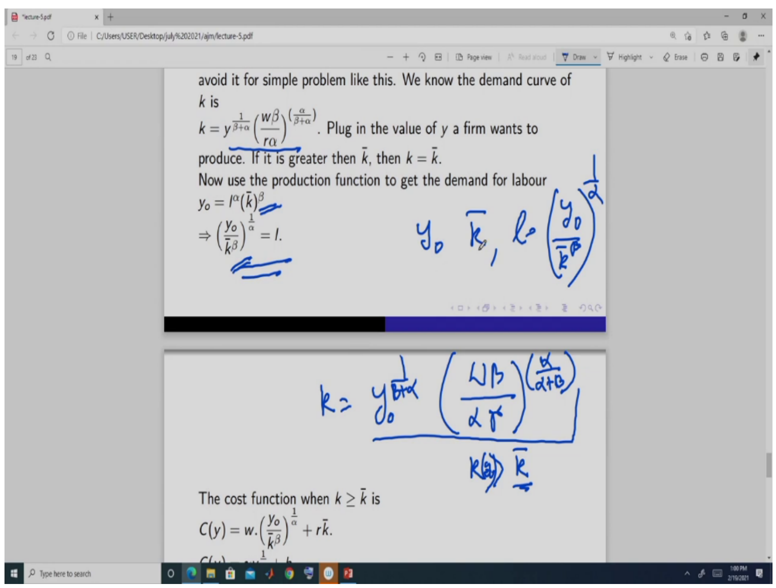

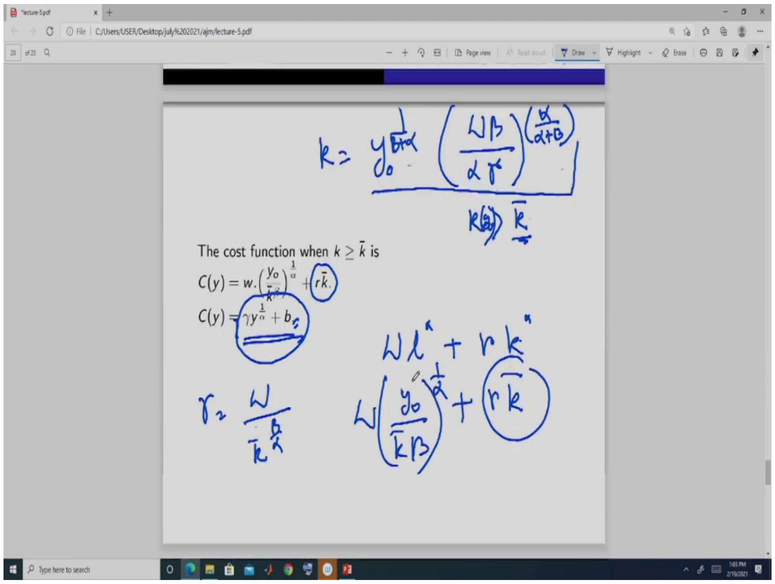

So, it is going to be something like this suppose this is the a, and suppose this is the optimal and suppose k is fixed here k bar. So, our optimal points would have been here, but we cannot have more capital than this k bar. So, we say that this is going to be our optimal point how do we find this we fix k bar and then we find the corresponding demand for labor from the production function fixing the production function at this y naught amount of output, right, this.

Now, what is possible we can choose any point from these points, right. Now, this point is at this isocost line, any point which is right to this point, only those points we can choose, right. So, it will be at isocost which is higher than this. So, that is why this point is going to be the point which will give us the minimum cost. So, that is why this k bar and l is equal to y naught divided by k bar to the power beta then whole to the power 1 by alpha, this is the demand for labor- $l = \left(\frac{y_o}{k}\right)^{\frac{1}{\alpha}}$ and based on this we will get the optimal combination of labor and capital, capital is going to be fixed.

But here in the same thing, suppose the situation is something like this, suppose, this and in this case, suppose this is the optimal point cost minimizing point when and suppose k bar is this. Now, this is going to be the demand for capital and this is going to be the demand for labor star. Now, here we can always employ this much amount of capital because our capacity is this. We already have this much amount of capital and we can take use this. But here this optimal point is not getting binded by this constraint. But here it is binding, optimal point is higher than what is possible for us to employ. So, that is why in this case we can go ahead with the previous thing.

Now, in this case what we will have to do when this is binding or not what how do we find? we in the demand curve we can plug in those specific output and from there we can find this specific demand for capital and then we compare it with our constraint amount and then if it is constraining, then we will fix it at that level and if we fixed it at that level then after that we will find the optimal amount of labor rate at this.

So, if we do that, now, our cost function is going to be what, cost function is this as the optimal point, optimal point, this optimal point is what? k bar so, this is already fixed and this is like this . So, this you can write it in this form- $C(y) = \gamma y^{\frac{1}{\alpha}} + b$ gamma into y to the power 1 by alpha plus 1 plus b, b is this portion and gamma is you can think gamma as this- $\gamma = W/k^{\frac{\beta}{\alpha}}$, right. So, our in the Cobb Douglas cost function now, become looks like this, okay.

And since alpha takes a value which is always less than 1 and greater than 0, so if we plot the cost function like this, then you will have a think if your output is here and your a, this is the cost function, okay, let us draw it in a different page.

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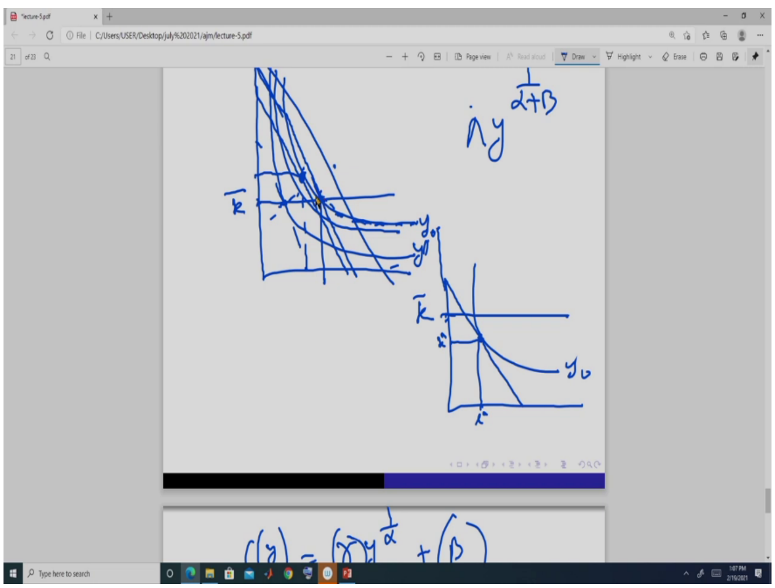

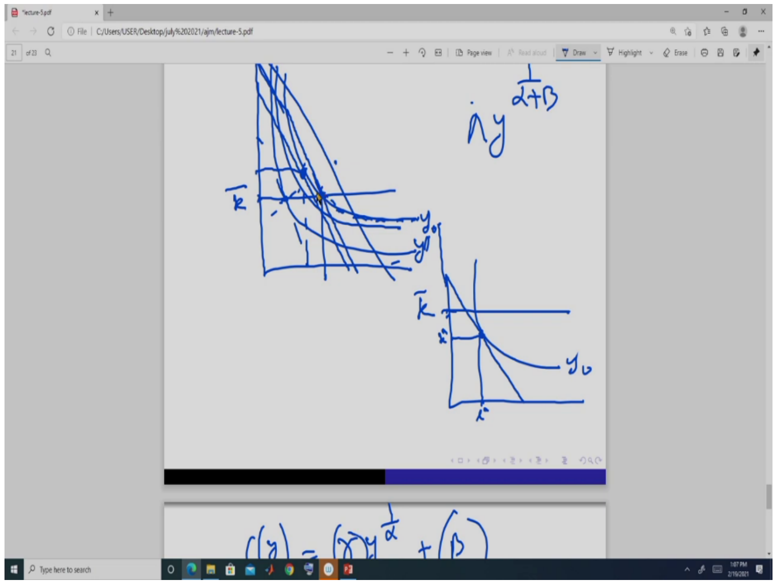

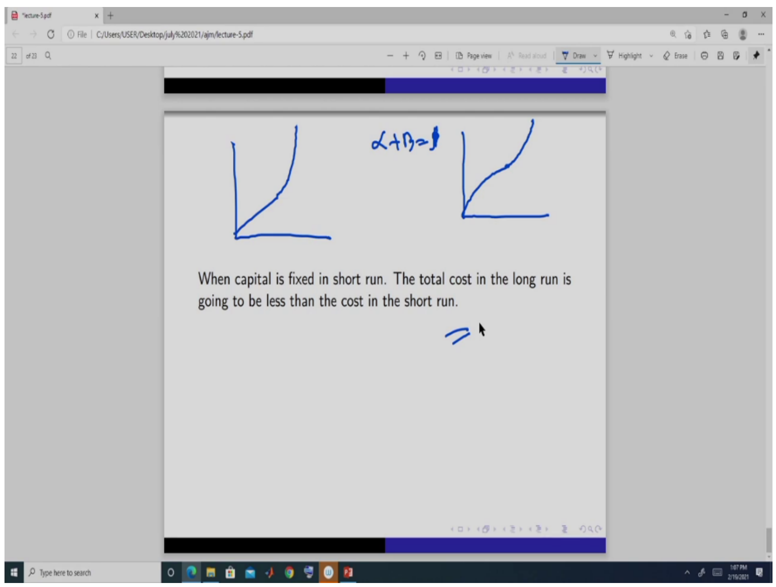

So, our cost function is now this- $C(y) = (\gamma)y + \beta$. So, this is a positive number, this is a positive number and if we take put here, so, when it is 0 it is this and as it goes on a. So, since alpha is less than 1, so, we will get a curve like this, this is our cost, total cost function. see how it got changed when we are employing this, okay. Now, here actually this cost function there is a see what is happening if we just take why I have taken see.

This cost function is only valid when our output is more than see in this case, when this is the binding point, right? if we are producing output which is less than this then what do we get? it is so, any point output which is less than this then only, right? because otherwise we will get the same cost function like this right? alpha plus beta till this y double dash level of output level.

So, actually our cost function should be if we do it properly should be like this it will be this nature right? and then it will be this nature depending on whether alpha plus beta is equal to if alpha plus beta is less than 1 and then after this it will be since it will be power is a, it will be further like this or if this is if one it will be like this and from this point it will be like this or it can be like this and after a point it will be like this.

So, our total production total cost function can be of this nature. But we will see encounter some form of costs function in this way, this form also when we do the actual industrial organization and there you can think that actually some problem like this we are facing some kind of problem like this, okay. So, from this what do we get? We get that in the short run, if we have a problem that is that the capital is fixed then we cannot go beyond it, so, we cannot use capital beyond it.

So, what may happen our optimal point which should have been this instead of that it becomes this which is fixed at this, but in the long run if we can vary the capital then it will be this, but this lies at a lower isocost line than this. So, in the long run the total cost will never exceed the short run cost. So, this is what we will, we get okay. So, the long run cost is always going to be less than or same as the short run cost, the short run cost it is possible to be greater than long run, but it will never be less than long run, okay. Thank you very much.