Introduction to Market Structures

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Module two: Production and Cost Curves
Lecture Production Function

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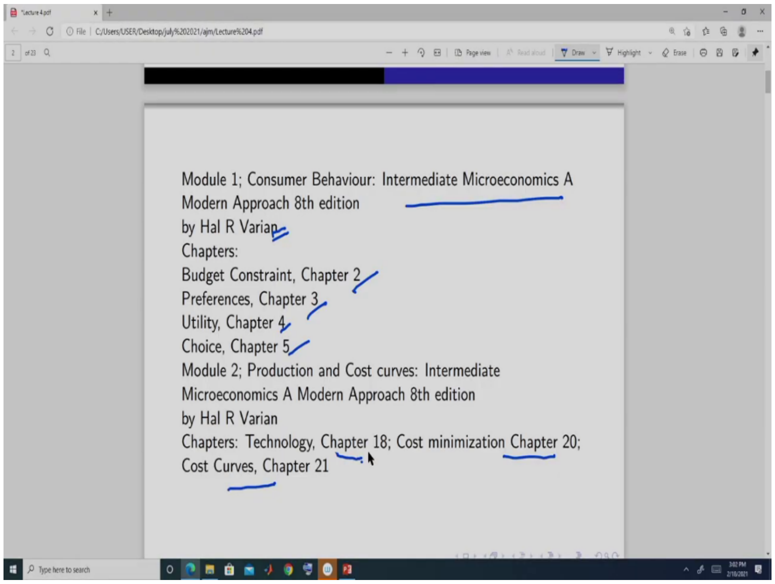

Welcome to the course Introduction to Market Structures and today we are going to start module two, that is cost and production cost function in that. So, before that, let us first give you, I will give you some references for consumer behavior that is module one, you follow this book if you want to follow a book that is Intermediate Microeconomics A Modern Approach by Hal Varian eighth edition and the specific chapters that you can go through from this book is like chapter two, chapter three, chapter four and chapter five from this book.

Or you can alternatively the power point slides that you will get it is sufficient for this portion. And today, we are going to start module two that is production and cost curve. So, for this also, you can follow the same book intermediate microeconomics a modern approach, eighth edition by Hal R, Varian and the chapters are, chapters are like technology, it is chapter *eighteen*, cost minimization chapter *twenty* and cost curves chapter *twenty one*. So, these three chapters, so, today we will do chapter *eighteen* that is technology.

So, when we say production, what do we have in mind? So, it is like this, suppose, think of a farmer who owns a plot of land and in that plot of land he uses his own labor along with some machine to cultivate this land and he produces a food grain or some vegetables or some other crops. So, this whole process is a production, like you plow the land using your labor and some machine you sow seeds and then you harvest once these plants are grown up and from that you get if it is food grain, then you get food grains, if it is vegetables you get the vegetables like that.

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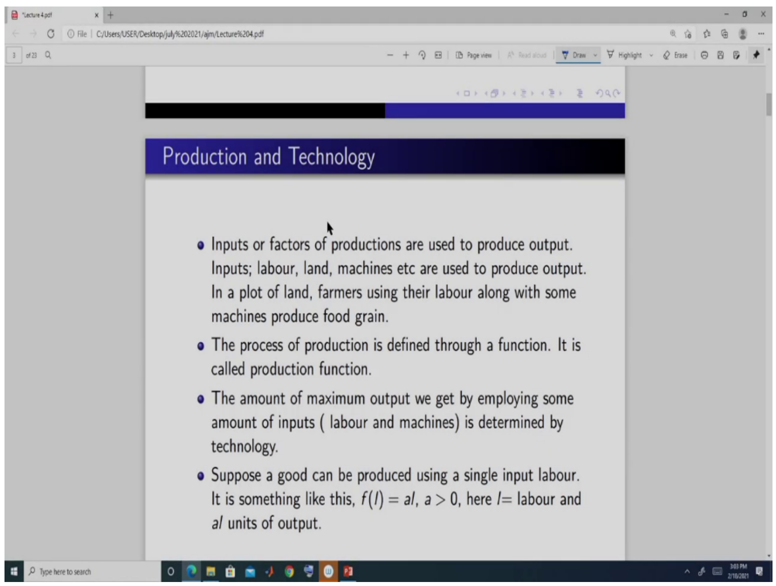

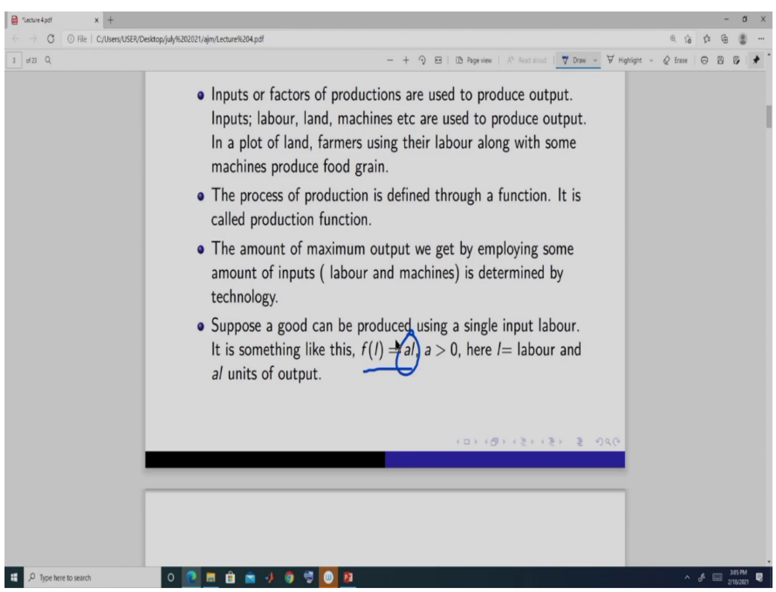

So, this whole process has some inputs, these are factors of production. So, in this example inputs are like land, labor, the machines like the tractor or the harvester or etc or Bullock and output is the food grain the crops those are outputs or alternatively you can think of suppose you are producing automobile. So, here the inputs are like iron and steel, some machines which you along with the workers, you use these machines to give different molds, different forms to the structure of the automobile.

And again using some steel and machine you use the machines and then you together assemble them and then you get a automobile. So, this whole process is determined or whole process is not determined it is represented as a function. It is like you provide input and you get some output and here output is our the goods that we want and inputs in the most general case are like labor, land, capital means machines, this three and the raw materials like the intermediate was like to produce machines, we need iron, iron and steel.

So, that is a raw material, we convert this into machines, right. So, those are raw materials. So, what do we do, we represent it as a function. So, for simplest case suppose we take the output which can be produced by a single input and that is suppose labor and then we can write it in as this way- $f(l)=al, a>0$. So, here a into l, l is denoting the labor, okay.

So, a is some positive number. So, you plug in labor and the amount of output you are going to get is this much, okay. And when you plug in some specific amount of input how much amount of maximum possible output that we are going to get that is determined by what is called a technology, okay.

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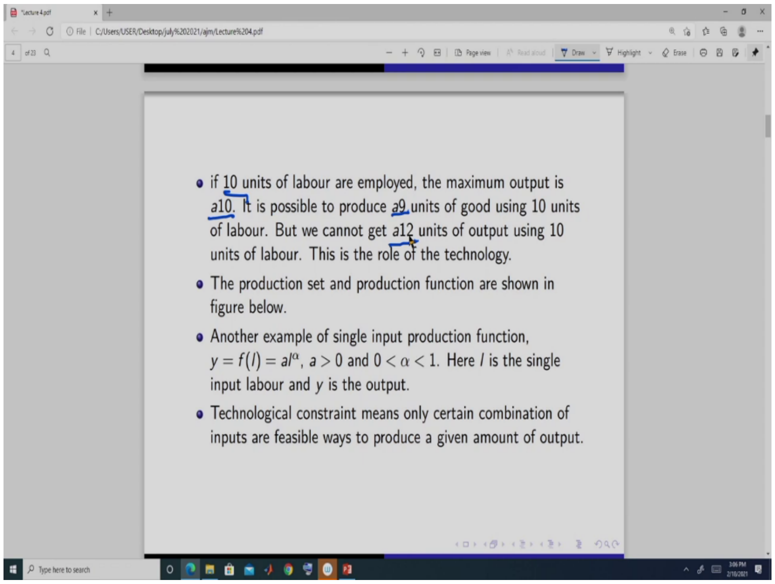

So, it is something like this that suppose we employ 10 units of labor. And if we employ, employ 10 units of labor then we will get using that above production function our output is going to be a into 10. So, now, it is possible to produce a into 9 that much units of output because it means we are not whatever maximum it is possible is a into 10. But still we can produce this a into 9.

But we will never be able to produce a into 12 units of output by employing 10 units of labor. So, this is the role of the technology. So, it defines or the determines the maximum possible output given input.

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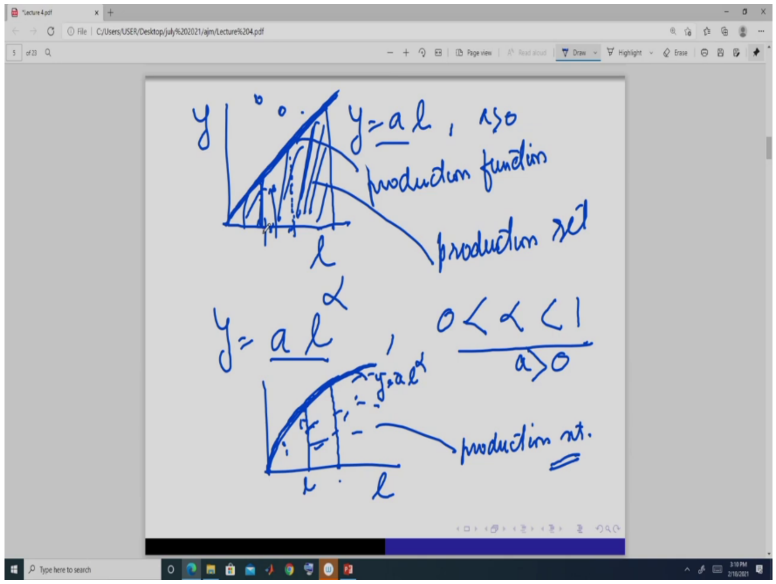

So, like here if we take in this a labor suppose, and here we take output and our production function is a into l where a is some positive number, then we get a line like this. So, we plug in this much amount of labor the output, maximum output is this much. But we can produce each of these units like if we employ this much amount of labor, the maximum output is this much. But we can produce any one of these possible levels of output.

But we will not be able to produce this much amount of output if we employ this, to produce this much amount of output we need to employ this much amount of labor okay. So, this is the production actually function okay. And this whole set is called the production, oh sorry, this is called the production function and this whole set this is called a production. So, anything here this point it is feasible because, if we employ this much amount of labor we can produce this.

But this is not feasible by employing this much amount of a. Another example of production function you can take like here-$y = al^{\alpha}$, where alpha takes a value between 0 and 1 this. So, if we plot labor here and alpha is some lies between a number between 0 and 1 and a is a positive number. So, we get a curve like this. So, this is like this, here if we employ this much amount of labor the maximum output possible is this much, if we employ this much amount of labor, maximum possible output is this much.

So, this curve is the production function and this whole set this is the production set in this case that is all these points are feasible okay. I hope it is clear. So, what we are doing we are plugging in input that is factor of production and we are getting output and we define this thing through a function and that is production function. So, we get when we are using only one way input, so, we are getting our production functions, these are some examples of production. Now, let us complicate this.

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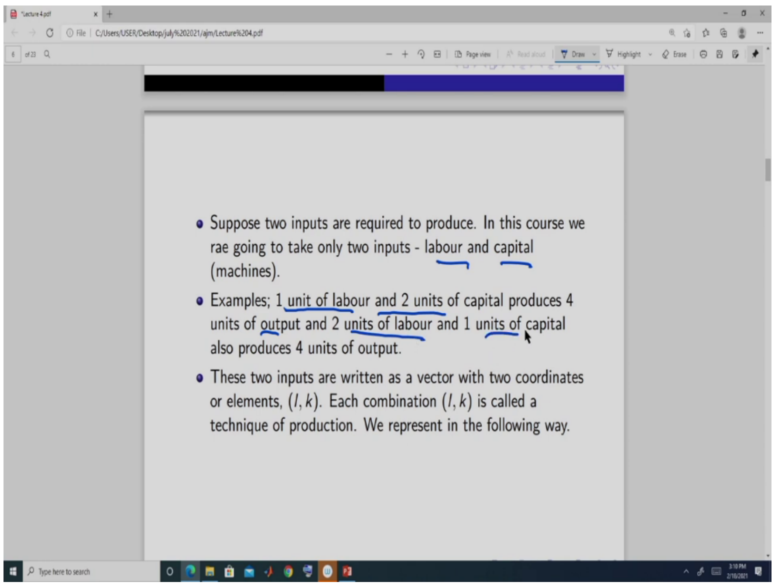

Suppose we have two inputs and these two inputs are labor and capital, capital means some machines. So, labor along with some machines produces output okay. For example, suppose we can do and now here since we have two inputs. So, now, we can combine them in different ways. For example, 1 unit of labor and 2 units of labor can produce 4 units of output okay.

So, this is one technology. So, the technology is this now, what we can do? We can also produce four units of output using 2 units of labor and 1 unit of capital. So, what we are getting? We are getting different combinations of these two inputs may give us same output.

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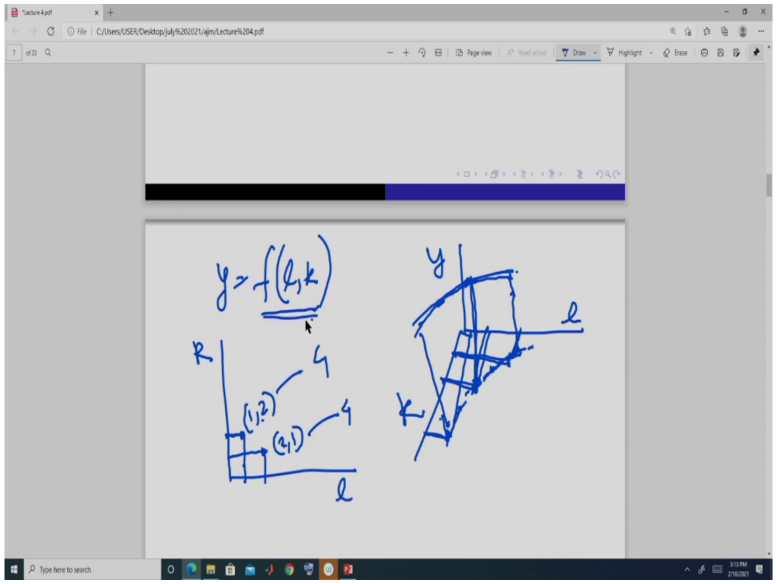

So, our production function is now actually something like this-y=f(l, k), okay. And what we do in this axis we plot labor in this axis we plot capital and suppose we are 1 unit of labor and 1 unit of labor and 2 units of capital give me 4 units. So, this point gives me, so this (1, 2)gives me 4 units of output. Same this point which is (2, 1) also gives me 4 units of output. So, these two if we take if we use the height of this a height to represent the output, then this two point should be at the same level, same height, okay.

So, this is our now technology. So, technology here means that this is giving me 4 units. This is giving me again 4 units. So, and this is a specific technique of production and this is also a specific technique of production okay. Now, here what we can do, if we plot labor here and capital here and output here. So, then suppose this point, this much amount of capital, this much amount of labor and height is giving me the amount of output.

So, if we have a combination of points like this and all of them are at suppose same height like this, then we are getting same output from these different combinations. So, each of this is a separate technique of production okay. So, here we are employing this much amount of capital and this much unit of labor. Here we are using this much unit of capital and this much unit of labor. Here we are using this much unit of capital and this unit of labor. But each of this technique is giving me same output. So, here we have more capital less labor, here we have less capital more labor like this. So, same thing like this.

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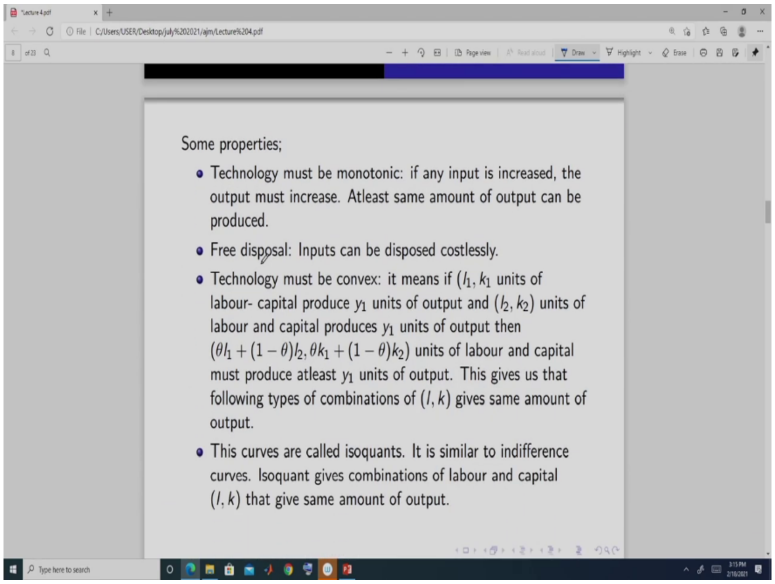

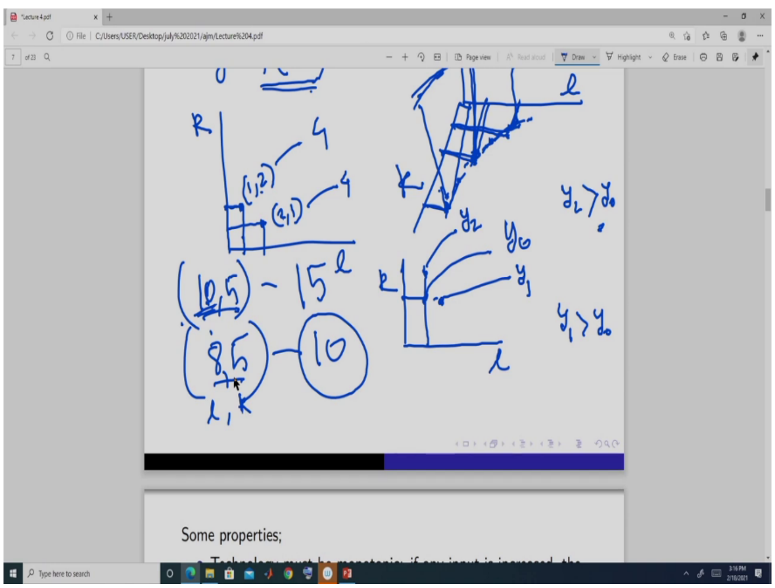

Now, in this production function, we impose certain conditions and these conditions are first that the technology must be monotonic, what it means that if we increase any input then the output must increase. So, it is something like this, if we take like this labor and capital and if we are at this point suppose, and at this point gives me suppose, y naught unit of output. If we combine this much amount of capital and this much amount of labor, then if I increase capital, so, this must give me some y 1 unit of output.

And y 1 must be greater than y naught, or if I move to any point like this, where we have kept the amount of labor same and increase the amount of capital and suppose, this point is y 2, then again y 2 must be greater than y naught because I am employing more of capital here than at this point. Here, I am employing more of labor than capital. So, again this output should be more than this okay. So, technology must be monotonic.

And second that we should have something called a free disposal. Free disposal means, that the input can be disposed costlessly. So, it means suppose, I have suppose 10 units of labor and suppose 5 units of capital. And I can combine suppose 5 units of capital with 8 units of labor to produce 10 units of output, okay.

And 10 units and 10 units suppose give always be more, it gives me suppose 15 units of output okay. Now, suppose I want to produce only 10 units and I already have 5 units of labor, then I will not cost anything extra, if I want to dispose 2 units of labor okay. So, I will not. So, I can easily switch from this combination to this combination. And I will not bear any extra cost for that okay. So, this is what free disposal means. And the second, the technology must be convex. What does it mean?

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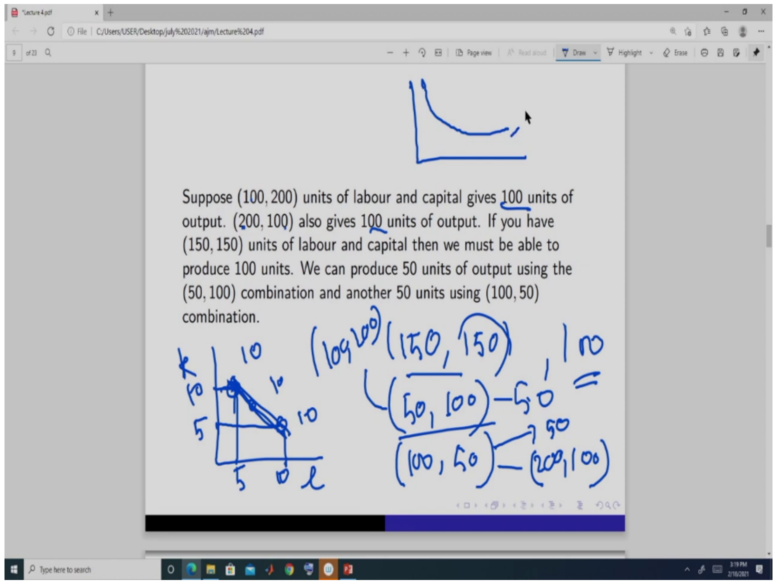

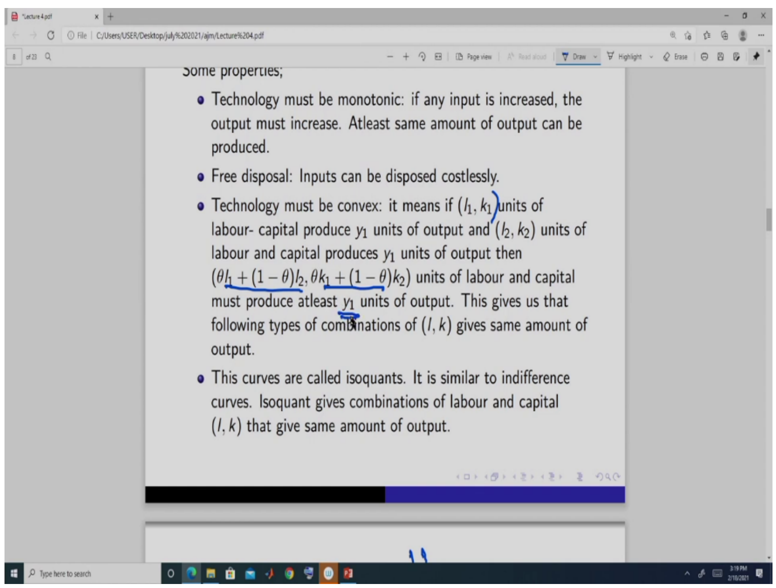

Convex means that if suppose, you take labor here capital here and suppose this point, this is suppose 5 units and 10 units. And suppose this is 5 and 10. And both of them give me suppose 10 units and 10 ten units. Then, if I take a combination of these points, then I should be able to produce at least 10. So, this is something like this suppose, I am given with 100 units of labor and 200 units of capital. I can produce 100 units of output.

And same with 200 units of labor and 100 units of capital, I can produce 100 units of output okay. Now, if I want to use 150 and 150 units of labor and units of capital, I should be able to produce 100 units of output, how? By taking 50 and 100 units of labor sorry, 50 units of labor and 100 units of capital, and 100 units of labor, 50 units of capital. So, this will allow me to use the first technique that is (100, 20).

So, this will allow me to produce 50 and this here, I am using 200 and 100. So, this will allow me to produce (100, 50) again. So, (50 50) we can produce 100 units, right, this is, is it clear? I think this, this is a simple example. So, graphically it means that if we take this point, this is one technique and this another technique.

And if we take a linear combination of these two techniques suppose, any point then I should be able to produce whatever I am being able to produce using this technique or this technique okay. So, this assumption, if we assume same we will give us level curves of this form. And this is something called isoquant, we will come to it now.

So, formally it is like this, that if we use l1 and k 1 units of labor to produce suppose y 1 units of output. And again we can use l 2 and k 2 units of labor and capital to produce y 1 units of output. Then any linear combination, that is theta l1 plus 1 minus theta l2 this linear combination of labor- $\theta l_1 + (1 - \theta)l_2, \theta k_1 + (1 - \theta)k_2)$. And again linear combination of capital, that is theta k 1 plus 1 minus theta k 2 will give us at least y1 units of output, okay. So, this is what is means that the technology is convex.

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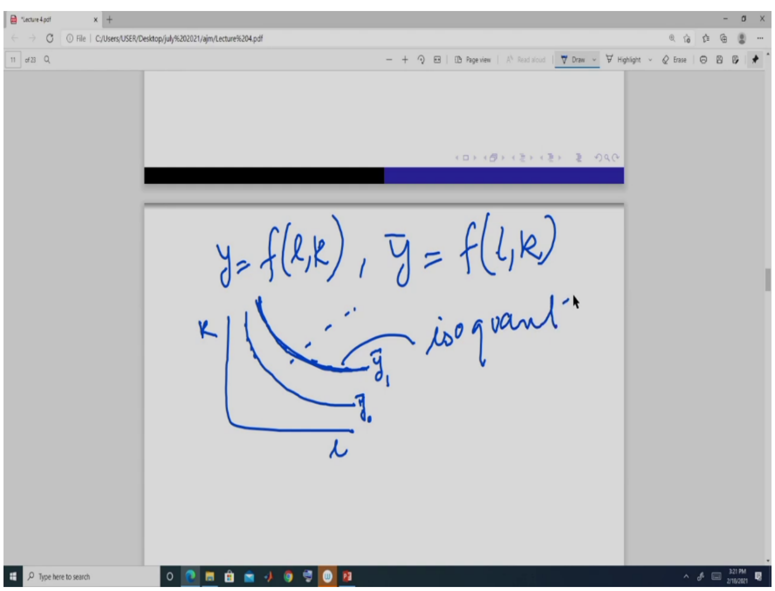

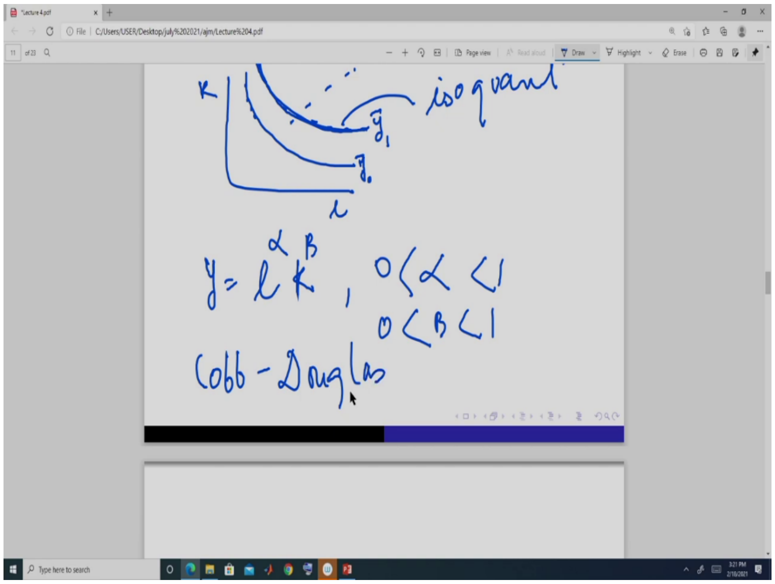

And what we can do since we have this production function like this- y=f(l,k)now if we fix the level of output k bar and we find out all the combination of l and k, which is giving me this here. Suppose, if we assume that the technology is convex we get a combination like this. So, each point here gives me y bar units of, if I increase this level curve, suppose this is y bar naught and this is y bar 1, here output is more and all these combinations of capital and labor give me y bar 1 units of output okay.

And as we move in this northeast direction, output is increasing. This is straight from monotonicity and convexity gives us this okay. Now, so, these are called isoquant, something similar to indifference curve. And we will require them while solving a optimization problem that the firm solve, okay.

Now, let us do some example. Suppose our production function is this- $y = l^{\alpha}k^{\beta}$. So, l to the power alpha, k to the power beta, where alpha takes a value (0, 1) beta takes a value (0 and 1), okay. This is a very famous production function and it is called a Cobb Douglas production function, one variation we have done in Cobb Douglas utility function okay.

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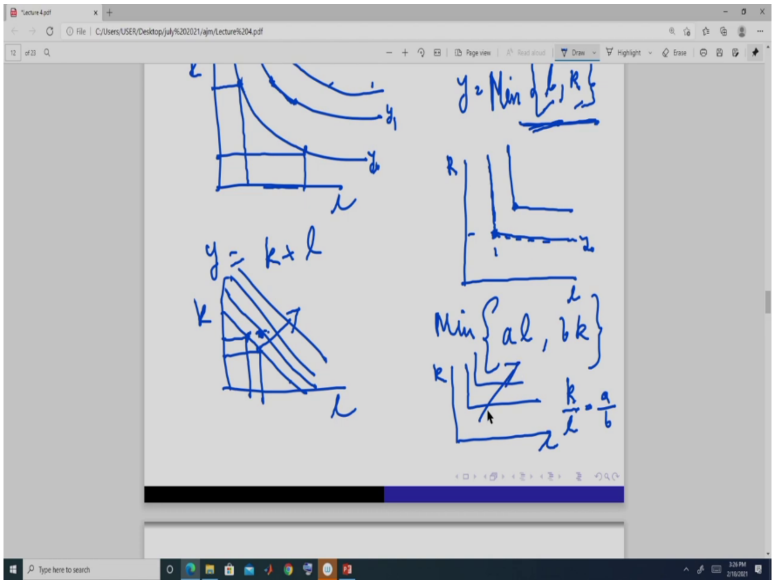

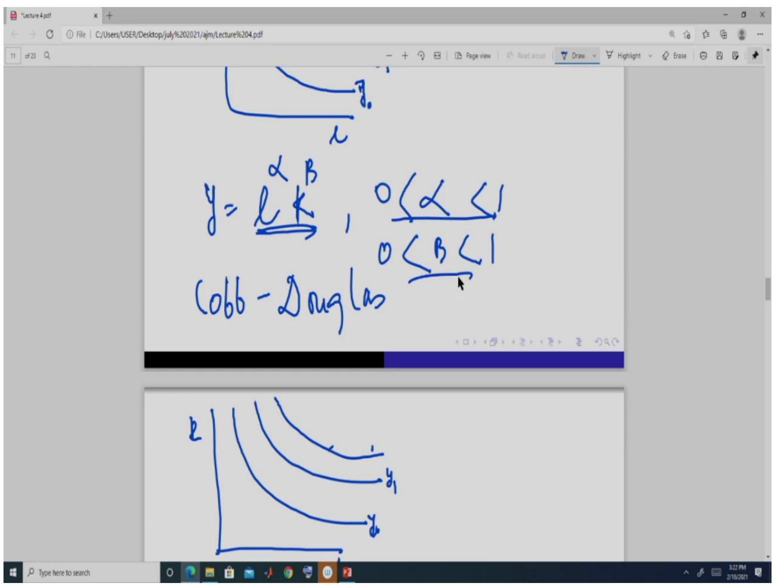

Now this function isoquant of these functions will be something like this. And here this is output is y naught here y, y 1 so output is increasing in this, okay, we will get curves. So, these are level curves or isoquant when our production function is this. And this alpha and beta takes the value in this range. So, here what do we see that we can substitute 1 capital and labor at some degree. So, this we are using this much capital and this much labor.

Here we are using this much capital and this much labor. So, if we move from this point to this point, what we are doing? We are substituting, we are reducing the use of capital and increasing the amount of labor. Same here, if we move from this to this, what we are doing? We are increasing the use of capital and decreasing the use of labor okay. So, there is some form of substitutability allowed in this production function.

So, this technology is allows us to substitute a bit of capital and labor. But suppose, our production function is this $y=\min\{l,k\}$ this. Now, if we plot l here and k here, if we look at this a, it will this, it will be suppose for. So, this production function are generally called a fixed proportion. So, we produce output using a fixed proportion at this. So, even if we keep on increasing the amount of capital, our output is not going to increase given a fixed amount of labor.

So, you can see that here this is violating monotonicity and also this is not a differentiable function okay. Here, if we fix this amount of capital and if we keep on increasing more labor, output is not going to increase it is going to remain fixed. So, that is why to produce a level of output we require labor and capital in fixed proportion and that is here (l k), okay. So, this would be equal proportion.

So, here this does not allow us to do any substitute. So, factors are not substitutable in this production function, we require both the inputs in fixed proportion. Here, it should be always equal. But instead suppose, our production function **is** suppose a l and suppose b k. Then here also it will be again, something like this. But, they will be required in the ratio suppose if we look at k sorry, we generally take k by l. If we look at this k by l ratio here, it should be always in the **a** by b ratios.

So, in this production function the inputs capital and labor should always be used in this ratio to get output okay. Now, suppose let us take another production function and this is suppose simply k plus l- $y=k+l$. So, here if this is labor and this is capital, it is going to be a straight line. So, these two inputs are perfectly substitutable. So, if I move from here to here, I produce the same level of output. So, to produce but if I increase the keeping same a I will increase the output.

So, I will be at a higher isoquant okay. So, isoquants here are increasing in this, production are increasing in this, direction, here production is increasing in this direction okay. So, these are some examples of production function okay. Now, here to keep our life simple, we generally assume that the production function is differentiable okay.

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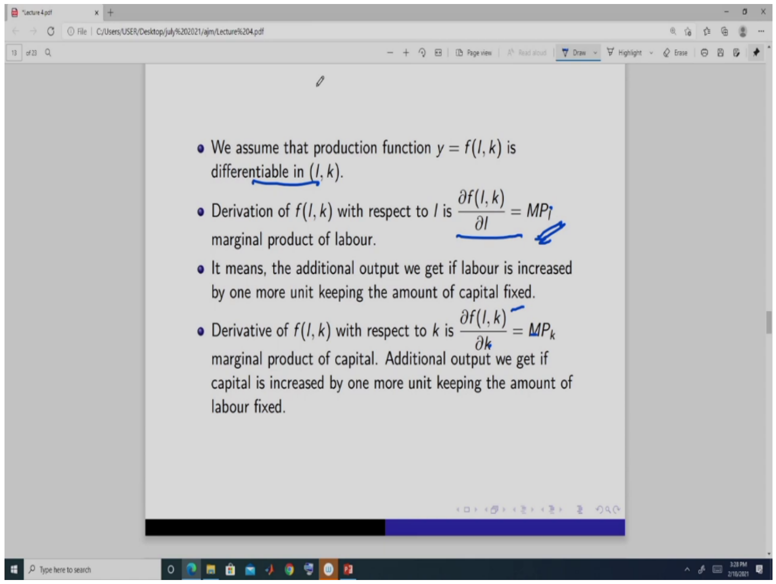

If they are differentiable in both labor and capital then these partial derivatives are possible. And this partial derivative that is our total output with respect to labor is called marginal product of labor, that is if we keep the amount of capital fixed and if we increase labor, how much amount of output, additional output we are going to get it is given by this. So, it is something like this. So, at the margin, if we increase one more unit of labor, how much additional output do we get, if we keep the level of capital fixed.

Here this- $\frac{\delta f(l,k)}{\delta k} = MP_k$, the partial of the production function with respect to capital is the marginal product of capital. And we write it MPk. So, it means that if we fix labor and if we increase capital then how much amount of additional output we are going to get? So, at the margin if we increase capital by 1 more unit, by how much unit the output is going to increase? okay.

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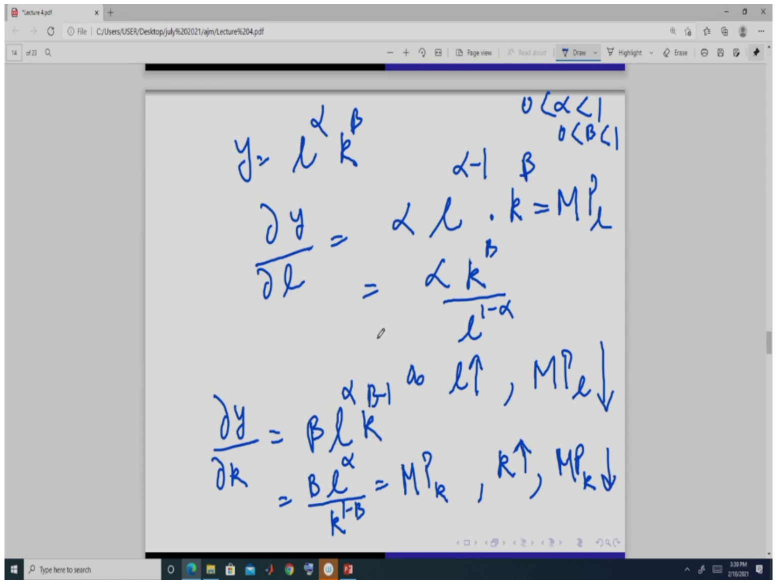

So, it is suppose our production function is the Cobb Douglas production function. So, this, like this- $y = l^{\alpha}k^{\beta}$, then this- $\frac{\delta y}{\delta l} = \alpha l^{\alpha-1}. k^{\beta} = MP_l$ is, right, and we know alpha lies between. So, this is actually like this- $\alpha k^{\beta}/l^{1-\alpha}$. So, it means what? That as labor increases, this is you can call marginal product of labor, marginal product of labor decreases, *okay*. Same, if we take this- $\frac{\delta y}{\delta k} = \beta l^{\alpha}. k^{\beta-1} = MP_k$, why? Because beta lies between again 0 and 1, right. So, this is marginal product of capital. So, as capitals increases, marginal product of capital decreases from here, right.

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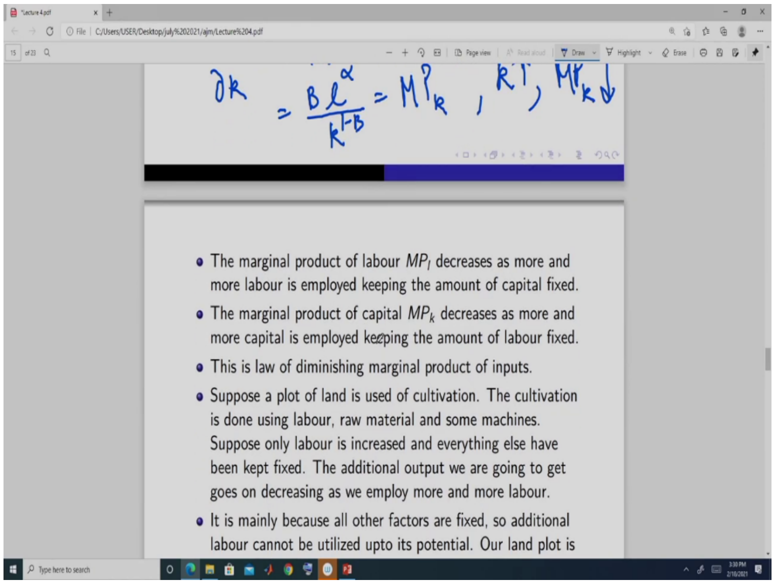

So, this thing is called the law of diminishing marginal product. So, it is something like this, that if we fixed one input and keep on increasing the other input, then the additional output that we are going to go, get it is going to increase. But it is going to go up, it is going to be positive. But it is the additional outputs are going to be go on decreasing.

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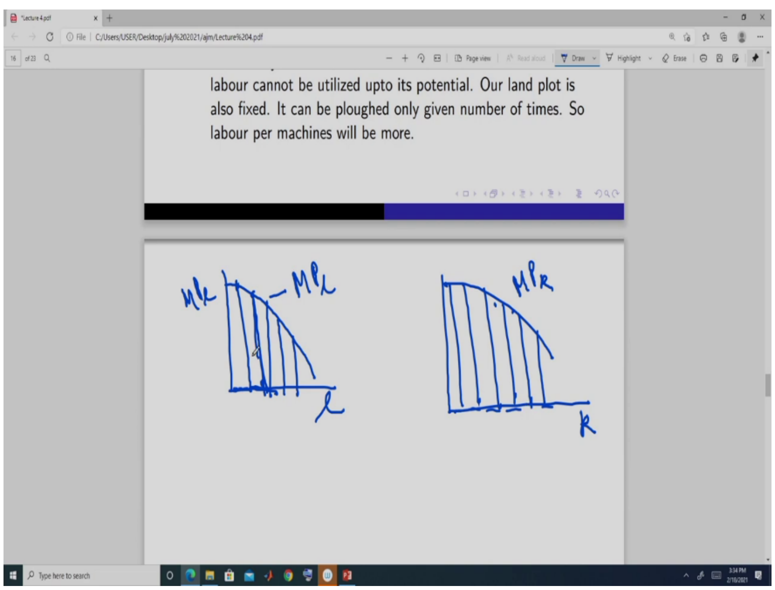

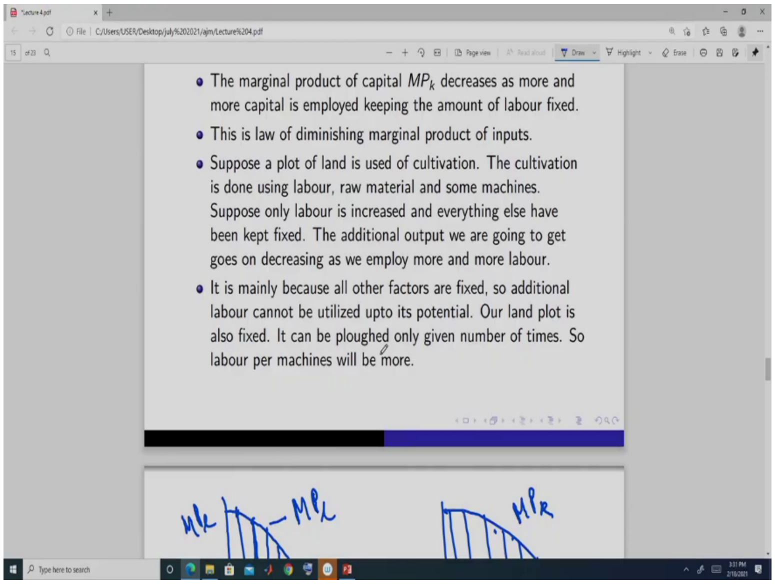

So, it is something like this, that if we plot labor here and marginal product of labor is going to be something like this. If we plot marginal product of labor here. So, that means, keeping capital fixed as we keep on increasing labor, marginal product is positive, but it is going down. So, this marginal product means the additional output that we are going to get. So, at this level of labor marginal product is this much.

If we increase further, marginal product is gone down, but it is still positive, okay. Similarly, marginal product of capital is going to be, going down as we go on increasing capital. So, here as we go on increasing capital marginal product is decreasing, it is positive but it is going down, like this. And this is what the law of diminishing marginal product says. The main idea is something like this. So, suppose, you fixed a plot of land and you are cultivating that land.

So, your land plot is fixed, you are only using labor. Now, if you keep on increasing the labor what is happening? And suppose your machines are also fixed. So, you can only plow that field only for a fixed number of times. You cannot go on plowing that land. And further machines can also be used only for plowing. Suppose only a few number of times. So, then the additional labor is not adding or you cannot use the additional labor to its potential.

So, if each labor can work for eight hours. So, you now you what you do each one is doing any at least fully for two hours or three hours. So, the additional output that you are going to get is less or you can think of suppose you have a place, you have a restaurant which uses one stove to produce some dish suppose but now, if you keep on increasing your staff and if you keep the number of gas stove same, then you will not be able to use increase output by use thing, additional output will, you will have some additional output.

But it will be going down as you keep on because this since you have your stove is fixed only now, what they can do? They can only reduce their labor, can reduce the amount of time they are working at to utilize that stove, and if you want to utilize all the workers together. Otherwise, they cannot use it because one person is already using and there is only one stove. So, other person will lie idle.

So, what he can do when this person can get tired so, other will substitute then the output is not going to go down. So, now the when the person gets tired so, he produces very at a lower rate. Now, since a new person will substitute him. So, that lower rate is not going to go down. But now if you keep on increasing the labor. So, this if you are using it for suppose you are opening that restaurant for eight hours, then you cannot employ all the people and produce at the same rate because your number of stove is same.

So, that is the idea that the factors get congested and that is why we marginal additional output is although it is positive, but it is less, it goes on decreasing okay. So, this is the idea. So, as we increase labor the additional is going down here, as we increase capital the additional output is going down okay.

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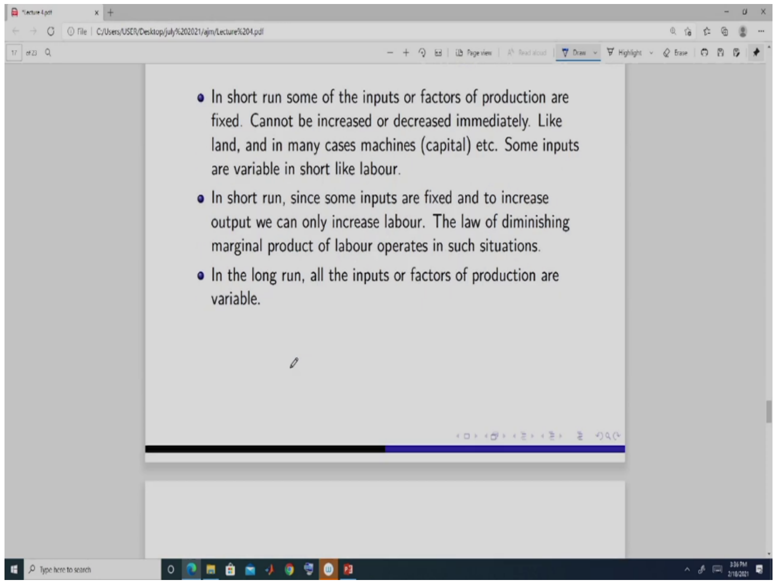

So, now, what do we see that in the short run some of these factors are going to be fixed, like machines or land. We cannot change them immediately, it takes some time. Suppose we want to use more machines. So, these machines need to be procured and again this needs to be installed and all those things requires a lot of time. So, immediately we cannot do this, cannot change the our level of machines that we are using.

So, some factors are fixed in the short run. But in the long run, all the factors of productions or inputs are variable, we can change as many machines we want to use we can do it. Because it is long run. So, here there is no distinct demarcation that this many time period denotes short run. And this many times, if it is more than this many time periods then it is long run. But the idea is something like this, that if some of these inputs cannot be varied, cannot be increased or cannot be decreased immediately then we say we are in a short run.

But if all the factors can be varied, can be changes, then we say we are in the long run okay. So, now since in a short run, some of these factors or inputs are going to be fixed. So, that is why we see the law of diminishing marginal product may operate, that is why it plays some important roles okay.

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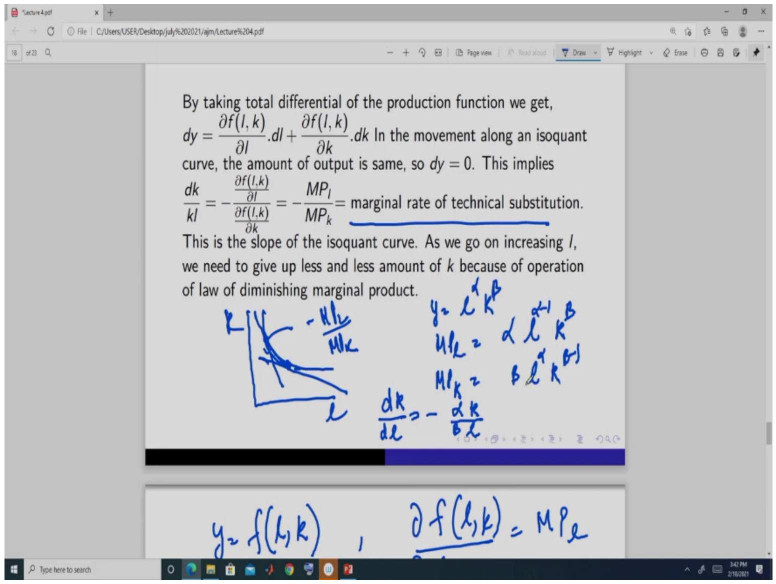

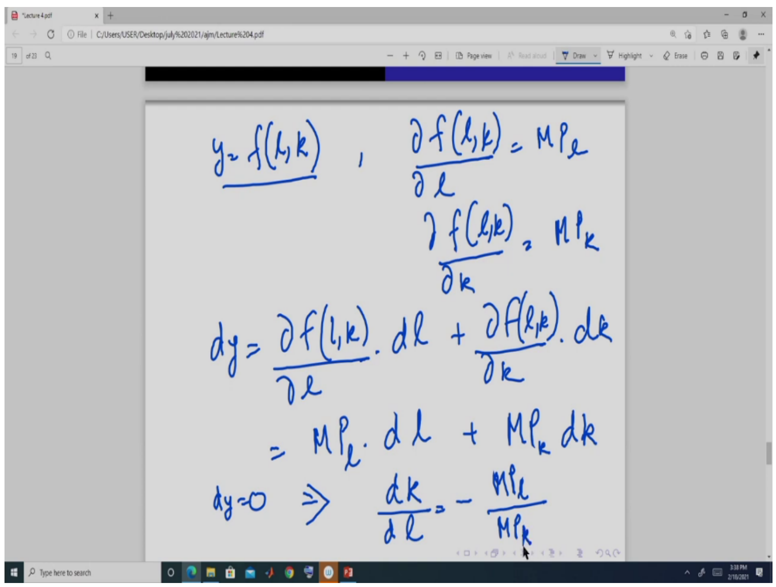

Now, we will study what happens when we move along a isoquant okay. So, suppose our production function is something like this- y=f(l,k), right. Now, it is differentiable. So, we have this thing- $\frac{\delta f(l,k)}{\delta l} = MP_l$. So, this is marginal product of labor, again, this is - $\frac{\delta f(l,k)}{\delta k} = MP_k$. marginal product of capital, right. Now, if we do what, take the total differentiation of this. So, this is going to be what, This- $dy = \frac{\delta f(l,k)}{\delta l}. dl + \frac{\delta f(l,k)}{\delta k}. dk$ now, this is you can write marginal product of labor- $dy = MP_l. dl + MP_k. dk$, this. Now, here suppose take our isoquants are something like this.

If we are moving from this point to suppose this point, what we are doing? We are increasing labor and decreasing capital, right, it is movement along an isoquant. right, So, what is happening? This changes in output is 0. So, this gives us this- $\frac{dk}{dl} = -\frac{MP_l}{MPk}$. So, this is what? This is the slope of isoquant. This, this is actually equal to minus of marginal product of labor by marginal product of capital. And if, this, this is the isoquant, slope of isoquant at this point.

And you will see that as we move in this way, that as we are going employing more and more labor to keep the output same, we have to give up less and less amount of capital. Why? Because of especially the law of diminishing marginal product. As we keep on increasing labor, the additional output that we are going to get, it is going to go down. If we keep the amount of capital fixed. So, now, so, this additional we do not want to increase output, we are moving along an isoquant. So, the amount of capital that I have to give up is going to be less.

Because the additional output is also less right. So, that is why what happens? The slope goes on decreasing as we go on increasing labor. And this is something called this ratio, slope is something called the marginal rate of technical substitution. So, it is marginal rate of technical substitution that is, as you go on increasing one input, how much amount of other input has to be decreased to stay at the same level of output.

So, it is that if you increase one unit of labor, how much amount of capital should be reduced to produce the same amount of output, okay. So, this is, this ratio is given by the actually the technology or the production function that you were using. So, in our case suppose, here we take the let us take the production function to be of this nature, that is Cobb Douglas. So, here we know marginal product of labor is this- $MP_l = \alpha l^{\alpha-1} k^{\beta}$, marginal product of capital is- $MP_k = \beta l^{\alpha} k^{\beta-1}$.

So, the slope of this isoquant is a ratio of these two things. So, it is going to be like this- $\frac{dk}{dl} = -\frac{\alpha k}{\beta l}$. So, what happens? So, as a goes up, and this needs to will going down, as we move here. So, this ratio is going to be smaller and smaller, since it is a negative a. So, it is going to be a bigger one, but we ignore the negative a, if we simply look at the absolute value, what do we get? We get that because the negative sign is mainly implying here, that as we increase labor capital has to be decreased. So, the negative relationship, but if we simply look at the absolute value, that then we see that it is it will go one decreasing, okay this is the main idea here.

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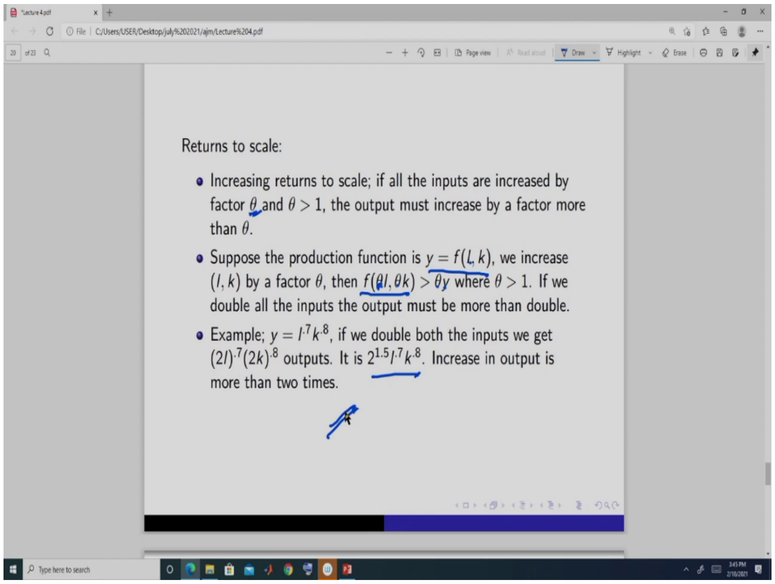

Now, we do another concept and that is returns to scale. Return to scale means, means mainly see in the long run all the inputs can be varied. And this plays an important role in the long run. So, now we differentiate the production function in three things and that is in terms of scale, returns to scale. So, that is increasing returns to scale, constant returns to scale and decreasing returns to scale.

What increasing returns to scale says? That suppose we are given a production function is like this- y=f(l,k), okay. Now, if we increase all the inputs by a factor theta (ϴ) and this theta must be greater than 1. So, if we suppose double all the inputs, then the output should be more than theta times the present output, okay. So, it is like this. So, the output must increase by a factor more than theta. So, if we increase all the inputs by a factor this theta and multiply all the inputs by factor theta, then the output, this output- f(ϴl, ϴk) should be greater than theta times y, where y is this.

For example, suppose production function is like this- $y = l^{.7}k^{.8}$ l to the power 0.7 k to the power 0.8. Now, here if we double labor and double capital, what do we get? So, it is this- $2^{1.5}l^{.7}k^{.8}$ so, it is 2 to the power 1.5, l to the power 0.7 k to the power 0.8. So, this means that the output has increased by a factor more than two times. So, when we have this kind of production function we say it exhibits increasing returns to scale. That is as we increase input and the output is going to be more than the multiple by which we have increased the inputs, okay. So, this is an example of increasing returns to scale.

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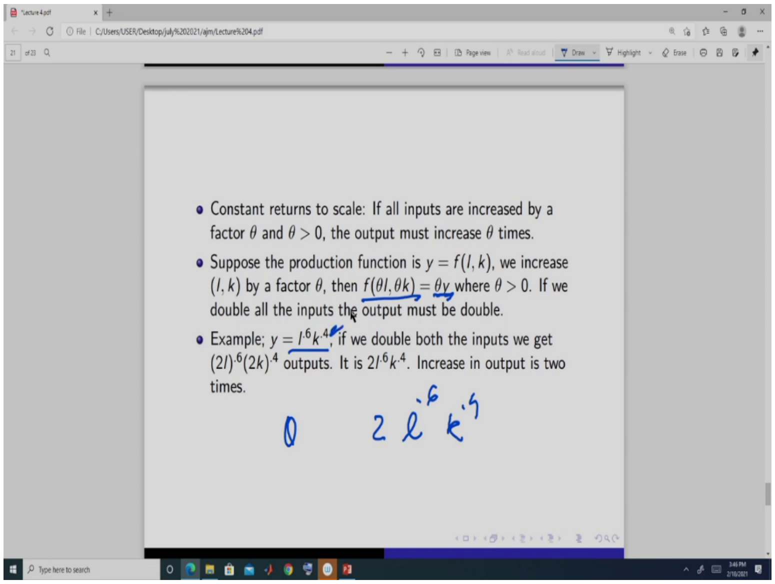

Next we have constant returns to scale, constant returns to scale says that if we increase, if we change the inputs by a multiple theta. Here theta can be any positive number then output should also change by only that theta factor. That is, if we increase all the inputs by a multiple of theta here this- - f(ϴl, ϴk)= ϴy, then the output should also increase by only this multiple, theta multiple. So, for example, suppose the production function is like this- $y = l^{.6}k^{.4}$ l to the power 0.6 and k to the power 0.4.

Here if we double all the inputs, that is 2 l and 2 k then we get the output is 2 l to the power 0.6 k to the power 0.4. So, output is exactly doubled now, because we have doubled the inputs. So, this is something called an constant returns to scale, okay. And this is one example of constant returns to scale production function.

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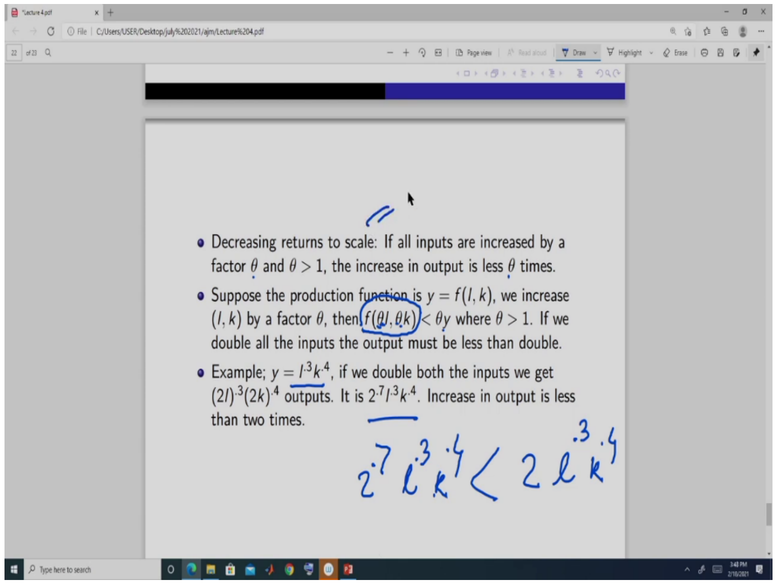

Next, something called decreasing returns to scale. What is the meaning of decreasing? It says that if all the inputs are increased by multiple theta. Here theta is a number which is greater than 1, then the input must, then the output must be increased by a factor which is less than theta. So, if we double all the inputs then the output should be less than double or if we triple all the inputs, the outputs should be less than triple.

So, it is like this, if all the inputs are increased by a multiple theta, that is theta greater than 1, then the output is this much, because this is the production function and it should be less than theta times y, where y is this- y=f (l,k), or it is this if we take this production function where l is to the power 0.3 and k is to the power 0.4, i.e $y = l^{.3}k^{.4}$. Now, if we double both the inputs labor and capital then the output is only 2 to the power 0.7 into l to the power 0.3 k to the power 0.4.

So, this 2 to the power 0.7, l to the power 0.3, k to the power 0.4. This is actually less than 2 to the power this-$2^{.7}l^{.3}k^{.4} < 2l^{.3}k^{.4}$,okay. So, output is now has not increased by, has not doubled it is something less than double, it is definitely it has increased but it is less than double. So, this is decreasing returns to scale, okay. So, today we will end at this only and next class we will see how firms decide on the amount of inputs, this l and k how much they are going to demand or how much they are going to use to produce a given level of output.

So, that decision of the firm we are going to study in the next module, in the next class. Today we have defined the technology that is a production function. So, firm when it is deciding how much amount of input of labor and capital to use, it is first given constant by a technology and that technology is given by this production function, okay. So, thank you very much.