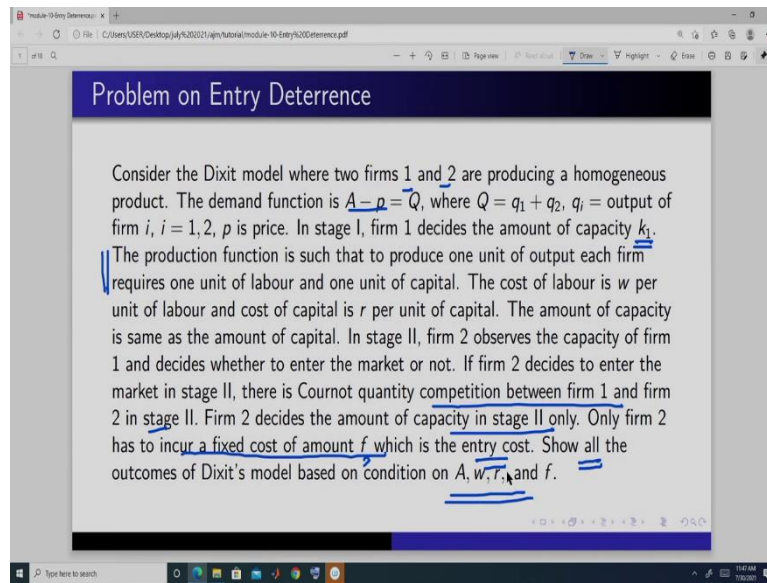


**Introduction to Market Structure**  
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**Module 12**  
**Lecture 43**  
**Tutorial on Dixit's Model of Entry Deterrence**

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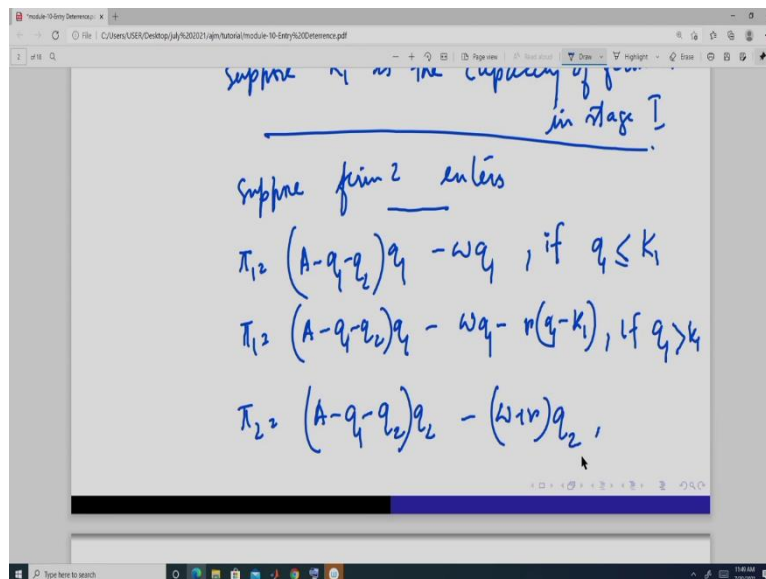
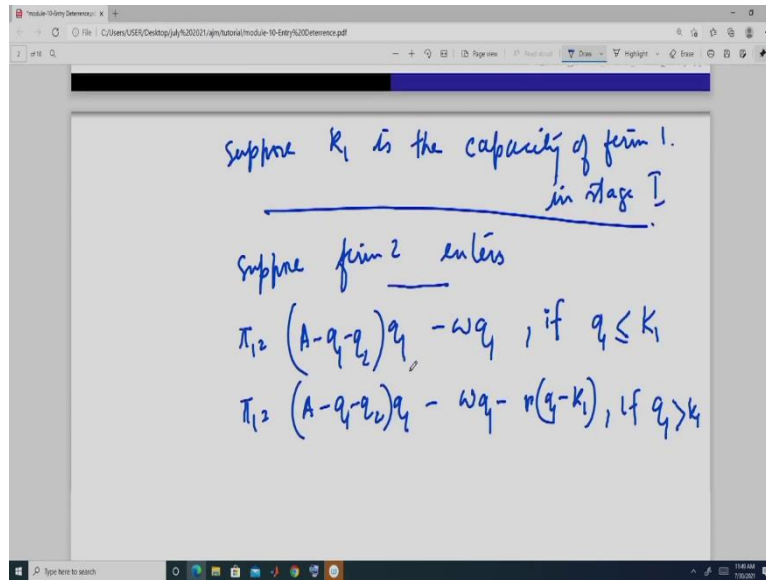
Let us solve one problem on Entry Deterrence or you can take this as an example. Now suppose there are two firms, firm 1 and firm 1 and firm 2, market demand function is this-  $A-p=Q$  and  $q_1$  and  $q_2$  are the outputs firm 1 and firm 2,  $p$  is the market price here in stage one firm decides the amount of capacity that is  $k_1$ , the production function is such that to produce one unit of output each firm requires one unit of labor and one unit of capital.

This is exactly same as the Dixit model, the cost of labor is  $w$  per unit of labor and the cost of capital is  $r$  per unit of capital, amount of capacity is same as the amount of capital, okay. And in stage two firm 2 observe the capacity of firm 1 and decides whether to enter the market or not to enter, if firm 2 decides to enter the market in stage two, there is cournot competition between firm 1 and firm 2 in stage two.

So, in stage two, there are two things that is being decided. So, first firm two decides whether to enter or not. And once it enters, then there is cournot competition between firm 1 and firm and firm 2 decides the amount of capacity in stage two only. So, only firm 2 has to incur a fixed cost of amount  $f$  which is that entry cost firm 1 has no entry cost because it is an incumbent firm and firm 2 has an entry cost because it is an entrant firm.

And now we will show all the possible outcomes in the Dixit model based on providing conditions on  $A$   $w$  because these are the parameters in this model. So, we will find them.

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So, first suppose  $k_1$  is the capacity of firm 1 in stage 1, so this is  $k_1$  okay. Now let us move and suppose firm 2 enters, okay then profit of firm 1 is  $A$  minus  $q_1$   $q_2$  this  $\pi_1 = (A - q_1 - q_2)q_1 - wq_1$  if  $q_1$  is less than or equal to  $k_1$  and otherwise, it is this  $\pi_1 = (A - q_1 - q_2)q_1 - wq_1 - r(q_1 - k_1)$  if  $q_1 > k_1$  because the expenditure on the capacity is already borne in stage 1, okay so we get this and profit of firm 2 is and this is the total revenue this  $\pi_2 = (A - q_1 - q_2)q_2 - (w + r)q_2$ .

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$$\frac{\partial \pi_2}{\partial q_2} = A - q_2 - w - 2q_1 = 0 \text{ FOC.} \quad \left| \text{if } q_1 \leq K \right.$$

$$\Rightarrow A - q_2 - w = 2q_1$$

$$\frac{\partial \pi_1}{\partial q_1} = A - q_2 - (w+r) - 2q_1 = 0, \text{ FOC}$$

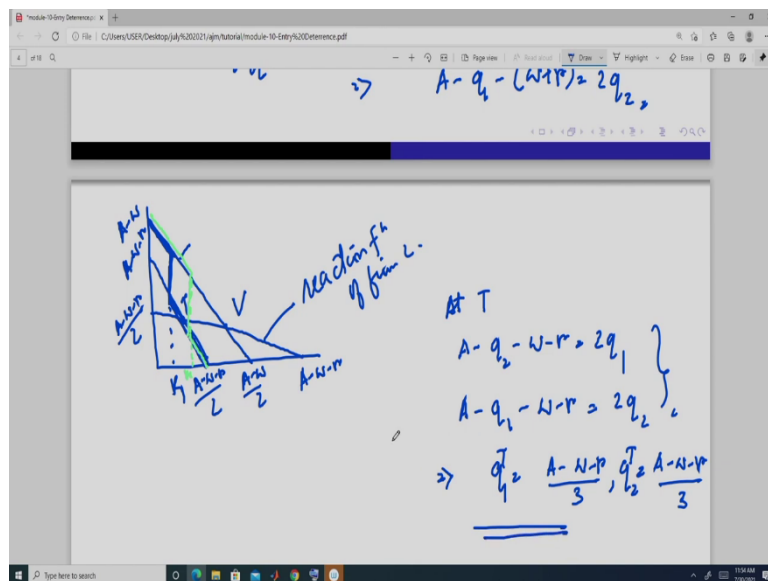
$$\Rightarrow A - q_2 - (w+r) = 2q_1, \text{ if } q_1 > K$$

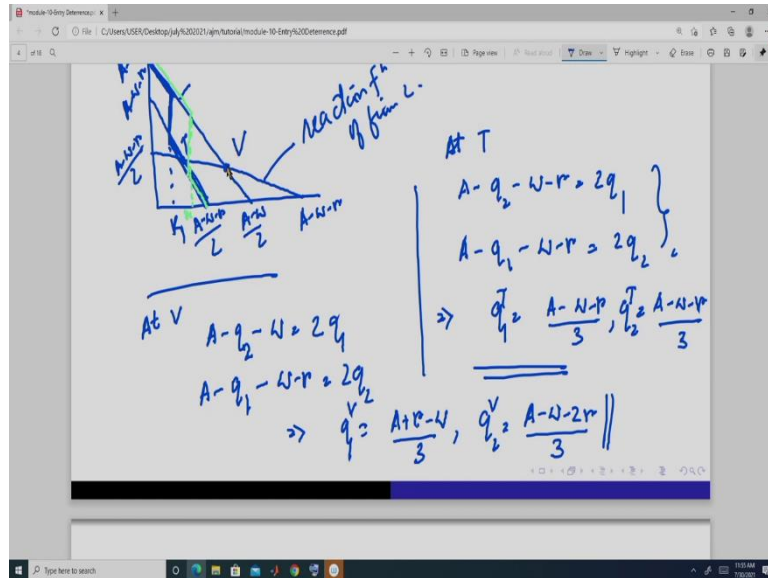
$$\frac{\partial \pi_2}{\partial q_2} = A - q_1 - 2q_2 - (w+r) = 0 \text{ FOC.}$$

$$\Rightarrow A - q_1 - (w+r) = 2q_2$$

Now from here we can derive the reaction function of firm 1 and firm 2. So from, so we get and this gives me, this is the first order condition- $A - q_2 - (w + r) - 2q_1 = 0$  implying and, so, these are the reaction functions- $A - q_2 - w - r = 2q_1$ ,  $A - q_1 - (w + r) = 2q_2$ .

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And if we plot them, we will get something like this, this, this is the reaction function of firm 2 this is point T, this is the point V that we have used and suppose this if this is the capacity, then the reaction function of firm 1 is this thick line, if this is the capacity, if this is k is the capacity then the reaction function of firm 1 is this green line this. So, we get the reaction function of this nature based on the exact value of capacity, okay.

Now, we will find out the output in this point T and V because our analysis is dependent on these two outputs. So, this is when this reaction function intersects with this reaction function. So, T point at T these two intersects-  $A - q_2 - w - r = 2q_1$ ,  $A - q_1 - (w + r) = 2q_2$ . So, from this we get that k1 is, we get this so, this is the point at T-  $q_1^T = \frac{A-w-r}{3}$ ,  $q_2^T = \frac{A-w-r}{3}$ , okay.

At V we get the reaction function of firm 1 is this  $A - q_2 - w - r = 2q_1$  and reaction function of firm 2 is this-  $A - q_1 - (w + r) = 2q_2$ . So, solving these two we get, so this is at the point V-  $q_1^V = \frac{A+r-w}{3}$ ,  $q_2^V = \frac{A-w-2r}{3}$ , okay. So, we get that this is less than this and this is greater than this so this point we have got.

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limit output of firm 1.  $q_1^L$   
 At  $q_1^L$ ,  
 $\pi_2 = (A - q_1 - q_2)q_2 - (w+r)q_2 - f$   
 the reaction f<sup>n</sup> of firm 2  
 $\Rightarrow q_2 = \frac{A - w - r - q_1}{2}$   
 $\pi_2 = (A - q_1 - q_2 - w - r)q_2 - f$

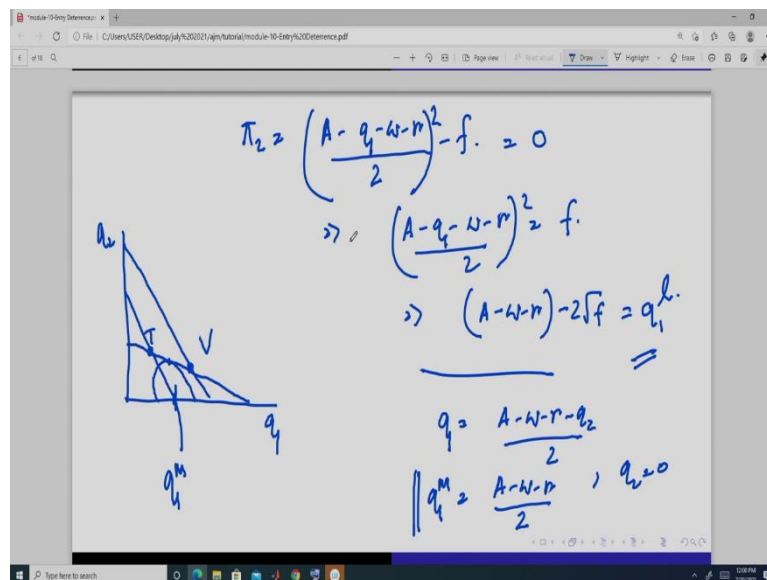
$\pi_2 = (A - q_1 - q_2)q_2 - wq_2 - r(q_2 - k)$ , if  $q_2 > k$   
 $\pi_2 = (A - q_1 - q_2)q_2 - (w+r)q_2 - f$   
 $\frac{\partial \pi_2}{\partial q_2} = A - q_1 - w - 2q_2 = 0$  FOC. | if  $q_2 \leq k$   
 $\Rightarrow A - q_1 - w = 2q_2$   
 $\frac{\partial \pi_1}{\partial q_1} = A - q_2 - (w+r) - 2q_1 = 0$ , FOC  
 $\Rightarrow A - q_2 - (w+r) = 2q_1$  if  $q_1 \leq k$

$\pi_2 = \left( \frac{A - q_1 - w - r}{2} \right) \frac{A - q_1 - w - r}{2} - f$   
 $\pi_2 = \left[ (A - q_1 - w - r) - \left( \frac{A - w - r - q_1}{2} \right) \right] \left( \frac{A - w - r - q_1}{2} \right) - f$   
 $\pi_2 = \left( \frac{A - q_1 - w - r}{2} \right)^2 - f = 0$   
 $\Rightarrow \left( \frac{A - q_1 - w - r}{2} \right)^2 = f$   
 $\Rightarrow (A - w - r) - 2\sqrt{f} = q_1^L$   
 $\equiv$

Now we will find the limit output of firm 1, limit output of firm 1 it such that it is  $q_1^l$ . So, at  $q_1^l$  I have made a mistake here. So, while specifying the profit function of firm 2 here I have to specify the entry cost  $f$  also, okay but this is a fixed cost so, it is not going to appear in the reaction function of firm 2, okay now into this and we know from the reaction function  $q_2$  is it is this- $q_2 = \frac{A-w-r-q_1}{2}$  so, there is this, this is the reaction function of firm 2 and we have got this.

So, plug in reaction function here we get this- $\pi_2 = \left[ (A - q_1 - w - r) - \left( \frac{A-w-r-q_1}{2} \right) \right] \left( \frac{A-w-r-q_1}{2} \right) - f$  and if we solve this we get, get this- $\left( \frac{A-w-r-q_1}{2} \right)^2 - f$ . Now, we have to find  $q_1$  such that this is equal to 0. So, we get equating this equal to 0 we get, so this is the limit output- $(A - w - r) - 2\sqrt{f} = q_1^l$ .

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Now, let us again draw the reaction curves. So, this is the point T this is point V and we know this is the monopoly outcome of a firm. So, this is it is in this reaction function. So,  $q_1$  is, it is this here, now,  $q_2$  is equal to 0 so,  $q_1^M$  is is this- $q_1^M = \frac{A-w-r}{2}$ . So, this is the monopoly output. This is the limit output. Now, we have to find out the stackelberg thing here, so stackelberg thing it is something like this. It is something like this so this point.

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$\Rightarrow (A - w - r) - 2q_1 = q_2$   
 $\underline{\hspace{2cm}} =$   
 $q_2 = \frac{A - w - r - q_1}{2}$   
 $\parallel q_1^M = \frac{A - w - r}{2}, q_2 = 0$

$\pi_1 = (A - q_1 - q_2)q_1 - (w + r)q_1 + rK_1$

$\Rightarrow q_2^S = \frac{A - w - r}{2}$   
 In this example  
 $q_1^M = q_2^S$

$\pi_1 = (A - q_1 - q_2)q_1 - (w + r)q_1 + rK_1$   
 $q_2 = \frac{A - w - r - q_1}{2}$   
 $= [A - q_1 - (\frac{A - w - r - q_1}{2})]q_1 - (w + r)q_1 + rK_1$   
 $\pi_1 = [\frac{A - w + r}{2}]q_1 - (w + r)q_1 + rK_1$   
 $\frac{\partial \pi_1}{\partial q_1} = \frac{A + w + r}{2} - q_1 - (w + r) = 0$

So, how do we find the stackelberg thing. So, this is the profit function of firm 1 you can write it in this form, if we take this reaction function because this reaction function means that its capacity is less than what is the amount of output it is producing, right. So, now here we know the reaction function of firm 2, it is this- $q_2 = \frac{A - w - r - q_1}{2}$ , plug in this here, we will get, now optimize this, this is done in the stackelberg we get equal to  $0 - \frac{d\pi_1}{dq_1} = \frac{A + w + r}{2} - q_1 - (w + r) = 0$ . So, from this we get so, in this example we get that  $q_1^M$  is equal to  $q_1^S$ , okay and we have got this.



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Handwritten mathematical derivation on a whiteboard. At the top, there is a small equation:  $0 = \frac{A - q_1 - q_2}{2} + rK$ . Below it, the main derivation starts with  $\pi_1^S = \frac{\left(\frac{A - q_1 - q_2}{2}\right)^2}{2} + rK$ . This is followed by  $\pi_1^S = (A - q_1 - q_2)q_1 - (k+n)q_1 + rK$ , with  $q_2 = 0$  written to the right. The final result is  $\pi_1^S$ .

Handwritten mathematical derivation on a whiteboard. It starts with  $\pi_1^S = (A - q_1 - q_2)q_1 - (k+n)q_1 + rK$  and  $q_2 = 0$ . This is simplified to  $\pi_1^S = (A - q_1)q_1 - (k+n)q_1 + rK$ . Then,  $\left(\frac{A - k - n}{2}\right)^2 + rK = Aq_1 - q_1^2 - (k+n)q_1 + rK$ . The final result is  $\Rightarrow q_1^2 - (A - k - n)q_1 + \left(\frac{A - k - n}{2}\right)^2$ .

Now, we have to find out this, this stackelberg we have got this to the same, we have to find out this part, this is  $S$  naught  $q_1$   $S$  naught so, how do we find  $q_1$   $S$  naught, so stackelberg profit if you substitute that here stackelberg profit is so,  $k$  here, if you look at this here  $k$  will not be more than, should not have excess capacity, because it is going to end soon. So, profit of firm 1 in this case you can, output is this.

So, this stackelberg profit can write this  $A$  plus,  $A$  minus because if you take this common you get this minus, this plus so, you can, you will get this only because actually if you look at this term what you will get, you will get that this  $k_1$  is actually equal to the stackelberg output and so, that cost is borne in stage 1 you will not get any additional cost here in terms of the  $A$  but if you remove it and add it together you will get the profit in this term only.



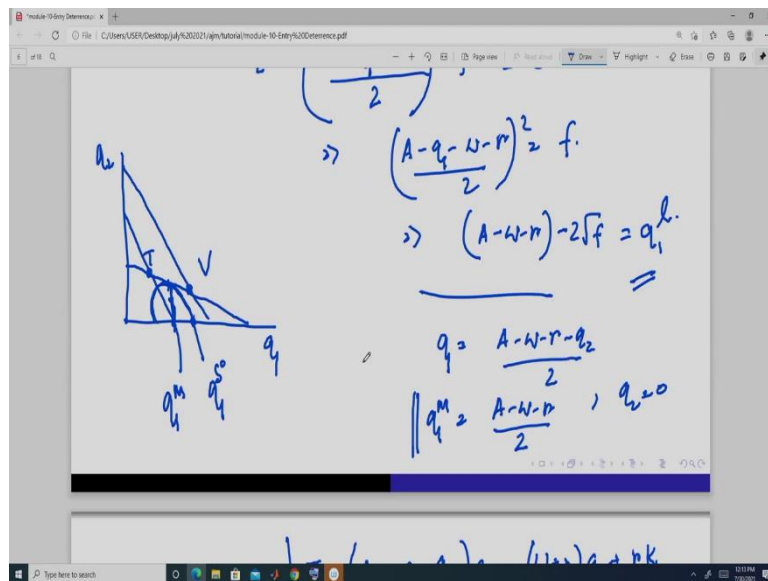
So, that in this term only because what you can do is you can write it in this term. If you simply take this position or simply hit this, okay stackelberg thing which we have got from this here, this point. Now how to find out this thing, this we can find out because here I have when I have got this stackelberg I have simply taken the stackelberg profit when we are not taking this capacity thing anymore consideration, okay.

Actually the capacity will be same as the output and so, we will get the same thing if we combine the cost of the stage 1 and the profit in stage two together, okay. So, this is going to be the total profit when we have the outcome as the stackelberg thing okay or you can simply take this and then go and this A is, this is the, now when q2 is 0 and fix this profit at the stackelberg thing, okay.

So, here I can write this in terms of this also if we strictly follow this A then we will get this, we will have, this is equal to 0. So, we will have, this term is cancels out, we get this-q<sub>1</sub><sup>2</sup> -

$$(A - w - r)q_1 + \left(\frac{A-w-r}{2}\right)^2 \cdot \frac{1}{2}$$

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$$\Rightarrow q_1^{S_0} = (A-N-P) \pm \sqrt{(A-N-P)^2 - 4 \frac{(A-N-P)^2}{8}}$$

$$= \frac{(A-N-P) + \frac{(A-N-P)^2}{\sqrt{2}}}{2\sqrt{2}} = \frac{(A-N-P)(\sqrt{2}+1)}{2\sqrt{2}}$$

$$\Rightarrow q_1^{S_0} = \frac{(A-N-P)(\sqrt{2}+1)}{2\sqrt{2}}$$

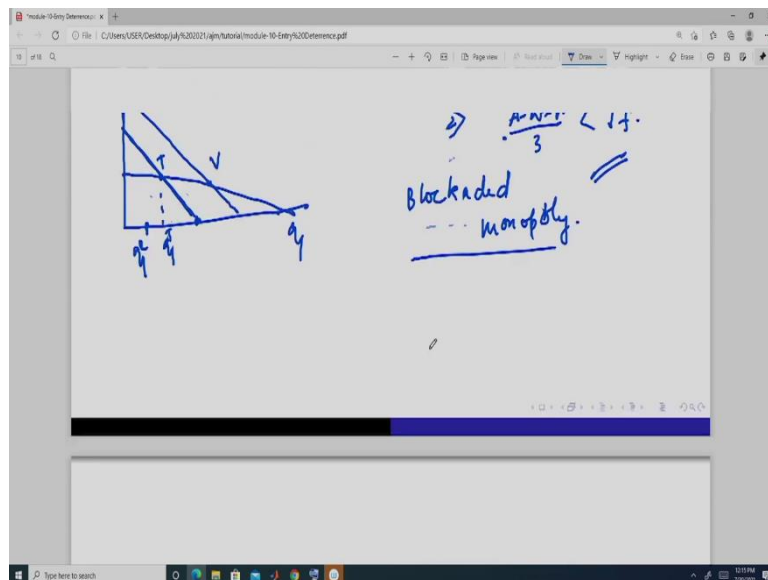
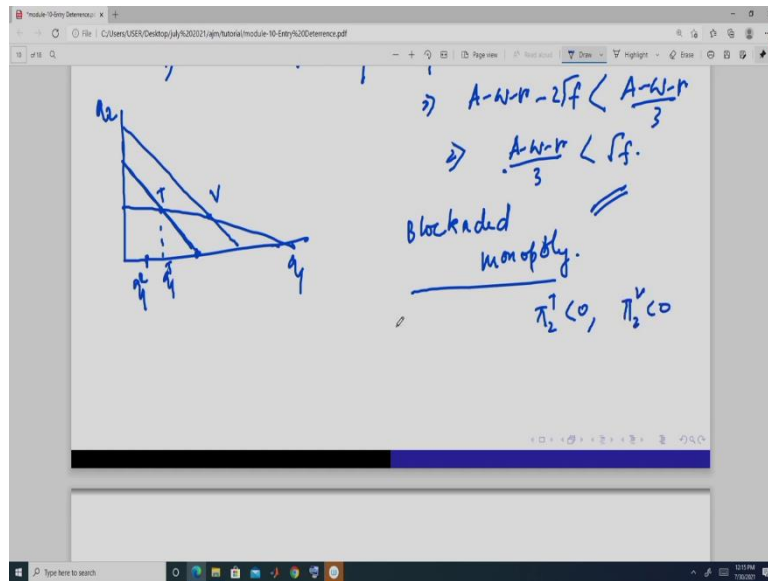
$$= \frac{(A-N-P) + \frac{(A-N-P)^2}{\sqrt{2}}}{2\sqrt{2}} = \frac{(A-N-P)(\sqrt{2}+1)}{2\sqrt{2}}$$

$$\Rightarrow q_1^{S_0} = \frac{(A-N-P)(\sqrt{2}+1)}{2\sqrt{2}}$$

$q_1^T, q_1^V, q_1^M, q_1^S, q_1^{S_0}, q_1^{L_1}$

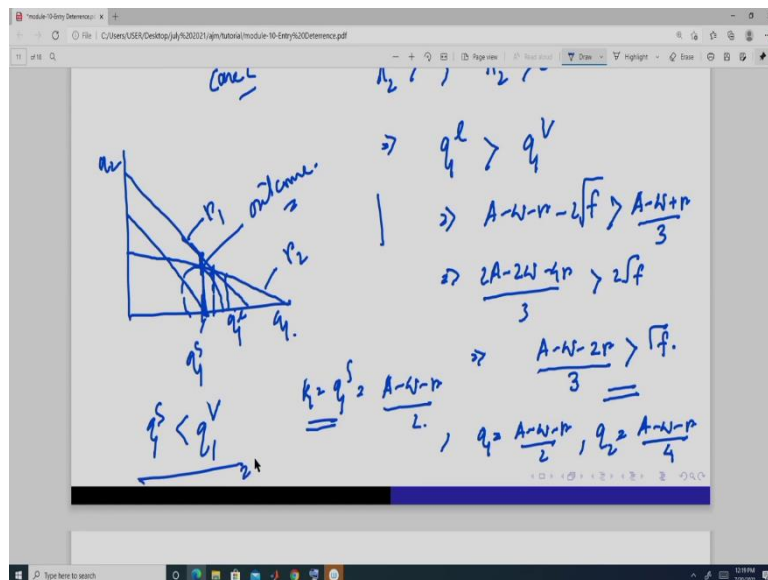
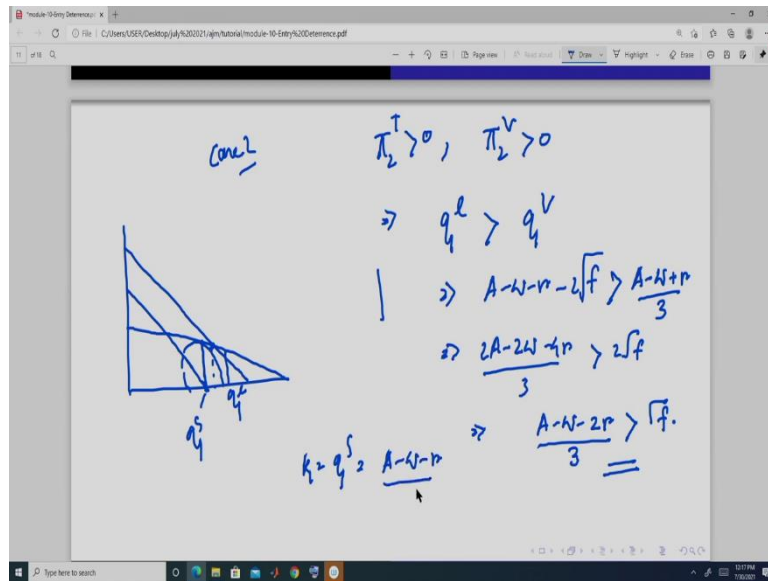
So, from this, let me get this divided by this which is this if we this is what, this is the, this q1 S naught, this, this output is this, this. Now, this minus A will be if we take it will be this one so, we do not bother we required this, okay. Now we we have derived all the outputs. So, what outputs we have? We have q1 T we have q1 V we have q1 M we have q1 S we have q1 S naught we have q1 l.

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Now let us do the case 1, if  $q_1^S$  is less than  $q_1^T$ . So, this implies  $A - w - r - 2\sqrt{f} < \frac{A - w - r}{3}$ . So, this means this is the case in this situation there is, if this is the case then what we have got, we have got that  $q_1$  this  $q_1$  is  $T$ , so firm 1 will so this is the case of blockaded monopoly. Next case 2 so, in this case what we had profit upon 2 at  $T$  is.

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Next case 2 profit upon 2 positive, this is also positive. So, this implies  $q_1^L$  is greater than  $q_1^V$ . So, this implies, it is this  $A - w - r - 2\sqrt{f} > \frac{A-w+r}{3}$  this so, if this is the case  $\frac{A-w-2r}{3} > \sqrt{f}$  then what do we see? Here  $q_1^L$  is somewhere here it is this, if this is the case and we know stackelberg is this. So, here in this case  $q_1$  is going to be  $q_1^S$  and it is going to be and here  $q_1$  is going to be the stackelberg outcome this  $\frac{A-w-r}{2}$  and  $q_2$  is going to be this  $\frac{A-w-r}{4}$  because here it is like this.

So, if it tries to it will not be able to deter firm 1 will not be able to deter the entry of firm 2 because if it has this much amount of capacity then output is going to be this, now, if it produces capacity like stackelberg it is here, reaction function is this. So, it is reaction function of firm 2 is this, this is  $r_2$  and this is  $r_1$ .

So, intersect and this is the outcome. So, it is the stackelberg outcome and firm 1 can ensure stackelberg outcome by having the capacity as the stackelberg outcome, capacity of the stackelberg outcome. So, firm 1 will do that, it will accommodate the entry of firm 2 and it will have a capacity of the stackelberg and outcome is going to be the stackelberg.

Now here we had two, we had, I had earlier while discussing this problem, not problem the in the main discuss and we had taken two cases where this point is greater than this or this point is less than but we will not require this because the stackelberg thing is less than the  $q$  this because we have seen that here. This is always less than this- $q_1^S < q_1^V$  because of this we do not require any further here, we will only get 1 case in 1 outcome in case 2.

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Case 3

$$\pi_2^T > 0, \pi_2^V < 0$$

$$q^T < q^L < q_i^m$$

$$A-w-r-2f < \frac{A-w-r}{2}$$

$$\Rightarrow \frac{A-w-r}{4} < \sqrt{f}$$

Blocked monopoly.

$$k_2 = \frac{A-w-r}{2}$$

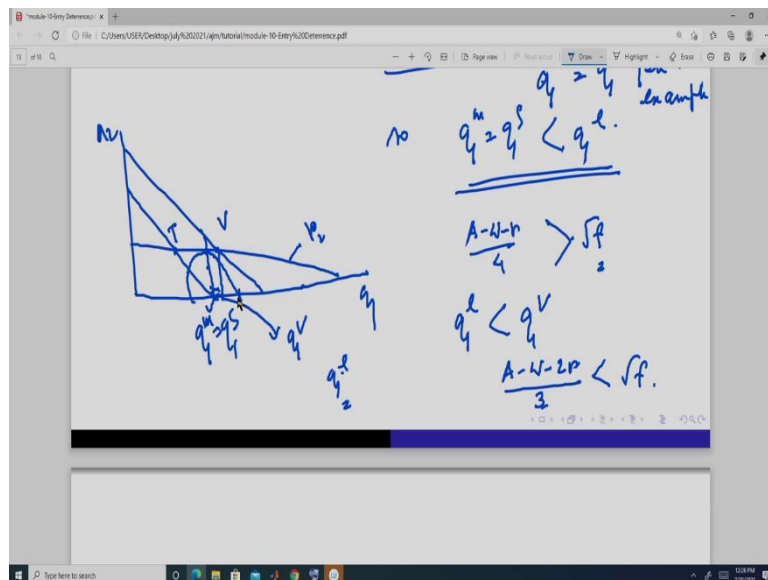
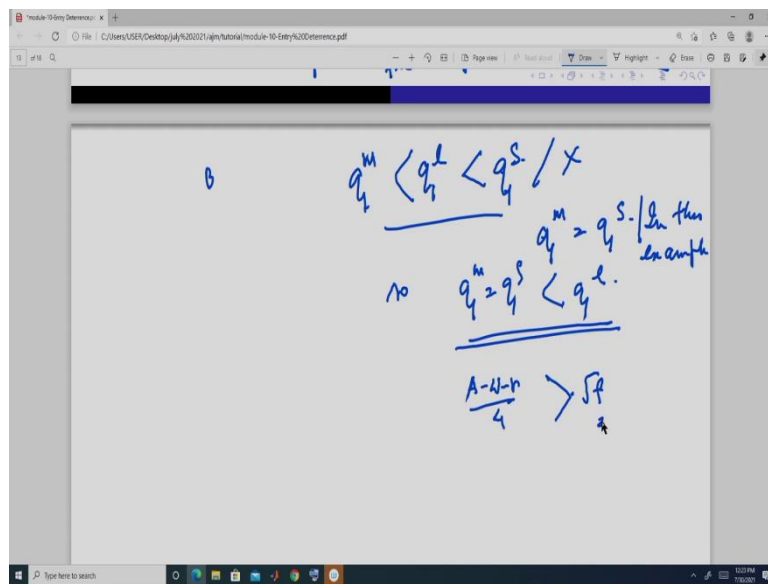
$$q_2 = \frac{A-w-r}{2}, q_2 > 0$$

firm 1 detains the entry of firm 2

$$\sqrt{f} < \frac{A-w-r}{3}$$

Now move to case 3, in case 3 we have profit of firm 2 is positive and T and it is negative and this so, in this case the first case A what we have is, so, this is, so, from this- $q_1^T < q_1^1 < q_1^m$  we get this- $A - w - r - 2\sqrt{f} < \frac{A-w-r}{2}$  and from this we know then it is this outcome- $\frac{A-w-r}{4} < \sqrt{f}$ , just opposite of that it is this-  $\frac{A-w-r}{3} > \sqrt{f}$ , so, absolute lie in this range to get this so, here again we have the blockaded monopoly and  $q_1$  is equal to a monopoly output and this is equal to and  $q_2$  is equal to 0 so firm 1 deters the entry of firm 2 okay, so this is 1.

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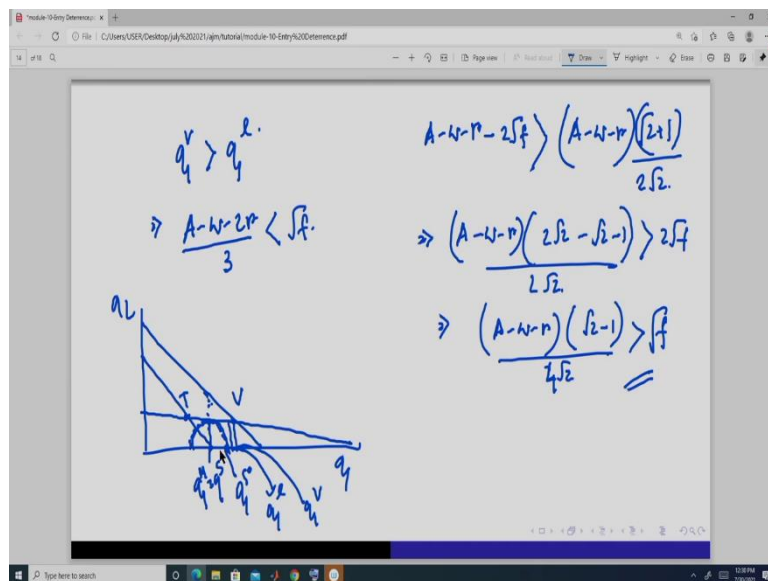
Next let us 3 B here  $q_1^1$  is greater than M, okay. Now, this has we had taken one more case it is like this, but we know in our case is equal to -  $q_1^m = q_1^s$  so, we will have only this case- $q_1^m = q_1^s < q_1^1$  because of this we will not have this case, okay. Because in this example we do not

get anything because this is equal to this. So, we get this, now here this implies that should be here, but we have 1 more condition that is  $q_1 < V$ .

So, from here we get this is less than this, right. So, here we get this will give me this condition this is the case so we get this. So, is just the sign is opposite, okay and you can see that this is less than this  $\frac{A-w-2r}{3} < \sqrt{f}$ . Now, here what we are getting, we are getting this is the reaction function of firm 2 and this is point T, this is point V, this is  $q_1$  M is equal to  $q_1$  S so  $q_1$  lie here, this is point is  $q_1$  V, this is this point.

So, in this range  $q_1$  lie, okay. Now if it lie here, this is the stackelberg thing, right. Now, if it produces the stackelberg thing, then it is going to be like this, right? and it will accommodate the entry. Now, if it gives the capacity as this then it is going to deter now, the A is whether it is going to ensure this or not, deter or it is going to accommodate because if it keeps  $k_1$  capacity as the A stackelberg thing then it is a commodity. So, that we can compare looking at this point.

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When  $q_1^L < q_1^S$

then  $K_1 = q_1^L \frac{(A-w-r) \cdot 2\sqrt{f}}{3}$

firm 1 deters the entry of firm 2

$\frac{(A-w-r) \frac{\sqrt{f}}{4\sqrt{2}}}{3} > \sqrt{f}$   
 $\frac{(A-w-r)}{3} < \sqrt{f}$

$q_1^L > q_1^S$

$\Rightarrow \frac{A-w-r}{3} < \sqrt{f}$

$A-w-r < 2\sqrt{f} < \frac{(A-w-r)(2\sqrt{f})}{2\sqrt{f}}$   
 $\Rightarrow \frac{(A-w-r)(2\sqrt{f} - \sqrt{f})}{2\sqrt{f}} < 2\sqrt{f}$   
 if  $\Rightarrow \frac{(A-w-r)(\sqrt{f}-1)}{2\sqrt{f}} < \sqrt{f}$   
 $K_1 = q_1^L$

The graph shows a downward-sloping demand curve and two horizontal lines representing price levels. The intersection points are labeled with  $q_1^L$  and  $q_1^S$ .

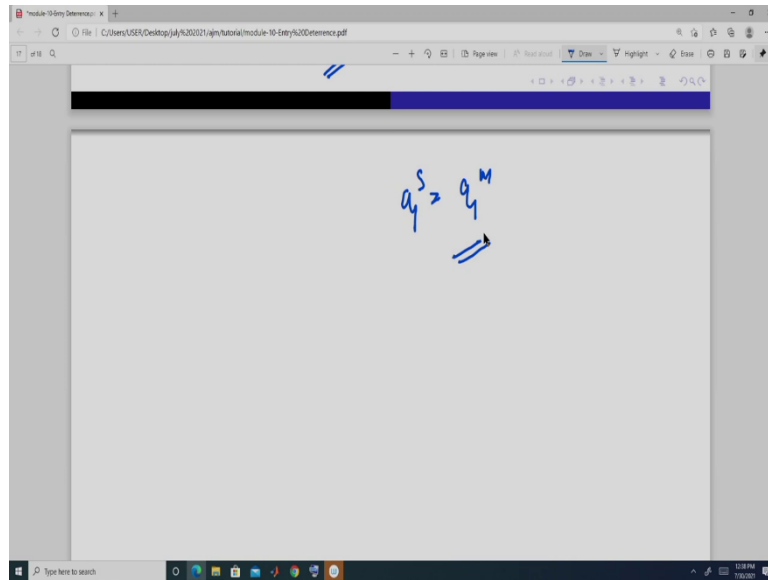
So, here in this situation when  $q_1^L$  is less than  $q_1^S$  then  $K_1$  is actually equal to  $q_1^L$  which is equal to this and firm 1 deters the entry of firm 2, but in this situation if we have, okay this we have this and we have this, this would be opposite side, okay. It should be this and so, here this is not binding and this is binding whichever you have to check that if this is true, if this is true, then it will choose this and it will deter the entry of this.

But if the case is opposite that is if  $q_1^L$  this is greater than  $q_1^S$  that means it will just change the sign of this that is  $2\sqrt{f} - \sqrt{f}$  should be this here. If this is the case and it has to be, okay I get this.



So, when this is the A, final outcome there is in the Cournot competition is going to be like this and this is going to be the outcome right? then to get this and  $q$ . So, these are all the possible outcomes that we get in this example of entry deterrence, here one, we do not get one possibility because we have this situation.

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Because of  $q_1^S$  is equal to  $q_1^M$ . So, in case 2 we are having only one case so, we do not have many possibilities in case 2, okay. So thank you.