**Introduction to Market Structures**
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**Module 12: Entry Deterrence, Bundling and Tying**
**Lecture 42**
**Bundling and Tying**

Hello and welcome to my course introduction to market structures. Today we are going to do bundling and tying, this is the last module of this course.

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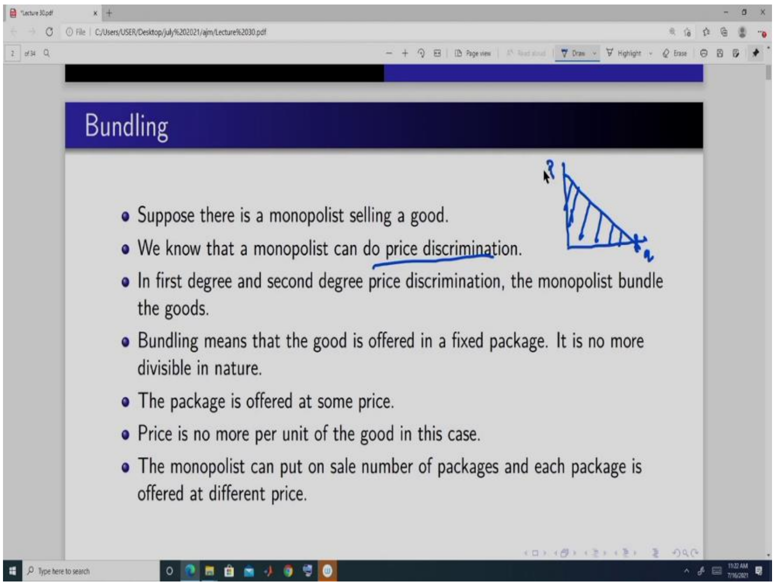


Bundling; one form of bundling we have already done, we have done the monopoly and in the monopoly we have seen that the monopolist can do a price discrimination and what do we mean by price discrimination? That means that they charge different prices to different quantities and that is in first degree price discrimination or they can charge different prices in different markets if the consumers cannot move around from market 1 to market 2.

Or they will package it in different way, packaging and that is a bundling like in our case, till now we have assumed that the goods are continuously divisible. So, you can get any quantity you want. But if it is packaged, then it is only available in fixed quantities, you cannot change that quantity, so that divisibility property that is gone.

So, monopolist can also do that, we have seen that in first degree price discrimination. So and we have derived in first degree price discrimination that the whole bundle, so suppose if we have a demand curve like this, then this whole amount and suppose there is no cost no margin, marginal cost is 0, then this whole amount, this amount will be packaged at a price which is given by this area below this curve demand curve when this is the quantity and this is the price. So, this is we have seen in first degree price discrimination, we have also done graphically the second degree price discrimination.

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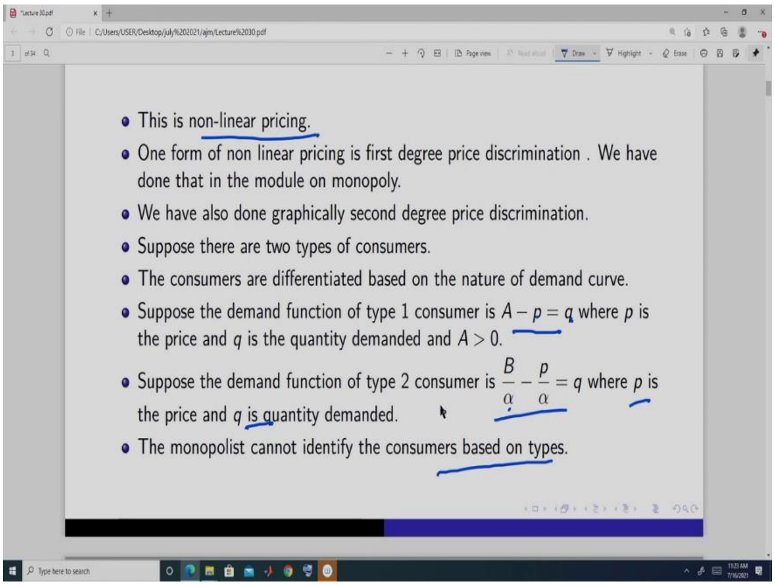


But we will do it algebraically today and we will consider only the monopoly because if we take the duopoly or oligopoly its calculations are slightly more involved. So, we are not doing it, okay. So and since if monopolist is doing bundling, so, this is a form of nonlinear pricing. And for simplicity, we assumed that there are two types of consumers. So, it can have more than two consumers; types of consumers.

And the demand function of type 1 consumer is this-A-p=q where p is the price and q is the quantity and demand function of consumer 2 is this- $\frac{B}{\alpha} - \frac{p}{\alpha} = q$, B by alpha minus p by alpha, where p is the price and q is the quantity demanded.

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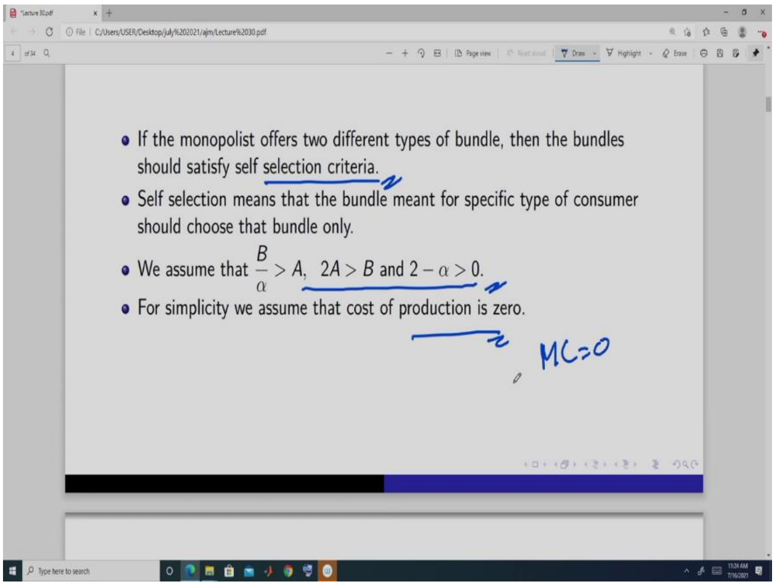


Now, the monopolist cannot identify these buyers, it knows that there are two types of buyers, but it does not know who is type 1 and who is type 2. So, it will choose or it will decide or it will design the package in such a way so that there is something called self-selection that is if I meant this bundle for type 1 consumer, then this type 1 consumer should choose that bundle only. And if I choose, if I design a bundle for type 2 consumers, then the type 2 consumers should select that only.

So, type 2 consumers should not select the bundle of type 1 consumer and type 1 consumers should not select the bundle of type 2 consumers, okay. So, this is the criteria of self-selection and we have to insure that. And we assume further these technical conditions and we will see why we need them, okay. And for simplicity we assume that there is 0 cost of production so that means marginal cost is 0 and there is no fixed cost, okay.

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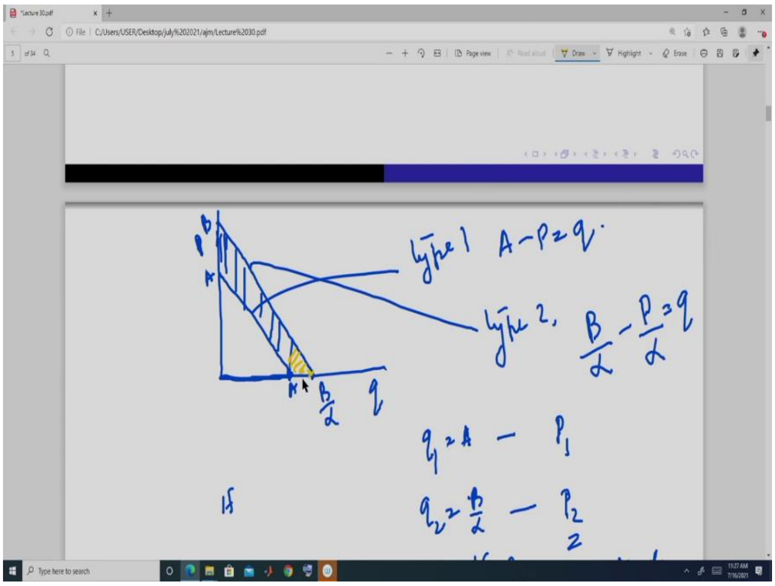

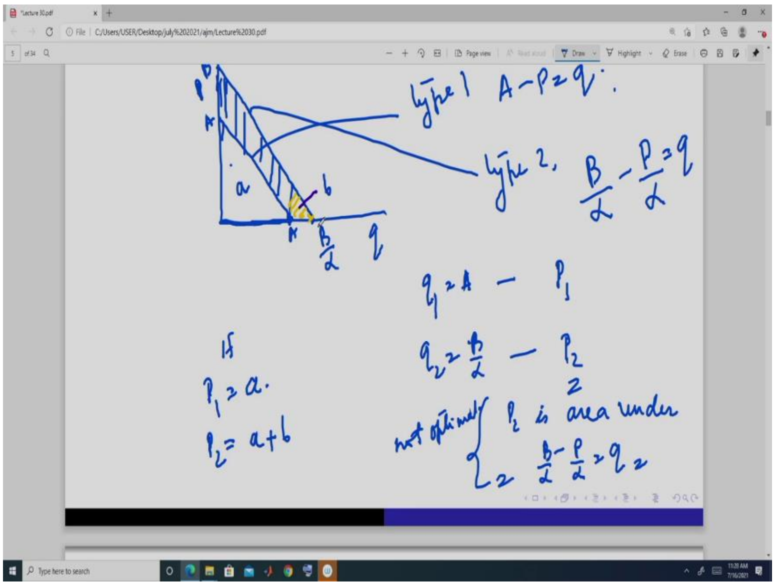

So now if we plot this, so this is quantity and this is price. This is suppose, this is the type 1 where A minus p. So, this is A and this is A and suppose this is type 2 B by alpha minus p by alpha is equal to q, sorry this is quantity, okay. So, this point is B and this point is B by alpha, okay. Now, the monopolist can sell this point is this, so, sell one bundle that is for type 1, type 1 it is bundle this at some price p1 and then for bundle B it can sell the amount this at some price p2, okay.

So, we for one bundle it has to be something like here. Now see if we see touch this and if p2 is this whole area, if p2 is this whole area, area under this demand curve if p2 is area under this demand curve-$\frac{B}{\alpha} - \frac{p}{\alpha} = q$ then p2 consumer 2, if it buys this bundle, then it gets this much amount of surplus. So, this is not an optimal strategy because it violates self selection, but here it can charge this much.

So, p2 and this area is suppose B and this area is suppose sorry let us denote it in different ways otherwise it will be confusing. This is suppose this and this is suppose small a. So, p1 can be equal to a, small a, this whole region and p2 can be a plus b, then the consumer 2, type 2 is indifferent between buying this bundle a and this bundle this and this and so, and we assume that the consumer, type 2 consumer buys this bundle when p2 is equal to this-a+b. Now, the question is whether this is the optimal bundle or not. So, how do we do this?

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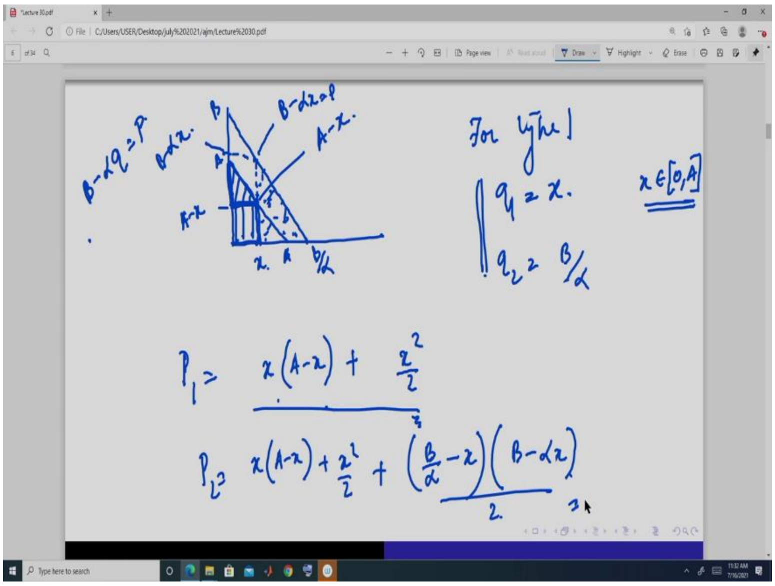


So, what do we do, we do, we solve it in this form. Suppose this is x, suppose for type 1 q1 is x and for type 2 it is b by alpha, okay, this, this is the package, okay. And here x would lie within this range, okay it can be A also. Now, within this, if this is the A, consumer 1 for type 1 it can charge this whole region for paying this. This region, this point is a minus x. So, this point is a minus x. So, this region is this rectangle into this triangle plus this triangle.

So, this area is x A minus this plus this distance is A minus this, so it is x, this distance is x so x squared, so this is the area of this triangle- $x(A-x)+\frac{x^2}{2}$, okay. And if A is equal to; x is equal to A then this is 0 and we have only this part, okay. And so this is p1, okay and p2 it can charge this much additional amount. So, this point is what? This point is B by alpha, sorry, this when we plug in this quantity and here this is we can write this as inverse demand curve and x is the A.

So, this point is B minus alpha is equal to P, this point is B minus alpha x. So, this triangle, so this triangle, area of that triangle, this dotted triangle is what? This height into base divided by 2. So, this is B by alpha minus x because this is x so this distance, this is the base and the height is this much. This is the height divided by 2- $(\frac{B}{\alpha} - x)(B - \alpha x)$. So, now p2 is this plus it can charge this triangle. Sorry, this area, not this triangle this whole area so that is the p1, it will get this- $P_2 = x(A - x) + \frac{x^2}{2} + (\frac{B}{\alpha} - x)(B - \alpha x)$.

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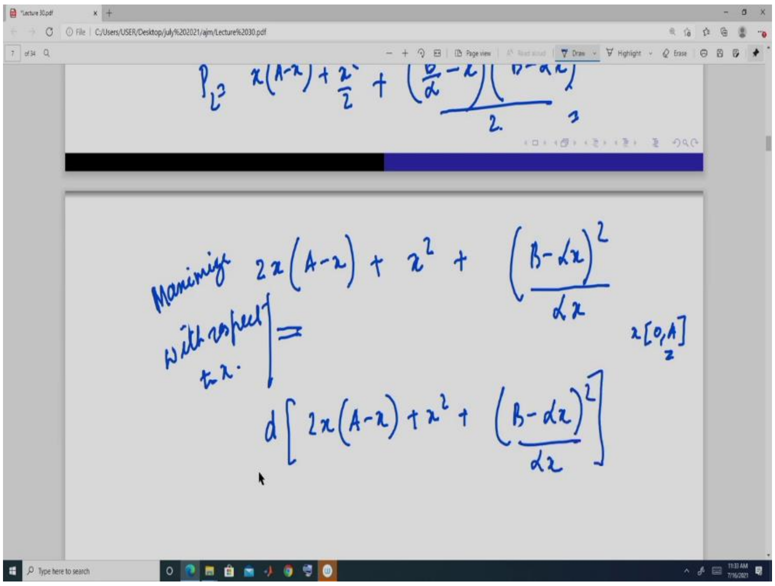


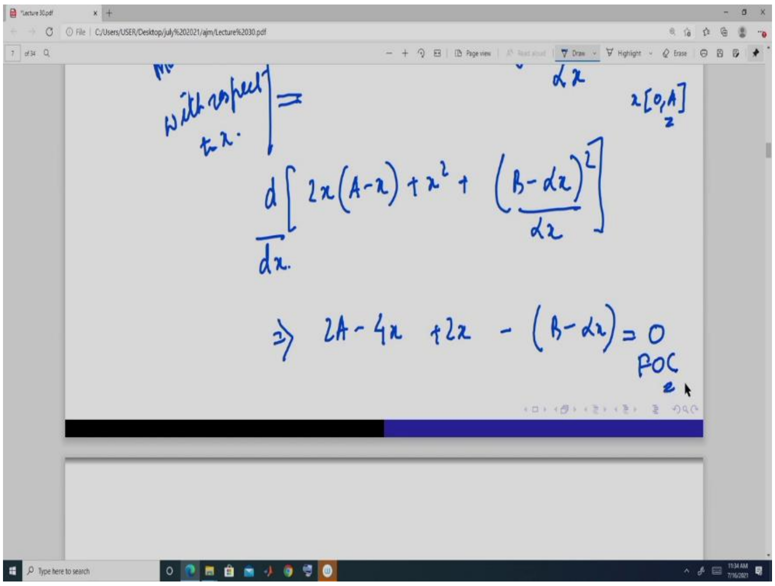


So, monopolist is going to choose x such that this is maximum. So, it will maximize this- $2x(A - x) + x^2 + \frac{B-\alpha x}{\alpha x}$ with respect to x and since this is differentiable function in x and x lies between 0 and A. So, this point is we differentiate this and equate it to 0 the first order condition and so what do we get? And this is equal to this, this is the first order condition-$2A - 4x + 2x - (B - \alpha x) = 0$. So, we solve this what do we get?

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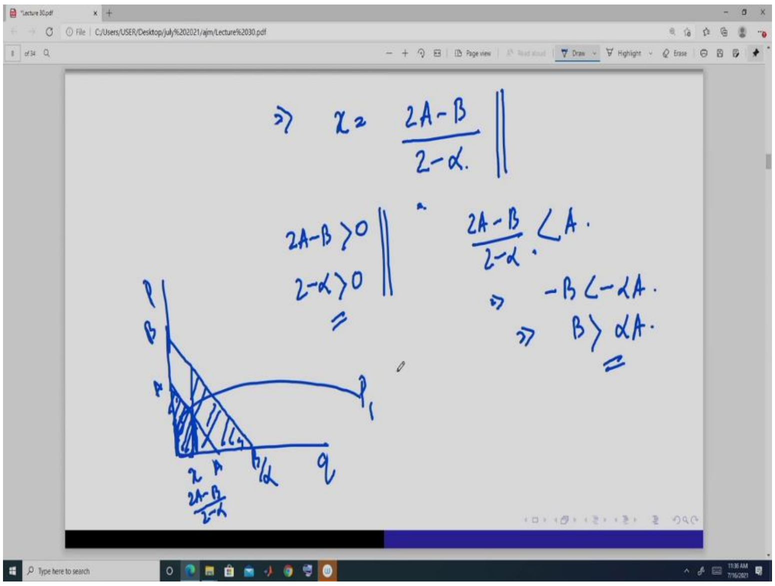


We get x is equal to 2A minus B divided by 2 minus alpha- $x = \frac{2A-B}{2-\alpha}$. So, that is why we assume this conditions, we require these conditions so that this takes a value like. Now, if you look at this, this point is actually less than A. So, this less than A means, so this means B should be greater than alpha A and this should be positive. So, this is positive-$2A - B > 0$ and this is positive-$2 - \alpha > 0$ both or both can be negative.

But, for simplicity assume both are positive. So, this ensures that we get this. Now here, this is the condition ensures that this point is somewhere here. This point is somewhere here x. So, this is the price p1 is equal to this region which is this and this region is p1 which is actually equal to since this x is equal to 2A minus B divided by 2 minus alpha. So, this is if we plug in that here in this portion, in this portion we will get it. So, and p2 is this region plus this region.

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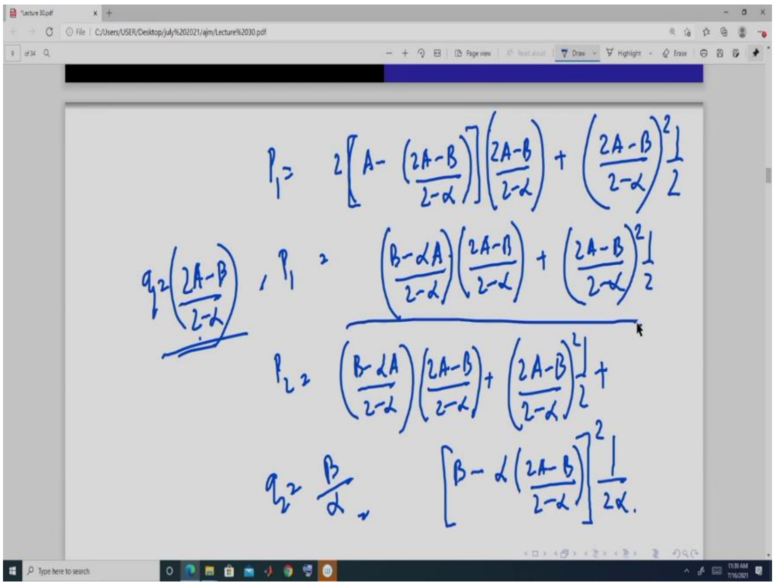

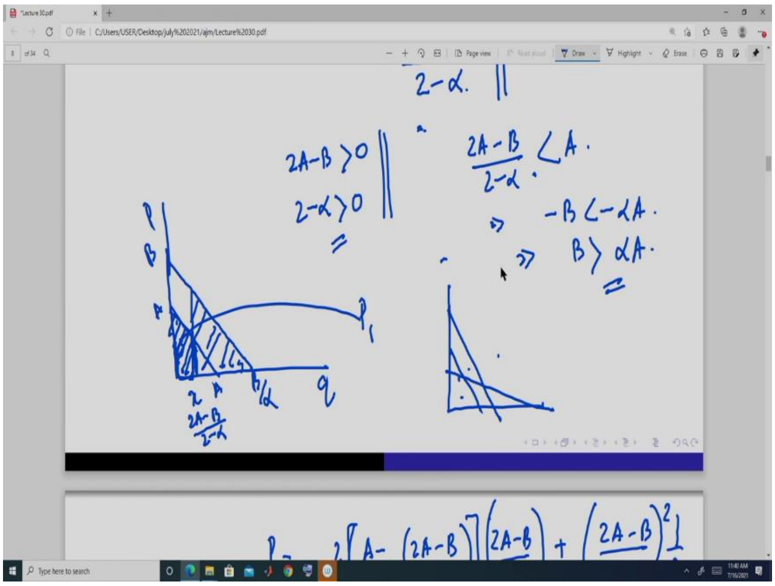

So, here we get that p1 is equal to 2 this-$P_1 = 2\left[A - \left(\frac{2A-B}{2-\alpha}\right)\right]\left(\frac{2A-B}{2-\alpha}\right) + \left(\frac{2A-B}{2-\alpha}\right)^2 \cdot \frac{1}{2}$, this and this is equal to, it is this-$\left(\frac{B-\alpha A}{2-\alpha}\right)\left(\frac{2A-B}{2-\alpha}\right) + \left(\frac{2A-B}{2-\alpha}\right)^2 \cdot \frac{1}{2}$ and p2 is this, this is at q1 is okay, this is the bundle-$\left(\frac{2A-B}{2-\alpha}\right) = q_1$. This is the package at this price, p2 is this whole thing plus one more additional term and that is, it is this-$P_2 = \left(\frac{B-\alpha A}{2-\alpha}\right)\left(\frac{2A-B}{2-\alpha}\right) + \left(\frac{2A-B}{2-\alpha}\right)^2 \cdot \frac{1}{2} + \left[B - \alpha\left(\frac{2A-B}{2-\alpha}\right)\right]$. So, q2 its bundle is B by alpha So, B by alpha at this price and q1 this at this price, this is the way the monopolist can package the product and it is like this.

So, this much amount at this price whole and this much amount at this area plus this area, okay. So, this is one way of bundling, okay. So, but if we have a, suppose if we have a demand curve of, suppose one demand curve is this, another demand curve is this, another demand curve is this, then also we can look whether, so there are three types of buyers, we can see whether we can bundle it or not.

So, here one possibility is that there may be three types of bundles or packages or there may be only two, okay . But here this will be much more complicated than this here, because there are three types of demand curves and we will have to look whether it is because we have to satisfy the self selection criteria. So, consumer 1 should not buy the package meant for consumer 2 or for consumer 3 and neither consumer 2 should buy the package of consumer 1 and consumer 3, either consumer 3 should by the practice of consumer 1 and consumer 2.

So, all these possibilities have checked. So, that is why if we simply increase one more type of consumers the complications increases by huge amount, but with two things it is still manageable but the idea is I think it is clear. So, the main idea is that if we can differentiate the demand curves, then it is possible to bundle the product, okay.

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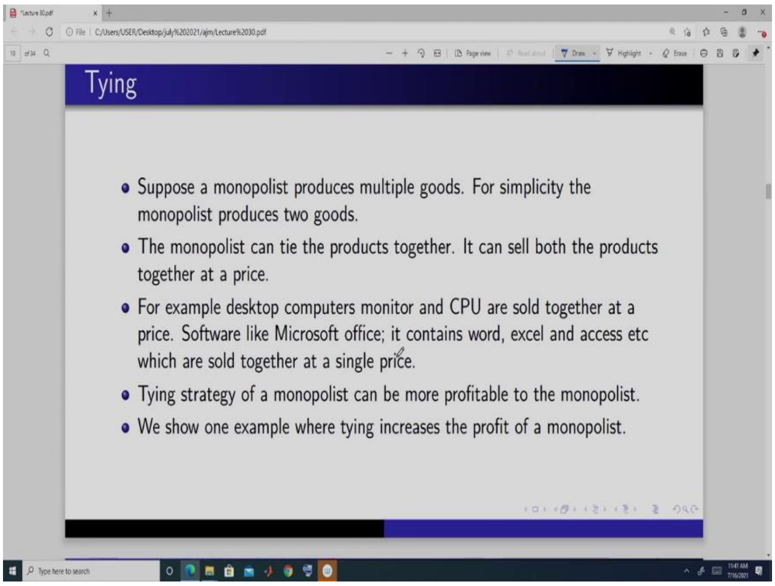


Now we switch to another topic and that is tying. What do we mean by tying? So, tying here again we take a monopolist, now monopolist can suppose a monopolist selling computer, desktop computer. So, desktop computer come as a two thing, one is the monitor and other is the CPU. So, this comes together. So, this is you can think of as a kind of a tying.

So, monitor is tied with the CPU or CPU is tied into a monitor or you can simply assemble this thing and you can buy them separately. So, this is or if you look at MS Office, so like MS Word, MS Excel, MS Access all these things are tied together. So you can buy only MS Office and all these three things are available, but each of them can be considered as separate product also, right?

So, this is a tying strategy and the monopolists can follow up tying strategy, okay. Now it can be shown that the tying is sometimes profitable and that is why the monopolists tie. Now here we are assuming that the monopolists are not a single good producer monopolist. So, they are producing multiple products, okay.

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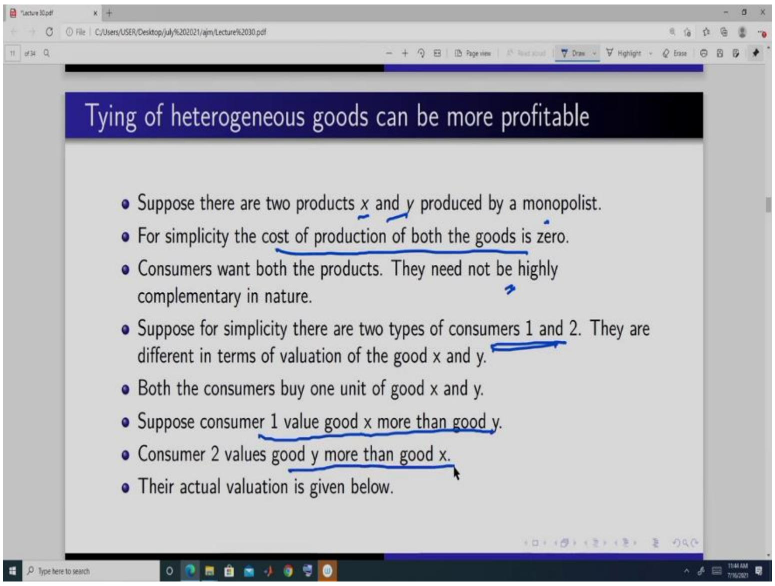


So, suppose there are two goods x and y and these two goods are produced by a monopolist, okay. And for simplicity, we again take the cost of production to be equal to 0. So, that is the marginal cost is 0 and there is no fixed cost, okay. And consumer, assume for simplicity that they want both the product, so we have some consumers and they want both the product.

And these products may not be strictly complementary in nature like in case of monitor and CPU they are complementary, you cannot have, the monitor is useless if you do not have a CPU and CPU is useless if you do not have a monitor, but in case of like MS Word, MS Excel, Access then they are not strictly complementary, although they have some degree of complementarity, but they are not highly complementary.

Like you can, if you are only working, if you do not work with any data or numbers then if you do not, you are not engaged in any form of calculation, only you require it for typing then MS Word is sufficient and you can do it that is okay. But if you are engaged in both calculations and writing, then you require both Excel and Word. But if you are only engaged in calculation and you do not require and need to do any writing, then Excel is sufficient for you, okay.

So, we assume that there is, both the products are required by the consumer but they are not highly complementary in nature. And for simplicity suppose we assume there are two types of consumers and the types of consumers are differentiated based on their valuation for this product and for simplicity what we have done we assumed that suppose consumer 1 value good x more than good y and consumer 2 values good y more than good x, okay. And their actual valuation is and is given like this.

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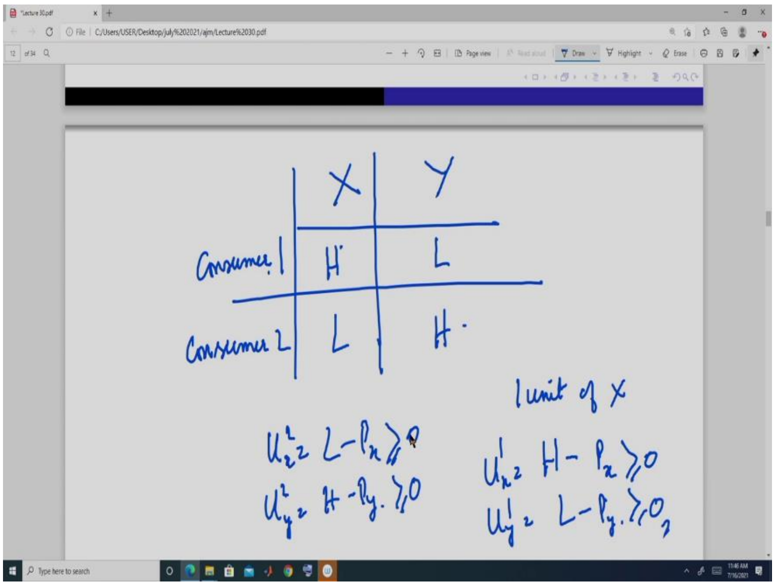


Suppose we have two goods X and Y okay and we have consumer 1 whose valuation is suppose high and this is low and consumer 2 is valuation is low and high and suppose these are same for to keep the things simple, otherwise it will create a lot of, we will have to do a lot of calculations and here my objective is to give you the idea not do the actual calculation.

So, we, so these are valuation. So, that means, if consumer 1 buy one unit of X, the utility it gets is H, net utility is this H minus Px and the utility it gets from y is H minus p, like this, okay. These are the price and both we assume that both of them consumes only one unit of good. So, consumer 1 each consumes one unit of X and one unit of Y if both of them are affordable.

Consumer 2 buys affordable or if they give them non negative, if this is they will consume for this consumer 1. And for consumer 2, utility is this- $U_x^2 = 2 - P_x, U_y^2 = H - P_y$. And he will buy both the product one unit of each if both of them are positive, okay. Now, how do the monopolist price this good? It has many strategies.

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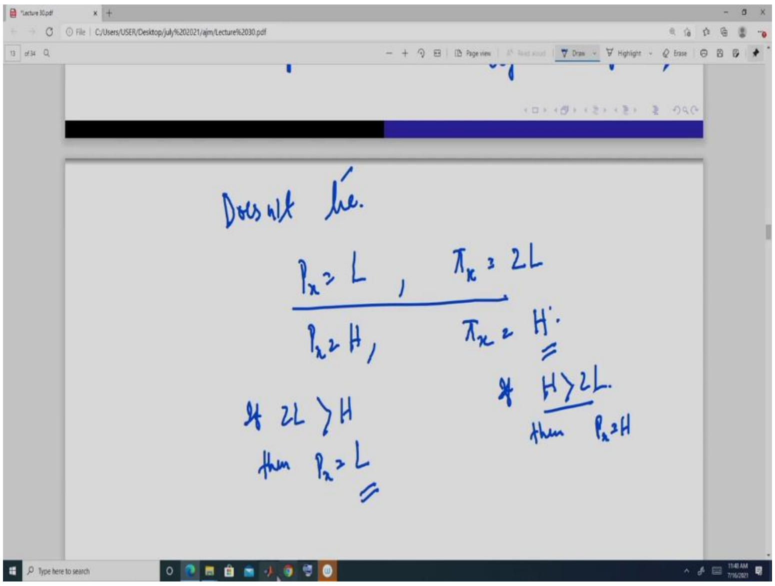

Suppose it does not tie, does not tie the product. So then it means what so Px it is, it can charge L. If it charges L, then profit from because it is a monopolist, so both the consumer can buy it, so it will have 2L. This is one possibility or it can charge this H and profit from x is only consumer 1 buys. So, this strategy is profitable when H is greater than 2L, right. Otherwise, so if this is the case, then Px is equal to H or if 2L is greater than this H then Px is equal to L, this.

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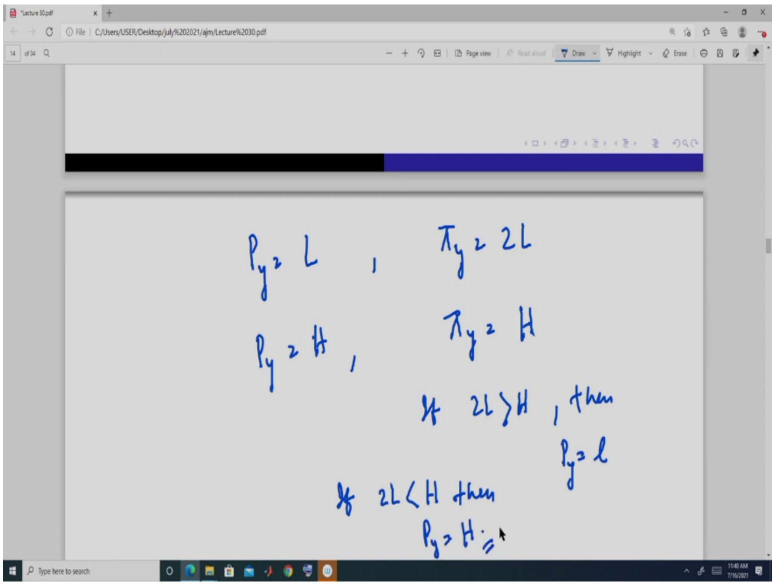

Similarly, for good y also. So, y is equal to L then profit of y is 2L if Py is equal to H, then consumer 2 buys, so, it is H only. So, if 2L is greater than H then price is, then is l and if 2L is less than H then Py is equal to h. So, these are the pricing strategies if it does not tie, okay if tying is not there.

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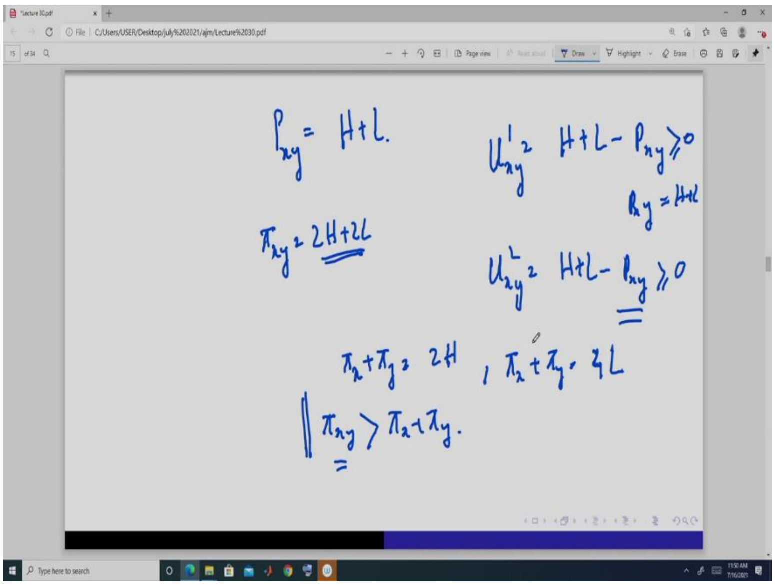


But if it ties it can charge a price, it can charge a price that is Pxy it is a tied product, but you are have to buy both the product and you have to pay only one price and that can be H plus L. Now here if this is the A consumer 1 utility of consumer 1 is H plus L is this-$U^1_{xy} = H + L - P_{xy}$, so it is going to be positive if this is, it is going to be non-negative if Pxy is equal to H plus L.

And this is again if we have this, so, profit sif it ties is 2H plus 2L-$U^2_{xy} = H + L - P_{xy}$ . Now here in this situation, if we have what are the possibilities? Possibilities are here, either it gets, if it does not tie, if its profit is x is 2H or it can be 4L, if you compare with this, you will get that Pxy is always greater than this-$\pi_{xy} > \pi_x + \pi_y$.

So, because of this reason, we get that the tying is a more profitable strategy then not tying. So in this situation, the monopolist is always going to tie this product x and y. So that is why we get certain products which are bundled together, which are tied together. We do not see, we do not get them as an individual product, but we get them as a composite product, where many things are together, okay. Even if the goods are not highly complementary, okay.

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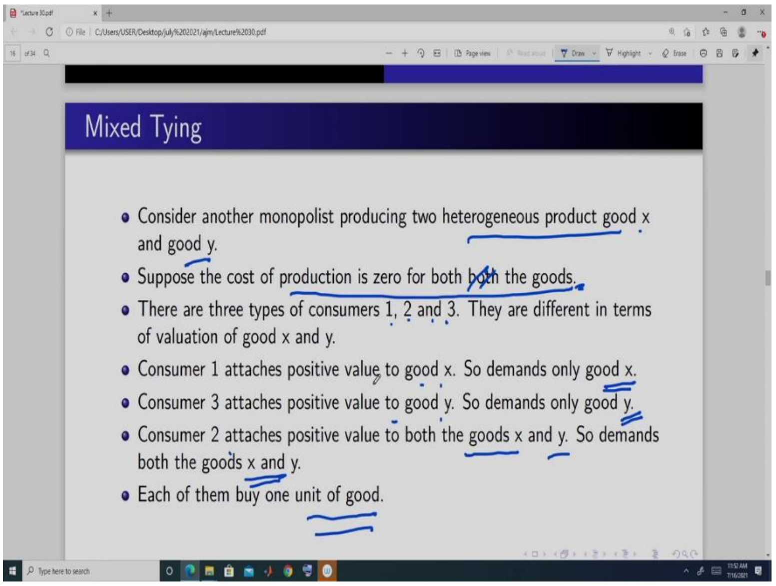


Next we can see that whether only tying is here or there can be other possible way of tying? So, we see something called a mix tying and when do we see mix tying? We will now do that. Again consider a monopolist which produces two heterogeneous product good X and good Y, okay. And suppose for simplicity cost of production is zero for both the product and there are three types of consumers 1, 2 and 3.

Now, we increase that type of consumer and they are different because their valuations are different. Consumer 1 attaches positive value to good X and so demands only good X. Consumer 1 does not demand good Y. Consumer 3 attaches positive value to good Y and so demands only good Y and consumer 2 attaches positive value to good X and positive value to good Y, so demands both the goods if it gets positive utility from or non-negative utility from them. And for simplicity each of them buys only one unit of good, okay. This is the setup.

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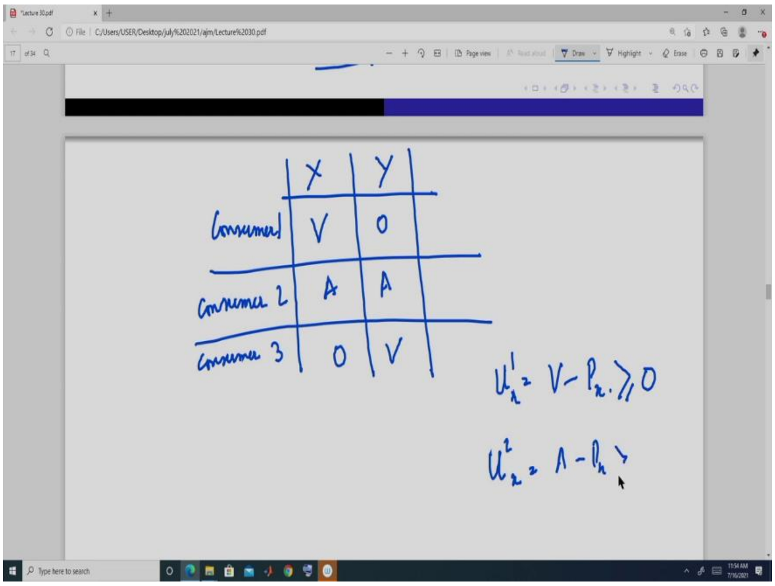

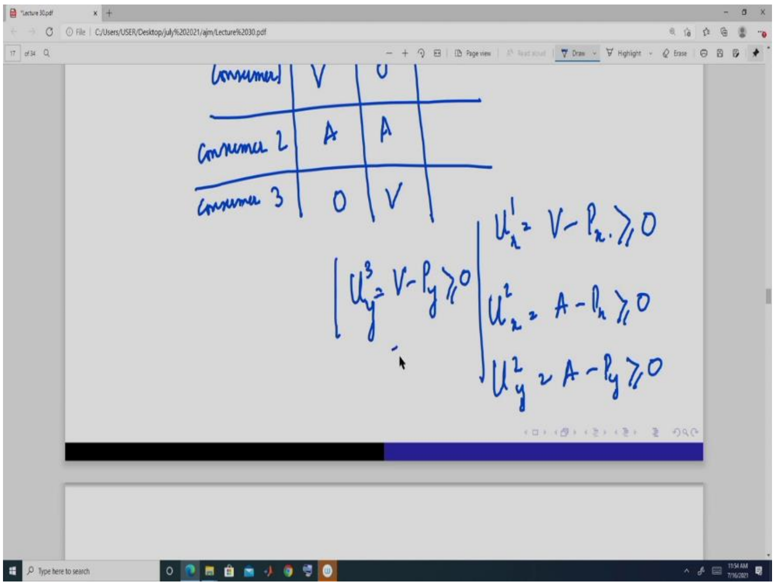

Now, we have to see how the, what is the strategy of the monopolist and before that let us look at the valuation. So, this is suppose X this is suppose Y. Consumer 1, consumer 2, consumer 3, suppose it values this and this is 0, and this is 0 and suppose this is A and this is A same. This is one simplest configuration that we can have, not even complicate this, only then the calculations will be more complicated, okay.

So, consumer 1 buys good X if V is this-$U_x^1 = V - P_x \geq 0$. Consumer 2 buy X if A minus Px-$U_x^2 = A - P_x \geq 0$, again consumer 2 buy good 2, if A minus Py gives this much-$U_y^2 = A - P_y \geq 0$ and consumer 3 buy only good A if V minus Py is greater 0-$U_y^3 = V - P_y \geq 0$. So, these are the possible A. Now, we have to see how the monopolist is going to price this and this information are there to the monopolist, okay.

And here even monopolist we do not need to specify whether monopolist can identify this types of consumers or not, even if the monopolist cannot identify, it will package the price itself will give **a** you, will lead to a self selection kind of thing, okay. So that in specification is not required.

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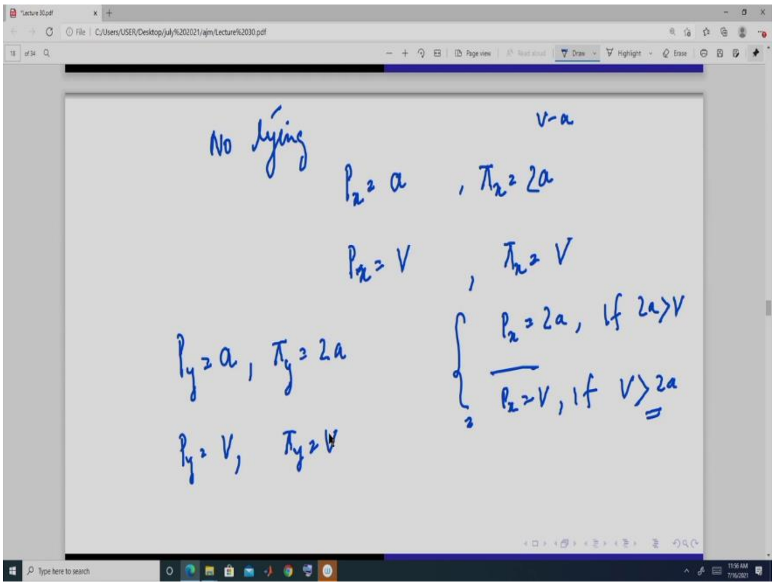


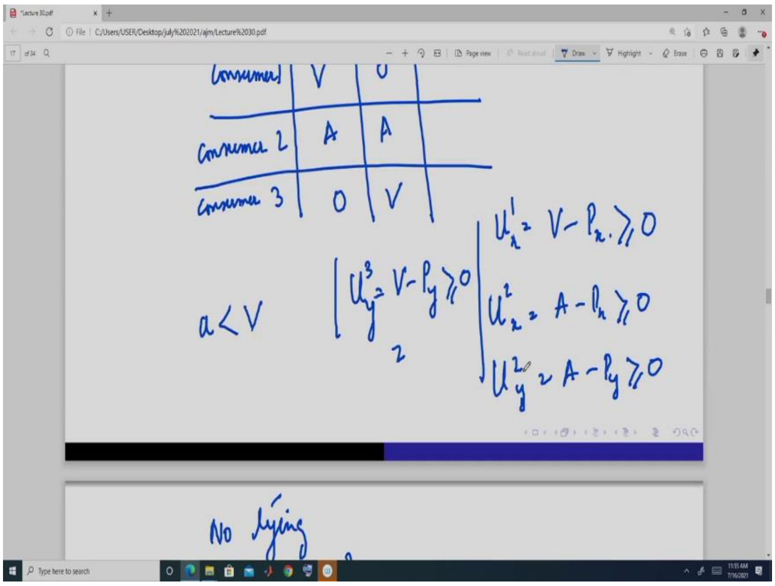


Now here, suppose no tying, okay. So then Px can be suppose a and further here we assume that suppose a is less than V, okay. So, if P is this α then profit from x is consumer 1 is going to buy because its utility is a minus V now, it is positive. So, it will get 1 and consumer 2 is also going to buy good a, so it is profit is this. Consumer 3 is not going to buy this and it can set this is equal to V also. Now then profit is this V.

So, the price here is going to be 2a, if 2a is greater than V or it is going to be this V if V is greater than 2a. So, it is same as the outcome we have got earlier and for price of good y, it can set the price a. Then consumer 2 is going to buy it, profit of firm monopolist is consumer 2 is going to buy and consumer 3 is going to buy. So, it is 2a if it sets the price this, only consumer 3 is going to buy, so it is going to this.

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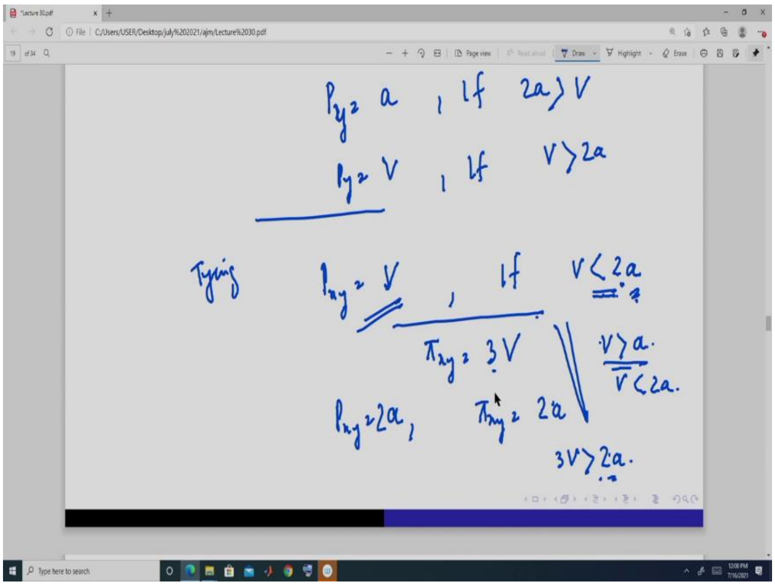

So, following this thing, we get that the Py is equal to a if 2a is greater than V and Py is equal to V if V is greater than 2a. If it is same, then they are indifferent you can charge anything, okay. Now, we have got this if there is no tying and if there is tying, suppose it does tie, then Pxy it can be V, if it is V then what it is? Then the profit and if it is V and if suppose V is less than 2a.

If this is the case, then profit from tying is, consumer 1 is going to get buy because it will get x, it is y is going to be useless for him but still he gets x and his valuation is non negative. So, his net utility is not negative if price is V. So, one consumer 1 will buy, consumer 2, since it is this he is going to get some positive utility if price is V of x y. So, this person is going to buy, so it is again V.

Third person, consumer 3 is also going to buy why because his utility from good V, good y is V and price is V. So, he is getting x but he has no value from x but it is still gets. So, the profit here is 3V, okay and if its price is suppose 2a, then this thing is going to only, if this is the case, then only consumer 2 is going to buy. So, profit here it is going to be 2a.

Now, if this is the case plus 2V and if we have suppose V is greater than a but V is less than V, right? then we will definitely have a situation where 3V is greater than 2a, okay. Because from this, from this condition- V>a, we get this, even if this is true we will get this, right. So, if they tie, then the price is always going to be this if this is true, okay and given this- V<2a. If this is not true, then it is not going to get this-3V. So, it will be only 2V, in that case also it is going to get 2V. So, it is going to be greater than this 2a.

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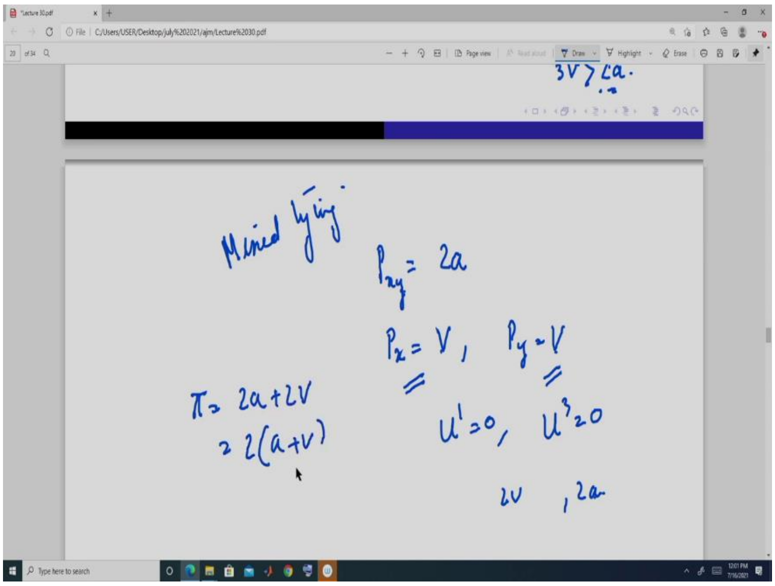

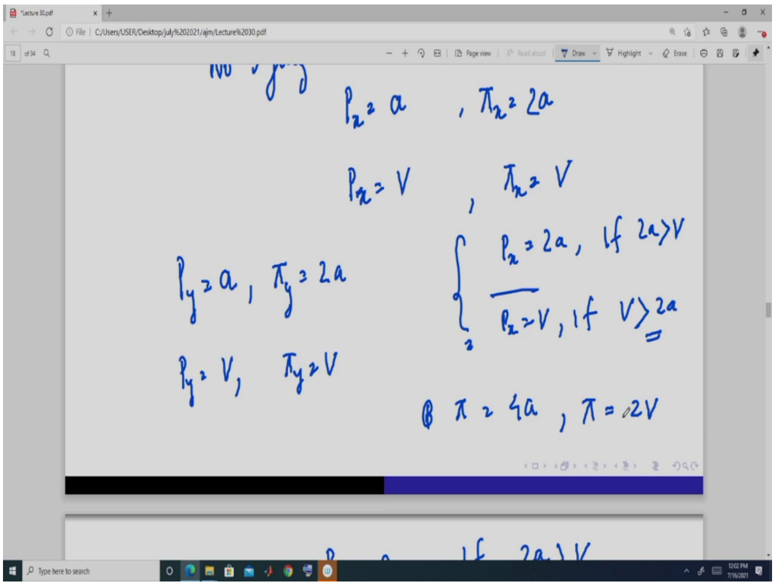

Now, so this is thing but monopolists can follow another strategy and that is the mixed tying. What it can do in mixed tying? It can set one price where it is getting both x and y at a price this-$P_{xy} = 2a$ and it can set a price V this. Now here in this equation-$P_x = V$ see if so what is going to happen? If consumer 1 will buy this, consumer 3 is going to buy this product, so at this, so their utility is going to be 0. It is going to be 0 and for this person for consumer 2, if it buys this and the separately its price is 2V, but if it buys together it is 2a, okay 2a is less than 2V.

So that is why it is going to buy that, so the profit in this situation to the monopolist is 2a plus 2V, which is 2 a plus V, this-2(a+V) is greater than both, this 3V here . And it is in this case profit is how much? Profit is this or this or it is this, so it is profit, here it can be 4a or it can be 2V, right.  And these are going to be less than this. So, that is why we get that mixed tying can also be one possible. So, the main idea is to extract as much consumer surplus as possible and so the monopolist can use different strategies to extract surplus.

So, one is the if it is possible, it will only do bundling to extract more, if it is not producing more than if it is only producing one good then it can do bundling. If it is producing more than one good, then again it can do bundling and it can do tying it can do mixed tying and again do can bundle the good also. So, there are so many possibilities it can, the idea is that the monopolist will always try to extract as much consumer surplus as possible.

Next topic we going to do is tying as product differentiations. Now here what do we mean by product so, product differentiation, we see that the products are different to a firm producing another type of, one type of product, another firm producing another type of product and they are different based on some characteristics or some attributes of this good.

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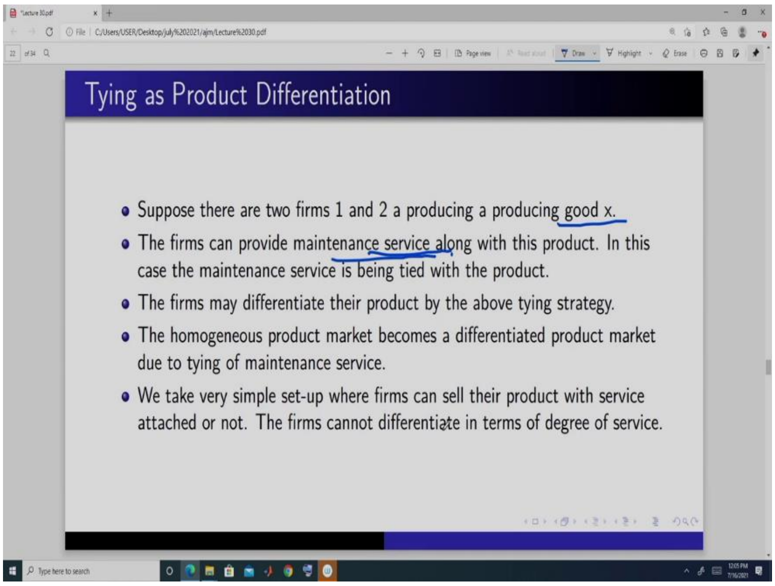


Now here we can see that suppose that both the firms produce a homogeneous product and that is suppose this x, but you require some kind of a maintenance service, maintenance service to for this good. So, by providing maintenance service and by not providing such maintenance service, you can differentiate this product. So, as a product itself it is homogeneous, but by additional this service, maintenance service, you are differentiating this.

So, for simplicity we take that the firm can either provide the service, maintenance service or it may not provide the maintenance service. So, these is a way of differentiating. So, you can have one product which you get the product and also along with it you get the maintenance service, another is you only get the product, okay.

So, in this case, we see that the products are now different; one comes along it maintenance services tied along with that product. In another it was only the product and if you want some service you will have to pay it separately. Now here we are assuming that the service is fixed, it is either you give service or you do not give service. So it is you cannot differentiate it in terms of degree of service that whether you will give 1-year service, 2-year service that is not possible, actually to keep the things simple.

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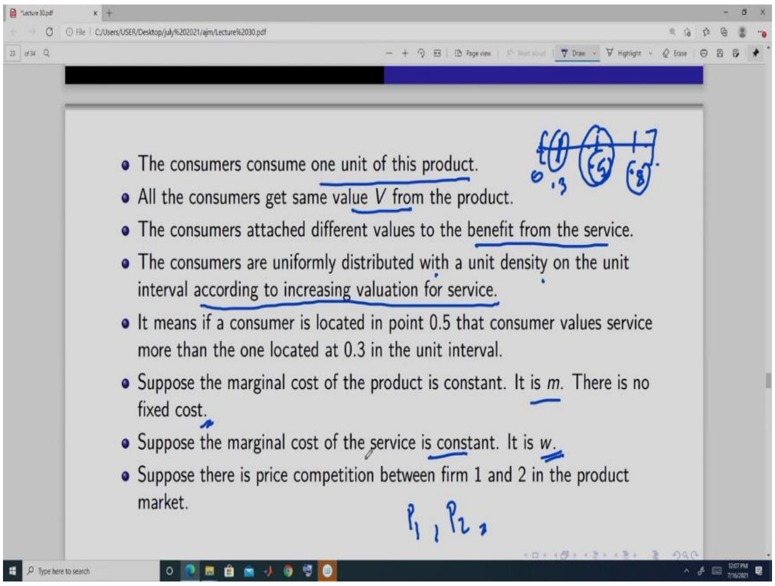


And consumers consume only one unit of good, they get the same value and that is this. Now but the consumers attach different values to the benefit from the service and how do we model it? We take this something similar to the hoteling model. And here we assume that the consumers are uniformly distributed with unit density on the unit interval according to increasing valuation of services.

So, if we take this 0 and this here, suppose this is consumer, so this consumer located here is going to value the service more than consumer this which is suppose located at this point, but this which is located at this point is going to value the service more than this, okay. So, this is how the consumers are located and there is you can say that the continuum of consumers located in this region or in this unit interval, each point or each dot represents one consumer, okay.

Now, suppose the marginal cost of this product is this-m and there is no fixed cost and suppose the marginal cost of the service, it is also constant and it is this w, okay. And in the product market for while selling this product that is there is price competition that means firm 1 decides a price p1 and firm 2 decides a price p2, okay.

So, you can say that there is a bertrand competition and so here we may get three possible outcomes and see and it is, you can say that it is a two stage game. What happens? okay We will discuss the game later.

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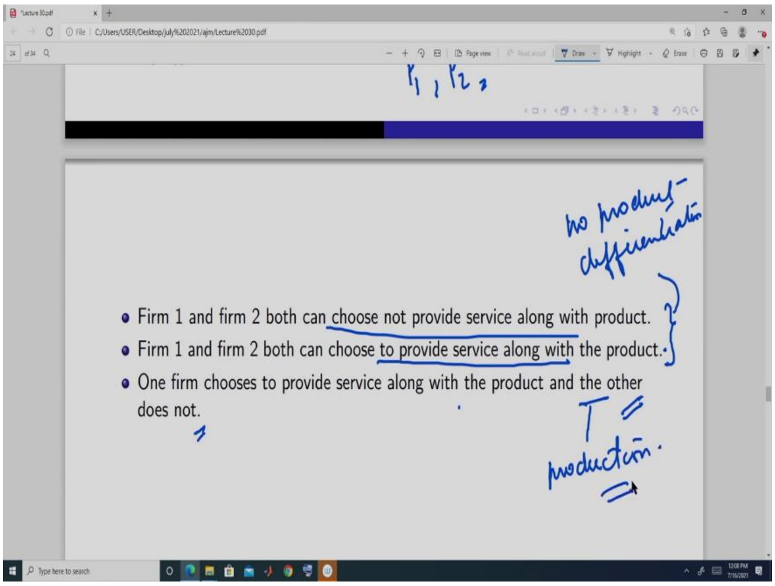

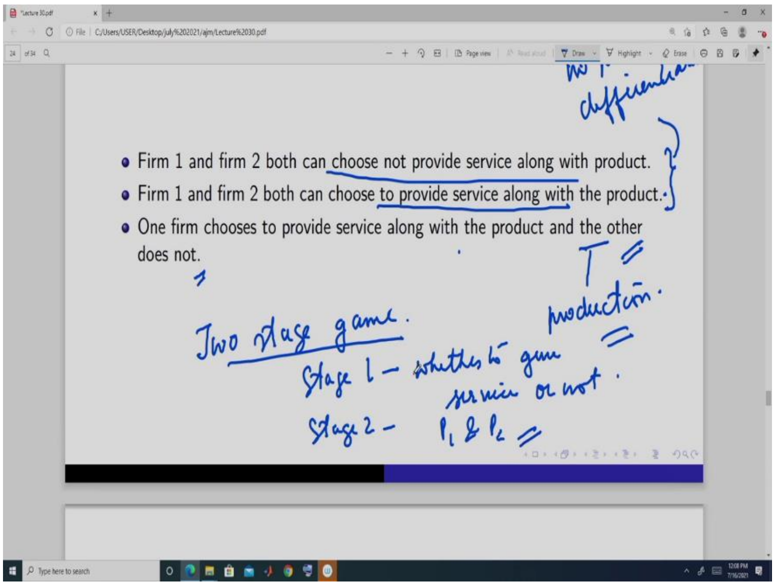


But first, what are the possible thing; one the both the firms choose, does not choose service along with the product, they give only the product that is one outcome. Another outcome is both the firms choose to give the or provide the service that is another outcome. Or another outcome is one firm chooses to give service and other firm chooses not to give the service, okay.

So, here we get the product differentiation. In this two case, there is no product differentiation and here we see that there is product differentiation, okay. And product differentiation is through tying a service and the game is a two stage game. Stage 1, it decides whether to give whether to give service or not and in stage 2 it decides p1 and p2, okay. So, we will consider it using subgame perfect Nash equilibrium.

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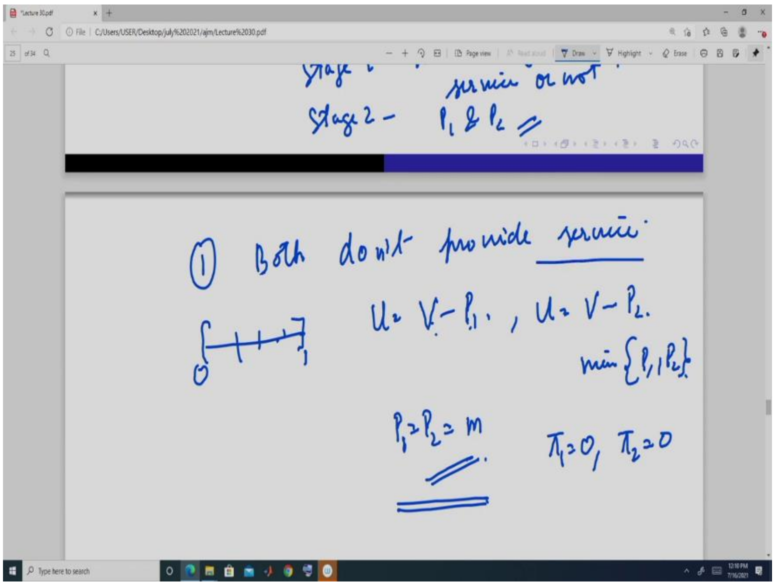

Suppose the first case, suppose both case 1; both provides, both do not provide the service. So, the utility that the consumer gets is V minus p1 if it buys from the A and p2, okay. V is the utility and it does not get any services, so it is same. So, here what is going to happen if this person is given any location, here or here any point it will choose that where p1 is which is lowest.

So, it is minimum of p1 and p2, right. So, we know here if it charges anything greater it is not going to buy. So, ultimately here p1 is equal to p2 and it is equal to m, this is the optimal strategy and profit of firm 1 is 0, profit of firm 2 is 0 in this case, when both do not provide service, okay.

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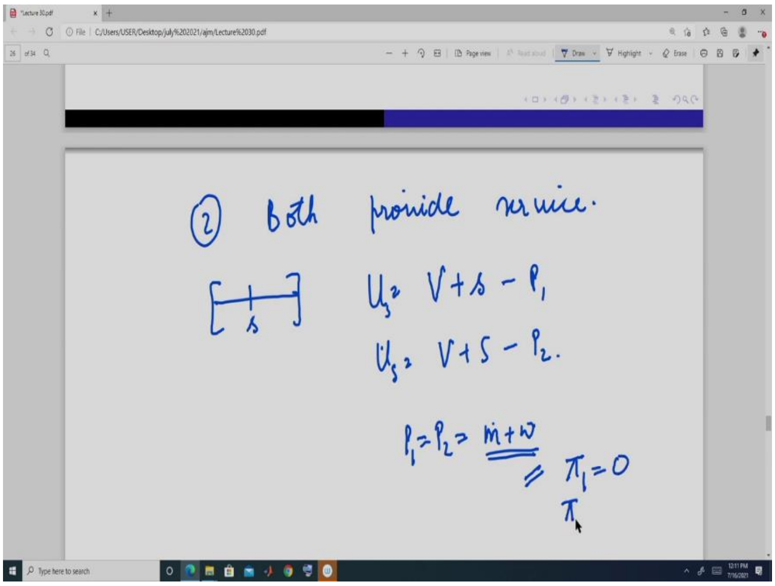

Now, suppose both provide service. When both provide service then what is going to happen? Utility of any consumer is going to be this plus this if this is the location of that consumer- U = V + s, consumer s minus p1 if it buys from here and it is going to be this if it is buys from firm 2. So, this remain same, so there this consumer is going to buy from s, from that firm whose price is less.

So, again here if you charge any higher price none is going to buy from that. So, that is why there is only one possibility that p1 is equal to p2 and that is m plus w because this is the marginal cost, sum of the marginal cost. This is the marginal cost of the product and this is the marginal cost of the service, okay. So, here again in this case profit of firm 1 is 0, profit of firm 2 is 0. So, bertrand paradox is fully operational here and here in this two case, in first case.

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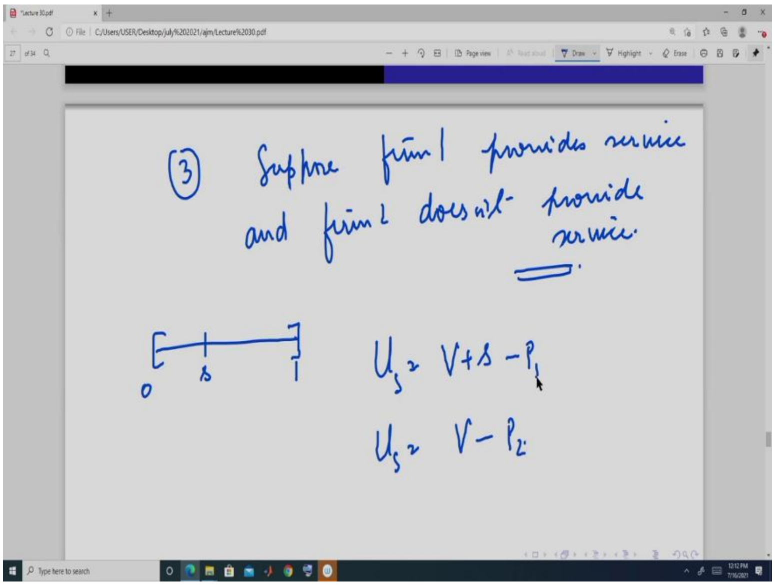


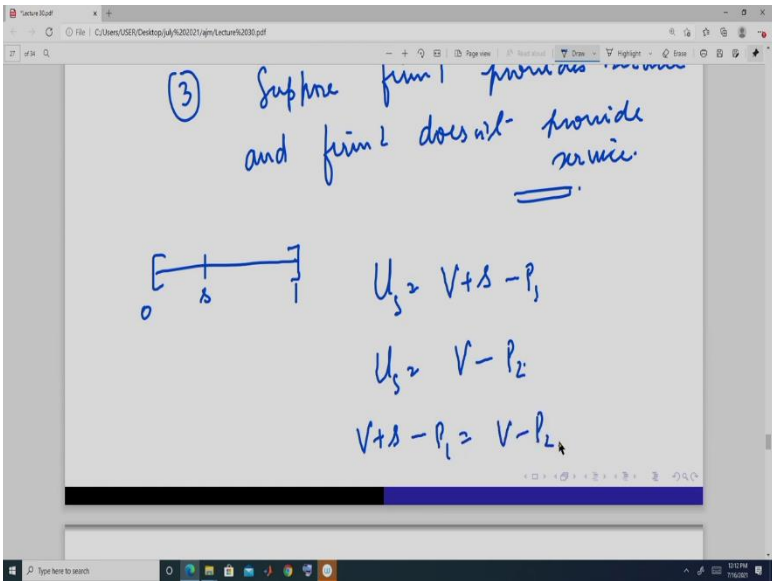


But in the case 3, suppose firm 1 provides service and firm 2 does not provide service. This is one possibility, okay. Now, let us look at these consumers. This is suppose consumer x, sorry consumer s, utility of consumer s is V if it buys from firm 1- $U = V + s - P_1$. And utility is this minus this, if it buys from firm 2 because it does not get the service, here it gets service. So, from these, these two valuations are not same. So, we get that now the price of firm 1 and firm 2 can be different. How can it be different?

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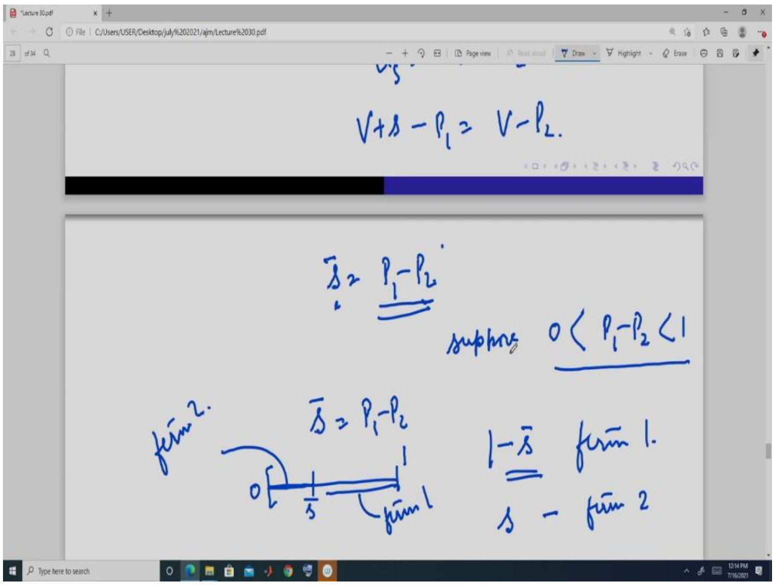

See we can find a value of s such that it is equal to P- $s = P_1 - P_2$ and here see all these person's s is more than this, valuation is more than the s, all this, all these person's valuation is less than s. So, in this case if this suppose this is s interior, so we assume this-$0 < P_1 - P_2 < 1$, okay. So, if this is the case then what do we get? We get that we have s bar where p1, given p1 and p2 and this person gets same utility from buying from firm 1 or firm 2.

But they are getting different product and in product of firm 1 is that the same product with service; product of firm 2 is only the product no service. So, the market of firm 1 is 1 minus this s bar, this-1-s, okay. And market of firm 2 is all these consumers are going to buy from firm 1 and all this, so, if this is s bar, then this region is for firm 2 and this region is for firm 1. So, this is the firm 2, okay now we define the profit function.

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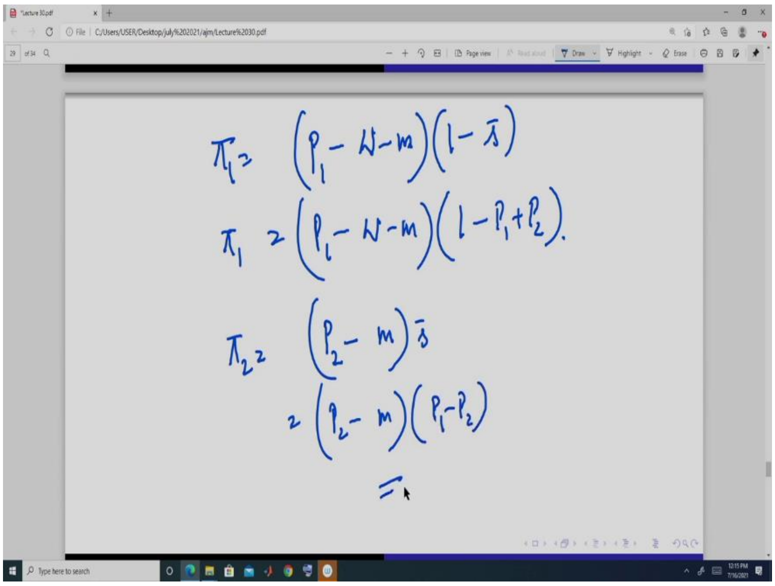

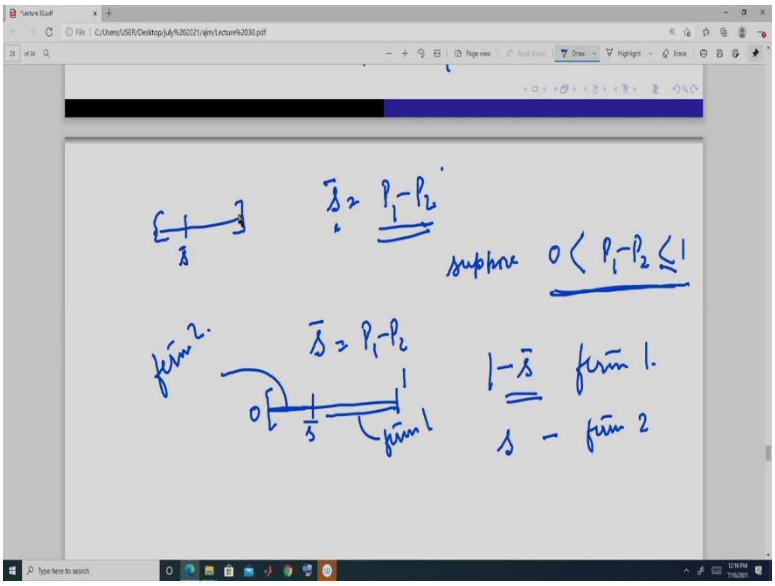


Profit function of firm 1 is it will charge the price p1 and it will marginal cost from service marginal cost from product into it will be serving this many people which is equal to this-$\pi_1 = (P_1 - w - m)(1 - P_1 + P_2)$ and profit of firm 2 is this-$\pi_2 = (P_2 - m)(P_1 - P_2)$. Here while doing this we have assumed this thing, now we can take this, okay. Why because if so that we get an interior thing as lies within, not here, neither here.

If it is at the boundary, then it is means either firm 1 is serving or firm 2 is serving. So, it is so that means what either only one type of product is there, we do not see any product differentiation in this case. So that is why we will assume this only, consider this thing.

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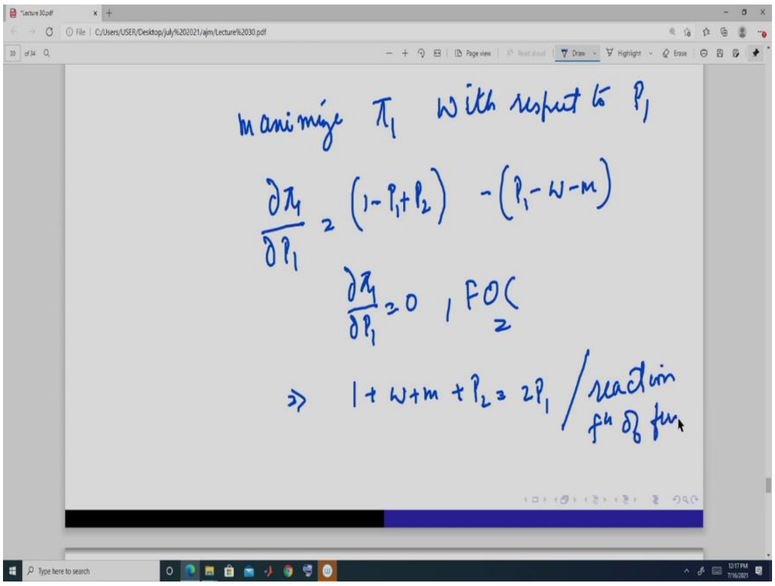


Now here we maximize profit, maximize p1 with respect to p1. So, we get what? This will give you this-$\frac{d\pi_1}{dP_1} = (1 - P_1 + P_2(+(P_1 - w - m)$ and first order condition gives this equal to 0 first order condition. So, we get the reaction function of firm 1 in this form. This is the reaction function of firm 1-$1 + w + m + P_2 = 2P_1$, okay.

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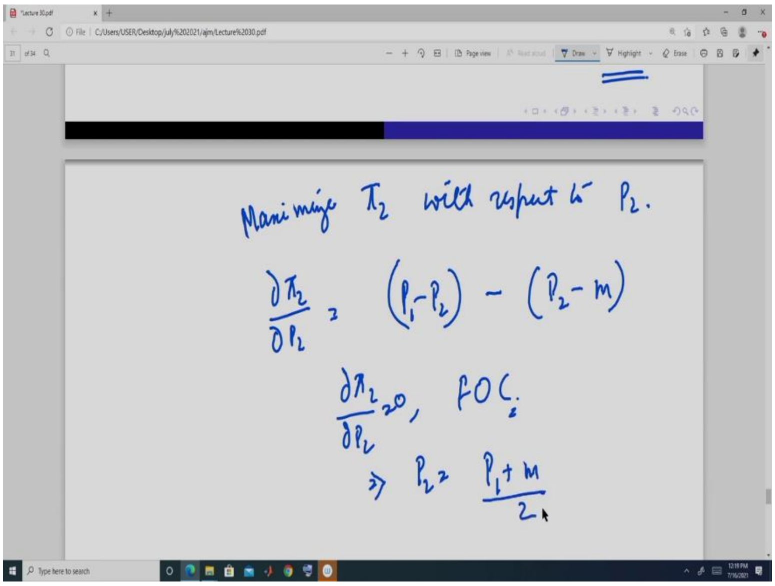

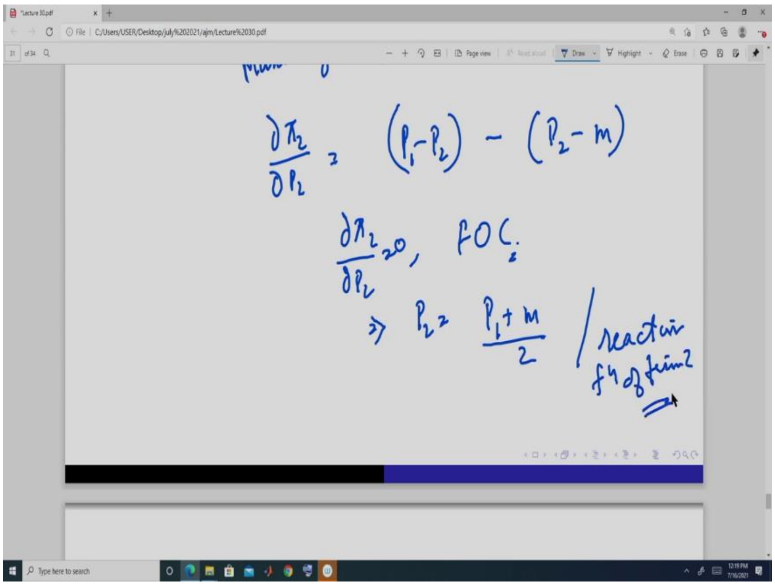

Similarly, firm 2 what it does maximize p2 with respect to p2. Now, here what is going to happen so, we have we know the a it is going to be this and this is going to this-$\frac{d\pi_2}{dP_2} = (P_1 - P_2) - (P_2 - m)$ first order condition gives me this is equal to 0 first order condition, so we get p2 is equal to p1 plus m divided by 2. This is the reaction function of firm 2-$P_2 = \frac{P_1+m}{2}$.

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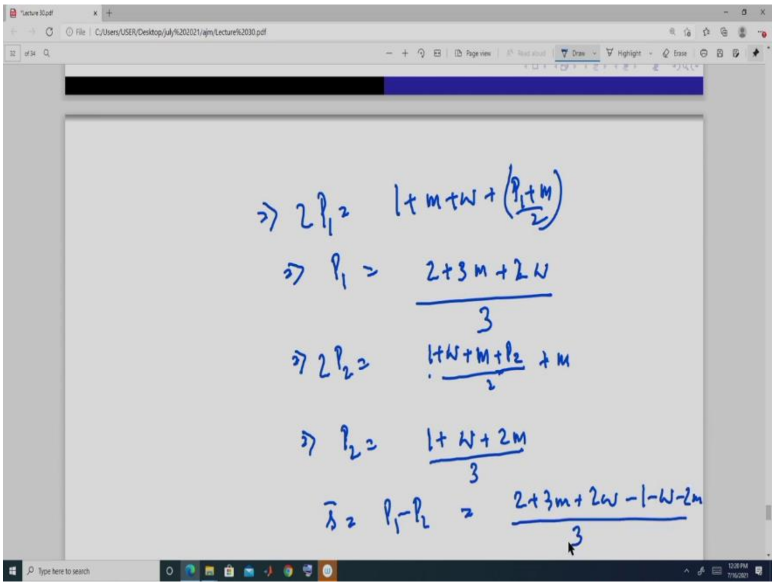

Now, we solve these two reaction functions what do we get p1 is equal to 1 plus m plus w by… So, this implies p1 is equal to 1 plus; 2 plus 2m plus not here it is also m. So, it will be 3m; 3m plus 2w divided by 3-$P_1 = \frac{2+3m+2w}{3}$ and p2 is what? So, 2p is 1 plus w plus twice m divided by this-$P_2 = \frac{1+w+2m}{3}$. Now, what is s bar? s bar is p1 minus p2 this is 2 plus 3m plus 2w minus 1 minus w minus 2m divided by 3.

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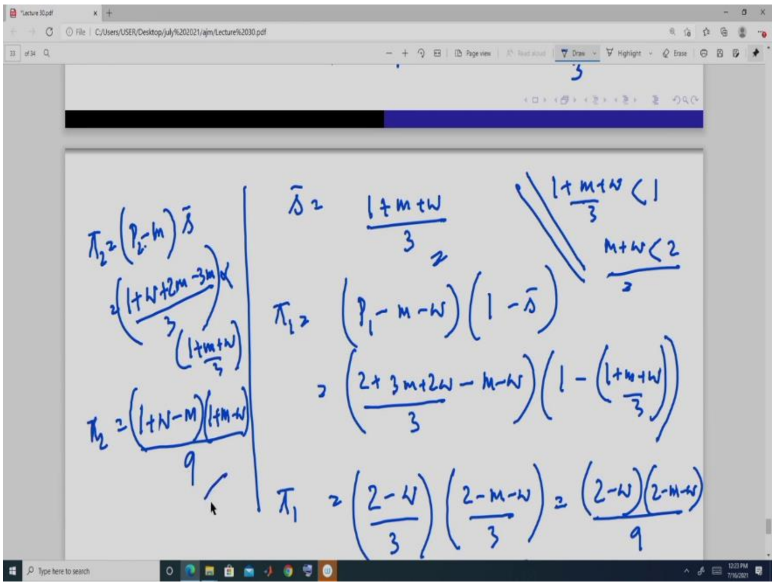

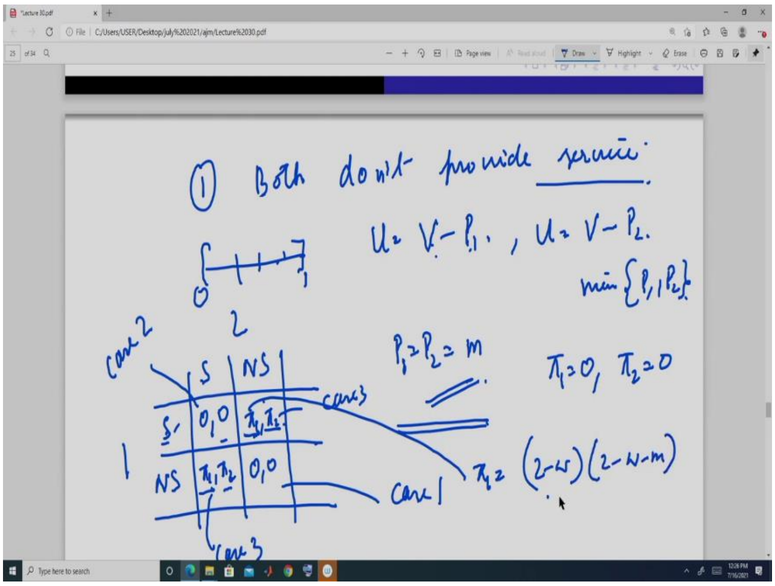


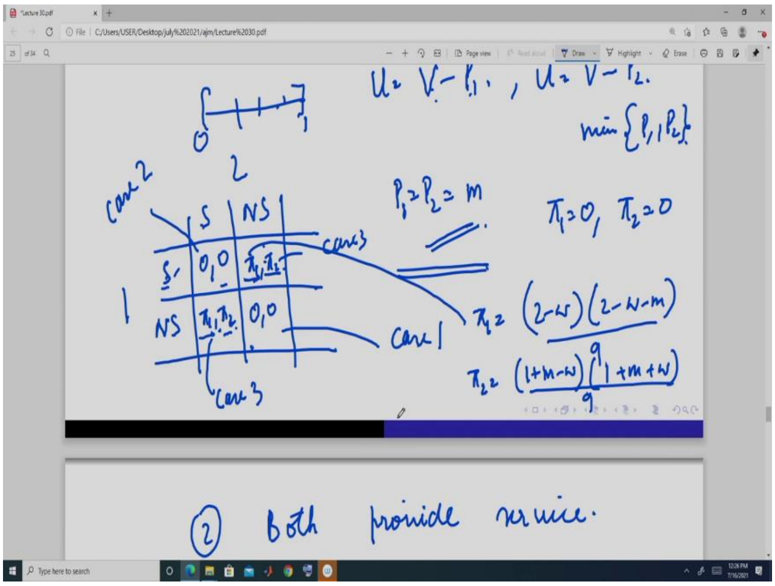

So, s bar is equal to 1 plus m plus w divided by 3-$\frac{1+m+w}{3}$. So, profit of firm 1 in this case is p1 minus w minus m minus w 1 minus s bar. So, this is 2 plus 3m plus 2w minus…-$\pi_1 = \left(\frac{2+3m+2w}{3} - m - w\right)\left(1 - \frac{1+m+w}{3}\right)$ , right? Now, here we have to assume that this is definitely greater than 0 and this is less than 1. So, we have to assume that this is less than 2- m+w<2, okay for this condition to satisfy this-$0 < P_1 - P_2 \leq 1$. So, for simplicity assume this okay, this condition needs to be satisfied.

Now, solving this we get 2 is this, this is the profit of firm 1-$\left(\frac{2-w}{3}\right)\left(\frac{2-m-w}{3}\right)$. So, this is 2 minus w 2 minus m minus w divided by 9. And profit of firm 2, sorry profit of firm 2 is p2 minus m s bar, p2 is this 1 plus w plus 2m minus 3m into this, this so this is 1 plus w minus m 1 plus n minus w divided by 9, this is the profit of firm **2**-$\pi_2 = \frac{(1+w-m)(1+m-w)}{9}$ and this is positive and this is also positive. But, in this situation what do we get 0 and 0.

Now, here we can do something. So, strategy, in stage 1 strategy of firm 1 and strategy of firm 2. So, it provides service, it does not provide service, it provides service, is no service firm 1, firm 2. If both provide service it is 0 0, if both does not provide service it is 0 0, these two outcome 1, 2, so this is case 2 and this is case 1, this is case 3 and this is case 3.

Here we know profit a firm 1 and firm 2 both are positive. So, if firm 1, we have computed this firm 1 provide a service firm 2 and we will get the symmetrically opposite here. So, we get two Nash equilibrium under this and this. So, this we have got, this is what, this profit now, this profit, so it is 2 minus w, so it is 2 minus w and then 2 minus w minus m divided by 9. And profit of 2 is 1 plus w 1 plus m divided by 9.

So, in this is and it will be opposite here. So, this will be firm 1s and this will be firm 2s. And here this since both of them are positive so either this is a Nash equilibrium or this is a Nash equilibrium. So, here what do we get, so the subgame perfect Nash equilibrium outcome is that either firm 1 does provide the product differentiated product and firm 2 does not provide the service or firm 1 does not provide the service and firm 2 provide the service.

So, this is our form of tying where we see that both the firms are not doing tying, only one of them is tying and the tying is allowing them to differentiate the product and that is a subgame perfect Nash equilibrium outcome we have got, okay. And so, tying can be helpful in this sense and that can be a strategy which can be used in not only by the monopolist only also in an duopoly to differentiate the product.

So, from a homogeneous product market, you can switch to a differentiated product market by tying these kind of services. So, this is a very important strategy that a firm may utilize, okay. So, this was our last class. So, we I end this here.

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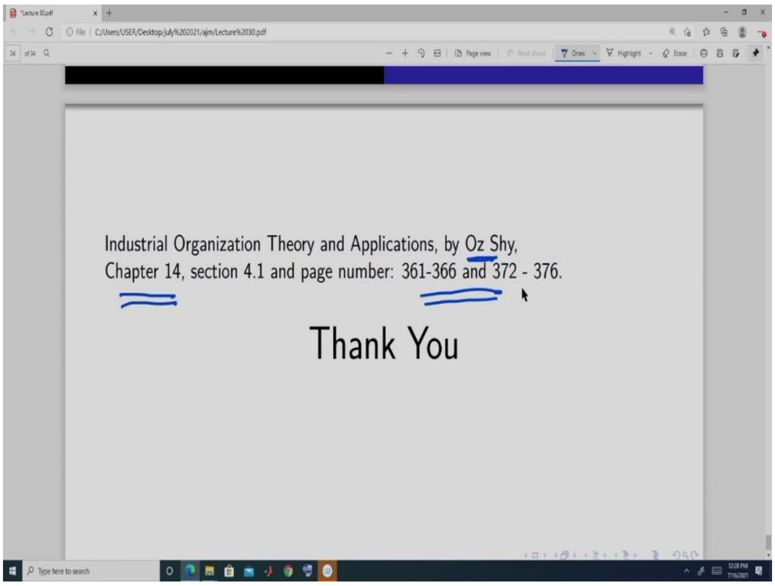


And for this portion tying and bundling, you can read this chapter 14 of this book by Industrial Organization Theory and Application by Oz Shy and these are the specific page numbers. So, this is a very long chapter and we have done a very small portion of it. And so that is why I have specified the pages.

So, here what we have done basically, in this A, we have done first we have mainly our objective was to look at how the interactions takes place in a market between the buyers and the sellers. So, from the buyer side we get the demand. So, that since they are buying, so we have derived the demand.

Firms they produce. So, they inquire incur some cost. So, we have done the cost minimization and from there we have derived the cost function. Now, then we have specified different setups under which these buyers and the firms interact. First is the competitive market where both the firms and the buyers takes the price as given and based on that, we get the market equilibrium we have shown that the perfectly competitive market is a Pareto optimal and also social welfare maximizing outcome.

Next, we have looked at monopoly; what is the monopoly price, what is the monopoly outcome? In the monopoly there is only one firm and we used the conventional downward sloping demand curve. And we have looked at the different price discrimination strategies of the monopolist also.

Then, we have introduced strategic interaction and to do that we have introduced game theory and we have done two types of, two forms of games; that is complete information static game, one shot simultaneous move. So, we know how to find a pure strategy and mix strategy of such games and then we have done dynamic games and we know we have used the solution concept that a subgame perfect Nash equilibrium.

And using these two we move to these two tools, we have done the analysis of the other forms of markets; the first Cournot duopoly where we have shown that it is not a pareto, it is something like a prisoner's dilemma kind of outcome, the Nash equilibrium outcome is and then we have looked at the Cournot oligopoly thing and we have also looked at the limit outcome of that.

Then we have introduced price competition that is the bertrand competition and we have discussed the bertrand paradox and their different setups. And then we have introduced capacity constraint and then we have seen that under certain condition the bertrand paradox may not be there, when capacities are not sufficiently big.

And then we have done in decreasing returns to scale in the production function. And then if we have price competition and decreasing returns to scale, then what is the outcome we see that there is a whole range of price that exist, okay. It is so, the Nash equilibrium, pure strategy Nash equilibrium is not unique.

Then we have done Stackelberg; both quantity competition and price competition and here we have androgenize, we have also looked not androgenize, we have look whether there is a first mover advantage or there is a second advantage. And we have got that in quantity competition there is first mover advantage and in a price competition if the goods are substitute theory the second mover advantage, but if the goods are complement then we see that there is a second mover advantage.

After that we have introduced a hoteling kind of model where we have done the product differentiation and we have done to under 2 setup that is, when the price is, when the firms choose the location simultaneously and when firms choose their locations sequentially. And we have found the pure strategy Nash equilibrium in the first case and the subgame perfect Nash equilibrium in the second case.

Next we have introduced entry deterrence; entry deterrence to see whether a firm is deterring because that may lead to a monopoly power in the firm. So, mainly it is important from the perspective of the regulatory authorities. Regulatory authorities always see whether the existing incumbent firm is engaged in some form of entry deterring strategies or not. And strategic investment can be a possible entry deterring strategy we have seen that, but it is not always, not in all the cases, only in certain cases.

Then, today we have done the bundling and the tying and there also we have shown the pure strategy Nash equilibrium, only in case of tying when there is a, we have introduced two firms, but in case of bundling we have only done it for monopoly thing, okay. So with this, I hope that you have enjoyed this course. Thank you.