Introduction to Market Structures
Professor Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati
Module 11
Lecture 40
Tutorial

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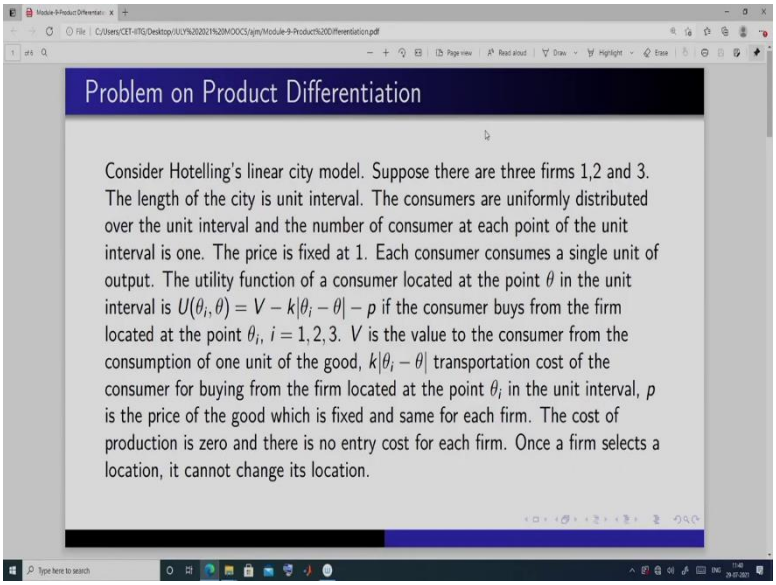

So, let us solve 1 problem on product differentiation and in this problem what I have done? I have combined both the sequential and the simultaneous decision. Because in the case of product differentiation we have done separately 1 section on where firm decides the location simultaneously. And in another section we have done firms decides the location sequentially, okay. So, now here in this question is these 2 things are combined.

So, let us look at the problem. So, consider Hotelling linear city model. Suppose there are 3 firms, firm 1, 2 and 3. The length of the city is unit interval. The consumers are uniformly distributed over the unit interval and the number of consumers at each point of the unit interval is, okay 1. So, it is specification is same till now. The price is fixed at 1 and each consumes a single unit of the output.

Utility function of a consumer located at the point theta in the unit interval is this-$U(\Theta_i, \Theta) = V - k|\Theta_i - \Theta| - p$ if this V that is the value the consumer gets from the consumption of that good and this is the, disutility that it derives from by travelling some or making some distance travelling some distance to buy that product.

And this is the price, price is 1, so it is equal to 1. If consumer buys from the firm located at point theta i when whereas the consumer is located at the point theta, okay. And theta I will take depending on the location from which firm it is buying it will give the utility. And V is the value to the consumer from the consumption of 1 unit of the good. And this is the transportation cost of the consumer for buying from the firm located at point theta i. Interval and p is the price 1 which is fixed and same for each firm, okay.

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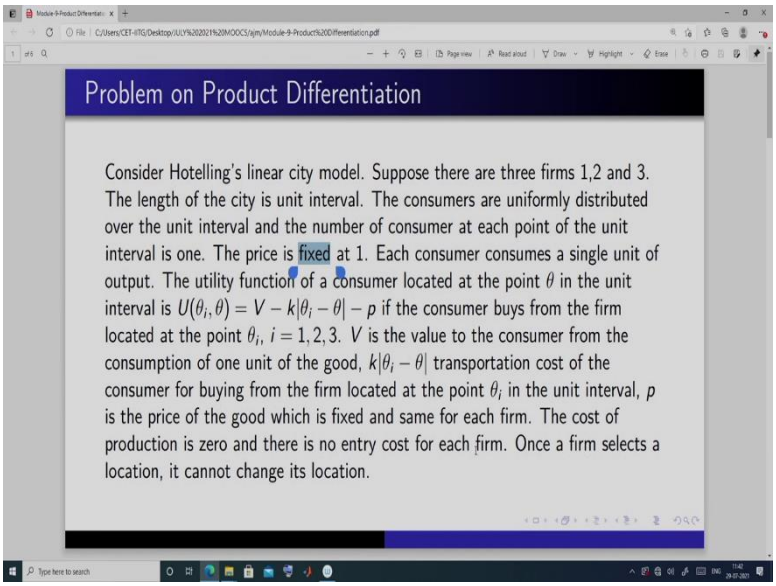

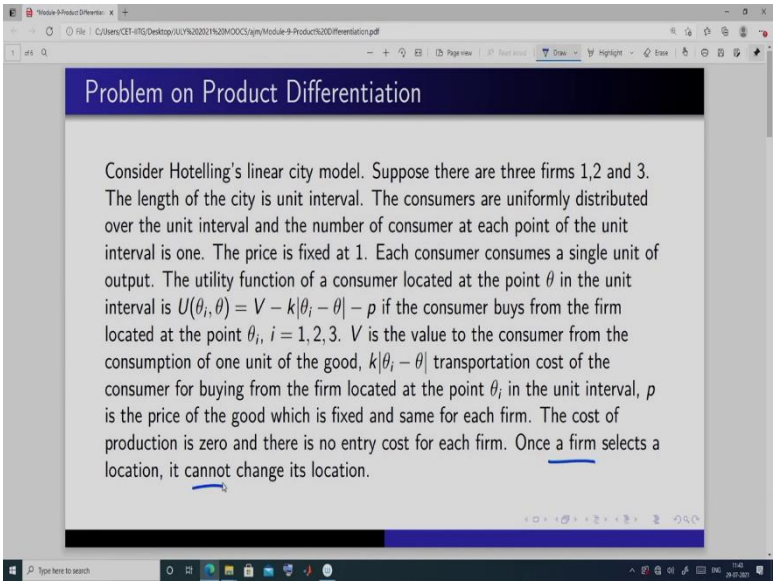

It is fixed at this and the cost of production is 0 and there is no entry cost because here the firms are also in entering sequentially 1 after another, so no entry cost. And once firm select a location it cannot change his location. So, once its chooses the location it fixed like firm 1 when it

moves fast, if it chooses the location and then when firm 2 and 3 they enters it cannot change his long it will be there.

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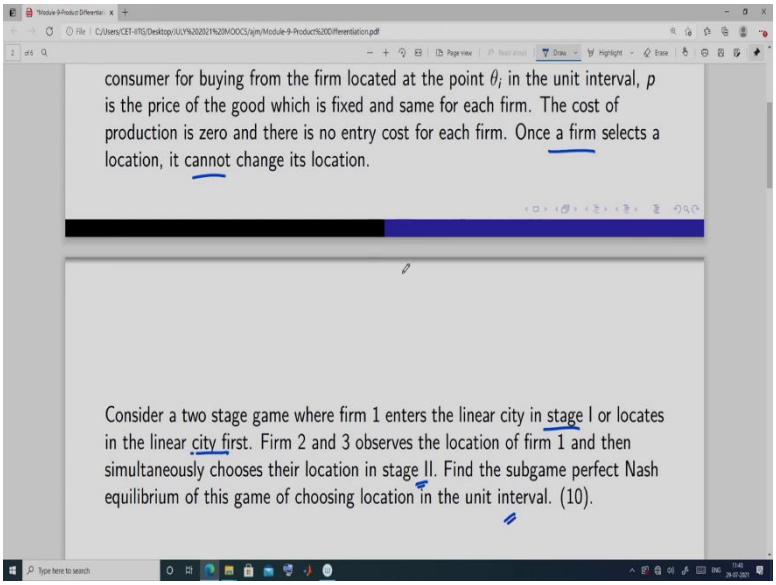

Now, consider a 2-stage game where firm 1 enters the linear city in stage 1 or locates in the linear city first firm 2 and 3 observes the location of firm 1 and then simultaneously choose their location in stage 2. So, firm 2 and 3 they are moving simultaneously. Find the subgame perfect Nash equilibrium of this game of choosing location in the unit interval, okay.

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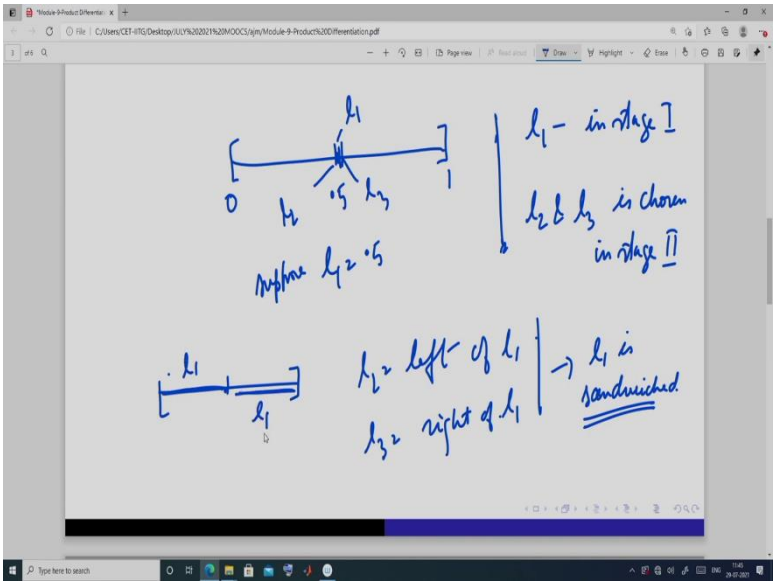

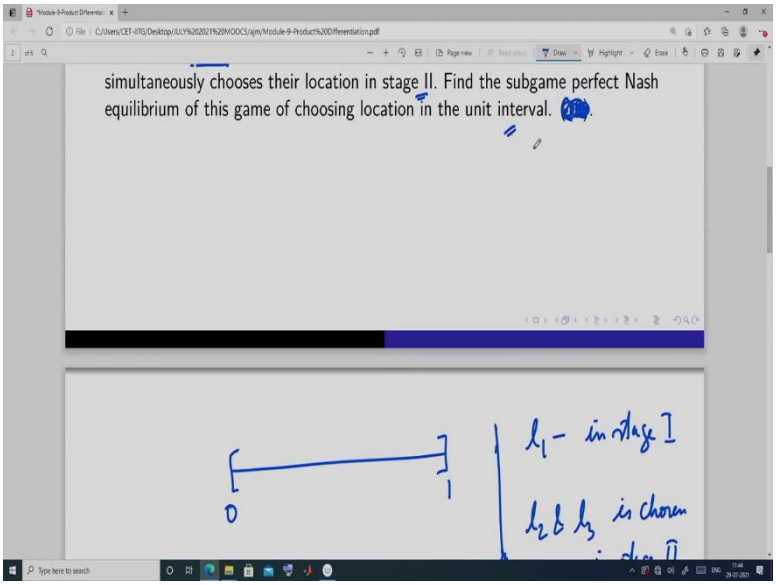

Now, let us solve this. So, this is the unit interval, okay. Now, what happens l1 is chosen in stage 1, l 2 and l 3 is in stage 2? okay So, we will solve this using backward induction and we have to find the subgame perfect Nash equilibrium, okay. So, what we are going to do? So, first suppose l 1 is this or let us solve this. Suppose this is suppose l 1 is at this point, 0.5 this is the middle and l 1 is here. Then in stage 2 firm 2 can choose this and firm 3 can choose this position.

So, this is l 2 and this is l 3. So, l 2 is to left of l 1 and l 3 is right of l 1. So, l 1 is sandwich, sandwich between these 2. So, when there it is sandwich, so it means what? It is not getting any consumers. So, its profit is 0, right? So, it will not select this. So, from here we get that it will be either in this or it will be in this, okay. It does not matter.

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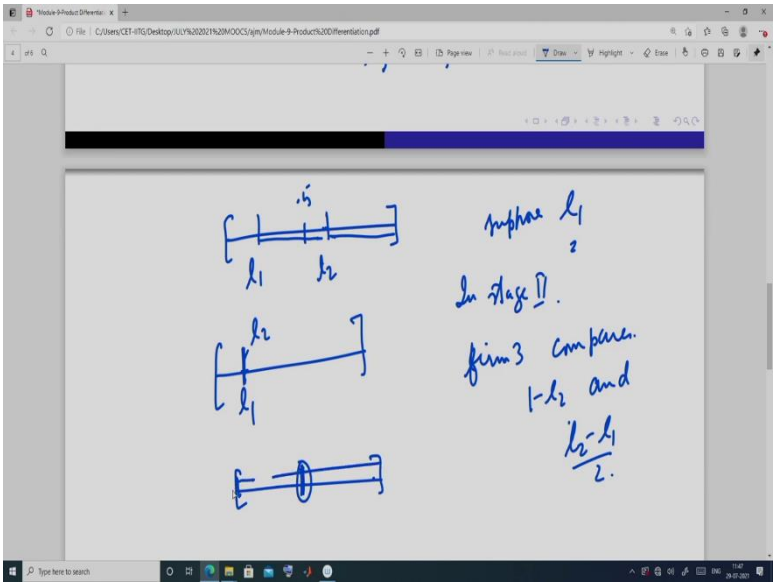

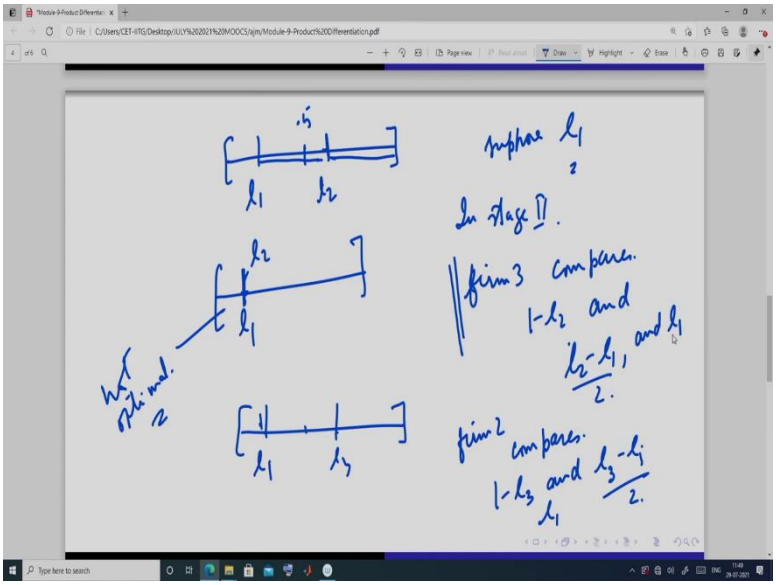

Now, suppose l 1 is here. Now, in stage 2 what firm what is happening firm 2 and 3 are entering simultaneously. Now, if suppose firm 2 selects this, okay firm 2 select this. And firm 3 while deciding since its output because it is deciding simultaneously along with it. So, it will select if its it will compare this half of this and it will compare this. So, it will firm 3 compares 1 minus l 2 and l 2 minus l 1 half of this because if firm 2 its locate suppose here, l 2 is here right of this. Then the best response of firm 3 is to locate here, okay.

Because you can simply use the argument that you have got from when there are 2 firms and suppose unit interval we have got this you get at the middle. Because if it locates here the other firm also locate here and it gets the whole. So, that is why this is not i. So, you can argue in that way. So, in this line here it is this is not optimal. So, if this is not optimal, then it will look at somewhere here. So, it will it has chosen suppose this l 2 is this. If l 2 is this, then firm 3 will compare this- $1 - l_2$ and $\frac{l_2 - l_1}{2}$.

Now, because l 2 and l 3 is being chosen simultaneously. So, if this is l 1 and suppose this is l 3 because l 3 is not going to be here again same using the same argument. Then firm 2 compares 1 minus l 3 this distance and it compares this half of this- $\frac{l_3 - l_1}{2}$ and it will also compare this because suppose it will compare this this and it will compare l 1. If l 1 is greater than it will locate here and if here also it will compare l 1. So, out of these 3 lengths it will choose the 1 which is the maximum.

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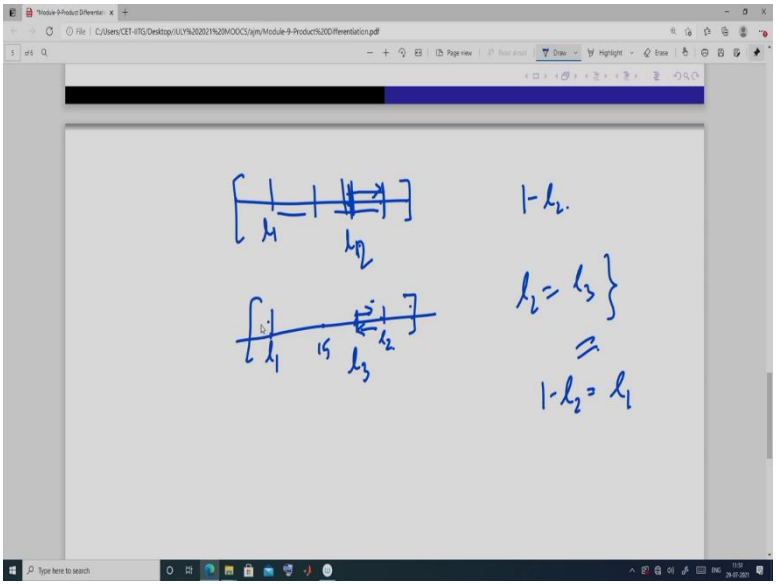

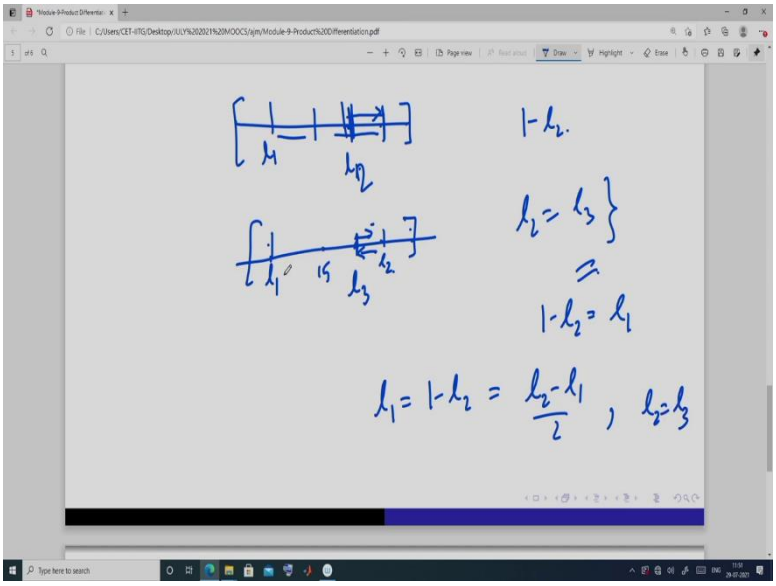

So, from this argument we get that firm 1, so given in location l 1. Firm 2 and 3 they will always try to see. If this is l 1 and see this is l 1, this is l 2 then firm 1, 2 will either look at here or it will look at here. Because if it locates here, then best response of firm 2 is not to here but to shift till this point, because it will get a more a. And so, l 1 is this, this is 0.5 and if this is l 3 then l 2 will locate here along with this l 3. So, this is and I hope it is clear.

Because if it is l 2 is here then firm 1, 3 will move this or if l 3 is there, firm 2 will move will gain by moving in this direction. So, that is why l 2 is equal to l 3, this is true. Now, when this is true? If we from this diagram because this length is very small, so we get that this should be true. Now, this would be true when? When 1 minus l2 this is equal to l 1 this distance is equal

to this distance-$1 - l_2 = l_1$. So, from this we get a condition that l 1 must be equal to 1 minus l 2 and l 2 minus l 1 and l 2 is equal to l 3.

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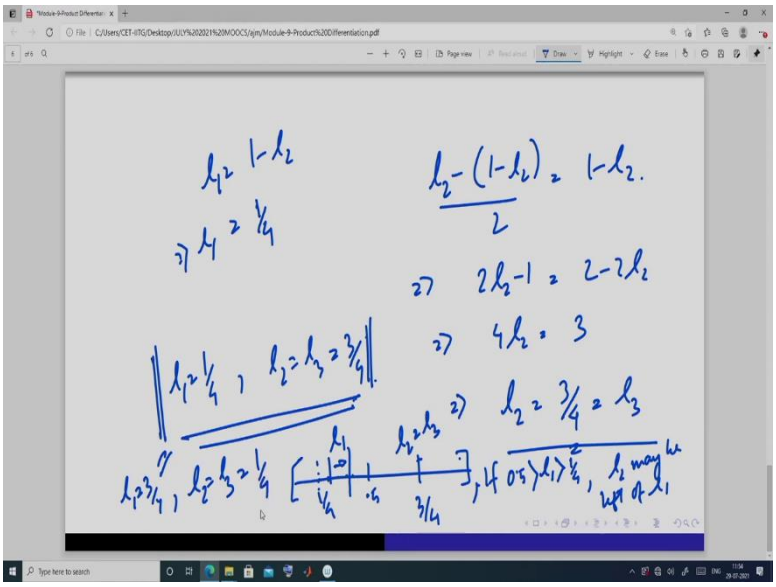

So, and when we solve this what do we get? We get this-$\frac{l_2-(1-l_2)}{2} = 1 - l_2$, so we get 3 by 4 and this is equal to l 3 and l 1 is equal to 1 minus l 2. So, l 1 is equal to 1 by 4. So, l 1 is equal to 1 by 4, l 2 is equal to equal to l 3 it is equal to, so this is another-$l_1 = \frac{1}{4}, l_2 = l_3 = \frac{3}{4}$. So, this is a subgame perfect Nash equilibrium. So, what do we get? We get this 1 by 4 is going to be l 1 is 3 by four is going to be equal to l 3. Because if you look at any other point you will see that there is it is not an Nash equilibrium, why?

Because if it moves here slightly less than this. So, then what is happening? l1 is reducing, so there is should not move here. But if it moves slightly here in this, so if it is here suppose l 1 then what is happening? This length is greater than this length, so this firm will shift here and there firm will shift here. So, it will be like this. So, if l 1 is greater than 1 by 4 and it is less than 0.5 then again what is going to happen? This l 2 will be either, so l 2 may be left of l 1.

So, again firm 1 may get sandwiched, so that is why it should not move here. So, that is why this is 1 subgame perfect Nash equilibrium. And we may have another subgame perfect Nash equilibrium which is l 1 is equal to 3 by four and l 2 is equal to l 3 and it is equal to 1 by 4. So, we will have only these 2. Now, here you will see what is happening? So, this model is about product differentiation and this is about horizontal product differentiation. So, here we will see that 1 firm is differentiating and the other 2 firms are producing homogeneous product. They

are not differentiating, but they are differentiating with respect to the product of firm 1, okay. Thank you.