Introduction to Market Structures

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Lecture 4
Tutorial

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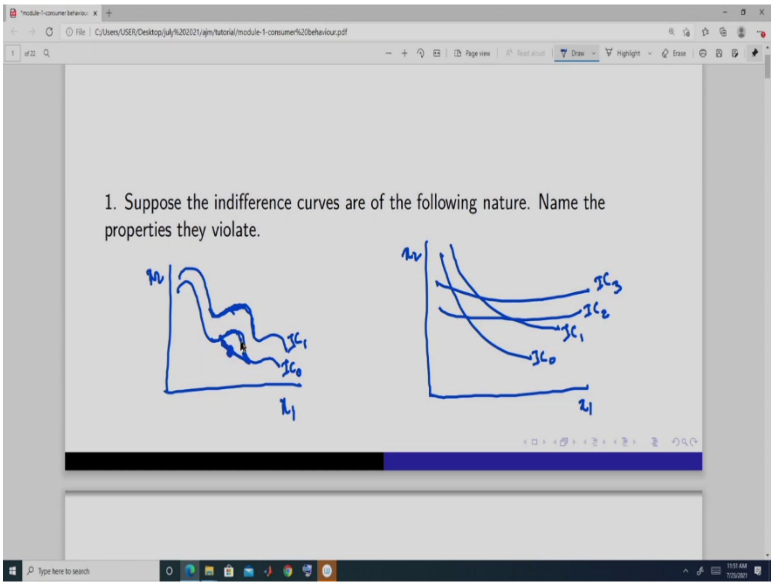

So, let us discuss few problems related to consumer behaviour and demand curve. We have already discussed this topic. So, now let us do some problems. Now, the first problem is suppose the indifference curves are of the following nature and name the properties they violate. So, there are two good x1, x2 and suppose the indifference curve is of this nature, this is IC_0, IC_1this is 1.

Another is it was again two good, good 1, good 2, and it is like this, this is IC_0, IC_1, $IC_{2,}$ IC_3. So, these are the indifference curves. So, we have to say or name the properties they violate. So, in this case what do we see? We see that the indifference curves is like this it is upward sloping, here also it is upward sloping and here it is like this. So, if you take suppose take this point and this point take any linear combination, we do get what this point is less preferred then this point or this point because it lies in a lower indifference. So, this violates 2 properties.

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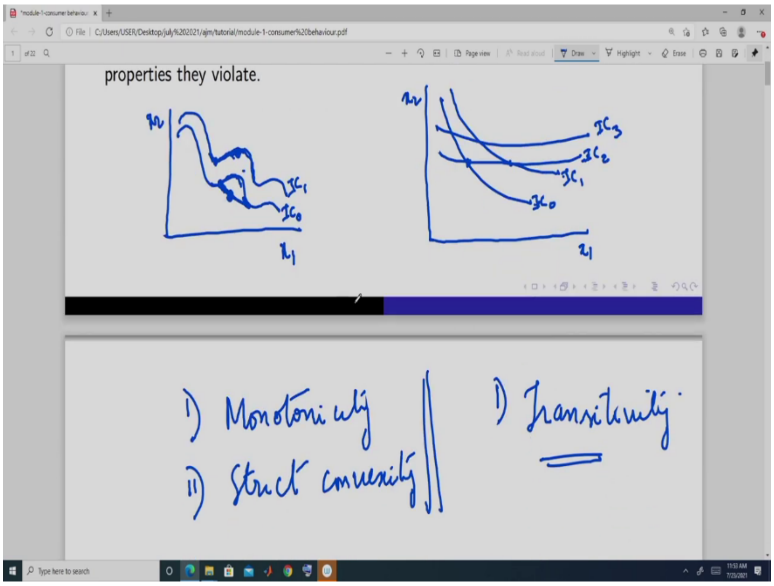

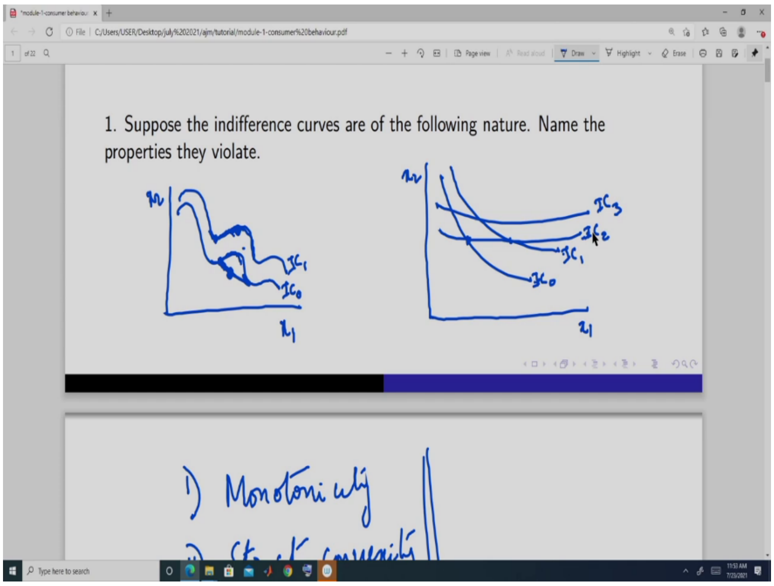

One, it is first is monotonicity, monotonicity means if X and Y if x1 and x2 both are higher than the utility should be higher, here in this point, both are higher than this point, but we are getting same utility. So that is why it violates monotonicity. Second here, it violates strict convexity of the preference, how? Here when we are taking a linear combination of this point and this point, we get any point in the straight line.

And those points are less preferred than these 2 extremes. So, it is just opposite of extremes are preferred than the average. So, that is why it violates strict convexity. So, these 2 are violated by this set of indifference curves. Now, here, if you look at these are the Ic curves and they gives different levels of utility and the utility is increasing in this direction, we know that.

So, this point, so these indifference curves intersects and we have seen, we have discussed this if they intersect, then they violate what is called transitivity. So, the second type of indifference curves violates transitivity.

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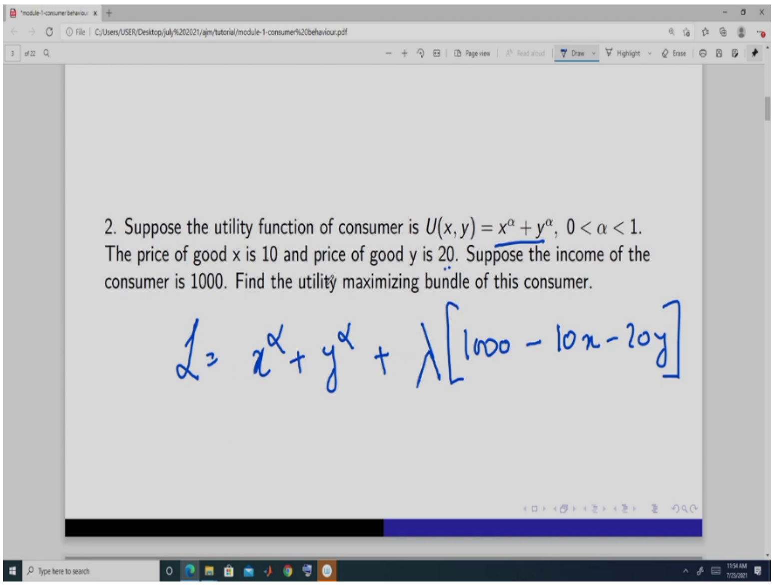

Next, let us look at this problem. This suppose we have a consumer and it consumes 2 good x and y and the utility function is this - $U(x, y) = x^{\alpha} + y^{\alpha}$. Price of good x is 10. Price of good y is 20. And income is 1000. We have to find the utility maximizing bundle of this consumer. So, let us since all of these are differentiable, so let us say the Lagrange and the Lagrange is this- $L = x^{\alpha} + y^{\alpha} + \lambda[1000 - 10x - 20y]$. This is the Lagrange multiplier. And we can write it, this is the budget constraint and this is the utility function. Now we maximize this.

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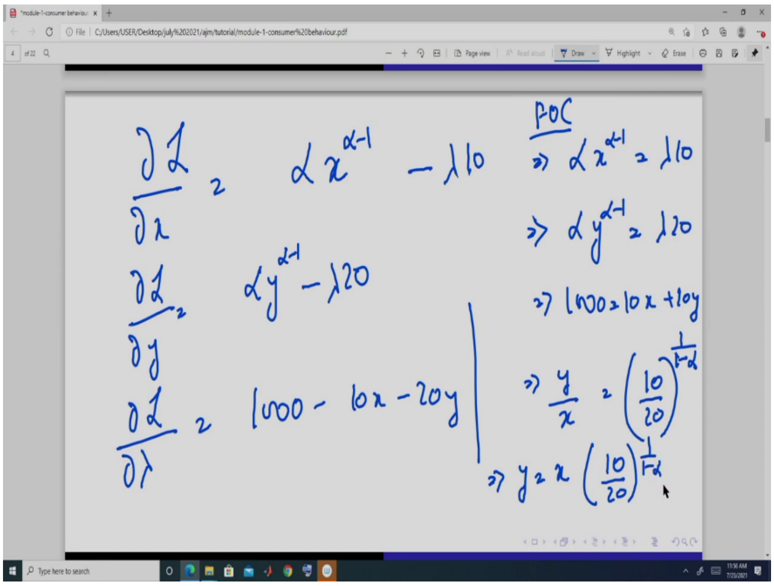

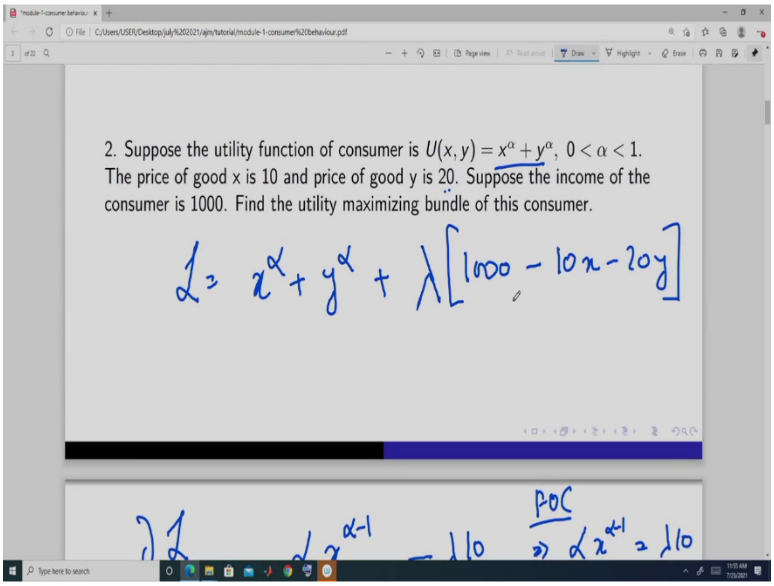

So, what do we do? We take the derivative with respect to x and what do we get? We get this, what do we get? It is 20 and now first order condition, first order condition will imply that this is equal to, i.e $\alpha x^{\alpha-1} = \lambda 10$, this will be equal to - $\alpha y^{\alpha-1} = \lambda 20$and this is equal to- 1000=10x+ 20y , get this. Now, from these two equations we get x, because here this alpha lies between 0 and 1. So, alpha minus 1 it is less than 1. So, I can write it in this way and this portion will be, sorry, this will be y by x actually, okay. So, now, here from this we get to y is equal to x , i.e $y = x\left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}$this.

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$$1000 = 10x + 20 \cdot x \cdot \left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}$$

$$= x\left[10 + 20 \cdot \left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}\right]$$

$$\frac{1000}{\left[10 + 20\left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}\right]} = x$$

$$y = \left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}} \left[\frac{1000}{10 + 20 \cdot \left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}}\right]$$

2. Suppose the utility function of consumer is $U(x, y) = x^\alpha + y^\alpha$, $0 < \alpha < 1$. The price of good x is 10 and price of good y is 20. Suppose the income of the consumer is 1000. Find the utility maximizing bundle of this consumer.

$$\mathcal{L} = x^\alpha + y^\alpha + \lambda\left[1000 - 10x - 20y\right]$$

$$\frac{\partial \mathcal{L}}{\partial x} = \alpha x^{\alpha-1} - \lambda 10 \quad \Rightarrow \alpha x^{\alpha-1} = \lambda 10$$

$$\frac{\partial \mathcal{L}}{\partial y} = \alpha y^{\alpha-1} - \lambda 20 \quad \Rightarrow \alpha y^{\alpha-1} = \lambda 20$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 1000 - 10x - 20y \quad \Rightarrow 1000 = 10x + 10y$$

$$\Rightarrow \frac{y}{x} = \left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}$$

$$\Rightarrow y = x\left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}$$

Plug in this in this budget constraint and you will get first order condition gives, it will be this x, and the next one, so, x is and this. So, this is the demand for x and demand for y you simply put the value we know this is 10 by this, this into this. You can simplify I am not simplifying this any further. So, this is the demand for y, okay. So, we can find out the utility maximizing bundle is given by these 2, x is equal to this - $\frac{1000}{10+20\left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}}$ and y is equal to this.-

$\left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}} \frac{1000}{10+20\left(\frac{10}{20}\right)^{\frac{1}{1-\alpha}}}$

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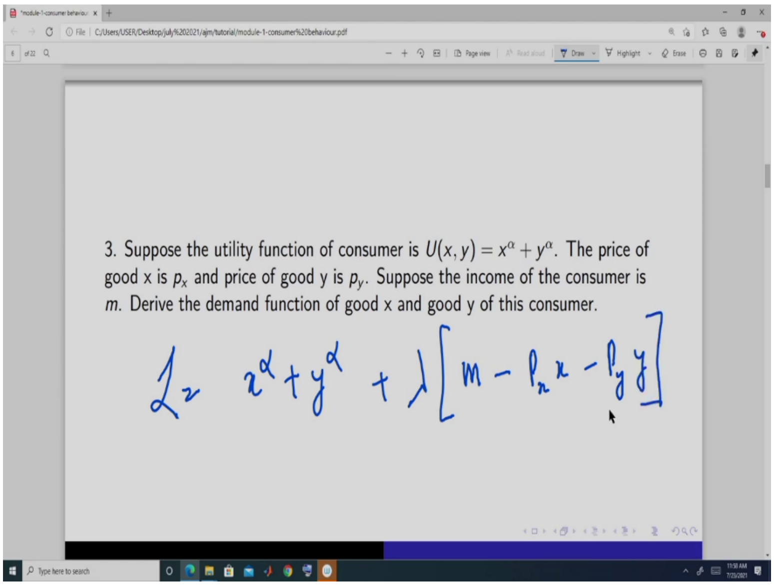

Now, let us take another example here. So, this we have kept the same utility function only we have now take, we have not taken any specific number for this prices and income, so, that we can derive the demand function. So, in this case our, we will follow the same thing Lagrange is going to be same. Here it will be- $L = x^{\alpha} + y^{\alpha} + \lambda[M - P_x x - P_y y]$

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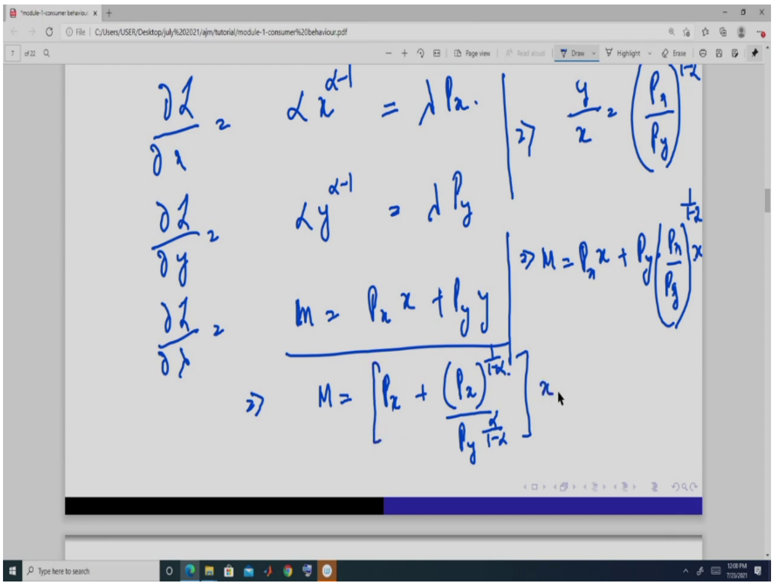

It is going to be this and the first order condition. This will give me- $\frac{\delta L}{\delta x} = \alpha x^{\alpha-1} = \lambda P_x$, $\frac{\delta L}{\delta y} = \alpha y^{\alpha-1} = \lambda P_y$, $\frac{\delta L}{\delta \lambda} = m = P_x x + P_y y$, , from these two I get this- $\frac{y}{x} = \left(\frac{P_x}{P_y}\right)^{\frac{1}{1-\alpha}}$. Next plug in this here, here, we will get m is equal to px plus py, i.e $M = P_x x + P_y y$.so, from here we will get, into x, i.e $M = P_x x + P_y \left(\frac{P_x}{P_y}\right)^{\frac{1}{1-\alpha}} . x$. So, this will give me m is equal to Px. So, Px here. So, it will be the Py. So, here Py is to the power 1 by 1 minus alpha and here it is power is 1. So, it will be we can write this alpha and this Py is this x- $M = \left[P_x + \frac{(P_x)^{\frac{1}{1-\alpha}}}{P_y^{\frac{\alpha}{1-\alpha}}}\right]. x$

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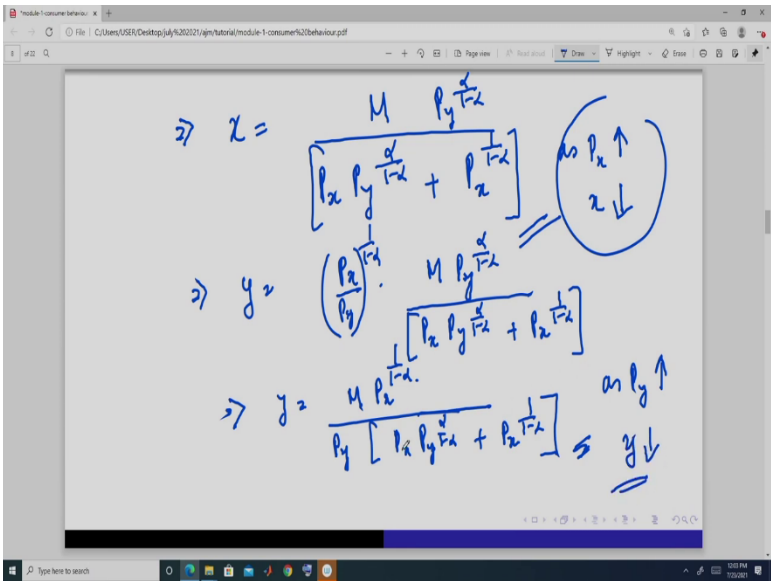

So, the demand function of x is m divided by Px to the power Py plus Py, Px to the power 1 minus alpha and Py,. So, this is the demand function of good x - $x = \frac{M P_y^{\frac{\alpha}{1-\alpha}}}{P_x P_y^{\frac{\alpha}{1-\alpha}} + P_x^{\frac{1}{1-\alpha}}}$ and we see as and it is downward sloping in Px. So, as Px increases, x falls because it is at the denominator of this expression that is why. Now, y we know is Px into x, so, x is this, it is this. Now, here from this we can get y, because here the power is 1 by 1 minus alpha and this is Py alpha to the power 1 by alpha.

So, this we can write m Px to the power 1 minus alpha. So, this will be Py. So, it is going to be this one, is the demand function of good y, i.e $y = \frac{M P_x^{\frac{\alpha}{1-\alpha}}}{P_y[P_x P_y^{\frac{\alpha}{1-\alpha}} + P_x^{\frac{1}{1-\alpha}}]}$, and you can see that the y's are in the denominator. Here Px are in the denominator, so, that is why it is this. Here again Py is in the denominator. So, that is why as Py increases demand also, it is again a downward sloping demand curve. So, we have got this.

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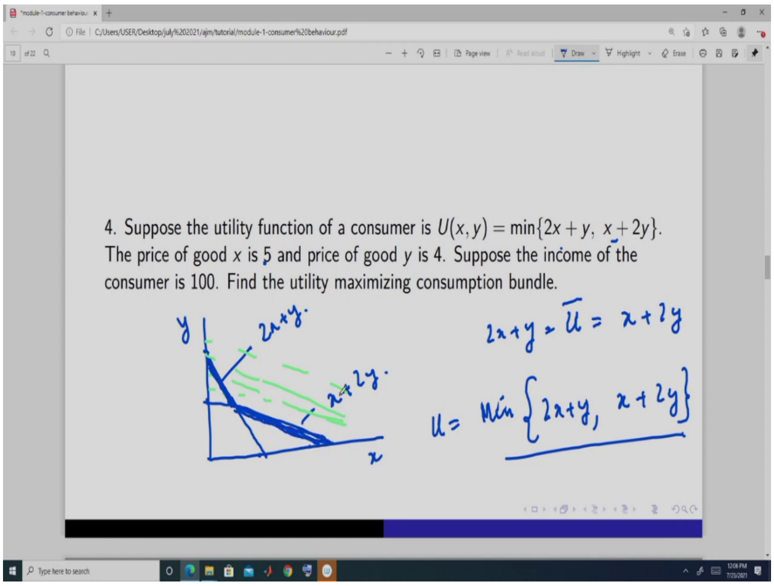

Now, let us do a slightly complicated utility maximizing problem. So, suppose the utility function is given in this form, price of good x is 5, price of good y is 4. If we are given this- $U(x, y) = min\{2x\ +\ y,\ x\ +\ 2y\}$, now this is mean of this expression. Now, if we take x, here y here, this suppose is equal to some u, and this is equal to some utility level fix. So, this is what? This is going to be this y is going to be u then this is going to be u bar by 2, this is if it is this is u.

So, it is this like this. Here, this is going to be u bar and this is going to be u bar by half. So, this line, this, so, this is 2x plus y and this is x plus 2y and we are given min of 2x plus y, utility function is this. Now, here if you look at these curves, these curves, so, what is happening? Level curves are increasing. So, if we are moving above this, this line, then min is given by this point, this line.

So, it is going to be this, because otherwise it is increasing this side is higher. Now, if we look at these curves like this its level curves are increasing, but this level curves is same as this point. So, that is why it is going to be like this. So, this utility function is can be graphically represented in this form.

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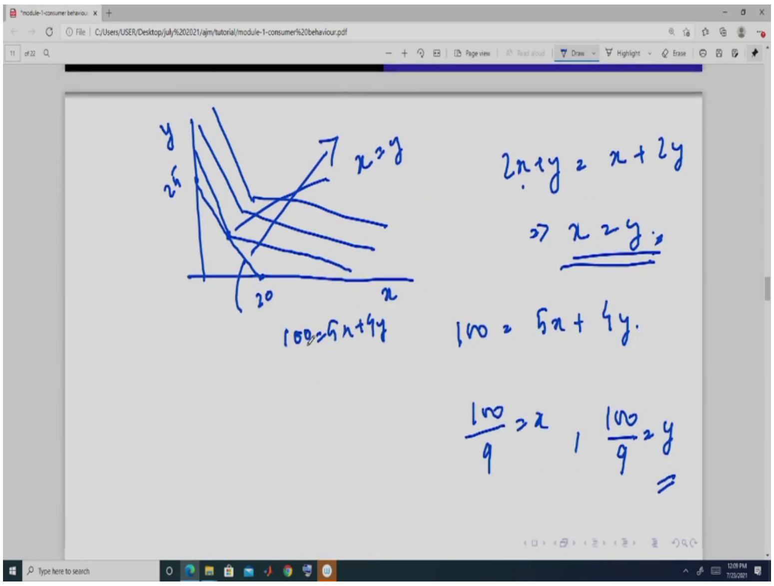

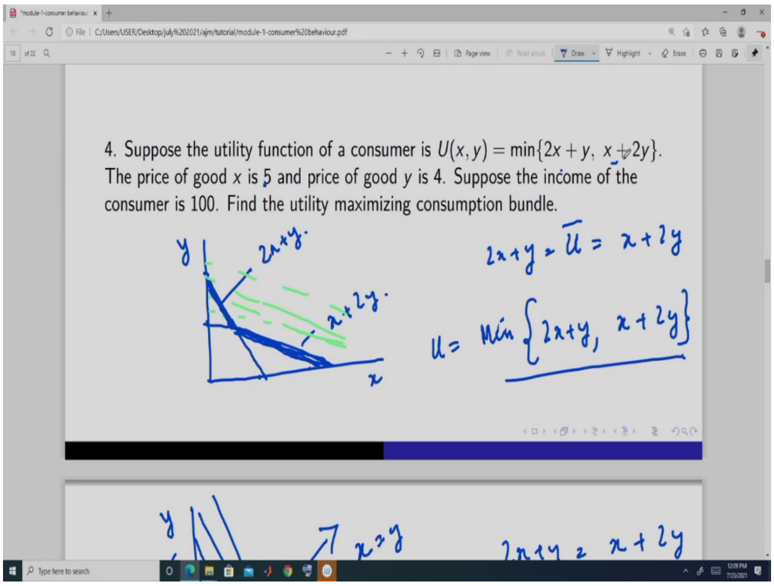

So, it is something like this. If it is x is here, y is here, it is this and it is this, this point is x is equal to y because 2x plus y is equal to x plus 2y and this implies x is equal to y, like this. So, utilities are increasing these are the Ic curves indifference curves. So, good is this 4, budget constraint is 100= 5x plus 4y. So, this point is 20 and this point is 25. So, we get a, if this is 20, then this is 25 this is, so, we will get a curve like this.

This is here, this is the and here, because if this budget line this slope is same as the slope of this then it would have match this, but this slope is less. If this slope is same as this then it would have match this, but this slope is higher than this slope. So, this point is only going to intersect here, not intersect it is going to be tangent to this point. But if this matches to any one of this then we will get different here, but here it is not matching.

So, in this case the optimal point is always going to be x is equal to y and here what we are going to get 10 by this and this is going to be the, this is 9, this is 9, okay. But here the main problem is or the difficult part is this how to draw the indifference curves, okay.

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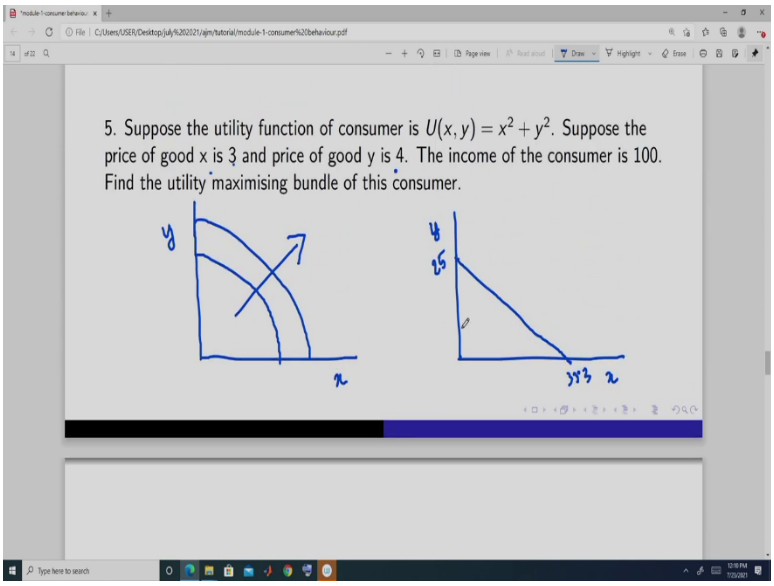

Once you can draw the indifference curve you will be able to find the a. Now, if we are given a utility function of this nature- $U(x, y) = x^2 + y^2$, x is here. If we are given this it is going to be somewhere here, utilities are increasing in this way. Price of good x is 3, price of good x is 4, income is 100. So, our budget it is 4, so, it is 25 and here it is 3, so, it is going to be 33 point something 33, this. Now, how to find the optimal point we see.

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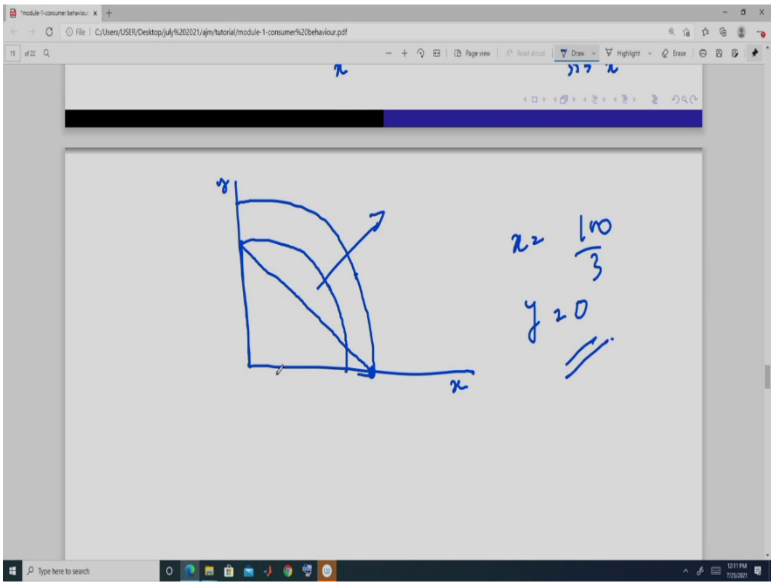

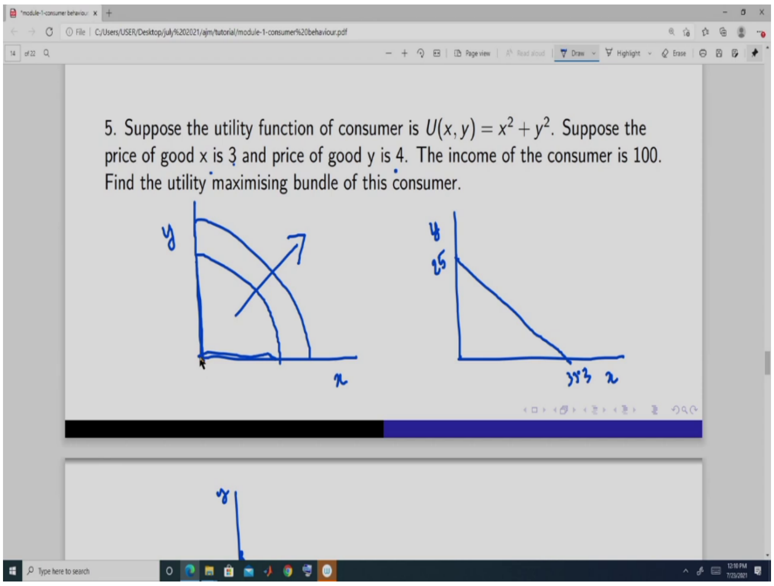

Here, so, the intercept this is less than the intercept here of the budget line, okay. And these indifference curves are such that these intercepts are seen. So, if I draw a, it will be somewhere here and if we draw here it will be here. So, this is at a higher level than this, since utility increases in this way, so, the optimal point is this. So, x is equal to 100 by 3 and y equal to 0, this is the optimal bundle in this case.

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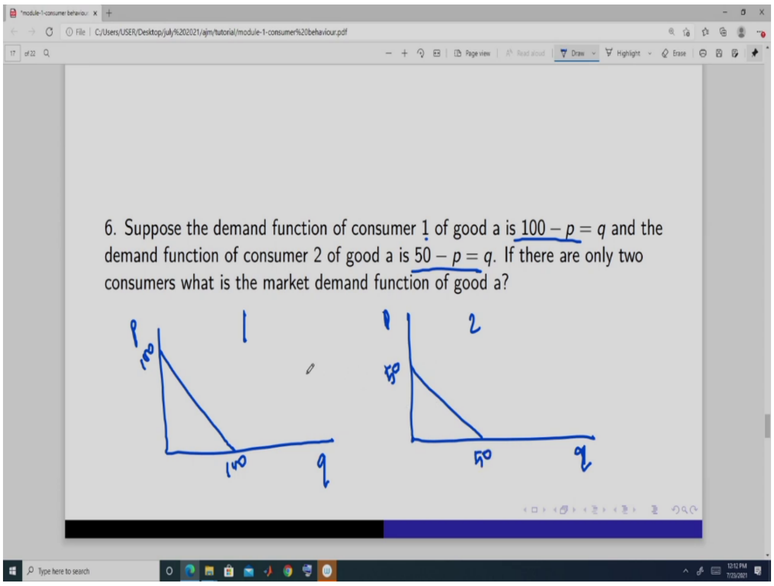

Now, suppose we are given 2 demand curves. This is for consumer – 100-p=q and this is for consumer 2- 50-p= q. Consumer one’s demand curve is you can say if you plot q here, p here it is 100, 100. Consumers 2, this is 1, consumer two’s is q here, it is 50 and 50, right. Now, what is going to be the market demand curve is there is only these 2 consumers?

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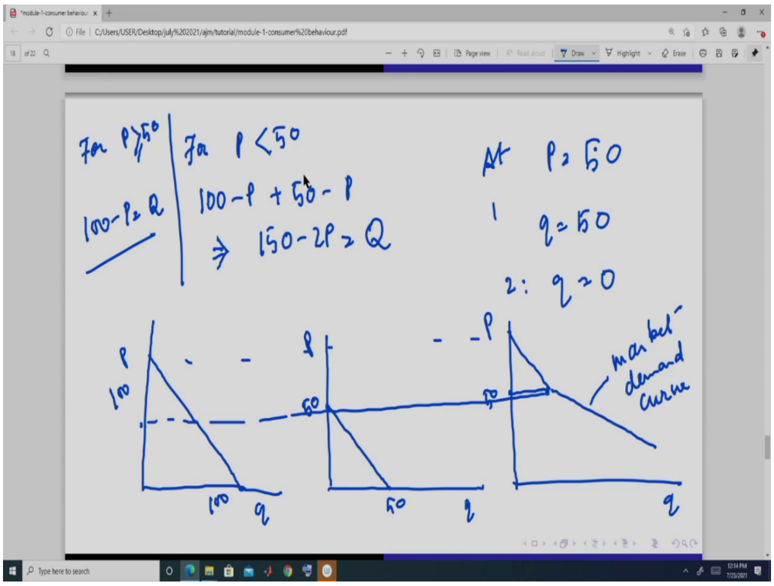

So, at P is equal to 50, what is the demand for consumer 1? Consumer one’s demand is 50 and what is the demand for consumer 2? It is 0. So, we are going to do simply a horizontal summation of the demand curves, 50. So, it is going to be 50. So, it is till this point it is this. Okay, I should have drawn this slightly, okay, like this and after this I am going to add this too. So, it is going to be so, for P less than 50 it is going to be 100 this plus 50, this is which is equal to 150.

This is going to be the total demand and if this we will get something like this. So, this is going to be the market demand curve. So, if we write it in a compact way, it is this. So, for this and for P greater than or equal to 50 it is, this is the market- 100-P= Q and for P less than this we are going to get this- 150-2P=Q, okay, this is the market demand.

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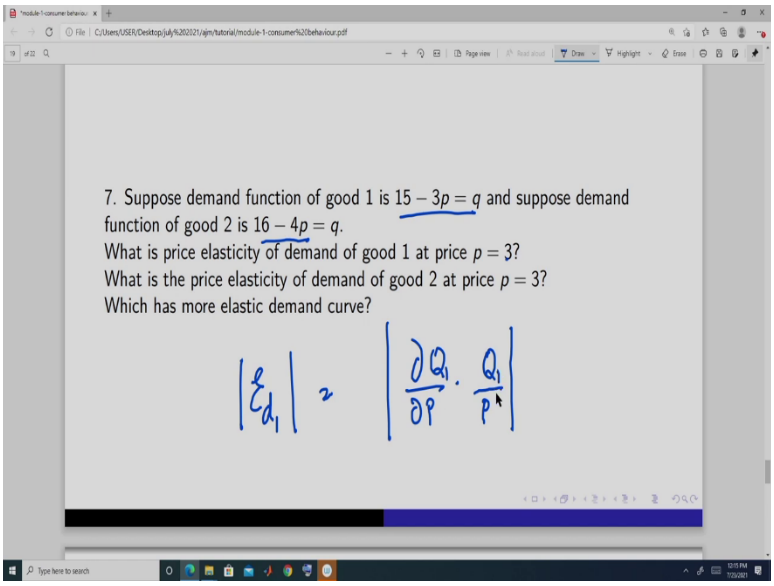

Now, suppose we are given a demand function this- 15-3p=q, this is of good 1. And suppose the demand function of good 2 is this- 16-4p=q. We have to find the price elasticity of demand of good 1 and good 2 and price and price history. So, we know the elasticity of demand is-$|E_{d_1}| = |\frac{\delta Q}{\delta P} . \frac{Q}{P}|$, get this, right. So, this if we simply take derivative of this, the demand function 1.

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$$15 - 3p = q \quad , \quad \text{At } p = 3, \; q = 6$$

$$\frac{\partial q}{\partial p} = -3$$

$$= -3 \cdot \frac{3}{6}$$

$$\frac{\partial q}{\partial p} = -3 \qquad q = 6$$

$$= \left| -3 \cdot \frac{3}{6} \right| = \underline{\underline{1.5}}$$

$$|\varepsilon_{d_1}| = 1.5$$

7. Suppose demand function of good 1 is $15 - 3p = q$ and suppose demand function of good 2 is $16 - 4p = q$.
What is price elasticity of demand of good 1 at price $p = 3$?
What is the price elasticity of demand of good 2 at price $p = 3$?
Which has more elastic demand curve?

$$|\varepsilon_{d_1}| = \left| \frac{\partial Q_1}{\partial P} \cdot \frac{P}{Q_1} \right| = \left| \frac{\frac{\partial Q}{Q_1}}{\frac{\partial P}{\partial P}} \right|$$

So, what do we get, demand function is this, right and we have to find that P is equal to 3. So, at P is equal to 3, q is, is equal to 6 and derivative of this q is equal to minus 3. So, plug in minus 3 at 3, 6. So, here it will be 3 and this will be, sorry, I have made a mistake in this is P divided by Q, because it is, this is percentage change in quantity and this is the percentage change in price, okay.

So, that is why we get this. So, this is equal to and when we take the modulus it will be equal to 3. So, one minute, it will be 3 by 2, so, it is 1.5. This is the elasticity of good 1 is equal to 1.5.

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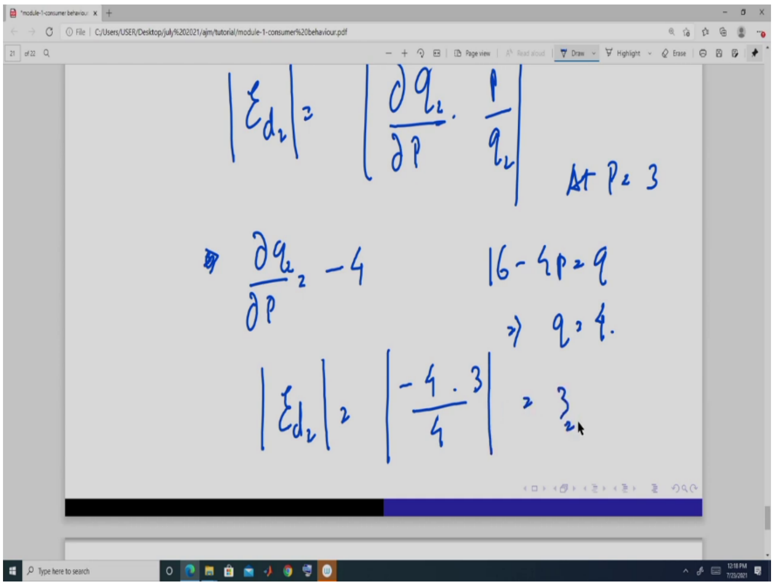

Similarly, elasticity of good 2 at this is, this- $|E_{d_2}| = |\frac{\delta Q_2}{\delta P} \cdot \frac{P}{Q_2}|$, at price is equal to 3, demand function is, so, it is q is equal to 4. So, and the slope is minus 4. So, this is.

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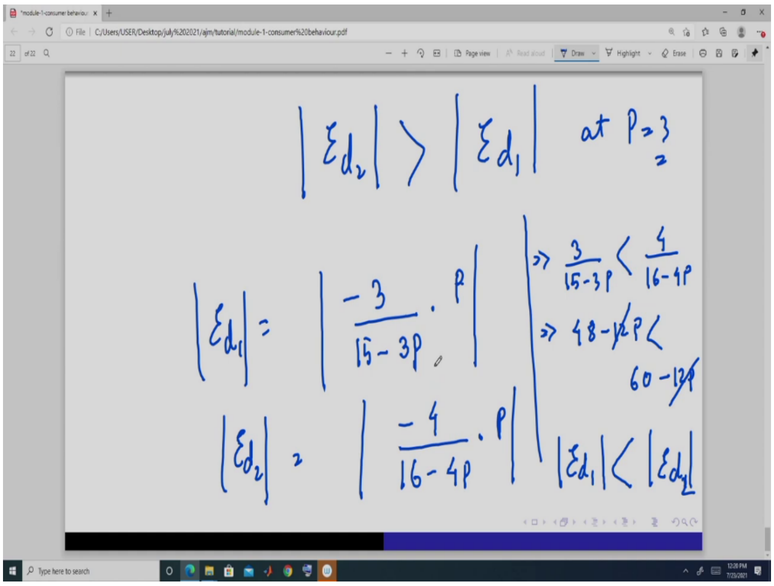

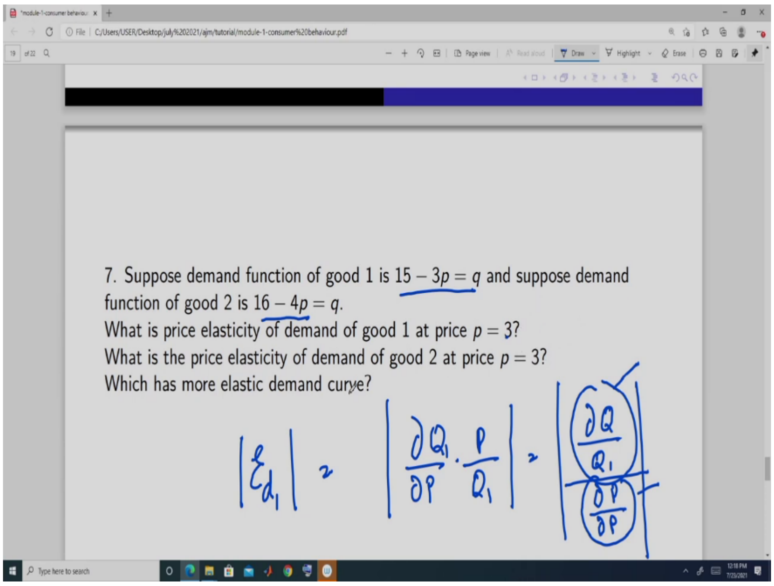

So, what do we get? We get elasticity of good 2 is greater than elasticity of good 1 and price is equal to 3. Now, can you say about the whole demand curve which one is more elastic? So, we have find out the elasticity. So, for this it is 1.5 for this it is 3 so, it is greater and which is more elastic. Now, these we can simply use the formula. So, slope of this is so, we can write this for a general price is this, which is this- $|E_{d_1}| = |\frac{-3}{15-3p}.P|$, it is this- $|E_{d_2}| = |\frac{-4}{16-4p}.P|$.

Now, we compare this, if we compare this what do we get? So, this removed the, so, it is 3 by, suppose, it is of this nature, okay, this, so, we get this is 48 minus 12P, 60 minus 12P this cancels and 60 is greater than 40, so, we get this is greater than this. So, we get that for each price elasticity of good 1 is less than the demand elasticity of good 2, we get this. So, these are few problems. So, you will get similar kind of problems in the assessment and question in the exam, **okay.** Thank you.