Introduction to Market Structures
Professor Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati
Module 11: Product Differentiation and Entry Deterrence
Lecture 38
Sequential Move Hotelling Model

Hello, welcome to my course, Introduction to Market Structures. So, we were doing product differentiation.

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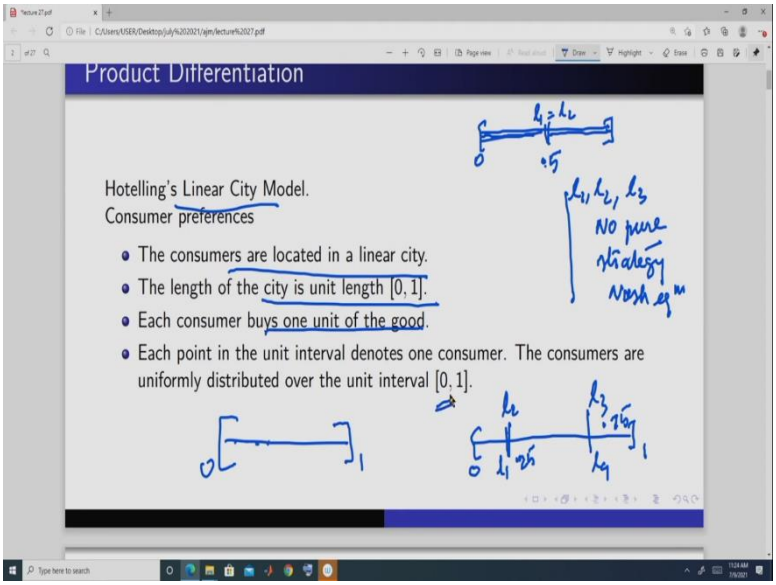

And we have done Hotelling's Linear City Model. And in the Hotelling's linear city model in the last class, what we have done? We have done that the firms enter, or firms chooses the location simultaneously. So, we have done 3 cases. When we have 2 firms, what we have got? When we have 2 firms, we have got that the firms choose the same location and that is in the unit interval. They will choose 0.5, 0.5. So, l1 is 0.5 and also l2 is 0.5.

So, firm 1 will share this, will get this market share and firm 2 will get this market share. And in case 3, when we have 3 firms, we have got that l1, l2, l3, we have got that there is no pure strategy Nash equilibrium, when we have 3 firms. And but when we have 4 firms, we have shown that when we have 4 firms, we get that firm 1 and firm 2 are going to choose the same location and that is 0.25 and firm 3 and firm 4, they are going to choose the same location and that is 0.75, in this 0 1-unit interval, okay.

So, this is one of the Nash equilibrium but there can be different combinations and you can get it. Different Nash equilibrium. So, this much we have done in the Hotelling model. Today we are going to do Hotelling model, but the firms are going to enter sequentially. So, we keep the specification same. So, we say that the consumers are located in a linear city and the length of the city is 0 1. Each consumer buys 1 unit of the good and each point in the interval, so this interval 0 1 interval, each point denotes 1 consumer. So, it means that the consumers are uniformly distributed within these 0 1 interval, okay. So, this is the specification, is same as the last one.

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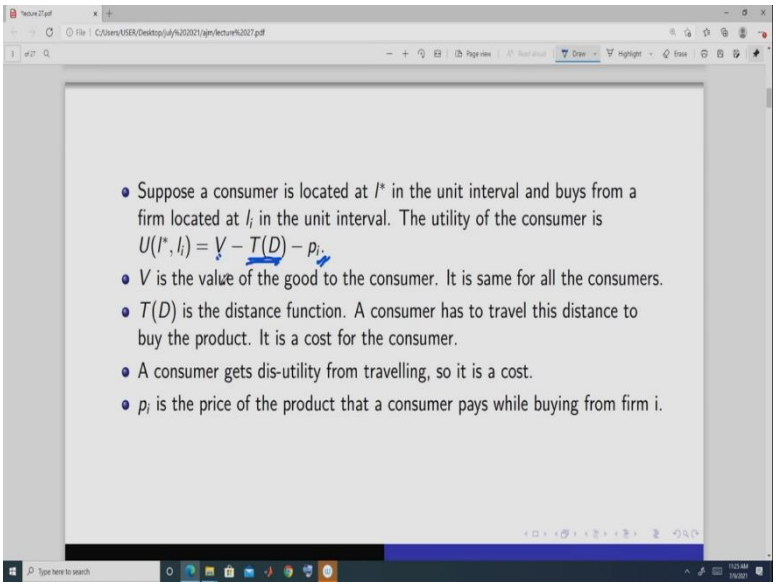

And we also keep the distance function same so the utility function, that the utility that a consumer gets from consuming this good is V. And this is the dis-utility from travelling a distance if that consumer has to travel some distance to buy from a firm. And this is the price. And for simplicity, we will see that the price is fixed and that is p. And V is the value of good to the consumer from the consumption of that good.

And T D is the distance function. And since travelling is a cost, so they get, it acts as a dis-utility for the consumer. And p is the price, so since it is, you are paying, so that is why it is a negative thing. So, you are, by consumption of that good, you get a value of V and then rest you are taking as a dis-utility, because you are paying, so it is going out from you.

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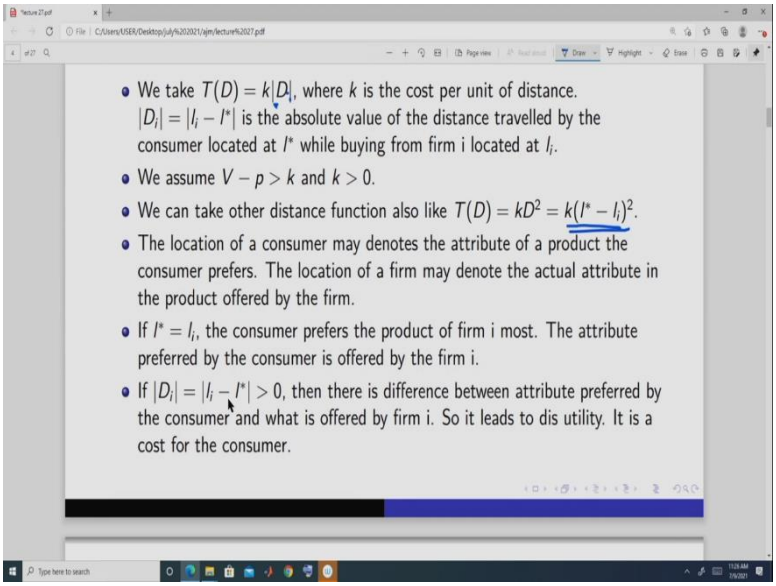

And the distance function is this-$T(D) = k|D|$, where k is the cost per unit of distance, and this is the modulus, absolute value, of the length of the, length between the location of the firm and the location of the consumer, okay. We can take any other form of distance function and it can be this quadratic form. But we will stick to this, as we have done in the last class.

So, these locations you can understand, locations in terms of the attributes, also different degree of attributes or different nature of attributes. And if a person, a consumer is located at the same place at the location of the firm, that means, you can say that the attributes are same. Whatever attribute a consumer wants, that attributes is being offered by the firm.

But if there is a difference, then it means the attribute in a product, that a consumer wants, that is not same as the attribute offered by the firm, and which is closest to the attributes of needs or attribute demanded by a consumer, okay. So, there is some difference, if this is, this distance is positive, okay. So, this is, this is same as what we have done earlier.

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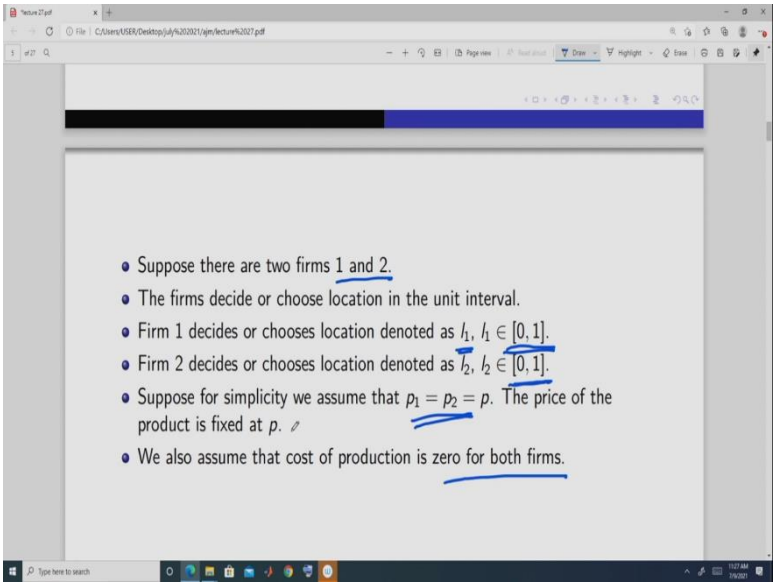

And for, at the beginning, we take 2 firms, firm 1 and firm 2 and firms choose the location in the unit interval. So, firm 1 choose l1 and l1 lies in this 0- and 1-unit interval. Firm 2 decides location and we denote it as l2 and it lies between 0 and 1. And for simplicity, we assume that there is price is fixed and it is same, okay. And we assumed also that there is 0 cost of production for both the firms. So, there is no cost of production, okay. So, this we have till now, it is same.

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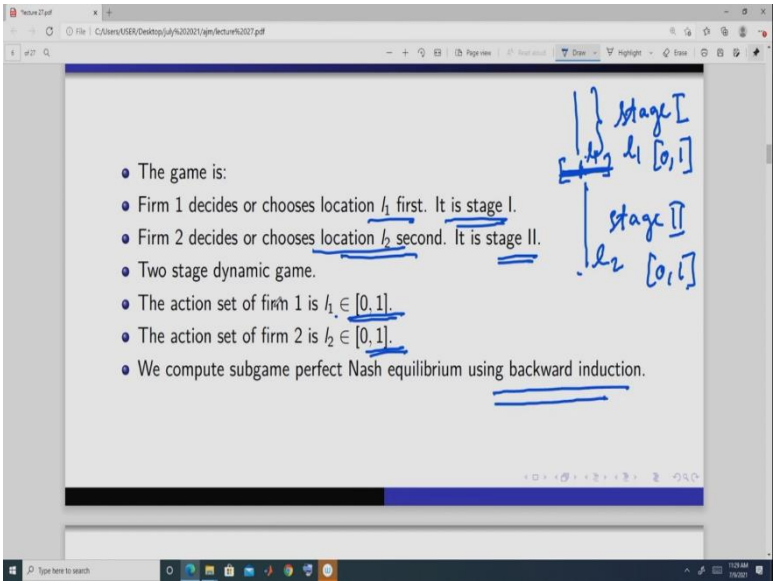

What is new? It is this. The game is new. What happens in this game? Firm 1 decides location l1 in first, so this is stage 1. Then after observing the location of firm 1, firm 2 decides its

location. This as l2 and this is stage 2. So, firm 2 locates after observing the location of firm 1, okay. So, this is a dynamic game or you can say this is a 2 stage dynamic game and action set because here action set and the since it is a dynamic game, so action set and the strategy sets are not same.

So, the action set of firm 1 is to choose a location in between this. And the action set of firm 2 is to choose a location between. So, if you look at this game then in stage 1, l1 is chosen from 0 and 1. So, this is stage 1. Then in stage 2, so here, you can say, it chooses a location from here, suppose somewhere l1 is here, then after observing this, firm 2 chooses and that is stage 2, location l2, lying between 0 and 1, okay. So, this is observed, l1 is observed, while l2 is being chosen, okay. So, we will compute the subgame perfect Nash equilibrium and we will use backward induction, okay.

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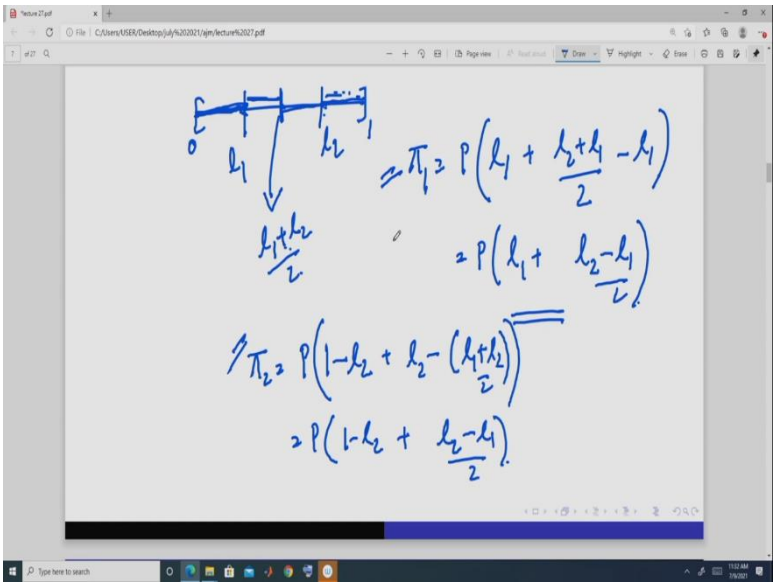

Now, how to payoffs? Let us define the payoff function. So, suppose this is the unit interval and suppose this is the location of firm 1 in stage 1. And this is the location of firm 2 in stage 2. So, the payoff of firm 1, we know, it will get this much 0 l1, because all these consumers nearest firm is firm 1. So, profit of firm 1 p l1. And we will get a consumer, who is going to be indifferent between buying from firm 1 and buying from firm 2. And this point we have derived it in the last class and this point is l1 plus l2 divided by 2.

So, this much distance again firm 2 is going to get. So, what is this distance? So, this distance is, so this point is this, l2 plus l1 divided by 2. And this distance, this point is l1 minus l1. So,

if this is the l1, so this holds also that this-this, because it is starting from 0 so this distance is l1. So, we get this. So, this is, we get this. So, this is the payoff of firm 1-$\pi_1 = P(l_1 + \frac{l_2 - l_1}{2})$.

And payoff of firm 2? Price is fixed, this. And this much, it is going to get, because all for these, all these consumers, firm 2 is the nearest one, because firm 1 is lying here, left of firm 2. So, this distance is 1 minus l2, this, and this distance we know, this location is, this point is this, so this distance is l2 minus, we get this. So, this is, this much it is going to get. And it is this- $\pi_2 = P(1 - l_2 + \frac{l_2 - l_1}{2})$. So, these two are the payoff functions. And we will have to find the location such that these payoffs are maximum. And mainly the subgame perfect Nash equilibrium also gives that.

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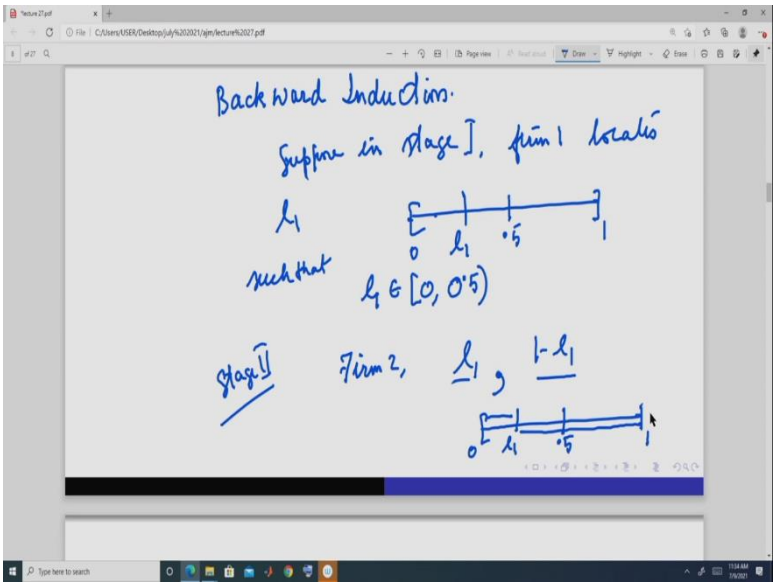

So, we will solve this. How we are going to solve it? We are going to solve using backward induction, okay. So, here, so we assume, suppose, in stage 1, firm 1 locates l1 as here. This is 0.5. This is 0. This is 1. And suppose l1 is. So, l1 such that l1 is this. So, this point is not included anywhere between 0 and 0.5, but 0.5 not included, okay anywhere. It can be this point, this point, this point, anywhere, okay.

Now, here if this is the case, so now, let us move to stage 2. So, this we assume suppose, this is in the stage 1. So, stage 2, what we will do? So, stage 2, firm 2 compares l1 and compares 1 minus l1, these 2 distances, it will compare first. Now, when it compares these 2 distances that is, this is l1, so it compares this distance and it compares this distance, okay. So, in this case, this distance is greater.

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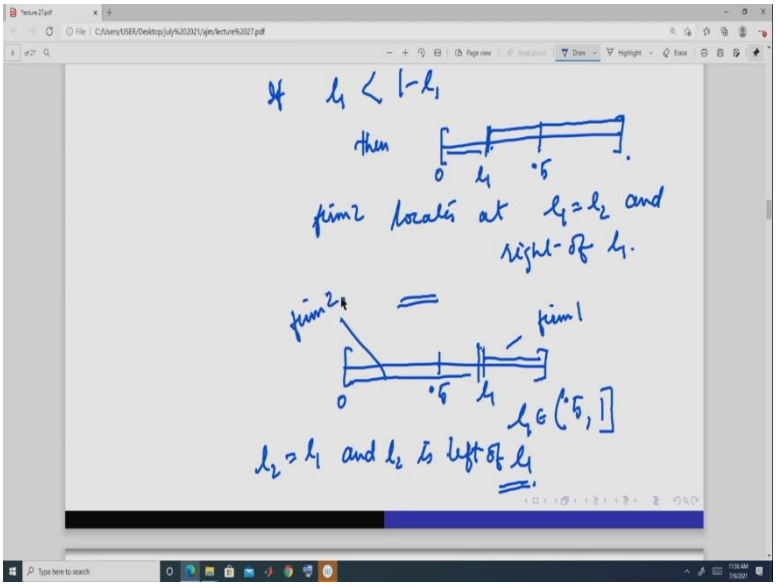

So, if l1 is less than 1 minus l1, then firm 2 will locate here, then firm 2 locates at, l1 is equal to l2 and right of l1, here. So, firm 2 gets this whole market and firm 1 gets this whole market, right? So, here this is the outcome in stage 2. So, now, so we know, how given a location of firm 1 that is l1, how firm 2 is going to behave now, in this case.

Now, again suppose, this is l1. So, this l1 belongs to this, so 0.5, it is not included. Then in this case also the argument is same as before, okay. So, here also, we will find that firm 2 is going to locate l2 is equal to l1 and l2 is left of l1. It is here. So, firm 1 will get this. This is firm 1 and this much is for firm 2, right?

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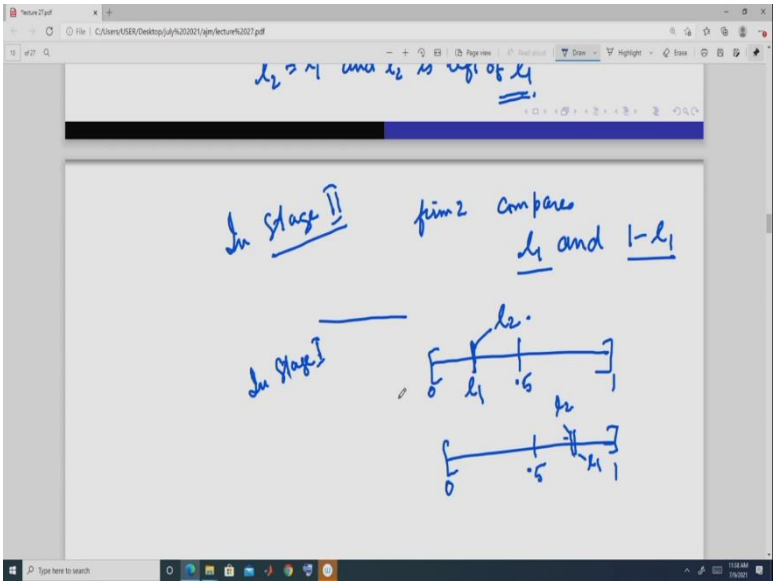

Now, so this is the way we know, how firm 2 is going to behave, given any location of firm 1, in stage 2. So, in stage 2, we have got that it compares, firm 2 compares l1 and 1 minus l1. Based on this it chooses, whether it will lie in the leftward of l1 or it is in the rightward of l1, okay. So, in stage 1, firm 1 knows this that, if so firm 1 knows that if it locates here firm 2 is going to locate here. If firm 1 here, then firm 2 in stage 2 is going to locate here. Again, firm 1 if it locates here, it knows firm 2 is going to locate here. So, it gets such that it gets this.

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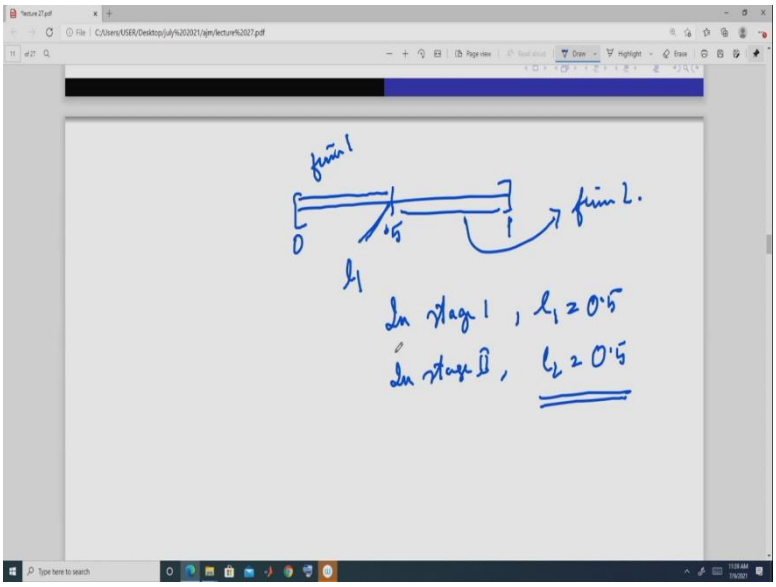

So, that is why in stage 1 the optimal location of firm 1 is to locate at 0.5. So, firm 1 gets this. And in stage 2, so in stage 1, l1 is equal to this- 0.5, and in stage 2, l2 is equal to 0.5. So, this much is for firm 1 and this much is for firm 2 or you can switch then it is same. So, we get that if we have 2 firms then the location is this.

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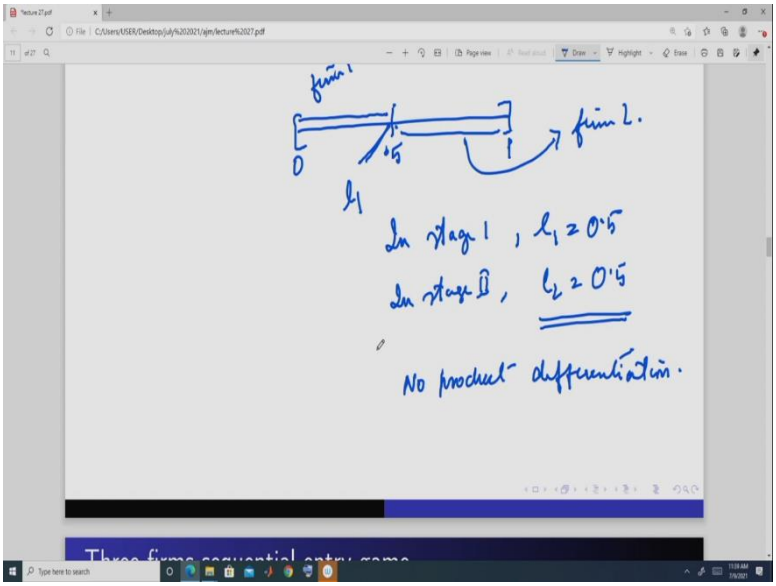

So, what is the implication of this? So, this means that there is no product differentiation, okay. Both of them are going to have same location. So, they are not going to differentiate. Or you can say that if you think it, in terms of attributes, so this much amount of attribute is going. So, it is going to be same for both the firms, okay.

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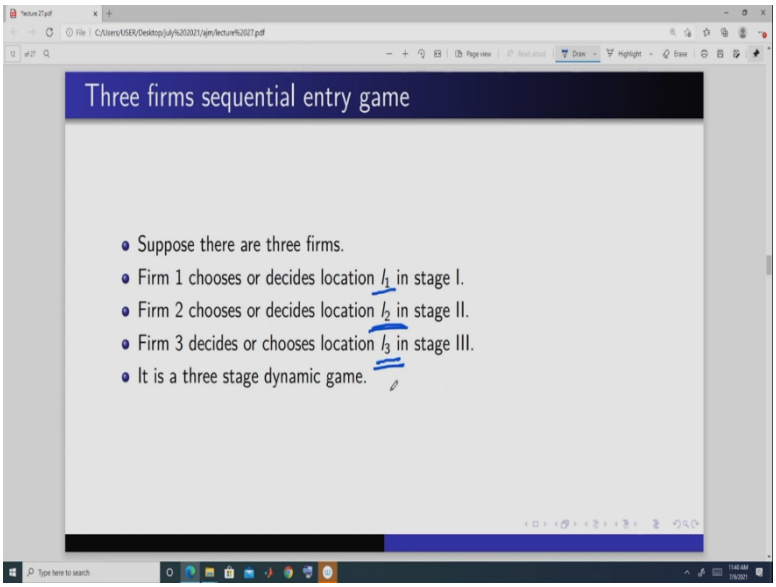

Now, let us move to 3 firms. So, we keep everything same, except we now increase 1 more firm. So, in stage 1, l1 is decided or it is chosen. In stage 2, l2 the location of firm 2 is chosen, and in stage 3 location of firm 3 is chosen. So, it is a 3-stage game now. Everything is same as earlier, okay.

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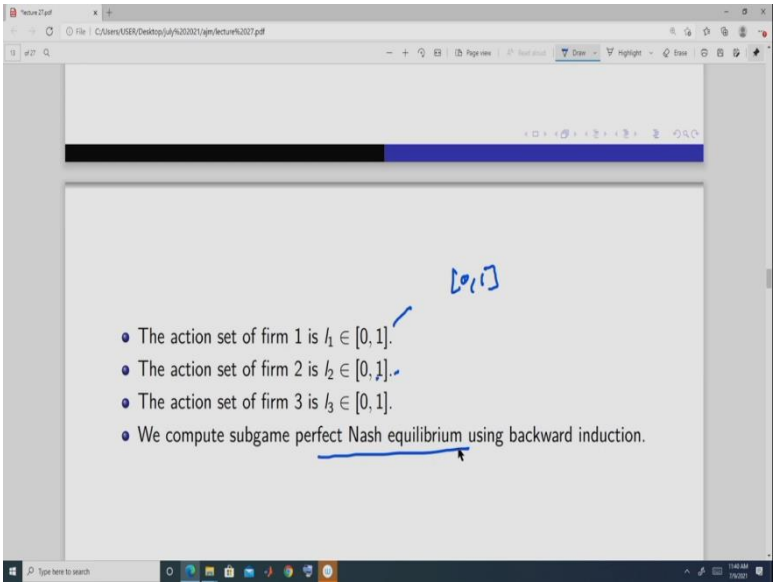

So, action set of firm 1 is l1, which lies in this, 0 and 1. Action set of firm 2 is again l2 and it lies in this, range 0 to 1-unit interval. And action set of firm 3 is again l3 and it lies between 0 and 1. And since it is a 3 stage game, so we use subgame perfect Nash equilibrium. And so we compute subgame perfect Nash equilibrium. And we use backward induction to solve it, okay. So, now let us first define the payoffs, okay.

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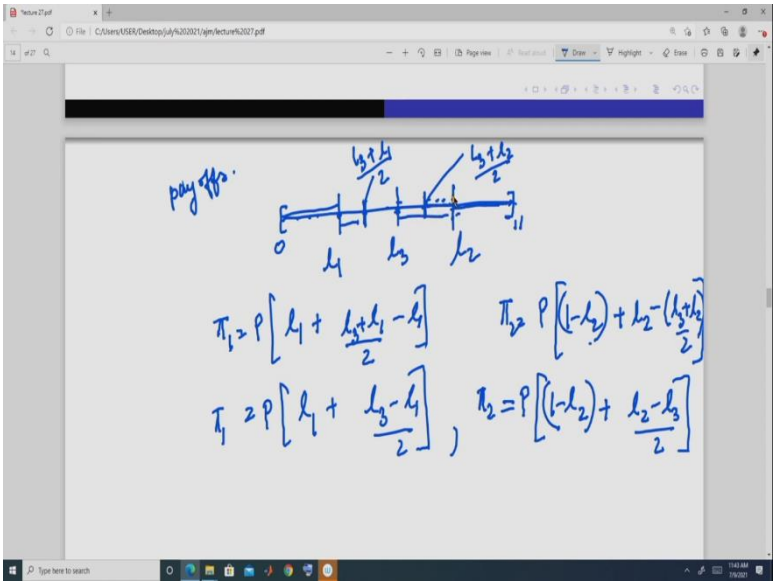

Suppose this is the unit interval, okay. This is the location of firm 1, this is the location of firm 2 suppose, and this is the location of firm 3, okay. Now, here, payoff of firm 1, price is fixed,

it is p. Since it is here, so all these consumers, firm 1 is the nearest to them. So, they are going to buy from firm 1. So, l1 is there.

Then here, in between this, so there is a point like this, and this point is, l3 plus l1 divided by 2. So, this person is indifferent from, between, buying from this firm and this firm. So, all these persons lying here are going to buy from firm 1 because firm 1 is nearest for them, nearest to them so it is going to be. So, the payoff function for firm 1 is this-P $\left[l_1 + \frac{l_3 - l_1}{2}\right]$. This much and half of this distance and this is this. That is this much.

Now, payoff of firm 2. Payoff of firm 2 is, again price is same, so p and this much, all these consumers are going to buy from firm 2 because they are nearest to the firm 2. So, 1 minus l2, this distance, this, this is 1, so l2 is this point, so this distance is, all the consumers lying here, are going to buy from firm 2. So, this is, plus there, we are going to get a point like this, who is indifferent between or this distance is same, so middle of this, middle of these 2 points, this point.

So, this point is again given by l3 plus l2 divided by 2. So, this many people are going to buy from firm 2. So, it is going to be l2 minus l3, this- $\pi_2 = P\left[(1 - l_2) + l_2 - \frac{l_3 + l_2}{2}\right]$. So, the payoff of firm 2 is going to be 1 minus l2, this much, plus l2 minus l3 divided by 2- P $\left[(1 - l_2) + \frac{l_2 - l_3}{2}\right]$. This distance. This distance is half of this distance. This distance is half of this distance. So, it is this.

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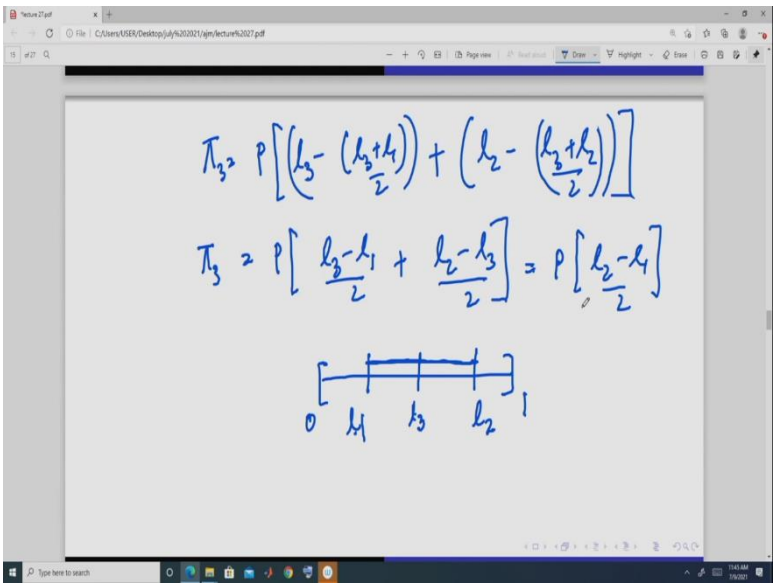

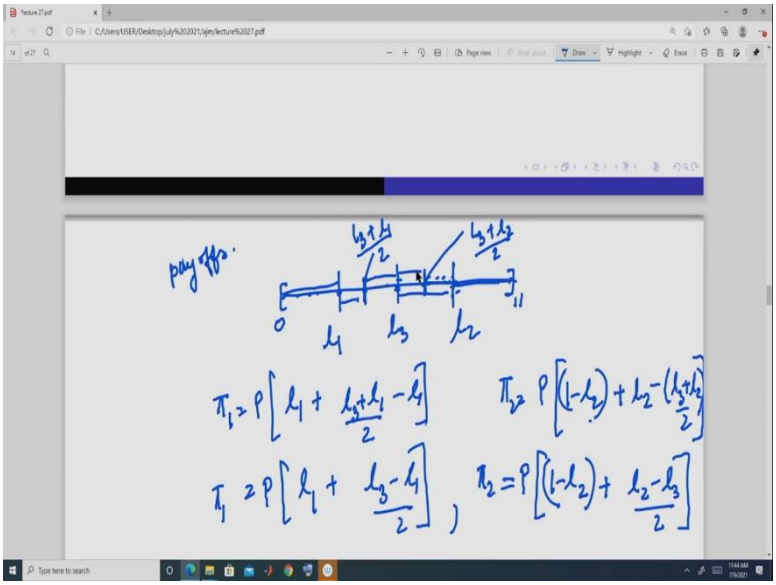

Now, the payoff of firm 3, price is same, this. Now, firm 3 gets this much and this much. So, this much is what? This much is l3 minus, l3 plus, this, plus l2 minus, l3 plus l2. So, this is the payoff of firm 3- $P\left[\frac{l_3-l_1}{2}+\frac{l_2-l_3}{2}\right]$. So, actually this is, you can see, half of this distance, this distance. Or here if you look at this position, this is 0, this is 1, location of firm 1, location of firm 2 and suppose, firm 3 here. So, firm 3's market share is half of this distance, okay l2 minus l1 divided by 2, this, ok. Remember this, we will use it. Now, let us find the solution. So, these are the payoff functions and firms are going to choose the location so that those are maximized. So, we will use the backward induction.

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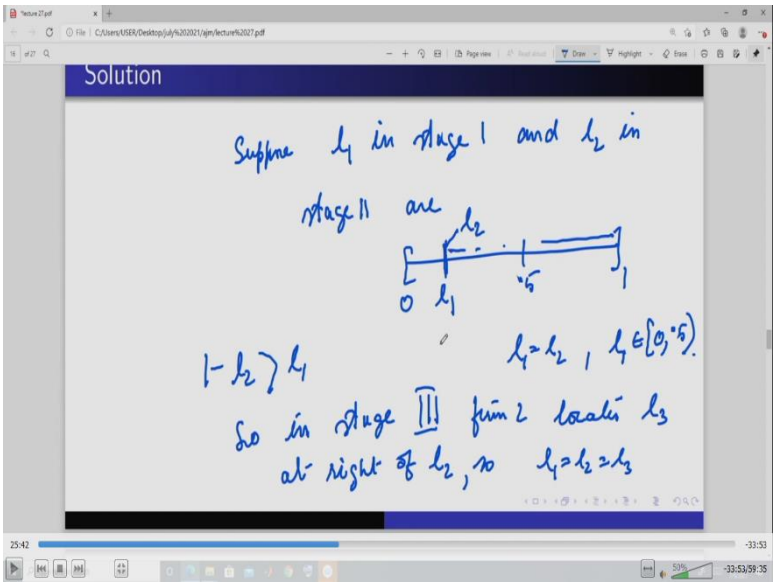

So, suppose, l1 in stage 1 and l2 in stage 2, are of this nature. This is 0.5. This is l1 and this is l2, okay of this nature. So, l1 is equal to l2 and l1 anywhere like this. Then if this is the case, then it is said, so this distance is so, so here, see 1 minus l2 is greater than l1. So, in stage 3, firm 3 locates l3 at right of l2 and so l1 is equal to l2 is equal to l3. So, firm 2 is now sandwiched. So, it will get 0 consumers. So, that is why firm 2, if given firm 1's location here, firm 2 will never look at the same position as firm 1's.

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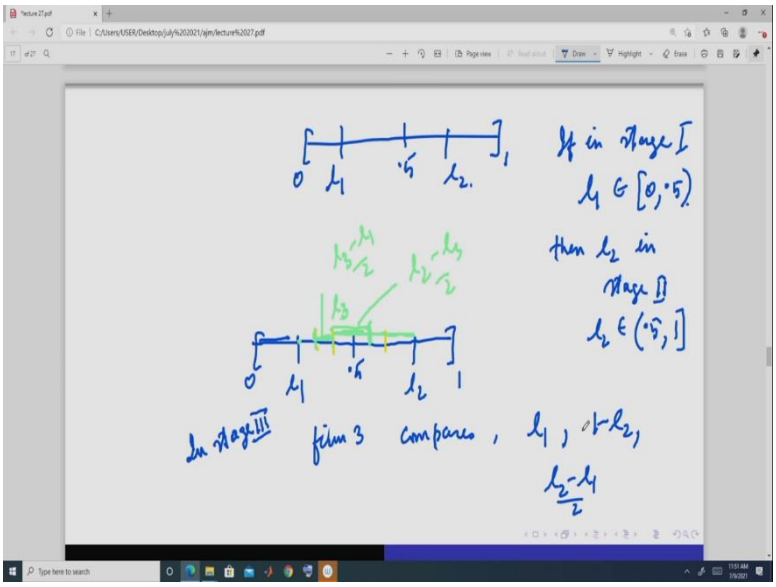

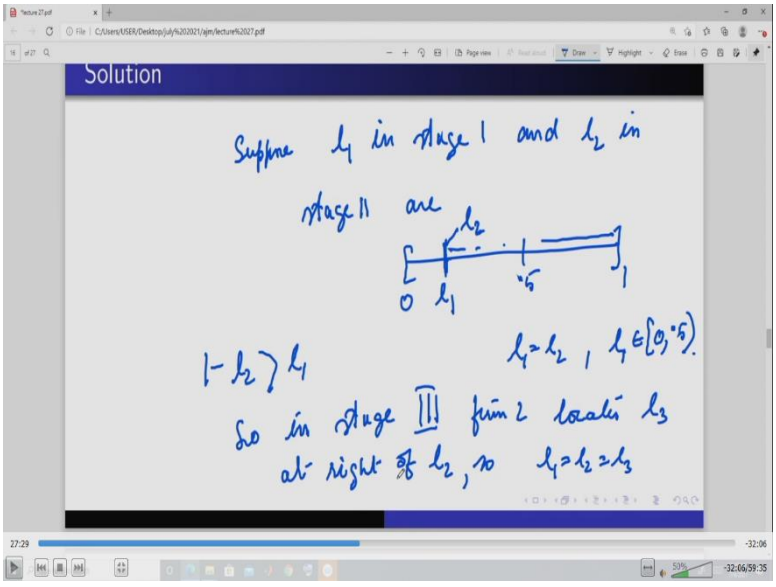

It will locate in some way like this. So, if l1 is here, somewhere here, then firm 2 will lie somewhere here, here in this region, here because otherwise firm 2, firm 3 in stage 3, we know, it is going to locate here, so l3 is here and it is going to be at right of l2, so it will get the whole market and l2 will not get anything. So, now so this is the case.

Another thing here, again in stage 3, so we get that it will be of this nature. So, if this is l1, this is l2, then this is the case in stage 3. Firm 3 compares, what, l1, this distance, it compares 1 minus l2, this distance. And if it lies anywhere here, here or here, anywhere in position like here, here or here, it is going to get, suppose it lies here, then its market share is, suppose this is l3, its market share is half of this distance, so this and it is going to get half of this distance, this.

So, we know, this half of this is, l3 minus l1 divided by, so this distance is l3 minus l1 divided by 2. This half of this is suppose this, so this distance is l2 minus l3, so, if we take this, so we get again what? So, this firm 2's a is l2 minus l1. So, it is going to compare this-l_1, this-$1 - l_2$and it is going to compare this length-$\frac{l_2 - l_1}{2}$, these three.

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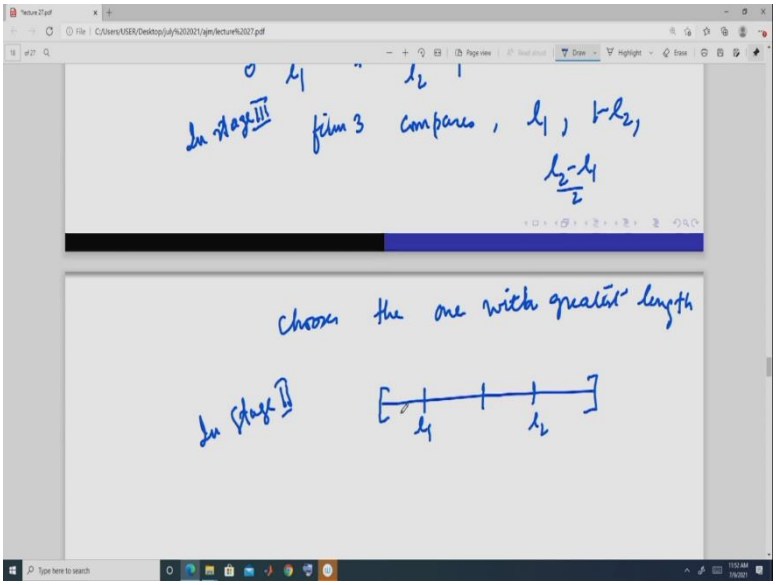

So, firm 3 compares this, it compares these 3 distances. And chooses the one with greatest length, whichever is the as greater, it will choose that, the length of which one is greater. So, in stage 2, firm 2 knows that firm 3 is going to compare these 3 things.

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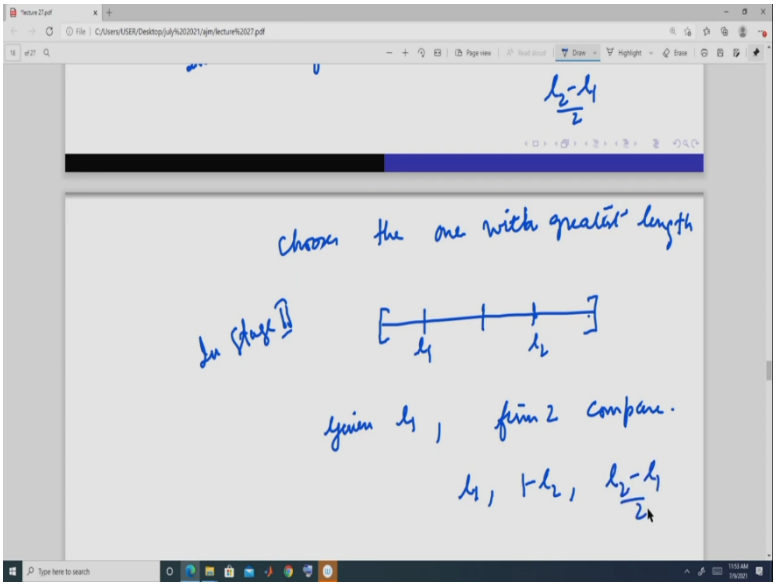

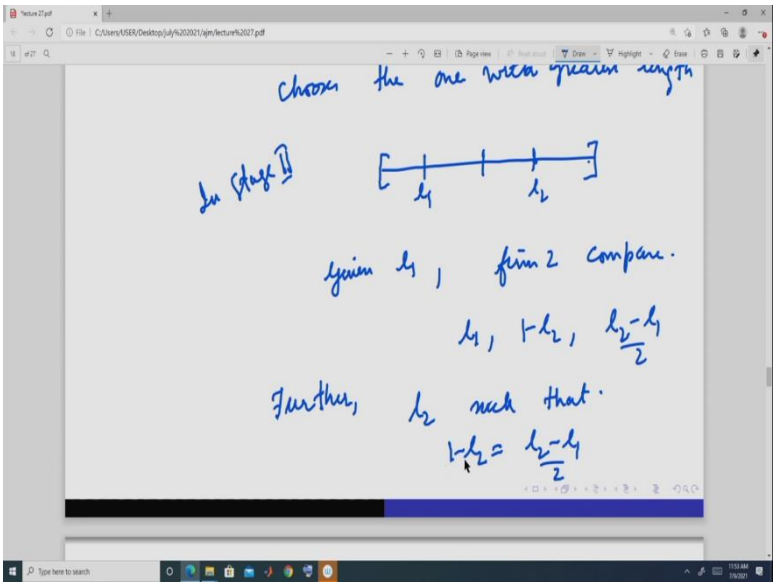

So, that is why given l1, firm 2 compares this. This, it can compare this location. So, firm 3 again can compare l1, 1 minus l2. And further firm 3 firm 2 will choose l2 such that 1 minus l2 is equal to-$\frac{l_2-l_1}{2}$. Because, because this much firm 2 can ensure, because given l1, it knows this.

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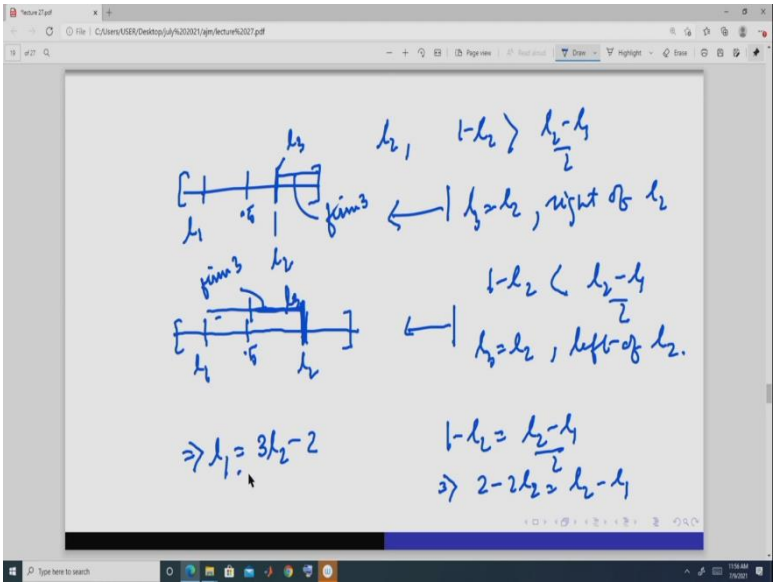

It can ensure this, because, if this distance, if suppose, it has chosen l2 and suppose 1 minus l2 is greater than this-$\frac{l_2-l_1}{2}$. Then firm 2 is going to choose l3 is going to be equal to l2 and right of l2. So, firm 2 will get less this much only. Now, if this is the case, then l3 is again equal to l2 and it is left of l2. So, here in this case it will be like this.

Suppose, this is l1 and suppose this is l2, then this is going to be l3 and this is going to be the firm 3's market share. In this situation, so firm 3 will be here and it will get the half of this distance. This is going to be firm 3, okay. So, that is why, for this reason, firm 2 will always ensure this-$1 - l_2 = \frac{l_2-l_1}{2}$, okay. Now, while ensuring this, we get the value of l1. Because see, from here we get, so if we get this condition that 1 minus l2 should be equal to this-$\frac{l_2-l_1}{2}$. So, this implies this-$2 - 2l_2 = l_2 - l_1$. So, then this implies what? l1 should be equal to 3 l2 minus 2, okay. So, now from here, here again, firm 1, firm 2 can do what?

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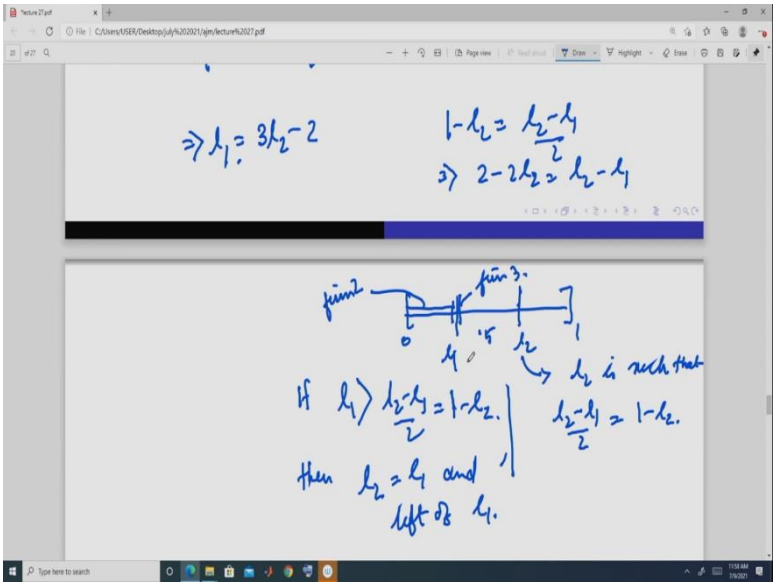

So, it will do this. Now, if suppose, this is 0.5, so suppose this is l1 and if suppose this is l2, so this l2 is such that l2 minus l1 half is equal to 1 minus l2, okay. Now, if l1 is greater than this- $l_1 > \frac{l_2 - l_1}{2} = 1 - l_2$, so firm 2 will locate here. So, if this, then l2, is equal to l1 and left of l1. So, this much is going to get by firm 2 and firm 3 will gets locate here, in stage 3, right? So, firm 2 will do this calculation in stage 2.

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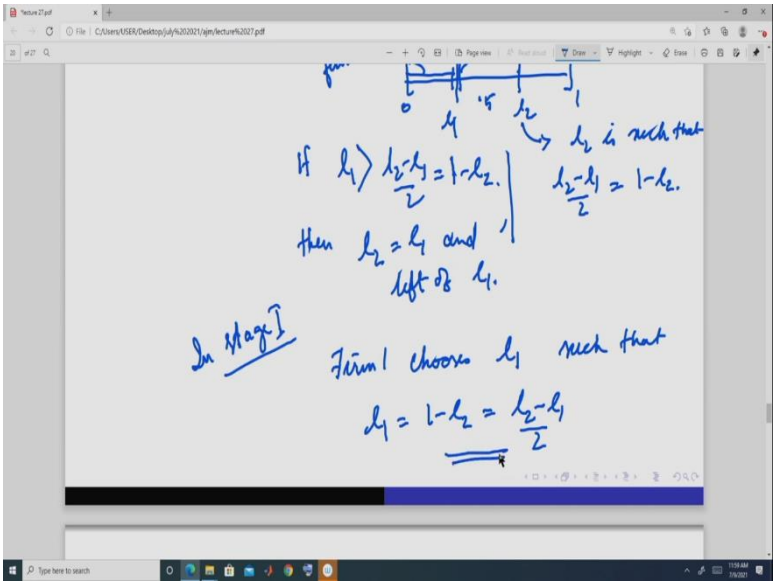

So, that is why, in stage 1, firm 1 will, firm 1 chooses l1 such that l1 is equal to 1 minus l2, which is equal to, given firm 2 is always going to ensure this- $l_1 = 1 - l_2 = \frac{l_2 - l_1}{2}$. So, if we get this, then we solve this.

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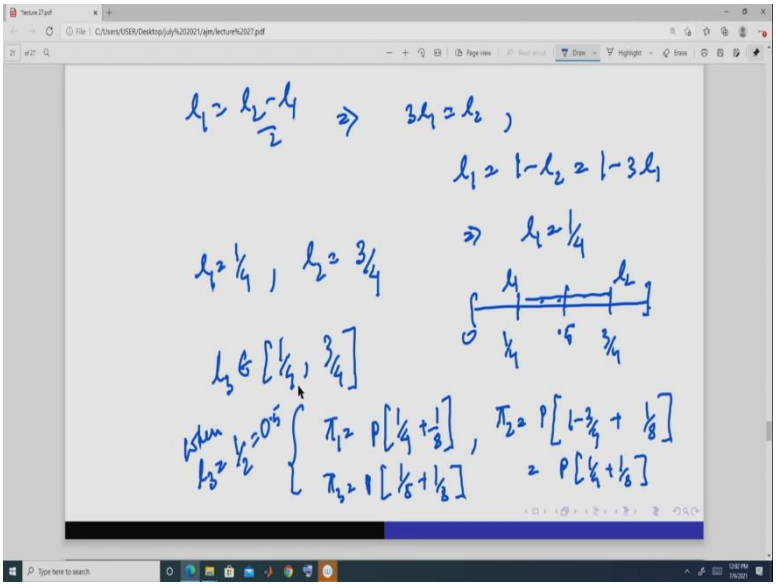

So, from here, we know that, if we solve this, we will get, so 3 l1 is equal to l2, from equating this to l1 is equal to l2 minus. This implies this, i.e.- $l_1 = \frac{l_2 - l_1}{2} \Rightarrow 3l_1 = l_2$. And further, when you plug in that here and l1 is equal to 1 minus l2 which is equal to 1 minus 3 l1 then we get that l1 is equal to 1 by 4. So, l1 is equal to 1 by 4, l2 is equal to 3 by 4 and then l3, see what is happening, this is 1 by 4, l1, this is 3 by 4, l2.

What is this distance? 3 by 4 minus 1 by 4, so this distance is, half and firm 2, wherever it locates, it gets half of this, so that is 1 by 4. So, firm 3 l3 is going to be indifferent lying between this and anywhere here-[1/4, ¾], anywhere. So, the payoff of firm 1, if firm 1, if it is going to be p, this plus suppose it is at the center, suppose it is 0.5, then it will be half minus 1 by 4, so it is again 1 by 8- $\pi_1 = P\left[\frac{1}{4} + \frac{1}{3}\right]$. Payoff of firm 2. It is this-$\pi_2 = P\left[1 - \frac{3}{4} + \frac{1}{8}\right]$. 1 minus 3 by 4 plus this, half of this, so this is 1 by 8. So, it is 1 by-$\pi_3 = P\left[\frac{1}{5} + \frac{1}{8}\right]$. And it is 1 by 8 plus 1 by 8. So, this is the case, when l3 is equal to half or it is 0.5. It can choose any point here. So, depending on that we will get different payoffs, okay.

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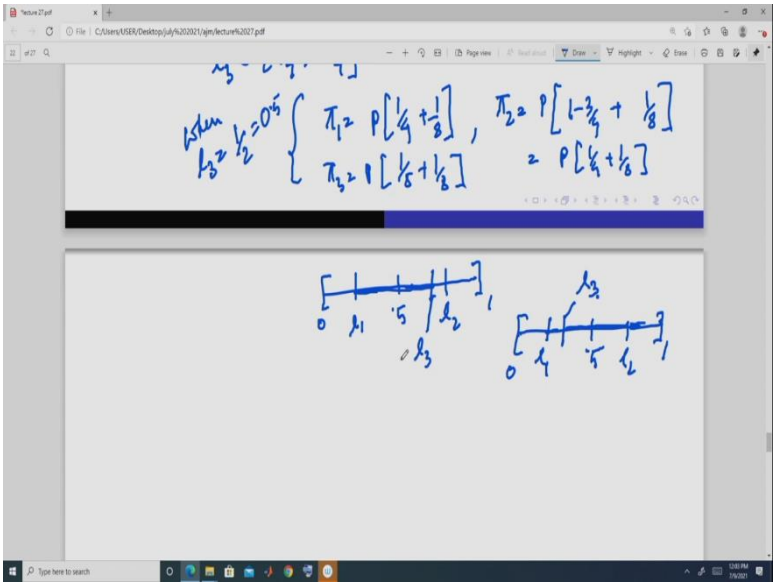

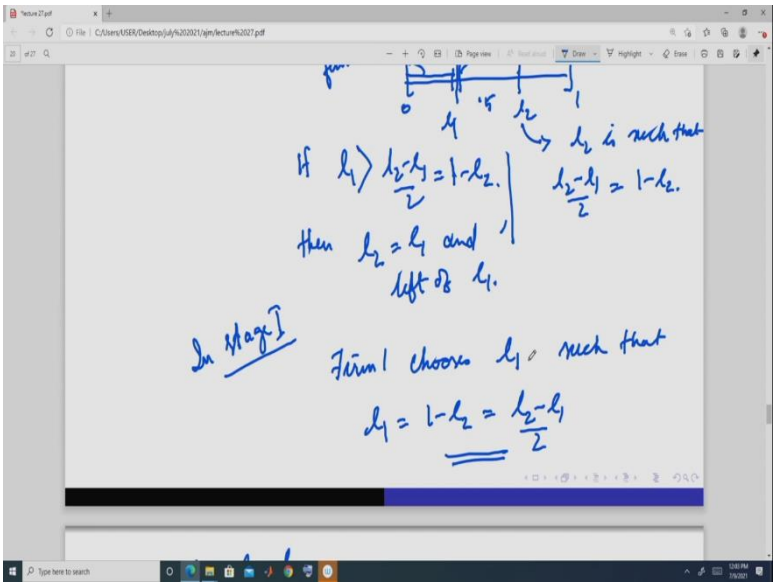

So, if it is here, now if it is here, then we will, firm 1 get, is going to get half of this. And if this is l3, if it is here, if l3 is here, then it is going to get half of this, and half of this is going to get by the firm 2. But firm 3 is always going to get 1 by 4 only. If it lies here also, it will get this much, 1 by 4. If it is here it will also get this, 1 by 4. So, firm 3's payoff is 1 by 4 only. But payoff of firm 1 and firm 2 can be different depending on the exact location of firm 2, firm 3, okay.

So, we get that actually there exist a subgame perfect Nash equilibrium, pure strategy subgame perfect Nash equilibrium or simply subsume perfect Nash equilibrium, when we have 3 firms, in this case. But when we had 3 firms and the firms are choosing the location simultaneously,

we found that there is no pure strategy Nash equilibrium, right. So, here we have found and it is not unique, okay.

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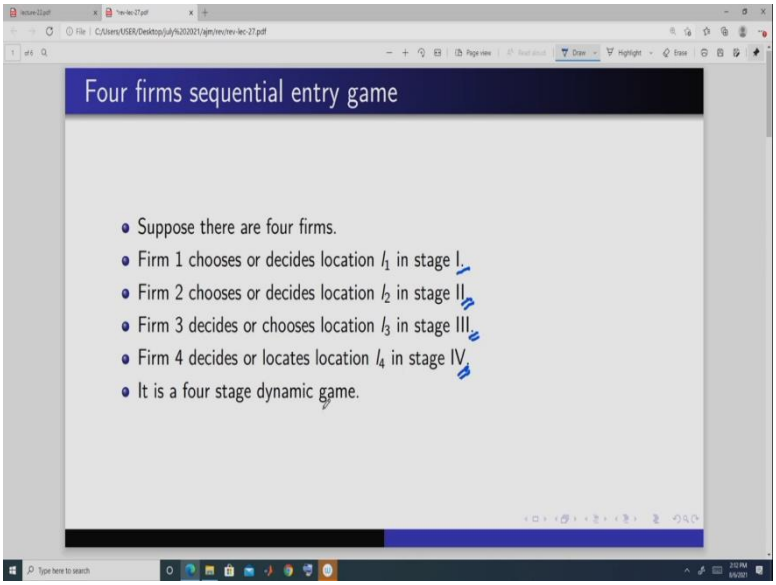

Now, suppose let us take the 4 firms sequential entry game. So, here we have 4 firms and firm 1 decides or locate in stage 1, firm 2 in stage 2, firm 3 in stage 3 and firm 4 in stage 4. So, this is a four-stage dynamic game, okay. And the payoffs are going to be similar as in the case of 2 stage or in the 3 stage that we have done just before this.

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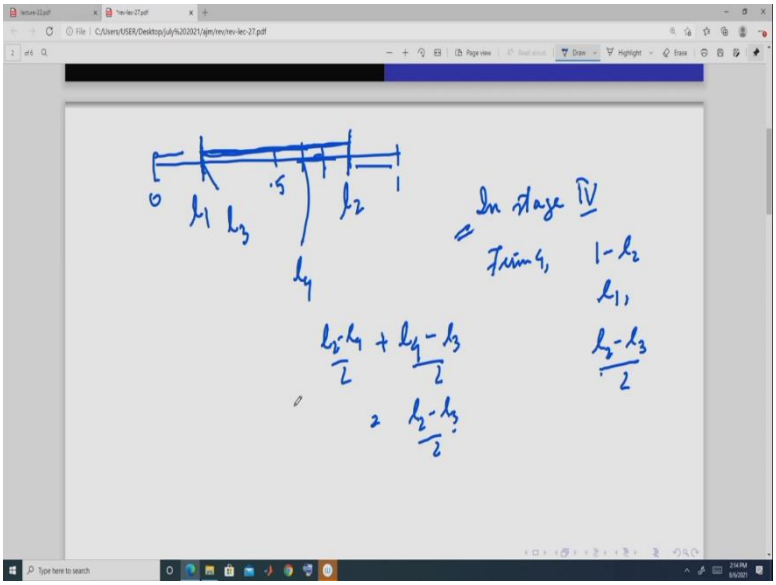

So, we will again use backward induction to solve this. So, suppose, this is 1 0 and suppose this is 0.5, this is suppose l1, this is suppose l2 and suppose this is l3, okay. Then in stage 4

firm 4 compares 1 minus l2 with l1 this distance-$1 - l_2$ this distance-l_1 and half of this distance-$\frac{l_2 - l_3}{2}$, will compare this, which is, because here, if it locates anywhere here, suppose, this is suppose l4, then what is the payoff of firm 4? So, what is the distance that the firm 4 is getting? What is the length it is getting?

So, it is getting half of this, so 2 minus this much, and again half of this, so plus l4 minus l3 by 2. So, this is-$\frac{l_2 - l_4}{2} + \frac{l_4 - l_3}{2} = (l_2 - l_3)/2$. So, this is the same way, whether it locates here, here any point, it will get only half of this whole this region, in this region, if this is l2 and this is l1 and this side it is, right of l1 is, l3. So, it will get this much. So, firm 4 going to compare this three, in stage 4.

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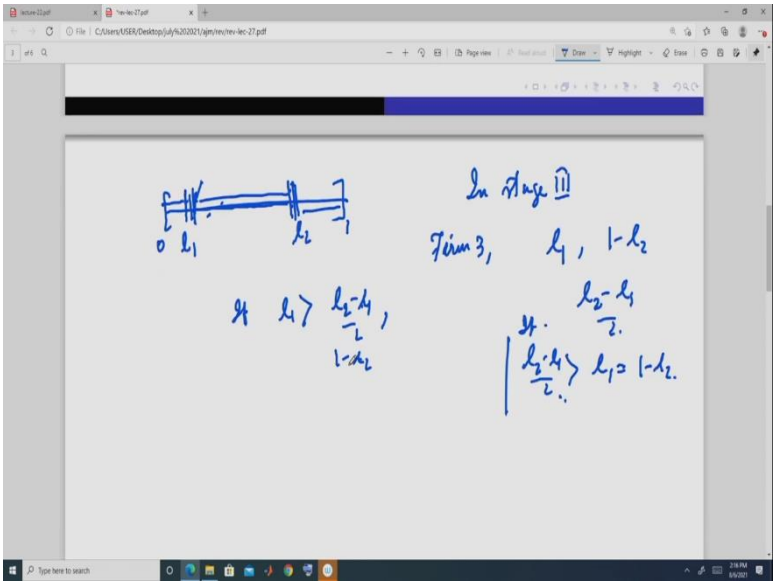

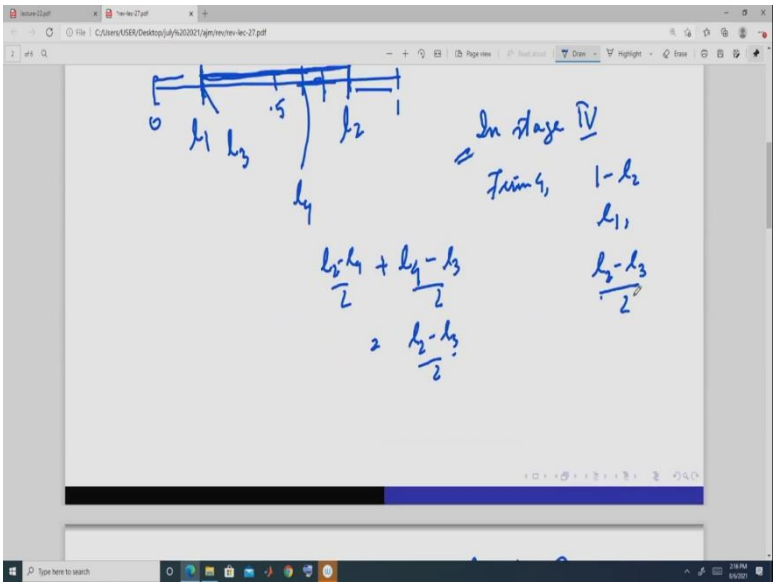

Now, in stage 3, firm 3 is given this situation that there is firm 3 is going to again compare this, going to compare this and is going to compare this, this distance. So, these 3 distance and it knows that if suppose, this is greater. Suppose, this is greater than-$\frac{l_2-l_1}{2} > l_1 = 1 - l_2$ and this is suppose equal to. So, if this is greater, then if it locates here any point, then firm 4 is also going to locate here and it will get the half of this a or will locate here, it will get the half of this a, anywhere firm 4.

We have already seen, it knows from, so and if so this is the a. If this is the case, then firm 3 will either locate here or it will locate here, if this is the case. So, and if suppose, based on this comparison, and if l1 is suppose greater than l2 minus l1 by 2. And it is also greater than 1 minus. Then it will locate here, but there is a chance that if it is greater than this, then firm 4 also locates here.

So, it will better compare between these two, these two in this situation. And whichever is greater, it will locate, try to, it will get, locate in such a way that it gets maximum of, from these two length, okay. So, from this, we get that the firm 3 is going to make this comparison, okay.

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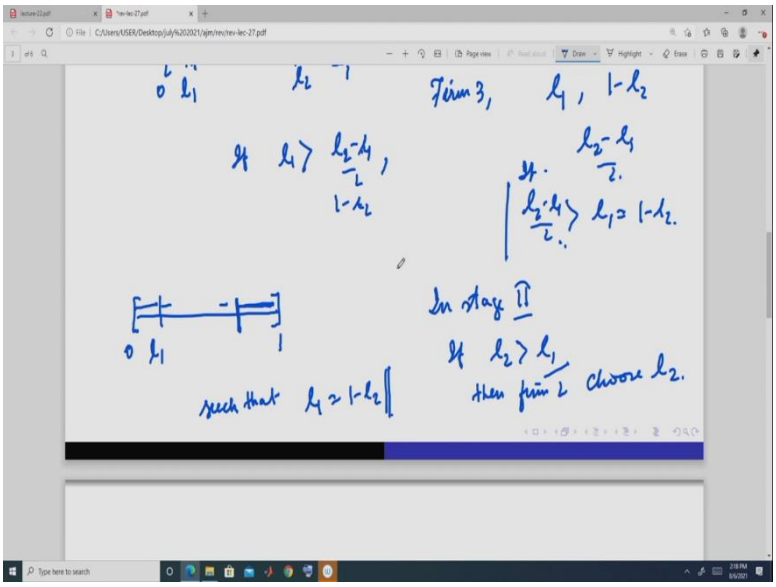

Now, in stage 2, firm 2 will know this that firm 3 is going to make this comparison, firm 4 is going to make this comparison. So, if this is l1 and if suppose l2 is greater than l1, or l2 is locating somewhere, somewhere here. Then it will at least ensure such that this distance is equal to this distance, okay. Or because at least it will, it can ensure this much-$l_2 > l_1$. So, firm

2, if this, then firm 2 will choose l2 such that this happens- $l_1 = 1 - l_2$, okay. Now, so firm we will do this. Because if suppose this is, suppose less than this.

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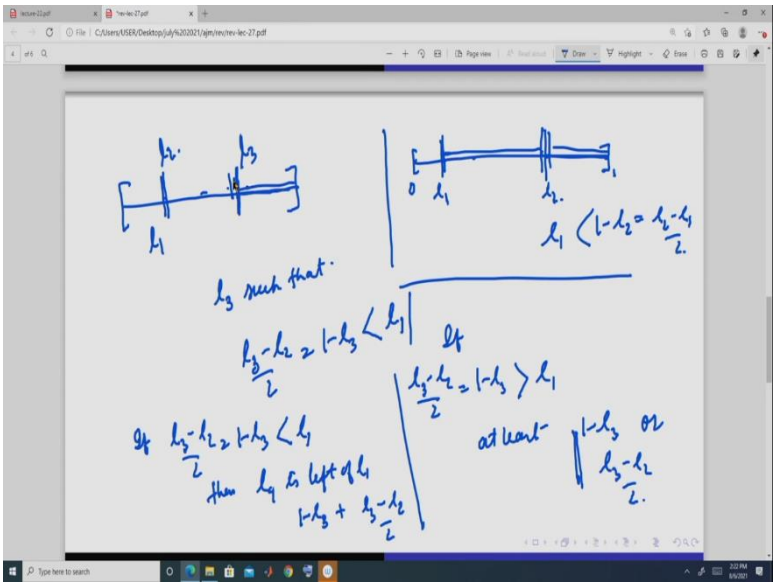

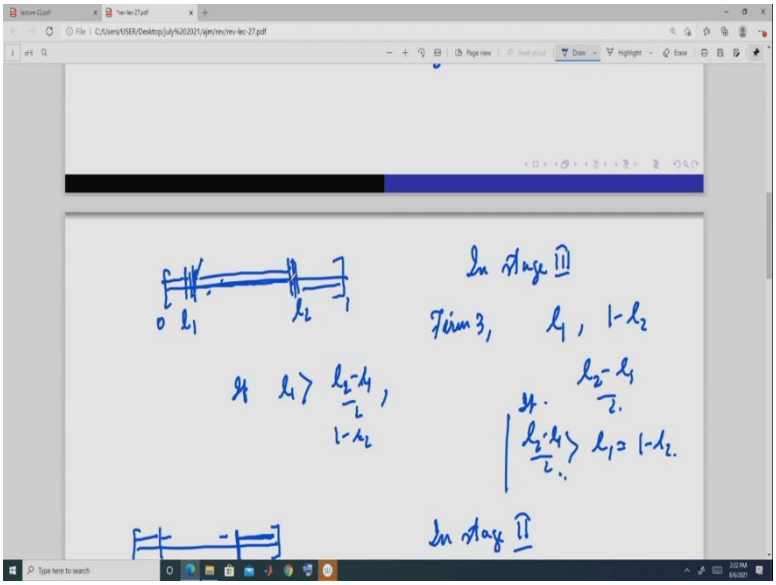

Now, it may happen that the situation is something like this that this distance is, okay. Now, if firm 2 chooses half of this, so it is chooses in such a way that this distance is, this is l2, so it is 1 minus l2 and it gets half of this-$1 - l_2 = \frac{l_2 - l_1}{2}$, okay. And this is greater than, suppose l1, okay. It can do this, firm 2 can do this.

Now, if in this case, what will happen? Firm 3 can locate here, it will get this much for sure, and firm 4 may locate here or may locate anywhere here and including this. So, firm 2 here, is not sure that it is going to get a none, it is going to get a positive length. So, that is why, at

least, if it ensures this much then it will always get this. So, if it does chooses a position which is greater than this, then it will ensure that this distance is equal to this distance, instead of looking at it in this way, okay.

But firm 2 has another possibility, see because there are now 4 firms, after firm 2 enters, there is, 2 more firms are going to enter. Firm 3 in stage 3 and firm 4. So, if this is l1 then if firm 2 chooses this l2, then what is going to happen? Then firm 3 is not going to choose this then, because firm 3 will also have a threat that the firm 4 is going to enter and then it will get this.

So, firm 3 will choose l3 such that in this situation, at least this is equal to-$\frac{l_3-l_2}{2} = 1 - l_3$. So, it ensures this, ok. Now, it may happen that this distance is less than l1-$\frac{l_3-l_2}{2} = 1 - l_3 < l_1$. If this happens then firm 4 is going to locate here, okay. So, firm 3 is going to get this and half of this. If this is the case then, then l4 is left of l1 and so firm a will get, the length of a is, firm, it will get this plus, so this is going to ensure, this much- $1 - l_3 + \frac{l_3-l_2}{2}$. So, that is why it is going to ensure.

Now, if is greater than l1 then, if this is the case, then firm 4 will either locate here and get this or it will locate here or anywhere here it will get. So, at least 1 minus l3 or this much-$\frac{l_3-l_2}{2}$ is ensured. So, that is why firm 3 will locate here, in this case. And firm 4 will locate either here or here. Now, firm 2 knows this. So, it can either choose in here or it can choose something like this. So, firm 2 has two possibilities here, okay given l1.

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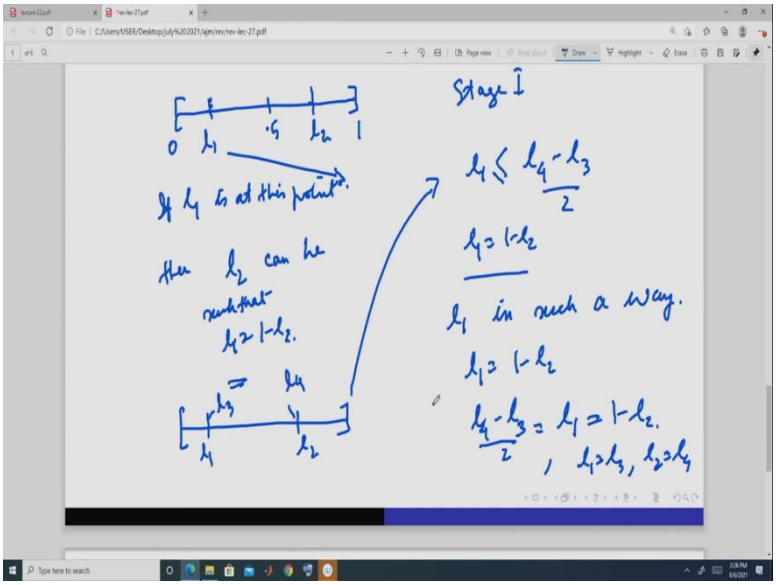

Now, firm 1 lets come to stage 1. So, firm 1, it knows that if it chooses a position like this then there is a possibility that firm is going to choose here, such that this is equal to this. So, if l1 is this, is suppose less than do not write, 0.5. But so, if l1 is this point, okay. Then l2 can be such that l1 is equal to 1 minus l2 or, okay so if it is this case, then what do we know? If this is the case, then firm 3 is going to either, firm 3 is going to choose here or it will choose here or it will here, depending on the sign, okay.

Now, what is going to happen here? So, this is going to be one situation, right. Another, then if firm 3 is here, then firm 4 is here. So, if this is the case, then we get, if l1 is here, l2 is here, then this position is l3 or this position is l4. We get this. When do we get this? We get this when l1 is less than equal to l2, this- $\frac{l_4 - l_3}{2}$. And we know that l1 is equal to. Because this is going to be ensured, in this case. So, firm 1 is going to do this calculation. It will choose l1, so l1 in such a way that it is going to solve these things. This- $l_1 = 1 - l_2$. And given this is equal to, which is equal. So, we get this- $\frac{l_3 - l_4}{2} = l_1 = 1 - l_2$. Now, from this, so here and further l1 is equal to l3, l2 is equal to l4.

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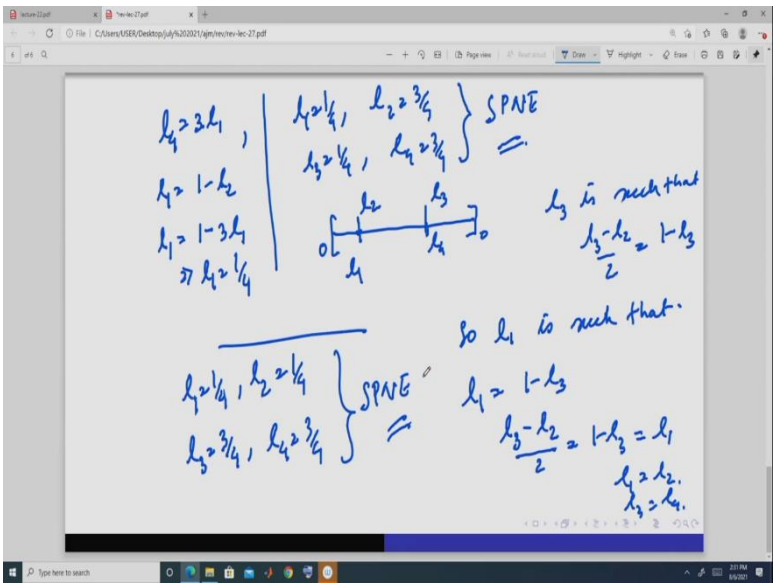

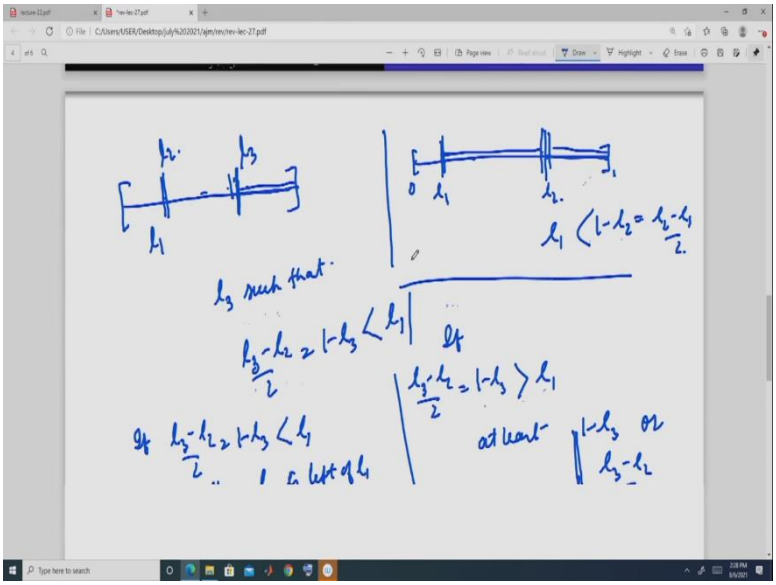

Now, from l4, so from this, we get that l1 is equal to 3 times l4, this. And l1 is equal to 1 minus l2, use this. This here a is 3 times, so from this we get l4 is 3 times l1. And we have this. So, plug in here, l1 is equal to 1, so here, it is l4 is a, so 3 l1. So, this will give me l1 is equal to 1 by 4.

Now, if l1 is equal to 1 by 4 then l2 is here, it is same as here, so it is 3 by 4. And l3 is 1 by 4, l4 is $\frac{3}{4}$. So, this is one subgame S P N E. Subgame Perfect Nash Equilibrium. Now, another thing that we have seen is, this possibility that this is l1 and firm 2 locates here, then l3 is this. So, l3 will be such as, such that l3 minus l2 is equal to 1 minus l3.

Now, firm 1, if this is greater, then we have seen, firm 4 is going to locate here, then this. So, l1 is again such that l1 is equal to 1 minus l3. l3 minus l2 is equal to 1 minus l3 is equal to l1 and l1 is equal to l2 and l3 is equal to l4. We can have that outcome. So, these equations are, if you look at them are same as this. So, again in this situation, we get that l1 is again 1 by 4, l2 is 1 by 4, l3 is 3 by 4 and l4 is 3 by 4. This is another Subgame Perfect Nash Equilibrium.

So, from this, we get that there are two this kind of this, but there are many, if you look at these things, then here l1 can be this position and then we can change that also, in this here. So, there are many possibilities. But the locations are going to be of this nature. So, we see that in this kind of product differentiation or that is the product differentiation through decision of or through the choice of location, then we see that there are two firms which are going to produce similar product and another two firms which are going to produce similar product but they are different from this.

So, we will see variety in the market but we will not see that much variety. So, there will be some firms producing similar homogenous product and there will be also some firms which and there will be difference between these two types of firms, okay. So, in when we have 4 firms they are entering sequentially, we find this as one of them.

These as the two Nash equilibriums, subgame perfect Nash equilibrium, where we find that there will be some firms producing homogenous product and some firms which are differentiating from those two set of firms, okay. So, this is the outcome in this situation, okay. So, thank you very much. And for this portion you can read class notes.