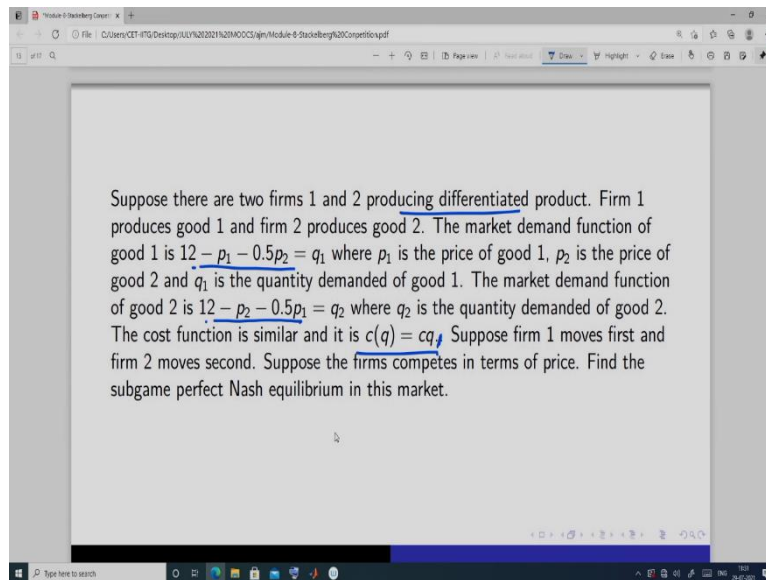


**Introduction to Market Structures**  
**Department of Humanities and Social Sciences**  
**Indian Institute of Technology Guwahati**  
**Professor Amarjyoti Mahanta**  
**Lecture 37**  
**Tutorial on Stackelberg Price Competition**

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Suppose the firm 1 and firm 2 produces differentiated good and the demand function of good 1 is this-  $12 - p_1 - 0.5p_2 = q_1$  which is produced by firm 1, demand function of good 2 is this-  $12 - p_2 - 0.5p_1 = q_2$  which is produced by firm 2 and the cost functions are same. It is like this-  $c(q) = cq$  suppose firm 1 moves first and firm 2 move second and find the subgame perfect Nash.

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$$\pi_2 = (12 - p_1 - 0.5p_2)p_2 - c(12 - p_1 - 0.5p_2)$$

$$\left| \pi_2 = (12 - p_2 - 0.5p_1)p_2 - c(12 - p_2 - 0.5p_1) \right.$$

$$\frac{\partial \pi_2}{\partial p_2} = 12 - 2p_2 - 0.5p_1 + c = 0 \quad \text{FOC.}$$

$$\Rightarrow \frac{12 - 0.5p_1 + c}{2} = p_2$$

$$\pi_2 = \left[ 12 - p_1 - 0.5 \left( \frac{12 - 0.5p_1 + c}{2} \right) \right] \left( \frac{12 - 0.5p_1 + c}{2} \right)$$

$$= \left( \frac{24 - 2p_1 - 6 + 2.5p_1 - 0.5c}{2} \right) \left( \frac{12 - 0.5p_1 + c}{2} \right)$$

$$\pi_1 = \left( 18 + 2.25p_1 - 0.5c \right) \left( \frac{12 - 0.5p_1 + c}{2} \right)$$

$$\frac{d\pi_1}{dp_1} = (2.25)(12 - 0.5p_1 + c) + 18 + 2.25p_1 - 0.5c$$

$$\pi_1 = \left( 18 + 2.25p_1 - 0.5c \right) \left( \frac{12 - 0.5p_1 + c}{2} \right)$$

$$= \left( \frac{24 - 2p_1 - 6 + 2.5p_1 - 0.5c}{2} \right) \left( \frac{12 - 0.5p_1 + c}{2} \right)$$

$$\pi_1 = \left( 18 + 2.25p_1 - 0.5c \right) \left( \frac{12 - 0.5p_1 + c}{2} \right)$$

$$\frac{d\pi_1}{dp_1} = (2.25)(12 - 0.5p_1 + c) + 18 + 2.25p_1 - 0.5c$$

$$\text{FOC} = 0$$

So, this is a standard problem and you can solve this. Profit of firm 1 is  $12 - P_1 - 0.5P_2$ , this is  $0.5P_2$ ,  $P_1 - C$  is this-  $\pi_1 = (12 - P_1 - 0.5P_2)P_1 - C(12 - P_1 - 0.5P_2)$  and profit of firm 2, so, there it is a negative sign. So, it means the products are if the price increases, then demand goes down or if  $P_2$  increases demand for good 1 goes down. So, it means goods are complimentary in nature, is this-  $\pi_2 = (12 - P_2 - 0.5P_1)P_2 - C(12 - P_2 - 0.5P_1)$ . So, we will use backward index and first solve this, respect to  $P_2$  and we will get equal to zero first order condition.

So, the reaction function of firm 2 is it is this-  $\frac{12 - 0.5P_1 + C}{2} = P_2$ . Now, you plug in this in the profit function of firm 1, take this. So, this is like 24 minus  $2P_1$  it is 6, it is this-  $\pi_1 = (18 + 2.25P_1 - 0.5c)(P_1 - c)$  and you will differentiate this with respect to  $P_1$ , you will get what. So, if you take the derivative of this part first, we will get this and first order condition will give that this is equal to zero-  $(2.25)(P_1 - c) + 18 + 2.25P_1 - 0.5c = 0$ .

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The image shows a whiteboard with handwritten mathematical work. The top part shows the derivative of the profit function for firm 1 with respect to price  $P_1$ , set equal to zero:

$$\frac{d\pi_1}{dP_1} = -(2.25)(P_1 - c) + 18 - 2.25P_1 - 0.5c$$

$$FOC = 0$$

The bottom part shows the steps to solve for  $P_1$ :

$$\Rightarrow 18 - 4.50P_1 + 1.75c = 0$$

$$\Rightarrow 18 + 1.75c = 4.50P_1$$

$$\Rightarrow \frac{18 + 1.75c}{4.50} = P_1$$

$$\Rightarrow \frac{18 + 1.75c}{4.50} = P_1$$

$$P_2 = 12 + c - 0.5 \left( \frac{18 + 1.75c}{4.50} \right)$$

$$\pi_2 = \left[ 12 - P_1 - 0.5 \left( \frac{12 - 0.5P_1 + c}{2} \right) \right] (P_1 - c)$$

$$= \left( \frac{24 - 2P_1 - 6 + 2.5P_1 - 0.5c}{2} \right) (P_1 - c)$$

$$\pi_1 = \left( 18 - 2.25P_1 - 0.5c \right) (P_1 - c)$$

$$\frac{d\pi_1}{dP_1} = (2.25)(P_1 - c) + 18 + 2.25P_1 - 0.5c$$

$$FOC = 0$$

So, we get this part is equal to 18 plus it is to 4.5 P1. Cost is 2.75 to be negative. So, it will be negative and positive. So, this term is actually negative, okay. So, this is equal to zero. So, 18 minus I think there is some problem here. This term is going to be plus and this is, so it is plus 2.7. So, it will be this part is 2.5 plus 2.25 and there is minus 0.5.

So, it will be 1.75 C with this and this will be so, this is the price and you plug in this price in this reaction function and you will get P2 is equal to 12 plus C minus 0.5 this, 18 plus 1.75 C-

$P_2 = \frac{12 + c - 0.5 \left( \frac{18 + 1.75c}{4.50} \right)}$  okay. So, we get this in the stage 2 and this in the stage 1. So, these are the subgame perfect Nash equilibrium prices in this price competition, okay.