Introduction to Market Structures
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Lecture 36
Simultaneous move Hotelling Model

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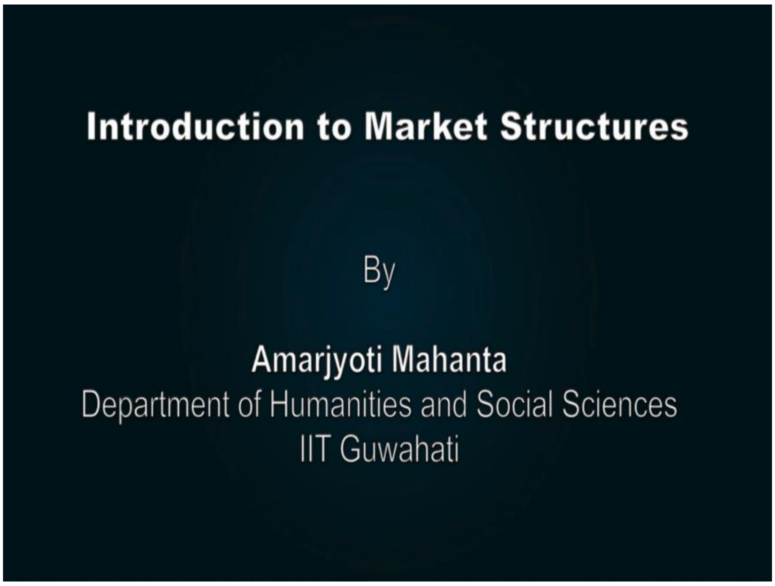

Hello, welcome to my course Introduction to Market Structures. Today, we are going to do product differentiation.

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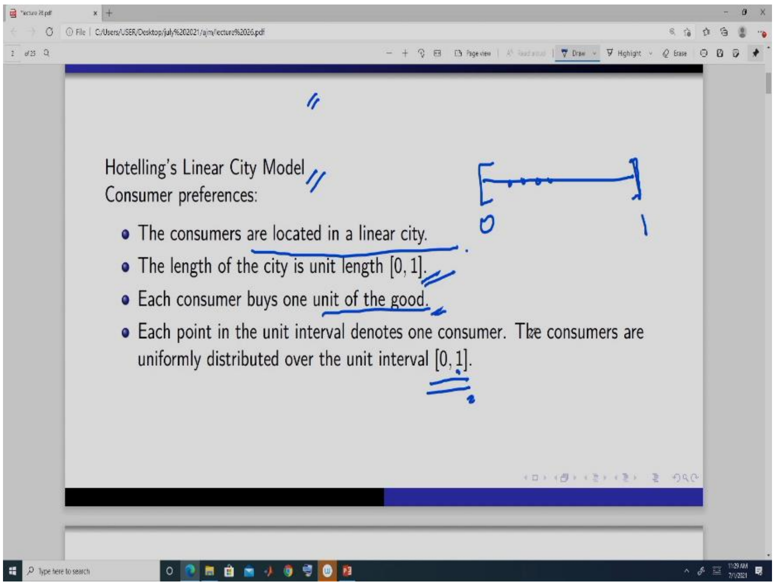

And we have already done a version of product differentiation while doing stackelberg price competition. In the stackelberg price competition, what we have done, we have assumed that the firm 1 and firm 2 can differentiate their product and that lead to either production of some complimentary goods or some substitute goods, it depends.

Now, we are going to do it completely in a different way and the model that we are going to do is a very famous model and that is Hotelling's Linear City Model. So, Hotelling's Linear City Model is mainly about a location, like think of stretch of the length of suppose unit length and there are some ice cream parlours. Now, the ice cream parlours they sell the similar kind of ice cream, but they can vary based on the location in that stretch, okay of one kilometre you can take.

So, depending on their different location what happens, people will go to that ice cream parlour which is nearer to that person. So, that is also a way of differentiating product or you can say that you have different attributes and these attributes can be mapped in a distance of 0 to 1. So, it is like it takes a value which lies between 0 and 1 okay.

So, let us do the specific model. So, what we do, we do that who what do we assume that the first consumers are located in a linear city, okay. So, it is a straight line and the length of the city is unit length. So, it can be any but for simplicity, we assume that it is unit length and each buyer buys only one unit of the good. So, this is you can say, it is an example of discrete good. So, here a consumer buys only one unit, okay and each point in the interval this unit interval, so, it is something like this.

So, this is suppose unit interval, here each point denotes one consumer, okay or you can say that the consumers are uniformly distributed over the unit interval [0, 1]. So, the distribution of consumers where each consumer buys one unit is uniformly distributed over [0,1], okay it is same thing, okay. But this is more technically correct, the consumers are uniformly distributed over the unit length, okay.

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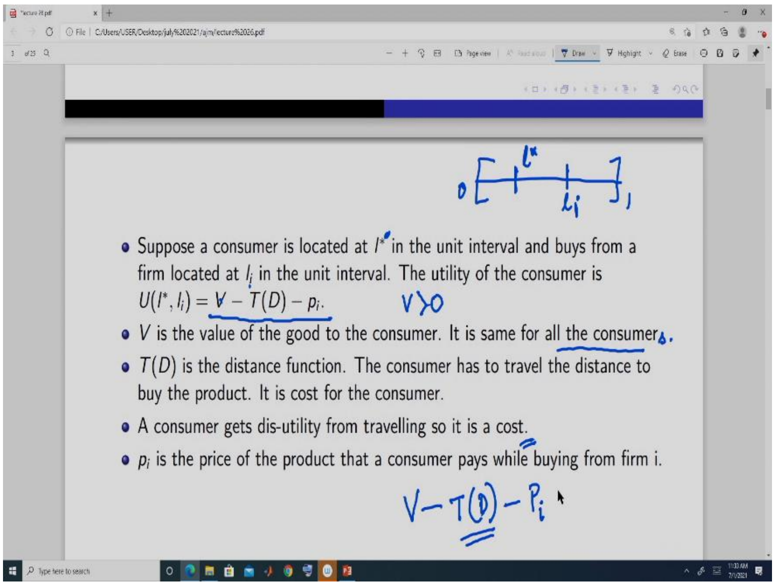

Now, suppose a consumer is located at this. So, here it is unit length. This is 1 and 1 and suppose the consumer is located here this is L star, this point, okay and buys from a firm located at Li and suppose this is another firm which is firm I and its location is Li. So, the utility of the consumer is given by this function- $U(l^*, l_i) = V - T(D) - p_i$, okay. So, what is this function? This function is, this V is a value that other consumer gets from the consumption of this good. So, each value is something like this that you have a utility function and in that utility function, you plug in the optimal value and you get the value function.

Now, here, when we say value function it means because the each consumer is buying only one unit, so this is you can say is the maximum amount a consumer is willing to pay to buy that. So, that is what we mean by the value, okay. So, this is V and we assume that V is a positive number, okay and again for simplicity, we assume that it is same for all the consumers, okay. Now, so, this portion is clear when we define the utility, we first find that there is a portion that is the value, okay. Now value means the maximum a consumer is willing to pay for that and this portion is the distance function. So, this is a kind of negative thing so, it is a kind of a cost or you are getting this utility from it.

So, the distance that a consumer has to travel from its own location and from the location where the firm is located so, that distance a consumer has to travel to buy that good. So, that is creating a kind of its disutility and so, it is taken as a cost, okay. So, or you can say it is the travel, you have to travel this portion and then while traveling you incur some cost and that is this cost, okay because this model has been used in many ways. If you study suppose R1 economics, how the city grows, how a city grow over time, then also you will find a Hotelling Model and this kind of the basic is this model, okay.

And since the consumer gets disutility from traveling so, it is taken as a cost and so, this portion is V that what I get from consuming goods or the maximum amount that I am willing to pay minus this. So, this D is the distance that one person has to travel or a consumer has to travel and this is given by a function. So, this is the cost that is the traveling cost minus the price of the goods set by firm I and that is Pi, okay. So, this is the utility. Why do we need this utility function? You will see why we need.

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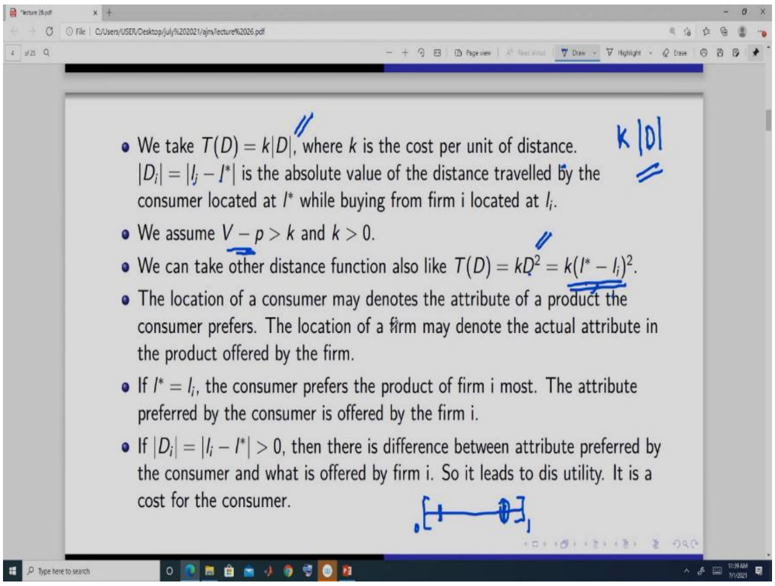

Now, let us define this distance function-$T(D) = k|D|$. So, this distance function is defined in this way k absolute value of D. Now here, k is the cost per unit of distance, okay and suppose the distance is D then it is k into D and this distance is defined in this way-$|D_i| = |l_i - l^*|$. So, it means that suppose take this 0, 1 suppose this is the location of a consumer that is L star and this is the location of a firm Li. So, then the distance is you can say D is absolute value of the distance is Li minus L star.

This distance this one. So, this into k gives you the total amount of cost incurred on traveling or the disutility that you get from traveling you can say that. Because this consumer is suppose buying from this, suppose then you get this much this small k into this amount of disutility or you can say it is a cost and then we specify further, you will see that this k we assumed in such a way that this is V minus p is this k is less than this amount-V–p>k and k>0, okay.

You will see why we need to assume this. So, this is one form of distance function and since this distance is always a positive value so, that is why we take it in terms of absolute value or you can take another form of distance function. Suppose you take the square of this. So, if you take the square of this, it is something like this-$T(D) = kD^2 = k(l^* - l_i)^2$, okay. So, the further away so, it is now this is no more linear in this. So, if you are further away then, your utilities this utility is much more compared to when you were nearer it, okay. Now this you can think of this linear city model in terms of like this.

Suppose you want to buy a product and that product has some attributes and that attribute you can give value to it and it like you can give value from 0 to 1 and suppose your preferred your location denotes the attribute that you want and the location of the firm denotes the attribute that is offered by the firm. Now, if there is a distance then it means what, that the exact kind of attribute that you want a product to have that is not being offered, okay. So, there is a mismatch. So, you get some amount of disutility from it but if suppose it is matched so, if your location and the location of the firm is same, then it means that the attribute that you want in that product, that same attribute is being offered by the firm.

So, it is something like this. So, you want suppose the speed of your mobile phone and you want a specific number to that, okay you want a specific speed, but the mobile phone companies are not offering that but it is offering something else. So, the difference between these two we can take that as a distance and that distance is giving you some form of a disutility because what you want you are not getting it, but if the distance is not there, then you prefer that product most. So, from here what do we get?

So, you will buy from that firm which is nearest to you because that means that firm is closest to you and so, that firm is giving you that attribute which is closer to the attribute you want, right? But here, if you think in terms of attribute as if you take attributes see there is again another problem here. So, we have to be very careful. So, like this so if we are talking about this point as one attribute and this point and another attribute, we do not want to imply that the quality here is better than the quality here.

It is not something like that. So, it simply it is, we are silent on the quality. What we are saying simply is that these are different nature or different kinds and the quality of these each kind is same or it is good, okay or simply you can think that when you are choosing from suppose, you are walking in sea beach and you want to buy an ice cream. So, you will look for that parlour, ice cream parlour which is nearest to you from where you are standing. So that idea, okay. So, consumers are going to buy from that firm which is nearest, okay. So that is why this distance function plays an important role. So, the way we define the distance function we can vary the results, okay.

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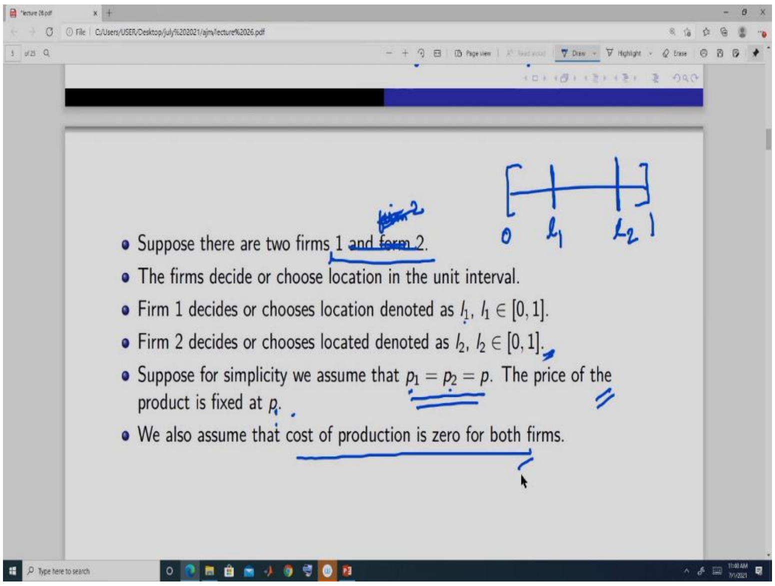

Now, we come to the firm. So, we first take the simplest firm and that is we have two firm. Firm 1 sorry two firms, 1 and 2, okay. And the firms decide or choose location in the unit. So here, in this unit location there is one location that is chosen by firm 1 and another is chosen by firm 2, okay. So, firm 1's location is denoted by L1 and the location of firm 2 is denoted by L2, okay. And for simplicity, we assume that the prices are same, p1 is equal to p2 they are same, okay and it remains fixed.

So, the firms in our model in the actual the original model hotelling and that which is further developed in a game theoretic form that includes this price. So, price is endogenously determined in that model but that is slightly complicated model. So, since this course is not that advanced course so, we will not take that second stage of the game in this model. We will only consider only this location thing. So, this price we will assume that it is fixed and it is same for each firm, okay and further to simplify the things, we assume that the cost of production is 0 for both the firms, okay.

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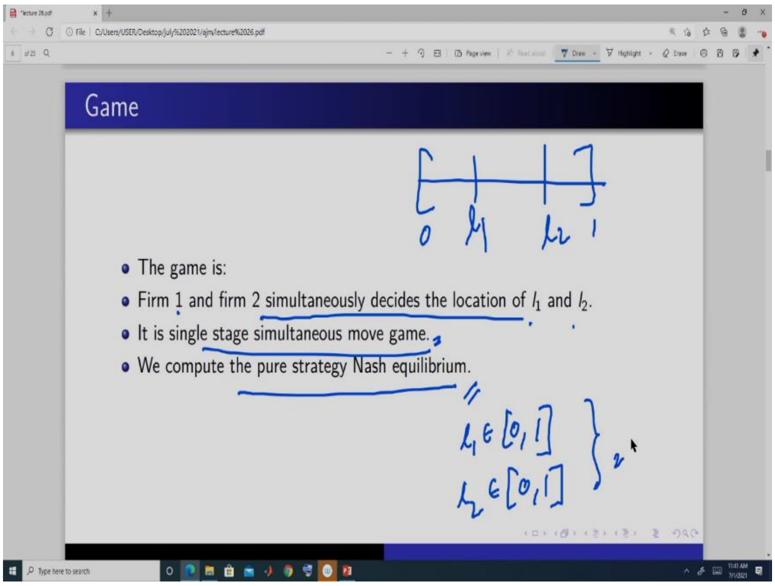

Now, what is the game here? So, the game is firm 1 and firm 2 simultaneously decides the location l1 and l2. So, in this unit interval 0 and 1, in this interval firm 1 chooses the location l1 and firm 2 chooses the location l2 and these decisions are taken simultaneously. So, this is a single stage simultaneous move game. So, in this game what do we do, we compute the pure strategy Nash equilibrium. Now here the strategy said you can say l1 is any location between 0 and l2 and strategy set of firm 2 is at any point in this unit interval 0 and 1, okay. So, this is the strategy. Now we move to the solution, okay. So, we find the pure strategy.

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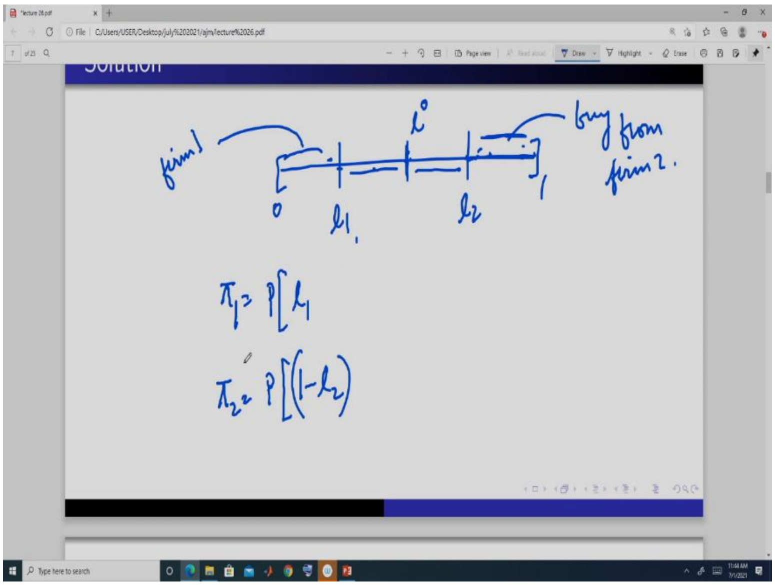

Now here, let us first define the payoff, okay. So, profit of firm 1, you can say suppose, okay before doing that, let us take one. Suppose, this is the unit interval and suppose this is the l1 and this is l2, this is the location of firm 1 and this is the location of firm 2. So, the profit of firm 1, so, these people are going to buy from firm 1, these people are definitely going to buy from firm 2 because if they move from here to here, this distance is quite bigger than simply this distance. So, this firm people are going to buy from this. So, price is fixed, cost there is no cost. So, it is going to get l1.

This is form and profit of firm 2 price is fixed, it is going to get 1 minus l2 because this distance any point here this whole distance is this is 1 from 0 to 1 and this is l1. So, this is left so it is 1 minus l1 this distance. So, this much it is going to get and it is going to firm 1 is going to get some portion of the people who are located in this region. So, we will get a person like this suppose L star or l dot or l not such that this person is going to be indifferent from buying from this and this because prices are same, value is same and if the distance is same, so, this distance must be equal to this distance for this person to be indifferent between buying from firm 1 and firm 2. So, how do we locate this l not. So, we have to complete this part. So, we have not yet done this.

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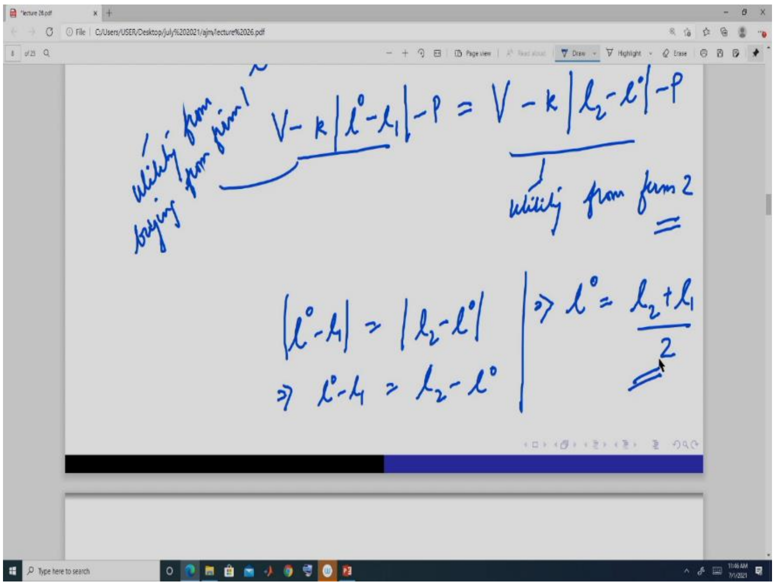

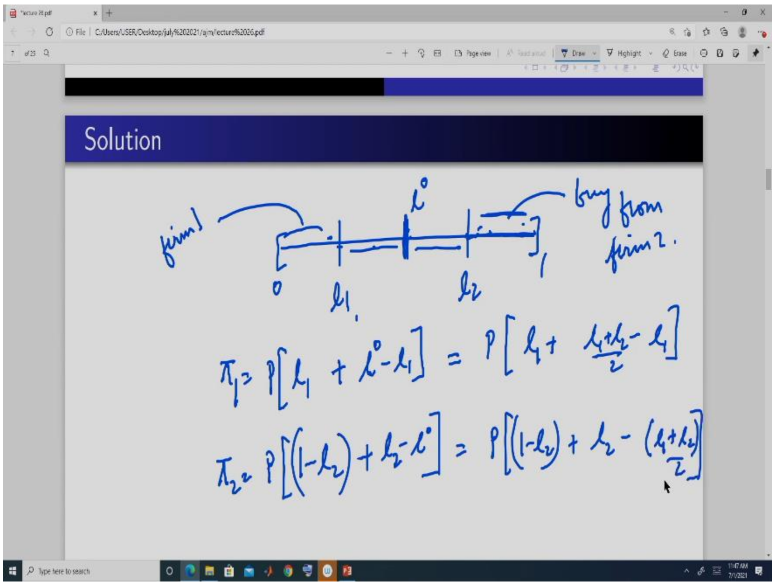

So, l not is l not is such that V minus k l **not** minus l1 should be equal to k l2 minus l dot minus P, i.e. $V - k|l^o - l_1| - P = V - k|l_2 - l^o| - P$ this is the utility from firm 1 and this is the utility from firm 2, okay. So, from this we have to find out this l not. So, this is you can say if we do the simple manipulations, we will get this is equal to this-$|l^o - l_1| = |l_2 - l^o|$ and since the absolute values are or this modulus are always positive, so, we can say l1 is l dot is going to lie somewhere here.

So, it is this point is greater than l1 and this point is less than l2. So, we take it this in this form, this distance is this-$l^o - l_1 = l_2 - l^o$. So, from here, what do we get? We get this-$l^o = \frac{l_2+l_1}{2}$. So, this l dot here, so, this person is who is indifferent and we have find out the location of that person. Location of that person is this, half of this distance between this. So, the profit of firm 1 here it should have been like this plus l2-$\pi_2 = P[(1 - l_2) + l_2 - l^o]$. So, this is equal to what is this? This is l1 plus l2 minus and profit of firm 2 is this- $P[(1 - l_2) + l_2 - \frac{l_1+l_2}{2}]$.

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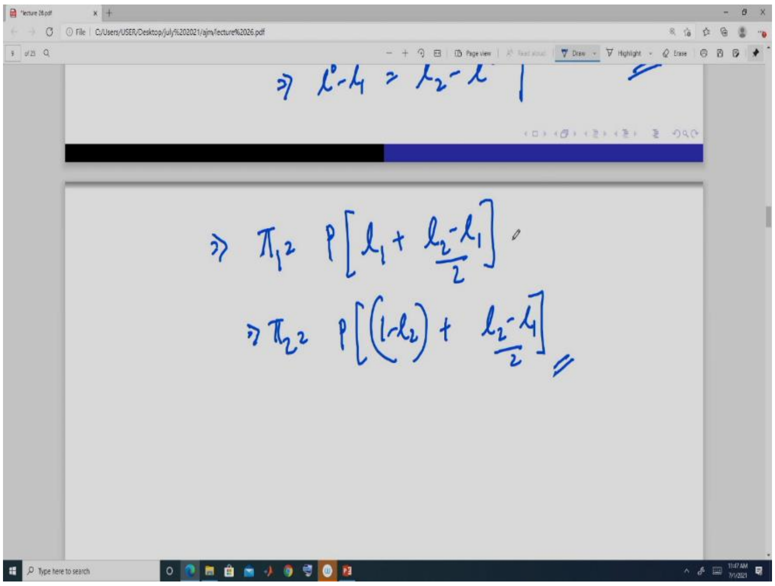

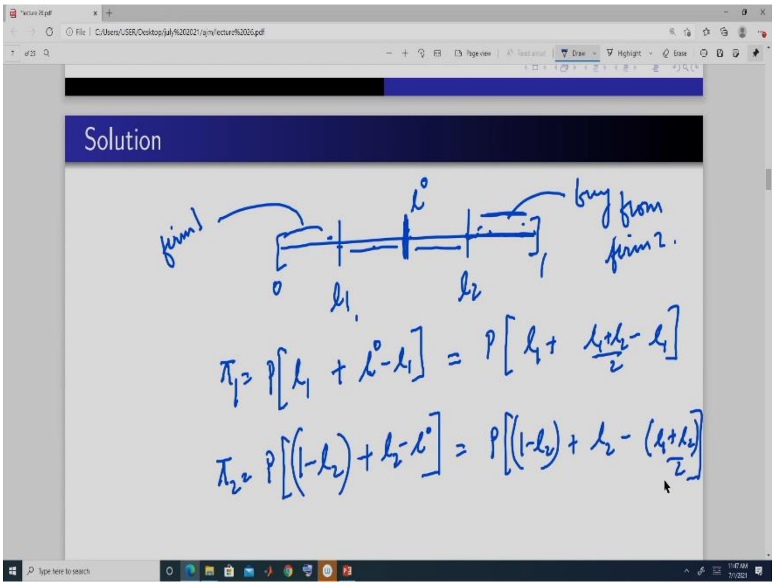

So, we can say that the profit of firm 1 is- $P\left[l_1 + \frac{l_2 - l_1}{2}\right]$ and profit of firm 2 is this- $P\left[(1 - l_2) + \frac{(l_2 - l_1)}{2}\right]$. Now the thing is we have to decide the optimal or the Nash pure strategy Nash equilibrium l1 and l2. So, how do we do that. So, we have on this this as the given some l1, l2, okay.

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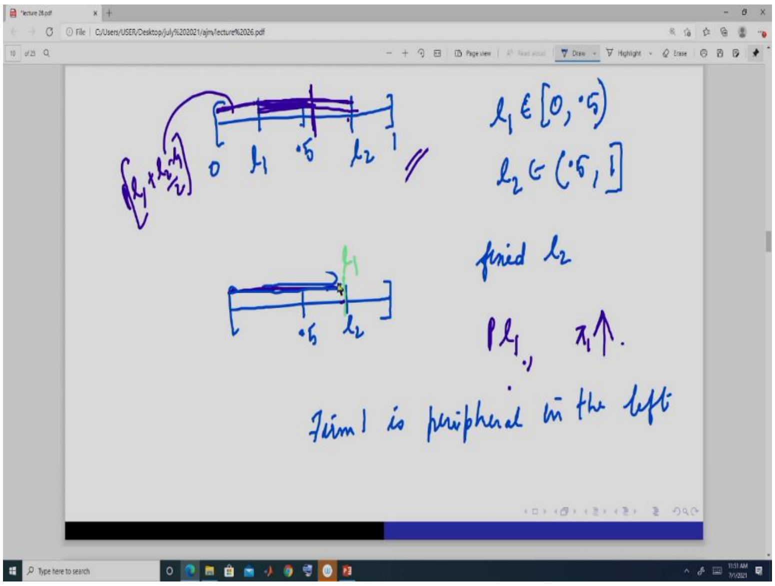

So, let us look at that. Suppose take this example, this case sorry not example, this case. This point is the half, okay suppose l1 is here and l2 is here, okay is this any point here lying between 0 and 0.5, any point here lying between 0.5 and 1. Is it optimal is any point? So, l1 lying between this point is suppose not included. Now, if we take this case now think of this equation. Here, fix l2, l2 is somewhere here because we have taken this. If you switch l1 here and suppose the l1 is in this same position, same position here and this is 11, okay.

What is the profit of firm 1? Now earlier profit of in this case, profit of firm 1 was this P into l1 plus half of this distance. This half of this, this-P $\left[l_1 + \frac{l_2 - l_1}{2}\right]$ now if it switches here in this position its profit is so, this whole is getting it is getting now earlier only this and half of this suppose this is the half. So, it was getting this much profit, now it is getting whole amount. So, what is happening? This gives the profit of firm 1 increases here.

So that is why this is not a Nash equilibrium, pure strategy Nash equilibrium. If firm 2 chooses this point, firm 1 choose point adjacent to it. Why? Because see if you look it from this side firm 1 is the peripheral firm, right? So, firm 1 is peripheral in the left, okay? Okay, So, there is no firm where it is going to lie here. So, if it lies adjacent, this whole portion is going to be its own market share. So that is why firm 1 is going to choose adjacent to firm 2. So, firm 1 locates to firm 2, okay.

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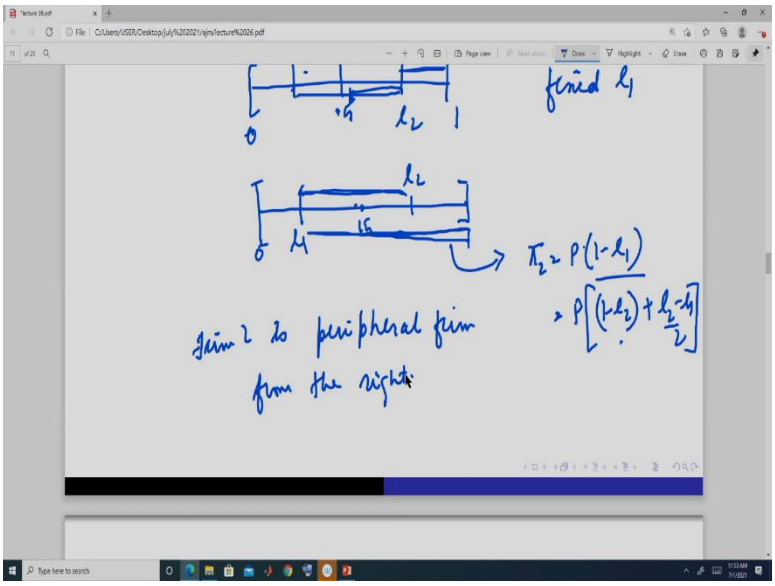

Now take this case, so, this is l1, this is l2. So, fix l1, okay. Now based on the same argument, you will see that if firm because now what is the profit of firm 2? Firm 2 is this length plus half of this length, half of this length which is suppose given by this amount so, it is this. Now, instead of that suppose it moves to this position. If we fixed l1, if it moves here this whole portion is the profit of firm 2. So, its profit now it is P into this distance, this whole distance.

So, this whole distance you can say is 1 minus l1 which is greater than P 1 minus l2 plus l2 minus l1, when l2 is this l1 is this fixing. So, this-$\pi_2 = P(1 - l_1)$ is greater than this-$P\left[(1 - l_2) + \frac{(l_2 - l_1)}{2}\right]$. So, that is why again firm 2 here from this argument, you will see firm 2 is peripheral firm from the right, okay.

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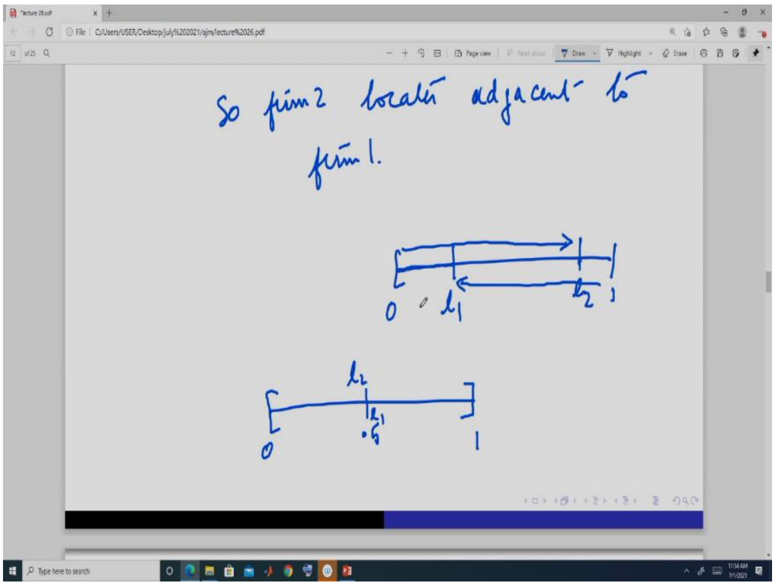

So, firm 2 locates adjacent to firm 1. So, from this argument we see that the peripheral firms will always located adjacent to the firm because this if you look at this position l1, l2 so, this is peripheral from left but this is not peripheral from left. This is peripheral from right. So, this firm will tendency will be to move here. This firm which is peripheral in the right will have a tendency to move here, okay. So, they will move. Now in this way what do we see that 0.5 which is the middle, centre of this unit length. So, l1 will locate here and l2 will also locate here.

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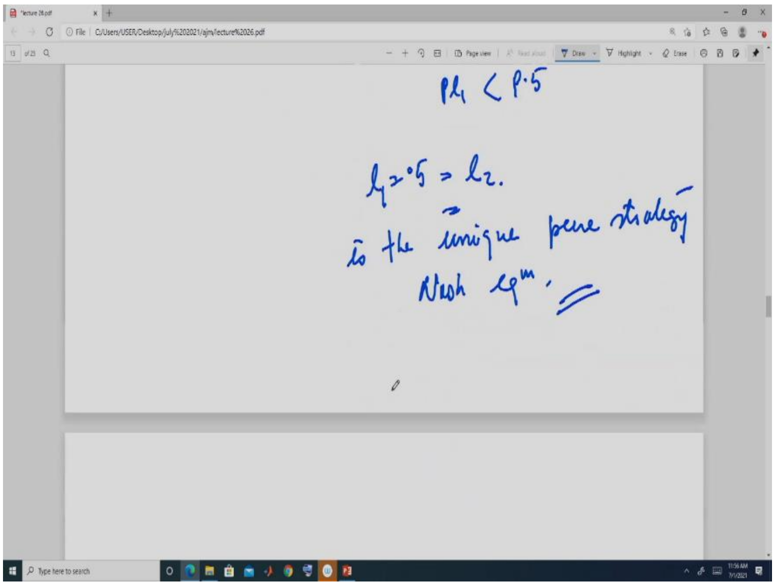

And so, in this situation what do we get? Because they will always locate at the adjacent position, right? Now, if you take any point other than this. Like take l1 here. So, if l1 is here then from that argument we know l2 is going to be here. So firm 1's profit is only this much which is less than half. So, profit is this- $\mathrm{pl}_1 < \mathrm{P}.5$. So, firm 1 if it moves here, it knows l2 is also going to because this l2 is greater than this here. So, profit of firm 2, this is greater than.

So, firm 1 if it locates here, this point firm 2 is also going to locate here, so it is better to locate here. So that is why l1 is equal to this. This- $\mathrm{l}_1 = 0.5 = \mathrm{l}_2$ is the unique pure strategy Nash equilibrium in this case when we have two firms and they decides the location simultaneously because any point other than this, there is a tendency to deviate, okay.

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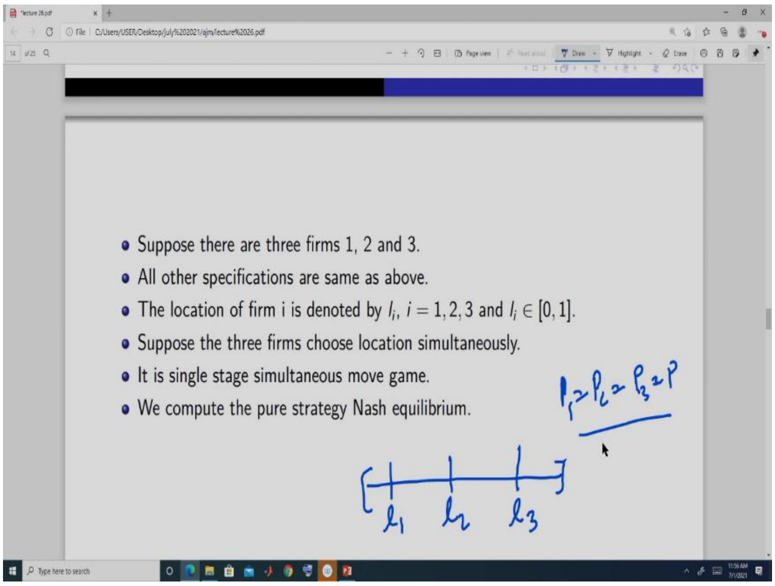

Now, let us introduce one more firm. So, suppose instead of two firms, we have now three firms and it is something like this. Firm 1 chooses the location l1. Firm 2 chooses the location l2 and firm 3chooses the location l3, okay. And rest of the specifications are same that the price is same that is p1 is equal to p2 is equal to p3 is equal to P. There is no cost of production and we have one consumer in each point. So, that is consumers are uniformly distributed in the in unit interval, okay.

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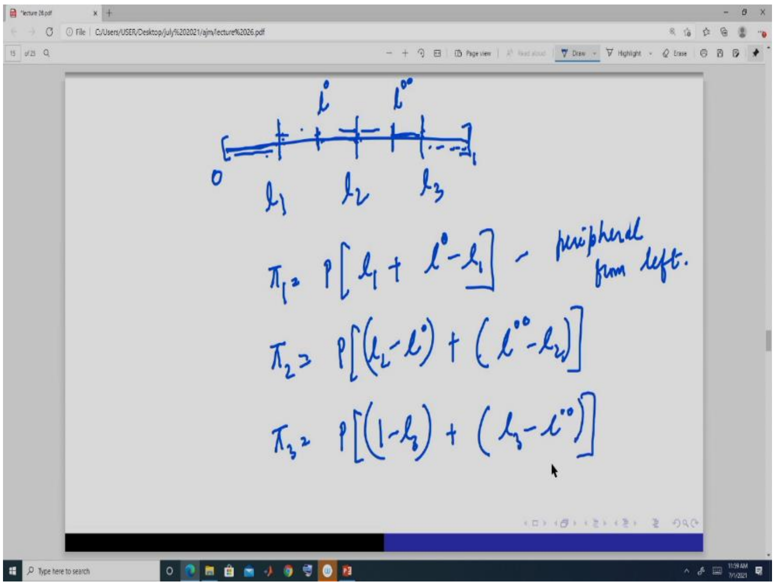

Now in this setup, see if we have a situation like this suppose this is 0 this is 1. So, this is the unit interval, suppose location of firm 1 is this, location of firm 2 is this, location of firm 3 is this. Now how do we define the payoff? we will have a person who is going to be indifferent between buying from l1, firm 1 and firm 2. Similarly, we will have a person here who is indifferent between buying from firm 2 and firm 3.

So, this location is suppose, l dot and this l not and this is suppose l double dot, okay. Now, how do we define the profit? Firm 1, firm 1's profit is this people are only going to buy from this firm only they because if they travel to any other firm their cost is going to be higher. So, profit of firm 1 this many people they are going to buy from firm 1 and so, these people are going to buy from firm 1. So, that is l not. This is the profit of firm 1-$\pi_1 = P[l_1 + l^o - l_1]$ if the locations are of this nature and then profit of firm 2 is it is in the middle, it is not a peripheral firm anymore because this is peripheral from left and profit of firm 1 is going to get half of this and sorry half of this and half of this right or these people.

So, its location is l2 minus plus this plus l double not minus l2- $\pi_2 = p[(l_2 - l^o) + (l^{oo} - l_2)]$ and profit of firm 3, firm 3 which is peripheral from right its profit is going to be this much- $\pi_3 = P[(1 - l_3) + (l_3 - l^{oo})]$. It is always going to get no one is going to buy from any other firms when firm 3 is located here. So, everyone is going to buy from firm 3 who are lying in this region and it is going to get this length because this person is indifferent between so it is this.

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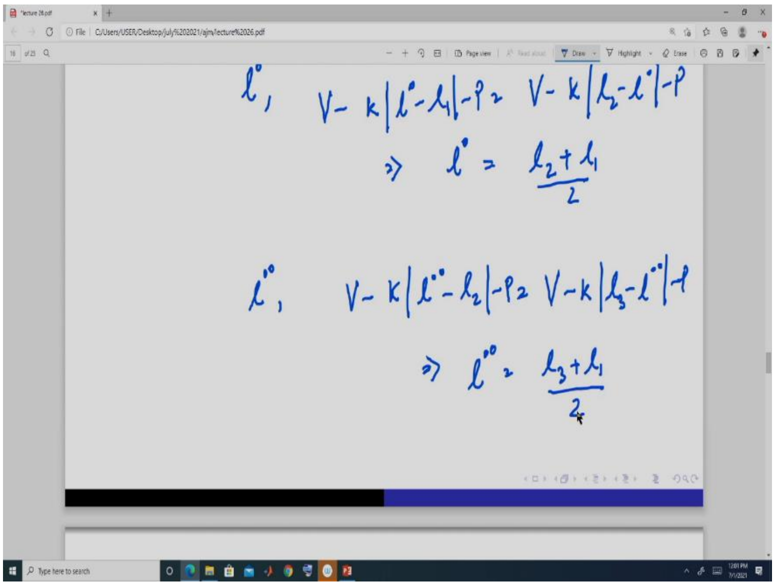

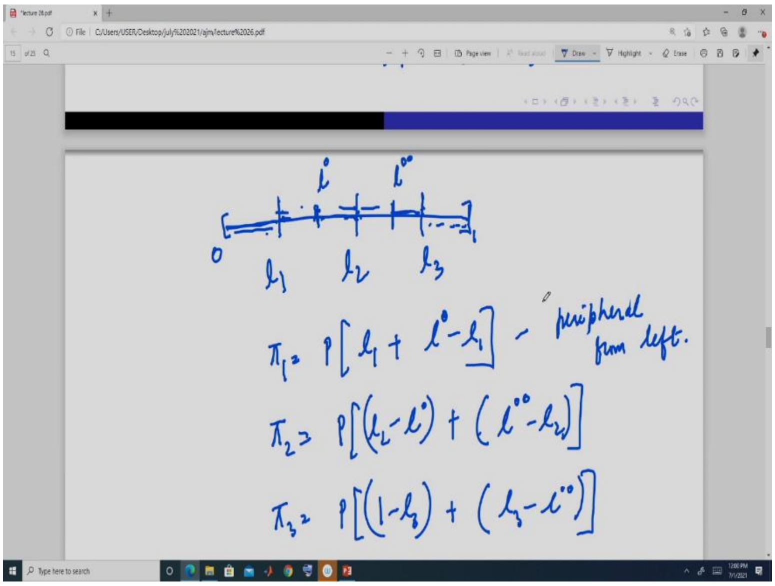

Now we have to find out these two l dot and l not and l double not. So, l double not is such that V is equal to this- $V - k|l^o - l_1| - P = V - k|l_2 - l^o| - P$. So, from here we get l this- $l^o = \frac{l_2+l_1}{2}$ again. So, it is going to be centre of this length, okay this. So, exact this much length is going to be how much? So, we will do that. Now, let us first find this l double not. l double not is such that l double not minus l2 is equal to l3. So, it is indifferent from buying from firm 2 and firm 3, it is this- $V - k|l^{oo} - l_2| - P = V - k|l_3 - l^{oo}| - P$. So, we get l this, this is the location of l double not- $l^{oo} = \frac{l_3+l_1}{2}$ and this is the location of l not.

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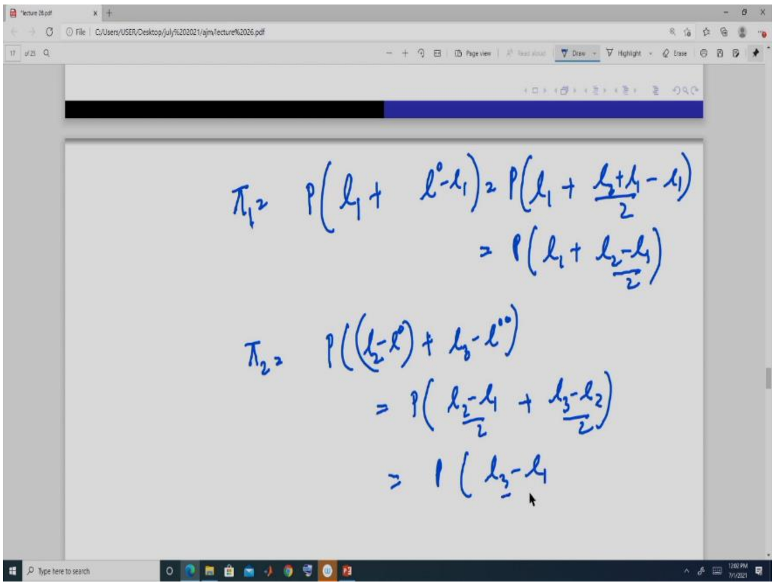

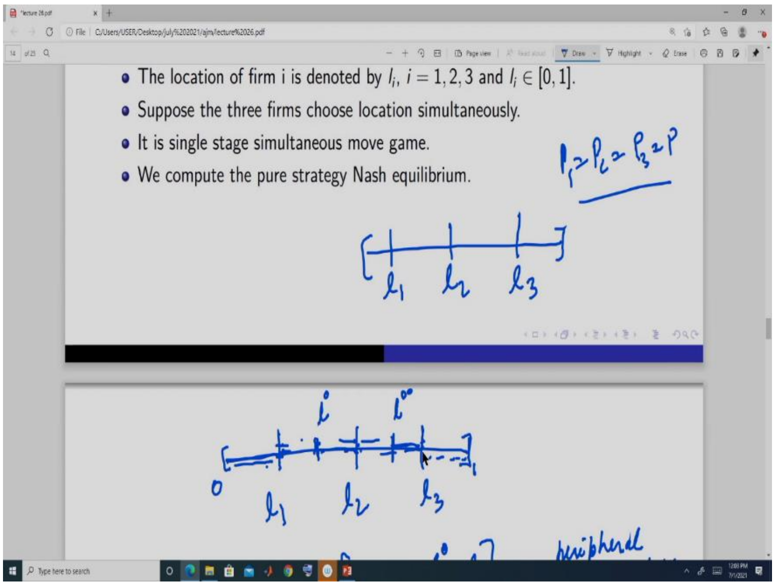

So, from here we get the profit of firm 1 is l1 plus. So, it is l dot minus l1, right? So, this is P into which is equal to-$\pi_1 = P(l_1 + l^o - l_1) = P\left(l_1 + \frac{l_2+l_1}{2} - l_1\right) = P(l_1 + \frac{l_2-l_1}{2})$ and this profit of firm 2 is so, this is equal to this portion is this only. So, this is plus here this is l3 minus l2. So, this is actually here this is farm two's profit is half of length is market share is half of this distance, okay, this- $P(\frac{l_3-l_1}{2})$.

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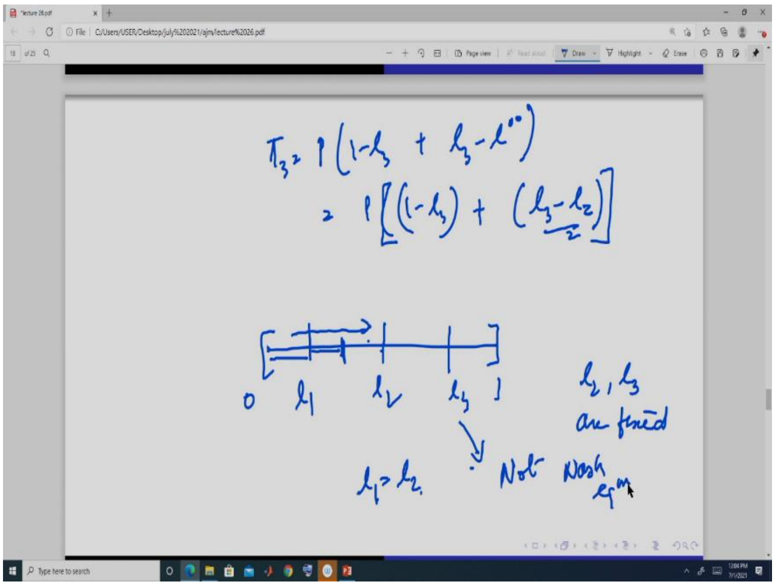

And profit of firm 3 is which is now equal to 1 minus l3 plus l3 minus l2 this- $\pi_3 = P[(1 - l_3) + (\frac{l_3-l_2}{2})$. Now, we have to find the exact location of l1, l2, l3. Now, if you take this and suppose this is l1 this is l2 and this is l3. Now if firm 1 fix this two suppose l2 and l3 are fixed. So, if it moves here what is happening? Earlier it was profit is this much plus half of this. So, it is this. Now, if you move what is happening?

Your V is getting profit is going higher because you are getting more length. So, your market share is increasing. So, it is best for firm 1to locate adjacent to firm 2. So, l1 is equal to l2 in this situation. So, that is why this is, this is not Nash equilibrium, right?

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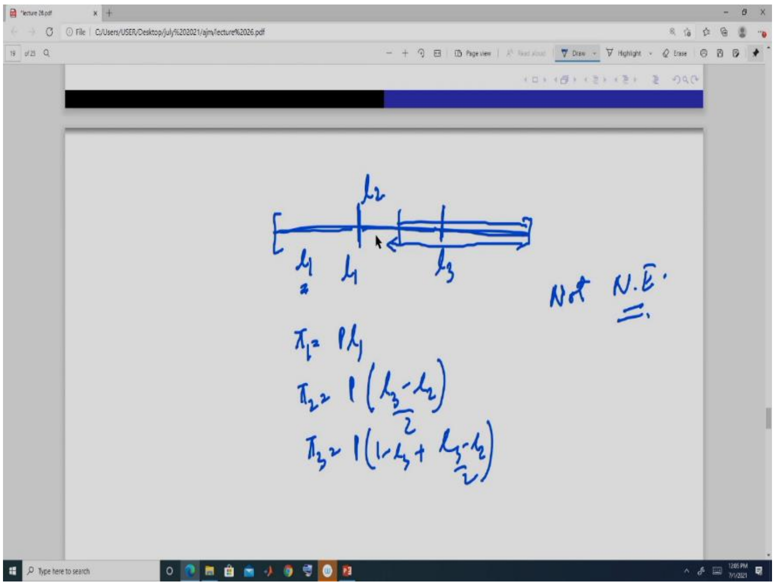

Similarly, if you look from the perspective of firm, so, now suppose l1 and l2 this is somewhere. Let us look at l3. It is here right. Now l3, so, what it is? This is suppose is got by l1 so, profit of is l1, profit of firm 2 is half of this because this side it is firm 1 and this side it is firm 2. So, it is this l3 minus l2 half of this and profit of firm 3 here if you look it is this 1 minus l3 plus half of this distance this. So, if so, it is getting some this much now, if it moves like this, then it is profit is going to increase. So, firm 3 so that is why (locate) locating here it is not Nash equilibrium if given this two a firm 3 moves like this its profit increases. So that is why it is not a Nash equilibrium.

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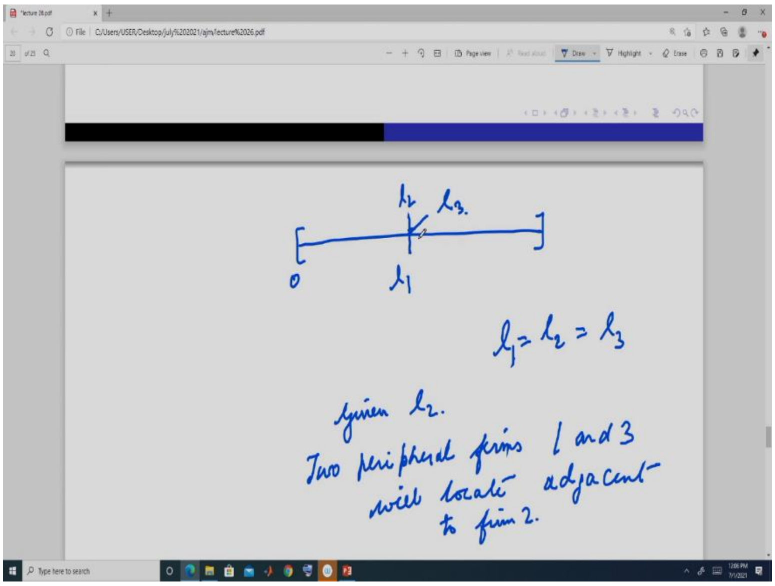

So, from here what do we get? We get that firm so l1 is going to be l2 from the first argument and from the second argument, we get that this it should also be adjacent. So, this should also be l3 should also be here. So, what that it means so given l2, two peripheral firms 1 and 3 will locate adjacent to firm 2 from 1 will from move from left and another will move from right.

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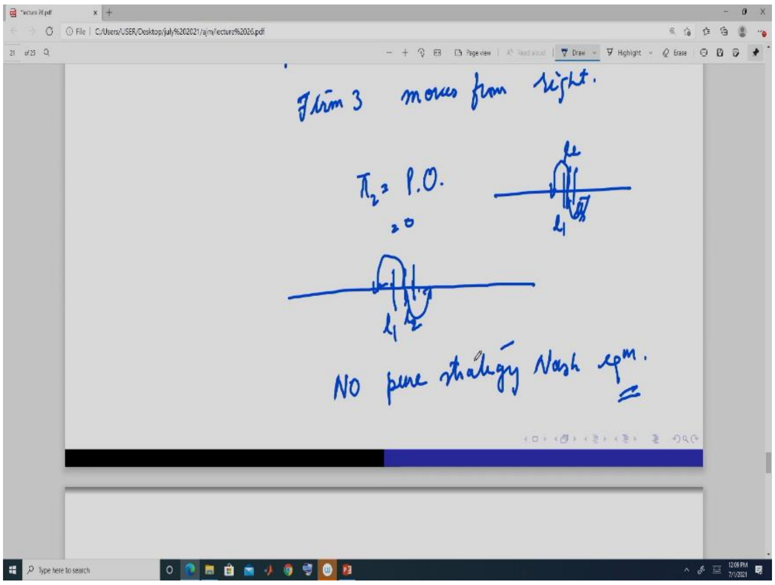

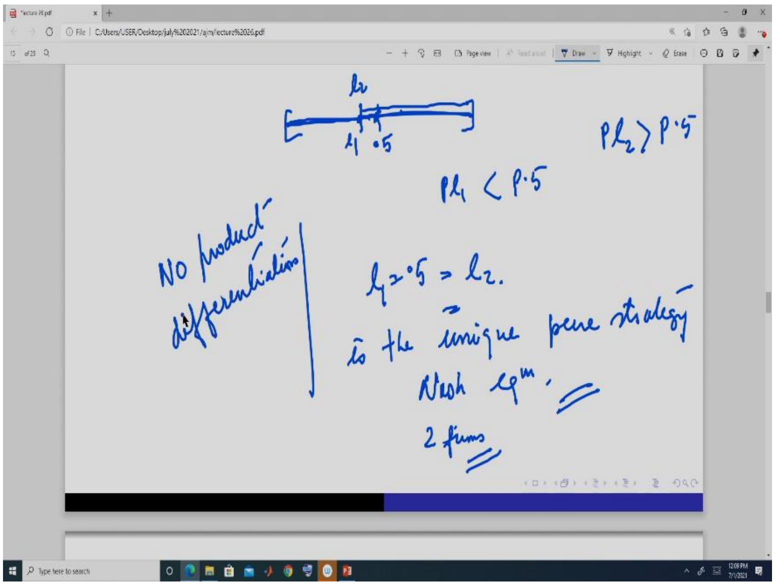

So, firm 1 moves from left and firm 3 moves from right and here we do not get any because then what is the profit of firm 2, profit of firm 2 is 0 because it is sandwiched between firm 1 and firm. So, it is here. So, it is sandwiched between firm 1 and firm 3. So, firm 2 will lie here or it will shift to this. So, if it is l1 is this and l two is and suppose this is adjacent so, firm 2 will think of moving here or it moving here. If it moves then firm 1 will be the centre. In this case, firm 3 will be the centre. So, then again it will try to move.

So, what do we get when we have three firms and they choose the location simultaneously then and price is fixed. So, then no pure strategy Nash equilibrium, okay. So, this is our when we have two firm we get that it is going to be at the centre of the this and so, it results into no product differentiation, right? So, it is a unique and so, it is since it is going to be the at the centre. So, in this situation we have no product differentiation, right? or you can say minimum product differentiation. And here we do not know what is going to happen because we do not have any pure strategy Nash equilibrium.

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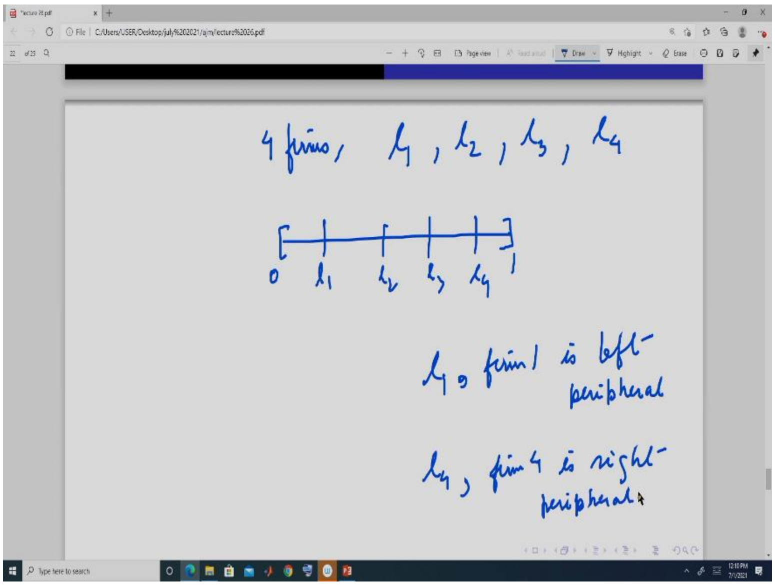

Now suppose we have four firms. Keep everything same and suppose the location of firm 1 is l1, location of firm 2 is l2, location of firm 3 is l3, the location of firm 4 is l4 and they choose from this and suppose this is l1, this is l2, this is l3, this is l4. Now from the previous argument, we know here l1 that is firm 1 is left peripheral firm and l4, firm 4 is right peripheral.

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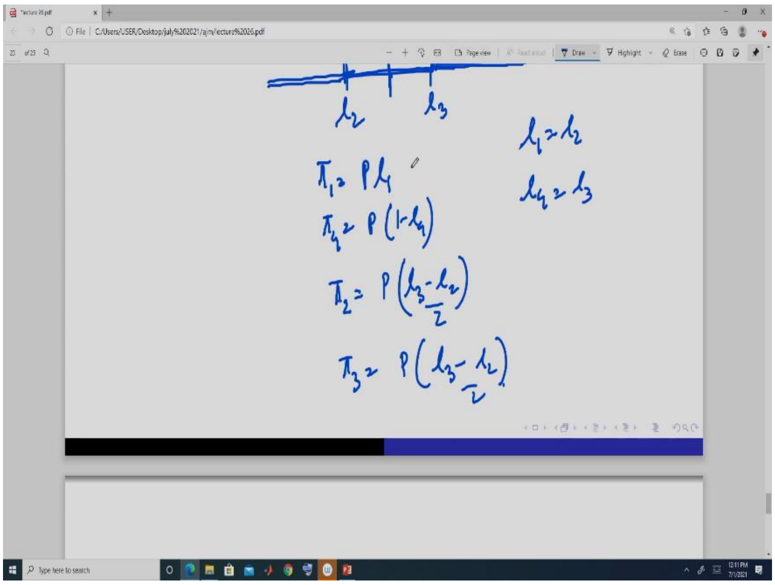

So, from the previous argument what do we get if l2 and l3 are fixed, here if l2 and l3 are fixed, so firm 1 will locate here adjacent and firm 3 firm 4 will locate here adjacent. So, in this case what is going to happen? Profit of firm 1 is going to be P into l1 this whole distance. Profit of firm 4 is going to be this length and we further we have l1 is equal to l2 and l4 is equal to l3. Now here if you look at what is going to be the profit of firm 2?

Profit of firm 2 is going to be so half of this distance. So, this is right because the we will get a person who is going to be indifferent between buying from l2 and l3, firm 2 or firm 3 and we can get this at what point this is the half of this distance. So, this is going to be this- $\pi_2 = P\left(\frac{l_3 - l_2}{2}\right)$ and for firm 3, it is going to be same, this. Now we can find out what is going to be the solution of this by solving this here. Is it possible to get a solution?

Now see, if this distance is less than this distance, then firm 2 will not lie here but it will lie right of firm 1 and we will take this. So that is why this distance must be equal to this distance. Similarly, if this distance is greater than this distance firm 3 will not lie (ri) left of firm 4 but will shift to the right of firm 4.

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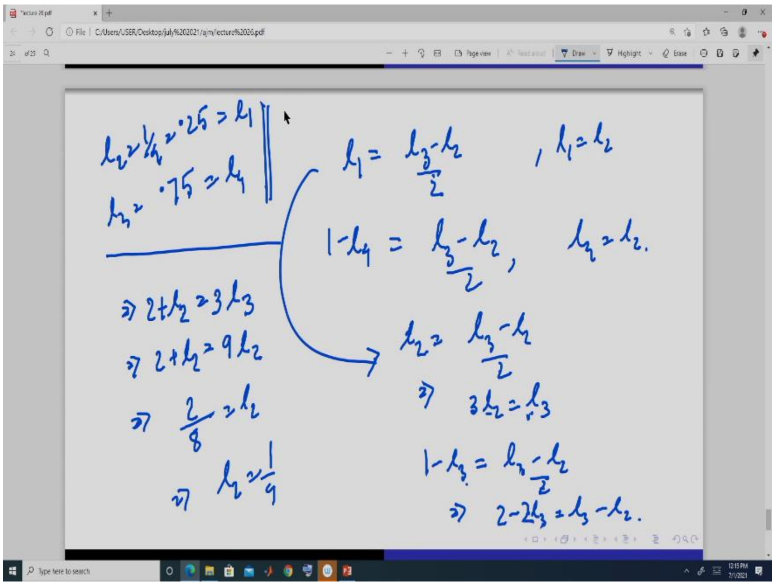
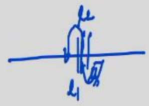


Firm 1 moves from left.

Firm 3 moves from right.

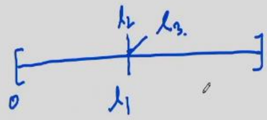
$\Pi_2 = P.O.$

$= 0$



No pure strategy Nash eqm.

$\pi_3 = 1\left(1 - l_3 + \frac{l_3 - l_2}{2}\right)$

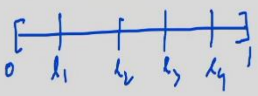


$l_1 = l_2 = l_3$

given l_2.

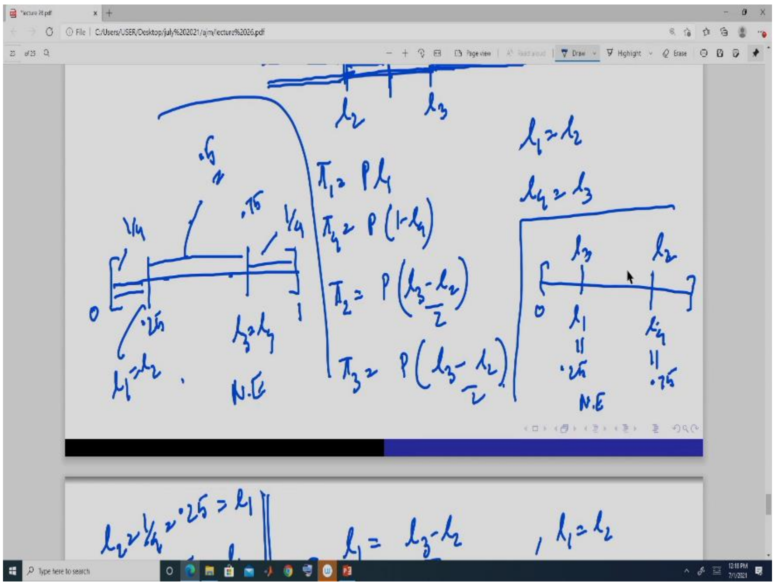
... firms 1 and 3

4 firms, l_1, l_2, l_3, l_4



l_1, firm 1 is left-peripheral

l_4, firm 4 is right-peripheral.



So, from this argument what do we get? We get that l1 must be equal to this-$\frac{l_3-l_2}{2}$, when l1 is equal to l2 and further, we get 1 minus l4 is equal to l3 minus l2 when l4 is equal to l2, right. So, from here, what do we get? So, l1 is equal to l2 from this. So, l2 should be equal to this from this here. So, this gives me this- $3l_2 = l_3$ and from here l3 and l4 are same. We get this so, from this what do we get? 2 minus 2 plus l2 should be equal to 4-2 $+ l_2 = 4l$. We have this, so it will go bring here should be 3 l this and from this, we have already this here. So, we get l2 in 3.

So, place here. So, it will be 9 l2. So, this is equal to 2 by 8 l2. So, l2 is 1 by 4. So, what do we get? We get here, we get that l**2** is 1 by 4 or what is 0.25. So, immediately l3 is 0.75 it is three times into this and this is equal to l1 and this is equal to l4. So, this is one pure strategy Nash equilibrium. You can take a different also combination of this. So, this is l2, l3 l1 is same and l2 and l4 is same. So, there you can get different combinations. So, this is one pure strategy- $l_2 = \frac{1}{2} = 0.25 = l_1$. So here, if you look at this diagram, it will be like this.

This distance is so this point is 0.25, so this distance is one-fourth and this is the location of firm 1 and firm 2, this is the location of firm 3 and this distance is again, fourth and this distance is and this distance is, you can say 0.5 distance so they are getting half-half, okay. So, this is a one pure strategy Nash equilibrium. So, in this case, when we have four firms, we get that there exists a pure strategy Nash equilibrium, but when we have three firms, we see that we do not have any pure strategy.

And again, when we have two firms, we see that there is a pure strategy and there is no product differentiation and when three, we do not know what is going to happen. When we have four, we get that the product differentiation is of this nature. Two firms who are going to be similar and another two firms are going to be similar but these two firms are different from these two firms, okay or another combination can be this that is l1, l3, this is equal to 0.25 and l2, l4 is equal to 0.75. This can be one another pure strategy Nash equilibrium. So, this is again a Nash equilibrium.

This is also this is also a Nash equilibrium. So, in this case we get this firm 1 and firm 3 they are similar, but and firm 2 and firm 4 are similar, but, these two are different. Here firm 1 and firm 2 are similar, firm 3 and firm 4 are similar, but this group is different from this group. So, this is a kind of product differentiation. So, we get that there are some firms which are similar but they are going to be different from the other set of the firms. So, we can go on trying this if we have suppose, five firms if we have six firms if we have seven firms and then we will see get a pattern like this that the peripheral firms are always going to lie adjacent to their nearest neighbour, okay.

And this we will not go further more than four firms because the calculations are slightly complicated, even five-six firms It is easy, but if you go beyond that, then you will have to solve many equations. So, you can try those on your own. And that is it.

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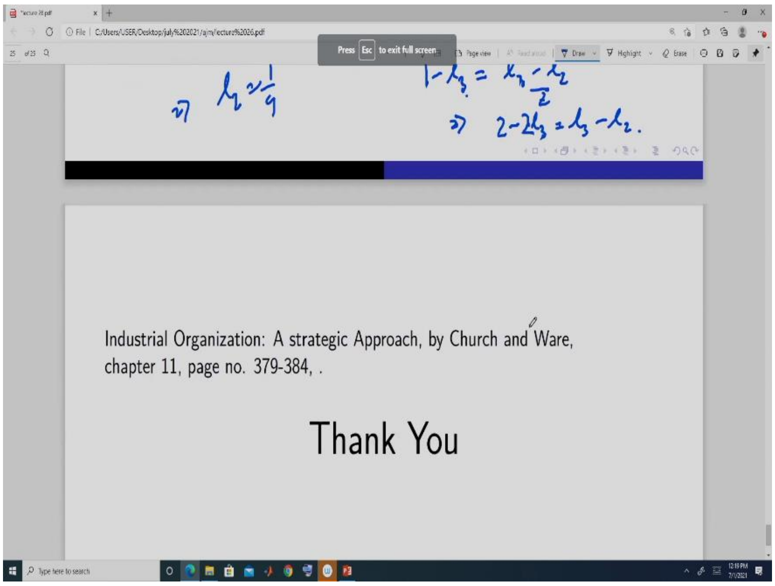

So, if you want to read from where this portion I have done, it is from this book Industrial Organization, a strategic approach by Church and Ware. These are the specific page numbers 379 to 384, chapter 11. Thank you.