

**Introduction to Market Structure**  
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**Lecture 35**  
**Stackelberg Price Competition**

Hello, welcome to my course introduction to market structures.

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• Suppose there are two firms 1 and 2. It is a duopoly market.

• Firm 1 and Firm 2 differentiate their product. The goods can be complementary or substitute in nature.

• Firm 1 produces good 1 and firm 2 produces good 2.

• The demand function of good 1 is  $A - p_1 + bp_2 = q_1$ ,  $p_1$  is the price of good 1,  $p_2$  is the price of good 2,  $b$  denotes the degree of product differentiation,  $A$  is a positive real number,  $q_1$  is the quantity demanded of good 1.

$q_1 = A - p_1 + bp_2$

So, in the last class we have done stackelberg quantity competition and today we are going to do stackelberg price competition. In stackelberg quantity competition, we have seen that if the reaction function are linear and downward sloping, then there is always a first towards first mover's advantage. So, it means that a firm which chooses output first that it has a, it makes higher profit then the firm which chooses output secondly, or in stage is two.

So, here today instead of price, instead of quantity firms are going to choose prices. So, again we assume more or less similar setup. So, there are two firms, firm 1 and, firm 1 and firm 2. And it is a duopoly market. Now, here what we have? What is different in this setup is that the products are differentiated, okay. So, differentiated product means they are not perfectly substitutable. So, the goods can be either complimentary or they are substitute in nature.

So, what does it mean? So, when I say substitute, it is not a very close substitute. It is something like, the brands the different brands, they substitute their product in terms of quality. So, in that sense we make this assumption, that they are substitutable. And complementarity, here it means

that goods are completely complimentary. That means, if I want this good 1, then I will also demand good 2.

But if it is substitute, then if I demand good 1 then I will not demand good 2, okay. Like, if we think of goods having differences say in terms of some attributes or in terms of some qualities, like this, okay. But that is not explicitly modeled here, we only model it in a very simplest form. So, firm 1 produces good 1 and firm 2 produces good 2. And the demand function of good 1 is of this nature- $A - p_1 + bp_2 = q_1$ .

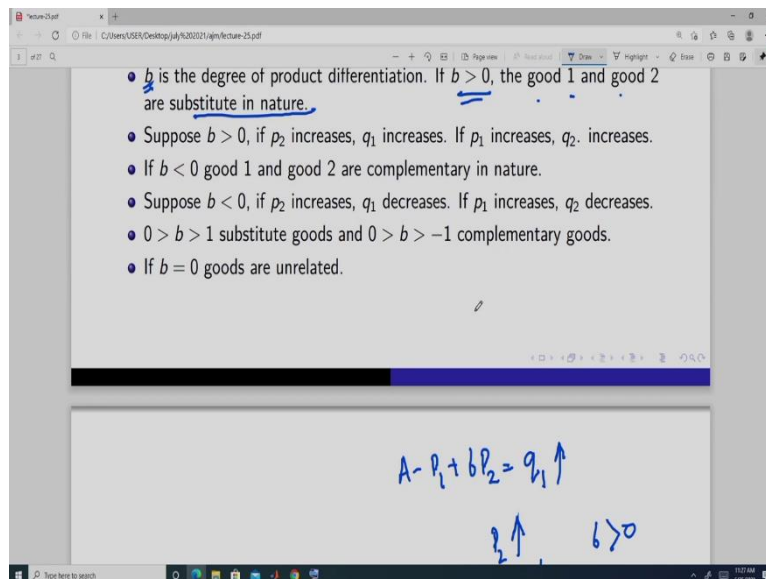
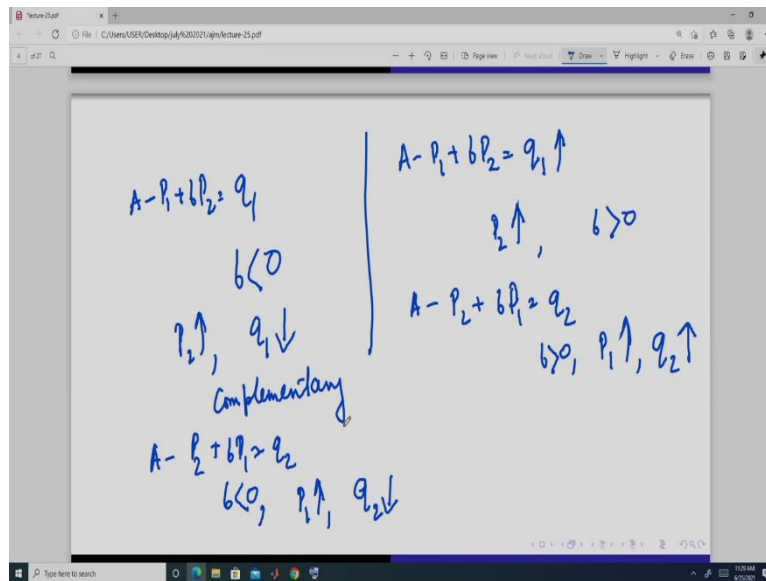
So, the demand function is A minus p1 plus b, p2 is equal to q1. So, p is the price of good 1, p2 is the price of good 2 and b denotes the degree of product differentiation, okay. And here A is a positive real number and q is the output. So, if you look at this demand curve, it is again downward sloping in this p1. This is the price of good 1. So, it is again downward sloping, we will come to this slightly later.

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The image shows a presentation slide with a white background and a blue header. At the top, the equation  $A - p_1 + bp_2 = q_1$  is written in blue. Below the equation, there is a list of bullet points explaining the parameters. The first bullet point states: "The demand function of good 2 is  $A - p_2 + bp_1 = q_2$ ,  $p_1$  is the price of good 1,  $p_2$  is the price of good 2,  $b$  denotes the degree of product differentiation,  $A$  is a positive real number,  $q_2$  is the quantity demanded of good 2." The second bullet point states: " $b$  is the degree of product differentiation. If  $b > 0$ , the good 1 and good 2 are substitute in nature." The third bullet point states: "Suppose  $b > 0$ , if  $p_2$  increases,  $q_1$  increases. If  $p_1$  increases,  $q_2$  increases." The fourth bullet point states: "If  $b < 0$  good 1 and good 2 are complementary in nature." The fifth bullet point states: "Suppose  $b < 0$ , if  $p_2$  increases,  $q_1$  decreases. If  $p_1$  increases,  $q_2$  decreases." The sixth bullet point states: " $0 > b > 1$  substitute goods and  $0 > b > -1$  complementary goods." The slide is displayed in a window titled "Notes2.pdf" with a file path "C:\Users\USER\Desktop\July 2021\ajm\lecture-25.pdf".

Next the demand for good 2 is again like this-  $A - p_2 + bp_1 = q_2$ , a minus p2 plus bp 1 is equal to q2. Here p1 is the price of good 1, p2 is the price of good 2, b denotes the degree of product differentiation. So, it is same as the b earlier for, in the, that we have, so, used in the demand for a good 1 and q is the quantity demanded of good 2. See, b is the degree of product differentiation. If b is positive, then good 1 and good 2 are substitute in nature.

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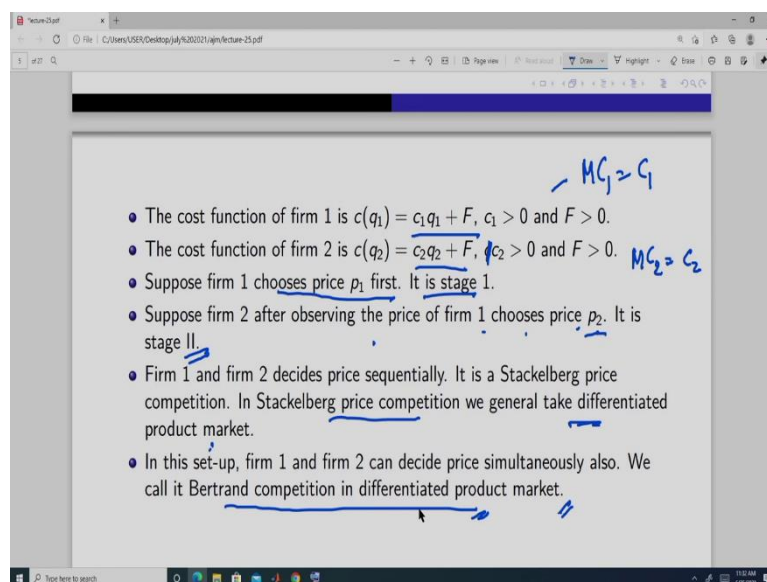
So, it, it is something like this. So, demand if, it is like this. Now here, if  $b$  is positive that means what? If  $p_2$  suppose increases, that is if  $p_2$  increases,  $q_1$  increases, demand for  $q_1$  increases, that is good 1 increases. So, good 2, when the price of good 2 increases, demand for good 1 increases. So, that means they are substitutes in nature, because the moment the price of a good increases, we reduce its demand.

So, that is why they are substitutes in this case, when  $b$  is positive. Similarly here, also we will get same thing. If  $b$  is positive and if price of good 1 increases, then quantity demanded of good 2, that is going to increase, okay. So, people will substitute, buy more of good 2 then good 1 when the price of good 1 increases, okay. So, this is the case. Now, if in the same demand function, if you take this and now suppose  $b$  is negative. So, that means what is happening?

If  $p_2$  increases, price of good 2 is increasing, then this portion is increasing so, this and it is negative, so,  $q_1$  falls. So, here when the price of, good 2 increases, demand for good 1 is also going, is going down. So, goods are complimentary here, okay. Similarly, if we take this good and suppose  $b$  is negative, then when price of good 1 increases  $q_2$  falls. Because the complementarity goods are, they are, they are going to be consumed together.

They required in certain ratio or in some here like pen and refill, or desktop and mouse, something like those, those kind of things. So, so since it is complimentary, so, if the price of one good increases, so, that is the demand for that good is going to go down so, the demand for other goods is also going to go down. So, in this way, there is product differentiation between firm 1 and firm 2. And how they are doing this product differentiation? We are not going to do that, okay. So, we will come to that later on when we do product differentiation.

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Now we specify the cost function. Cost function of firm 1 is  $c_1$  into  $q_1$  plus  $F$  -  $c(q_1) = c_1q_1 + F$ . So,  $c_1$  is some positive number. So, if we have a cost function like this, then we know the marginal cost of firm 1 is a constant and it is  $c$  and it is  $c$  as positive number. Similarly, the cost function of firm 2 is of this nature,  $c_2$  into  $q_2$  plus  $F$  -  $c(q_2) = c_2q_2 + F$ ,  $c_2$  is a positive real number sorry, is a positive real number. So the marginal cost of firm 2 is  $c_2$ .

So, it is a constant marginal cost, okay. Now we define the game, how? What actually the firms chooses? So, firm 1 chooses pricing first. So, it is stage 1, and firm 2 after observing the price of firm 1 chooses price  $p_2$ , so it is stage 2. So, here firm 1 and firm 2, decides price sequentially.

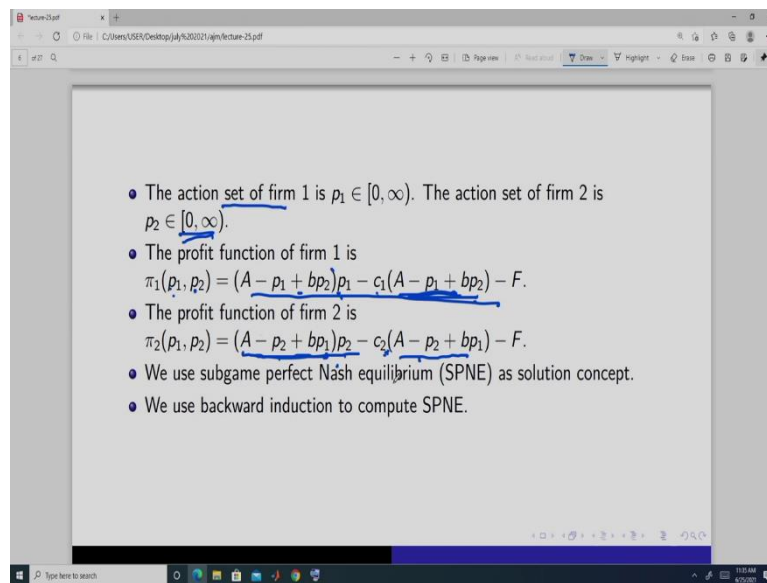
So, firm 1 chooses price, price first and then firm 2 chooses price, okay. And so that is why it is a stackelberg price competition.

Because we are choosing price, so price and since it is, decisions are taken sequentially so, that is why it is stackelberg. Now, generally when we do stackelberg price competition, we take, take differentiated product market, why? Because we have seen, that if we take homogeneous good market and if there is a price competition, then we also have seen something called a bertrand paradox. So only bertrand paradox when it, when we do not see any bertrand paradox.

When either there is a capacity constraint or there is decreasing returns to scale. So, that is why we bring in product differentiation to move out of bertrand paradox, okay. Now, in this setup, what we have defined now, like the cost function and the demand function, the way the products are differentiated, that can be studied as a bertrand competition in where? Prices are taken, are taken simultaneously or the firms choose their price simultaneously.

So, then it becomes a bertrand competition in differentiated product market. But we are not going to do that. And it is same, almost same there is no difference. So, it is simply more or less same as the Cournot competition, only here it is price and in Cournot it is quantity, okay. But in this stackelberg price competition, why we are going to do it? We will see that, the end that we get a very interesting result, okay. So, we are going to do stackelberg price competition.

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The screenshot shows a presentation slide with the following content:

- The action set of firm 1 is  $p_1 \in [0, \infty)$ . The action set of firm 2 is  $p_2 \in [0, \infty)$ .
- The profit function of firm 1 is  $\pi_1(p_1, p_2) = (A - p_1 + bp_2)p_1 - c_1(A - p_1 + bp_2) - F$ .
- The profit function of firm 2 is  $\pi_2(p_1, p_2) = (A - p_2 + bp_1)p_2 - c_2(A - p_2 + bp_1) - F$ .
- We use subgame perfect Nash equilibrium (SPNE) as solution concept.
- We use backward induction to compute SPNE.

So, it is prices are decided sequentially. So, that is why, since it is an extensive game, so, we are defining the action set. Now the action set is not same as the strategy set. Your strategies are mainly in a dynamic game, we define a strategy for an act, strategy is a set of complete set of action in all contingencies. So, here since it is a continuous state we cannot define contingency in that way.

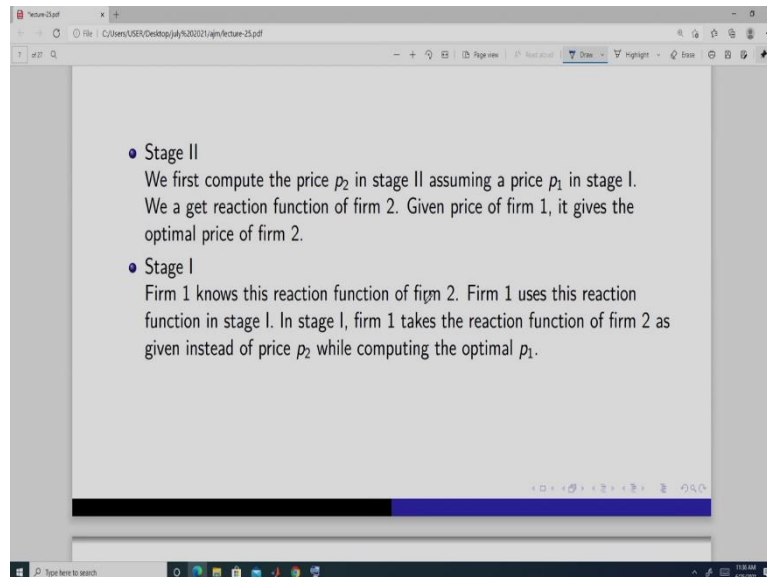
Because each price in a range are going to be, because  $p_1$  can take many possible values. And then we have to define  $p_2$  for each such contingency. So, that is it is not possible or we can do it using something called the reaction function and we will do that actually, okay. So, the action set of firm 2 is this, okay. So, firm 1, now the profit of firm 1 it is, it is going to be function of  $p_1$   $p_2$  and it is, this  $-(A - p_1 + bp_2)p_1 - c_1(A - p_1 + bp_2) - F$ . And profit of firm 2 it is this, this is what? This is the demand function.

So, it is quantity  $q_1$  into  $p_1$ , so, this is the total revenue minus this is the total cost. Because this is the  $q_1$ , quantity of good 1,  $c_1$  into this portion, this is the quantity. Now, so, total revenue minus total cost so, gives me the profit. Here, this portion is the quantity demanded of good 2 into price. So, total revenue, this is  $c_2$  marginal cost into the total output or the total quantity demanded or that is being supplied by firm 2.

So, it is a minus  $p_2$  plus  $bp_1$ , this is the demand function of firm 2 or good 2. So, this is a total cost, this portion is the total cost. So, total revenue minus total cost gives me the profit  $-\pi_2 = (A - p_2 + bp_1)p_2 - c_2(A - p_2 + bp_1) - F$ . Now, so, these profit functions are the  $p$  of

functions of firms. So, we will use sub game perfect Nash equilibrium as a solution concept in this case and we will find out the SPNE through backward induction.

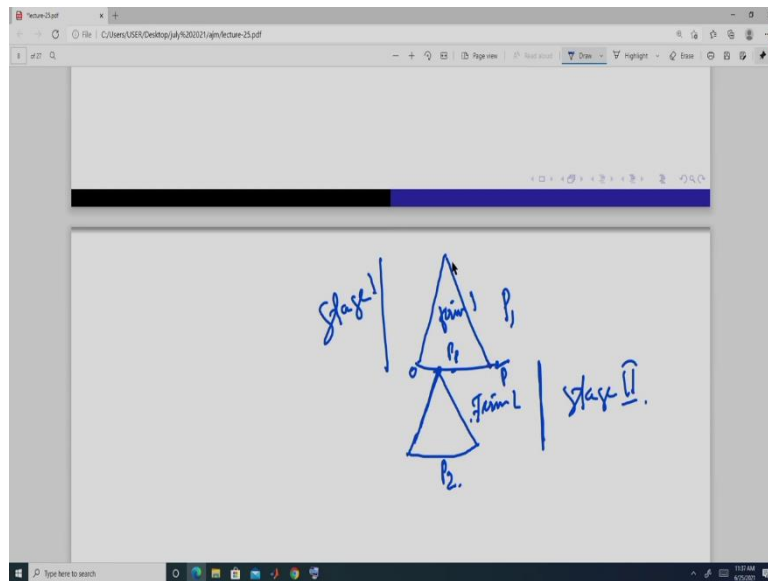
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So, what we do? So, first we will solve the stage 2, which is going to be, going to be the last stage. Assuming that there is a  $p_1$  in stage 1. So, we will get a reaction function of firm 2? In that when we optimize  $p_2$ ? So, we will know that if  $p_1$  is some, takes a specific value we will know what is the optimal value of  $p_2$  based on that reaction function. Now firm 1 will know this reaction function.

A firm 1 can compute this reaction function and firm 2 instead of taking  $p_2$  as given, what we will do? It will take that reaction function as given. And based on that, it is going to decide its optimal  $p_1$ .

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So, it is something like this, that in stage 1, in this stage, stage 1  $p_1$  is decided, okay. So,  $p_1$  takes some value here to come very, some positive number like this any point. And suppose this is and then after observing this, firm 2 is going to decide. So, this is firm 1 and this is firm 2. And this is stage 2.

So, here  $p_2$  is going to be decided, here  $p_1$  has been decided. So, this is stage 2. So, we solved first this and taking the optimal solution in this stage, we solved this, this is the way we use the way we solve the dynamic game, okay.



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Stage II Suppose firm 1 sets  $P_1$

$$\pi_2 = (A - P_2 + bP_1)P_2 - c_2(A - P_2 + bP_1)$$
$$\frac{\partial \pi_2}{\partial P_2} = A + bP_1 - 2P_2 - c_2$$

FOC,  $\frac{\partial \pi_2}{\partial P_2} = 0$

$$\pi_2 = (A - P_2 + bP_1)P_2 - c_2(A - P_2 + bP_1)$$
$$\frac{\partial \pi_2}{\partial P_2} = A + bP_1 - 2P_2 + c_2$$

FOC,  $\frac{\partial \pi_2}{\partial P_2} = 0$

$$\Rightarrow \frac{A + bP_1 + c_2}{2} = P_2$$

reaction fn of firm 2

So, so let us solve this stage 2,  $p_1$ . Since, this is a differentiable function so, we can take the partial with respect to  $p_2$  and we will get, this is firm 2, so, it will be  $p_2$ . Now, first order condition gives me, that is equal to 0. so, I get so, this is the reaction function of firm 2, we have got this  $\frac{A+bP_1+c_2}{2} = P_2$ . Now, based on this, we will so, this is the stage 2. In stage 2, we know that how firm 2 is going to react.

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$\frac{\partial \pi_2}{\partial P_2}$  reaction of firm 2

$$\Rightarrow \frac{A + bP_1 + C_2}{2} = P_2$$


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Stage I

$$\pi_1(P_1, P_2) = (A - P_1 + bP_2)P_1 - C_1(A - P_1 + bP_2) - F$$

$$\pi_1 = \left[ A - P_1 + b \left( \frac{A + C_2 + bP_1}{2} \right) \right] P_1 - C_1 \left( A - P_1 + b \left( \frac{A + C_2 + bP_1}{2} \right) \right) - F$$

$$P_2 = \frac{(2+b)A + bC_2 - P_1(2-b^2)}{2}$$

$$\pi_1 = \left[ A - P_1 + b \left( \frac{A + C_2 + bP_1}{2} \right) \right] P_1 - C_1 \left( A - P_1 + b \left( \frac{A + C_2 + bP_1}{2} \right) \right) - F$$

$$\pi_1(P_1) = \left[ \frac{(2+b)A + bC_2 - P_1(2-b^2)}{2} \right] P_1 - C_1 \left[ \frac{(2+b)A + bC_2 - P_1(2-b^2)}{2} \right]$$

Stage II Suppose firm 1 sets  $P_1$

$$\pi_2 = (A - P_2 + bP_1)P_2 - C_2(A - P_2 + bP_1) - F$$

$$\frac{\partial \pi_2}{\partial P_2} = A + bP_1 - 2P_2 + C_2$$

FOC,  $\frac{\partial \pi_2}{\partial P_2} = 0$  reaction of firm 2

$$\Rightarrow \frac{A + bP_1 + C_2}{2} = P_2$$

So, stage 1, firm 1 is going to use this. Firm 1, so profit function is this-  $(A - p_1 + bp_2)p_1 - c_1(A - p_1 + bp_2) - F$ . Fixed cost I have forgotten to put the fixed cost, okay this. Now firm 1 knows how  $p_2$  is going to be decided. If it chooses  $p_1$  what is going to be the  $p_2$ , it knows from this reaction function. So, firm 1 will plug in that value here, that function here instead. It will be this, then this portion will remain same bc 2 this is going to be plus b square and there it is going to be minus.

So, it is going to be minus 2,  $p_1$  sorry. And is going to be this, minus  $c_1$ . So, again here it is going to be the same thing. So, a plus b sorry, 2 A plus bc 2 minus this, okay profit of firm 1 is this-  $\left[ \frac{(2+b)A + bC_2 - P_1(2-b^2)}{2} \right] P_1 - c_1 \left[ \frac{(2+b)A + bC_2 - P_1(2-b^2)}{2} \right]$ . So, here it was this function of  $p_1$  and  $p_2$ . If you look, it is a function of  $p_1$  and  $p_2$ ,  $p_1$  and  $p_2$ ,  $p_1$  and  $p_2$ ,  $p_1$ . Now,  $p_2$  it knows it is given by this reaction function, so, this becomes only a function of  $p_1$ .

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$$\frac{\partial \pi_1}{\partial P_1} = \frac{A(2+b) + bC_2 - P_1(2-b^2)}{2} + \frac{C_1(2-b^2)}{2}$$

FOC,  $\frac{\partial \pi_1}{\partial P_1} = 0$

Stage 1  $\Rightarrow \frac{A(2+b) + bC_2 + C_1(2-b^2)}{2(2-b^2)} = P_1$

Stage 1  $\pi_1(P_1, P_2) = (A - P_1 + bP_2)P_1 - C_1(A - P_1 + bP_2) - F$

$$\pi_1 = \left[ A - P_1 + b \left( \frac{A + C_2 + bP_1}{2} \right) \right] P_1 - C_1 \left( A - P_1 + b \left( \frac{A + C_2 + bP_1}{2} \right) \right) - F$$

$$\pi_1(P_1) = \left[ \frac{(2+b)A + bC_2 - P_1(2-b^2)}{2} \right] P_1 - C_1 \left[ \frac{(2+b)A + bC_2 - P_1(2-b^2)}{2} \right] - F$$

$$\pi_2 = (A - P_2 + bP_1)P_2 - C_2(A - P_2 + bP_1) - F$$

$$\frac{\partial \pi_2}{\partial P_2} = A + bP_1 - 2P_2 + C_2$$

FOC,  $\frac{\partial \pi_2}{\partial P_2} = 0$

$\Rightarrow \frac{A + bP_1 + C_2}{2} = P_2$  *reaction fn of firm 2*

Stage 1  $\pi_2(P_1) = (A - P_2 + bP_1)P_2 - C_2(A - P_2 + bP_1) - F$

Now, we know again this is a differentiable function. So, this gives me, this  $\frac{d\pi_1}{dP_1} = \frac{A(2+b)+bC_2}{2} - P_1(2-b^2) + \frac{c_1(2-b^2)}{2}$ . And first order condition gives me, this is equal to 0. So, we get is equal to p1 this, this is stage 1  $\frac{A(2+b)+bC_2+c_1(2-b^2)}{2(2-b^2)} = P_1$ . In stage 1, we get this p1 and in stage 2 we get, this.

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stage II

$$P_2 = \frac{A + C_2 + bP_1}{2}$$

$$= A + C_2 + b \left[ \frac{A(2+b) + bC_2 + c_1(2-b^2)}{2(2-b^2)} \right]$$

$$= \frac{A(4-2b^2 + 2b+6^2) + C_2(4-b^2) + C_1b(2-b^2)}{4(2-b^2)}$$

Stage I  $\Rightarrow \frac{A(2+b) + bC_2 + c_1(2-b^2)}{2(2-b^2)} = P_1$

stage II

$$P_2 = \frac{A + C_2 + bP_1}{2}$$

So, in stage 2 now, we will get p2 as so, this is the reaction function of firm 1, firm 2. So, plug in the value of optimal value of p1. So, it is, it is this. So, it will be, is divided by 4, this.

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The image shows a handwritten derivation on a whiteboard. At the top, the equation for  $p_2$  is given as 
$$p_2 = \frac{A(4+2b-b^2) + c_2(4-b^2) + c_1b(2-b^2)}{4(2-b^2)}$$
. Below this, under the heading "SPNE", the equation for  $p_1$  is 
$$p_1 = \frac{A(2+b) + bC_2 + c_1(2-b^2)}{2(2-b^2)}$$
. Finally, the equation for  $p_2$  is simplified to 
$$p_2 = \frac{A + c_2 + bP_1}{2}$$
.

So,  $p_2$  is actually equal to, is this  $P_2 = \frac{A(4+2b-b^2) + c_2(4-b^2) + c_1b(2-b^2)}{4(2-b^2)}$ . And here, when we state SPNE, we say that  $p_1$  is equal to  $A$  this  $\frac{A(2+b) + bC_2 + c_1(2-b^2)}{2(2-b^2)} = P_1$ ,  $p_2$  is given by this reaction function  $P_2 = \frac{A + c_2 + bP_1}{2}$ , SPNE. And the actual outcome is actually this,  $p_2$  is this and  $p_1$  is this. Because when you plug in this here, we get this.

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$$\pi_1 = (P_1 - c_1)(A - P_1 + bP_2) - F$$

$$\pi_2 = (P_2 - c_2)(A - P_2 + bP_1) - F$$

$$P_2 = \frac{A(4 + 2b - b^2) + c_2(4 - b^2) + c_1 b(2 - b^2)}{4(2 - b^2)}$$

SPNE

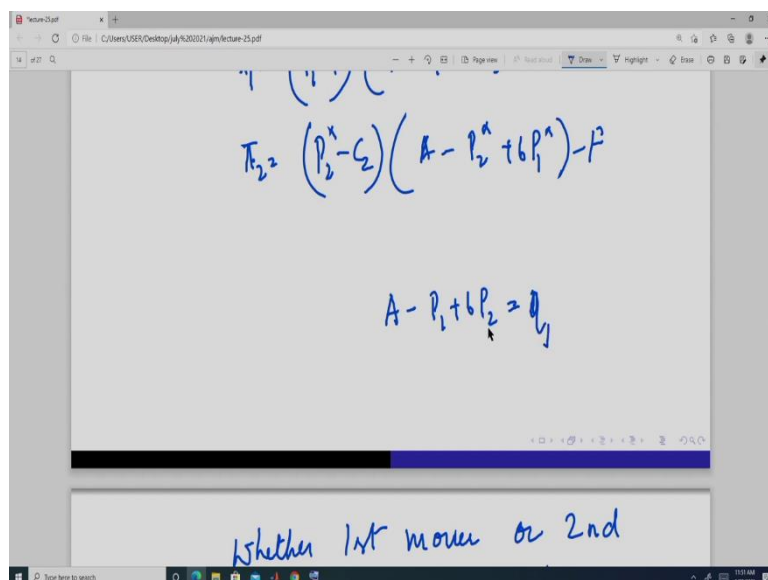
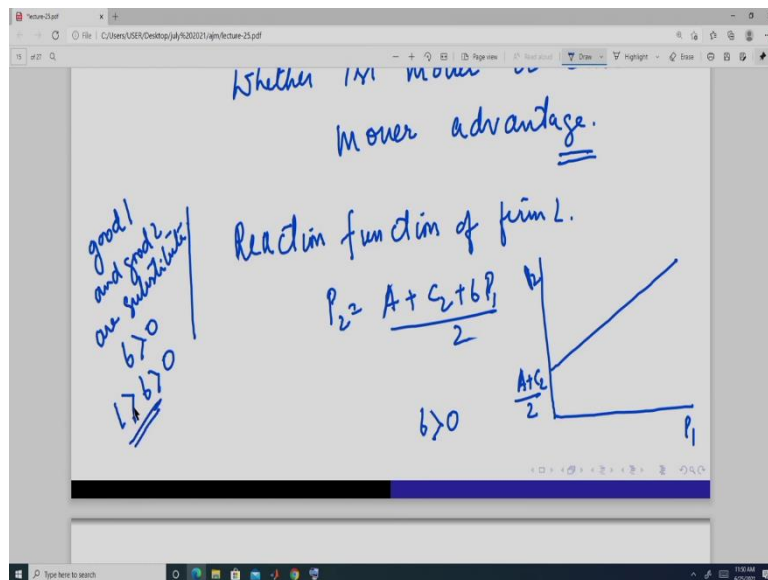
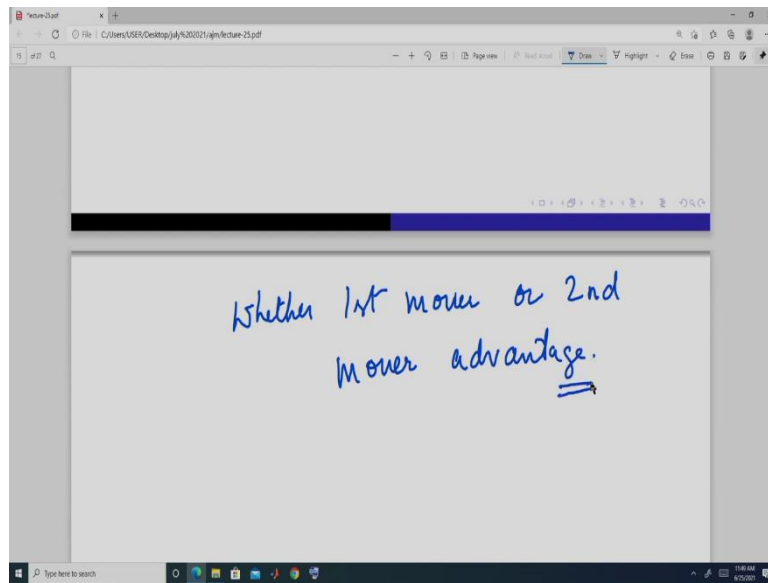
$$P_2 = \frac{A(2 + b) + b c_2 + c_1(2 - b^2)}{2(2 - b^2)}$$

$$P_2 = \frac{A + c_2 + b c_1}{2}$$

Now we can compute the profit function, but profit function is going to be very messy thing here. It will look a very messy, because profit function now it is you can write it in this form. So, plug in the optimal values, sub game perfect Nash equilibrium. And this is going to be  $c_2$ , this, where  $p_1$  is,  $p_1$  takes this value,  $p_2$  takes this value. So, I am not putting these value because it will look very messy.

But you can do it is, but how do we compare this? We are not going to compare profit of firm 1 and firm 2, what we are going to do? In fact you can also do it for but it will be better if you take the  $p$ ,  $c_1$  and  $c_2$  as same, then it will make sense otherwise it is, again it will be very conditional statement. But we will see whether there is a first mover advantage or a second mover advantage. How do we do that?

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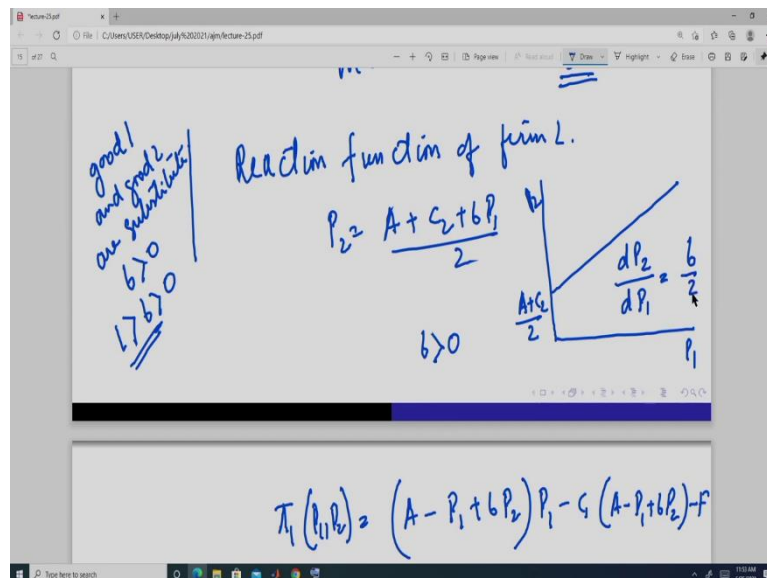
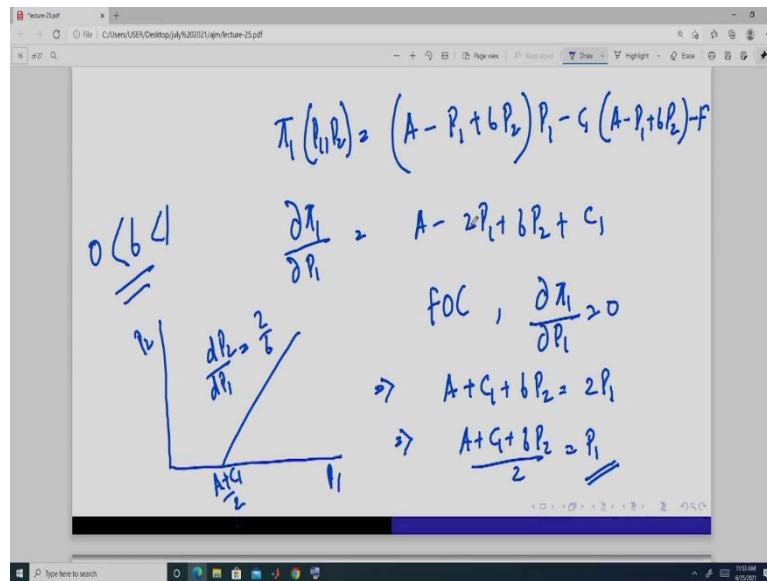




So, we now check whether first mover or second mover advantage, okay we do that. How do we do it? We, see we have already know the reaction function of firm 2. We have derived it, we what do we have got? Reaction function of firm 2 is, is this. So, if we plot this, this is  $p_1$  and this is  $p_2$ , we get a curve like this where this point is this, when? When  $b$  is positive. So, first let us take the case when good 1 and good 2 are substitute.

So,  $b$  is positive and further  $b$ , why it is less than 1? Because if it is greater than 1, then see in, in this demand function, if it is greater than 1 then the effect of  $p_2$  will be more on good 1 than  $p_2$ ,  $p_1$ , effect of  $p_2$ . So, that is why this is always taken as this as less than 1, okay. And when it is 0, there is completely unrelated they are not. So, we take this here and we get this reaction function which we have already done.

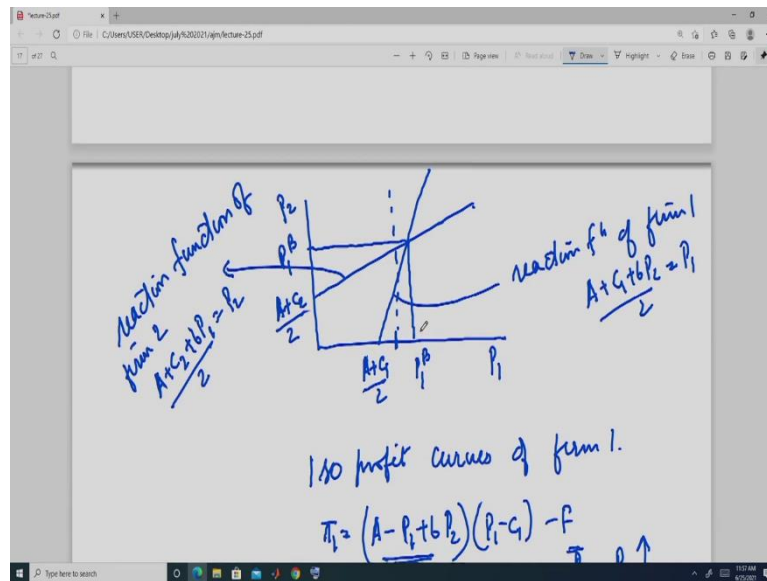
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Again we know the profit of firm 1, when firm 1 is this, this. Taking  $p_2$  as given, we get this. Again, the first order condition, first order condition gives me that this is equal to 0. So, this gives me, it is this  $\frac{A+c_1+bp_2}{2} = p_1$ . So, this is the reaction function of firm 1. And we have assumed that  $b$  is this. So, goods are substitute, so this is going to be, this is going to look like, where this.

And in this case the slope of this is going to be what?  $2$  by  $b$ , it is positive. And in this case slope is going to be  $dp_2$  by  $dp_1$ . It is going to be  $b$  by  $2$ , right? So, it is positive again because  $b$  is positive, it is this, But slope of this is less than this.

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So, we get that this and this is, this, this is the outcome when these two reaction functions intersect. So, we say this to be, when both the price are decided simultaneously, so, this is the bertrand competition when there is product, when there is differentiated product market, okay this. But, what we are dealing here? We are dealing with stackelberg. So, stackelberg case is when the firm 1 decides its output first and firm to decides its output second.

But you can reverse the sequence also it does not matter, okay. Now, but we will compare, we were trying to find out whether there is first mover advantage or second mover advantage. In the last class also, when we did the quantity competition, we did that analyze based, analysis based on the Cournot reaction function. So, here we will do the analysis based on the bertrand reaction function. Now, in this case the iso profit curves of firm 1.

Because firm 1, knows the reaction function of firm 2, firm 2 also knows this reaction function of firm 1. So, this you can say this is the reaction function of firm 1, which is  $A + c_1 + b_2 P_2$  this, and this is the reaction function of firm 1, where it is sorry, okay this. Now, iso profit curve, function is this, profit when we fix the level of profit and then look at the combination of  $p_1$  and  $p_2$ . So, this you can, this  $\pi_1 = (A - P_1 + b_2 P_2)(P_1 - c_1) - F$ .

Now, if we fix  $p_1$ , fix  $p_1$  and if you keep on increasing  $p_2$ . Since  $b$  is positive so, this portion, this is fixed, this portion is increasing, this portion. So, that means, if I fix  $p_1$  suppose here and then keep on increasing  $p_2$  like this, what is going to happen? Profit is increasing, right? So, profit increases in this horizon, in this vertical direction, okay. Because goods are substitute,

right? because when the price of good 2, keeping the price of good 1 fixed the demand for good 1 is going to increase.

Because people will shift from good 2 to good 1, because price is increasing. So, that is why its profit is going to be increased. Because more, more quantity will be demanded.

Now, so we know this, this diagram.

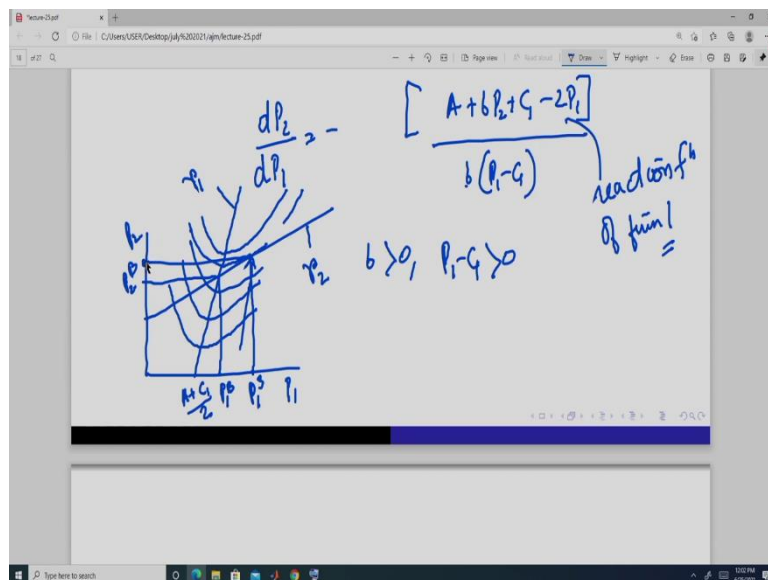
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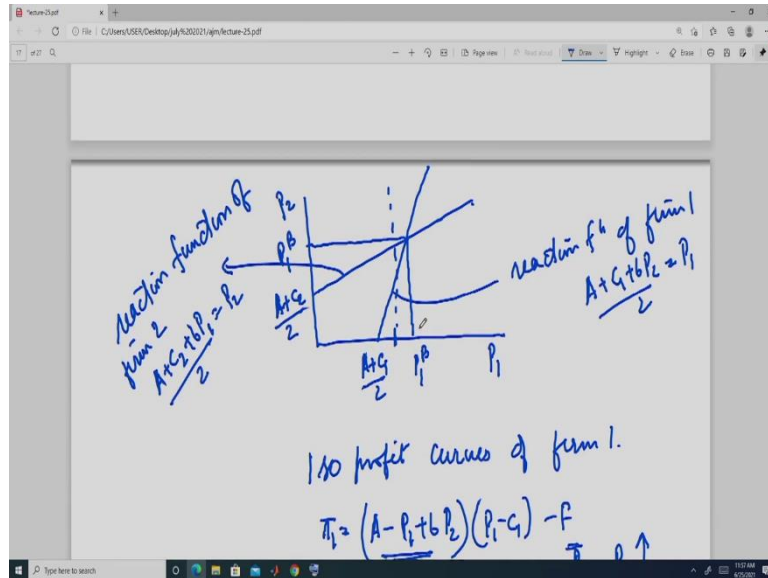
Handwritten mathematical derivation showing the reaction function for price  $p_1$ . The derivation starts with the profit function  $\pi_1$  and its differential  $d\pi_1$ . The reaction function is derived as  $\frac{dp_2}{dp_1} = - \frac{A + b p_2 + c - 2 p_1}{b(p_1 - c)}$ . The conditions  $b > 0$  and  $p_1 - c > 0$  are noted, along with the label "reaction fun of firm 1".

$$0 > d\pi_1 = [A + b p_2 + c - 2 p_1] d p_1 + [b(p_1 - c)] d p_2$$

$$\frac{d p_2}{d p_1} = - \frac{[A + b p_2 + c - 2 p_1]}{b(p_1 - c)}$$

$b > 0, p_1 - c > 0$       reaction fun of firm 1 =





Now, how, this function how it looks? So, if we would find the total differentiation of this, we will get this what? This and when we are moving along an iso profit curve, this is equal to 0. So, the level, profit level is fixed. So, we get this-  $\frac{dP_2}{dP_1} = -\frac{A+bP_2+c_1-2P_1}{b(P_1-c_1)}$ . Now, here if you look at this, this is what? This is the reaction function of firm 1, of firm 1. And this portion, since  $p_1$  is always going to be greater than  $c_1$  because  $c_1$  is a marginal cost.

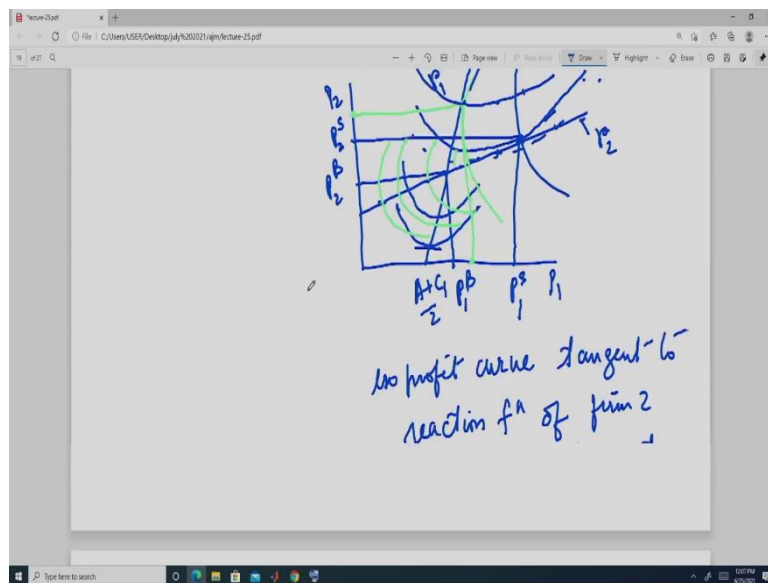
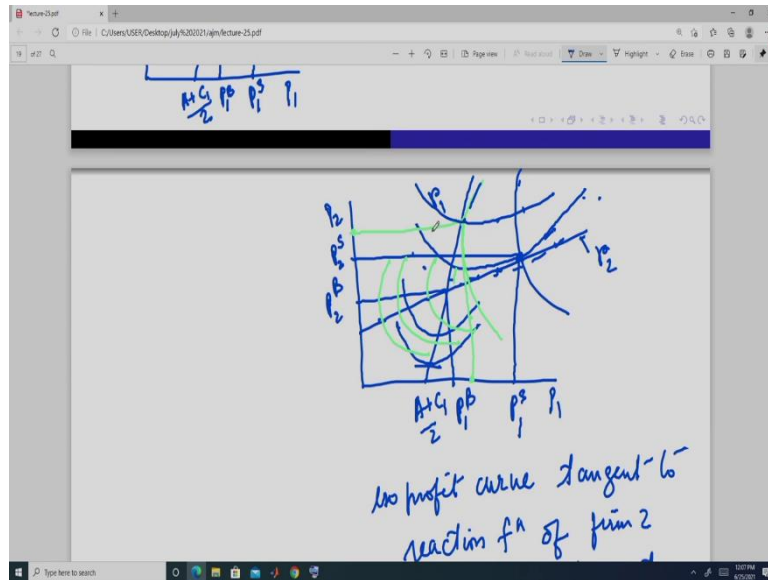
Otherwise 2 we will get the bertrand paradox. So, this portion is always positive because, because  $b$  is positive and  $c$  minus,  $p$  minus  $c_1$  is positive so, this portion is always positive. So, the sign of this how  $p_1$  as  $p_1$  varies, how the curves look like this iso profit curve this depends on the sign of this reaction function. And we know this reaction function in this portion, it is negative.

Because  $p_1$  is, at this point if we fix the  $p_2$ , at this point  $p_1$  is then that reaction function is this function is. Because this is equal to this. So, if we fix  $p_2$ ,  $p_1$  at this level it is going to be such that this takes a value 0. But if it is less, it is going to be what? positive. And since this is positive and this, there is a negative sign so, it is going to be negative. But if  $p$ , here it is this is greater than this.

So, this is going to this minus this portion, this portion is going to take a negative value. So, here it is a negative sign so, it is going to be positive. So, we get, so, suppose this is sorry, suppose this is the reaction function of firm 1, firm 1 and this is the reaction of firm 2. So, the iso profit curves are going to be like this, and they are increasing.

So, profits are increasing in this direction. This is the reaction function of firm 1 and this is the reaction function of firm 2. This is the bertrand outcome. And the stackelberg outcome is such that it is going to be tangent. So, it is suppose this point. So, stackelberg is going to be this.

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So, what do we get? I will draw this again, this is the reaction function of firm2. This is the reaction function for firm 1, and iso profit curves are like this, so it is here. And we will get a curve like this and this is suppose, this is the bertrand outcome, when both the firms decides the price simultaneously, this is stackelberg. When firm 1 moves first and firm 2 moves second, like this.

So, now, if it chooses any other point here, because firm 1 knows firm 2 is going to choose based on this reaction function. So, it will choose that point in this reaction function which will

give it the maximum profit and profits are increasing in this direction, right. So, if you take here, so, it will be below, if you take here it will again be below this. And as this is the, iso profit curves which is tangent.

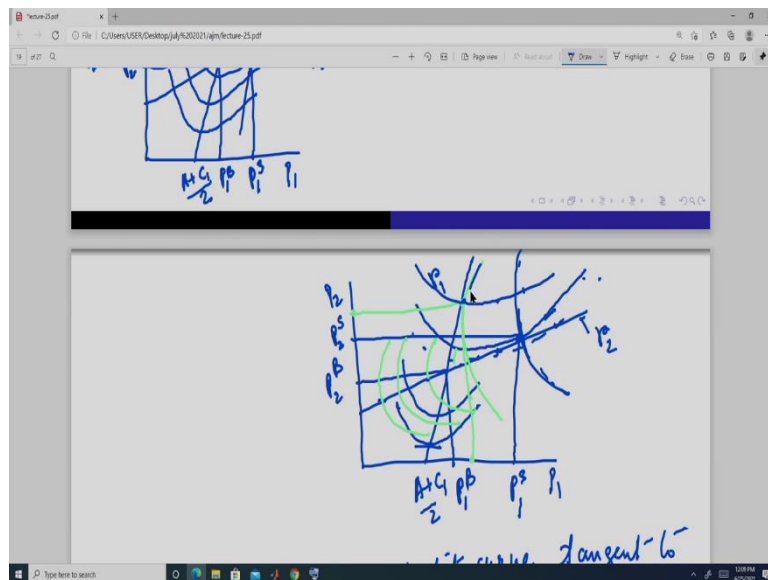
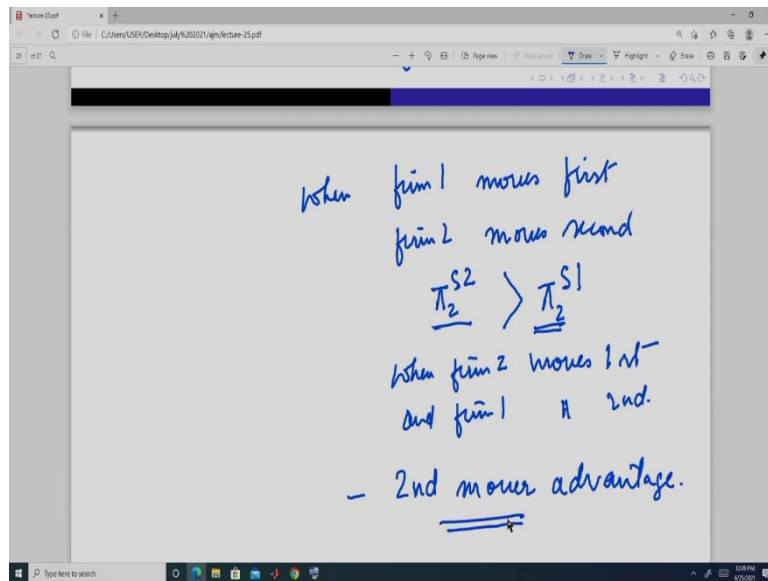
So, any other point it will be such that, that iso profit curve passing through that point will be lying below this. So, the profit is going to be less. So, that is why the iso profit curve tangent to reaction function of firm 2 gives the, stackelberg outcome, okay. So, the stackelberg outcome that we have computed, we have got it in this way. When, when  $b$  is positive, that is goods are substitutes. Now, look in this way.

So, we have got this. Similarly, the iso profit curves of firm 2 is going to be something like this. Yeah, it will be like this, it will be like this and it will be somewhere sorry, it will be something like this. Then this is the stackelberg outcome when so, this green one, this gives me the stackelberg outcome when firm 2 moves first and firm 1 moves second, okay. Now, if you look at this see what is happening. It will, it will be such that it will, it is going to be tangent, right. The moment it is tangent to this, what is happening?

It will lie above this, because there is a highly unlikely chance that it is going to be tangent. So, if it is like this, then it means what? The iso profit curve of firm 1 is here, but when it moves first it is here. So, this is above this. So, what happens? When firm 2 moves second and, firm 1 move second and firm 2 moves first, then actually firm 1 gets a higher profit. So, now it is here, right? and if you look at these here are increasing.

Here it is this, but if it moves first, firm 1, firm 2 moves first, then it is A will be somewhere here. Because it will be based on this reaction function. So, that is why firm 2 get higher profit when it moves second, and when firm 2, firm 1 moves first.

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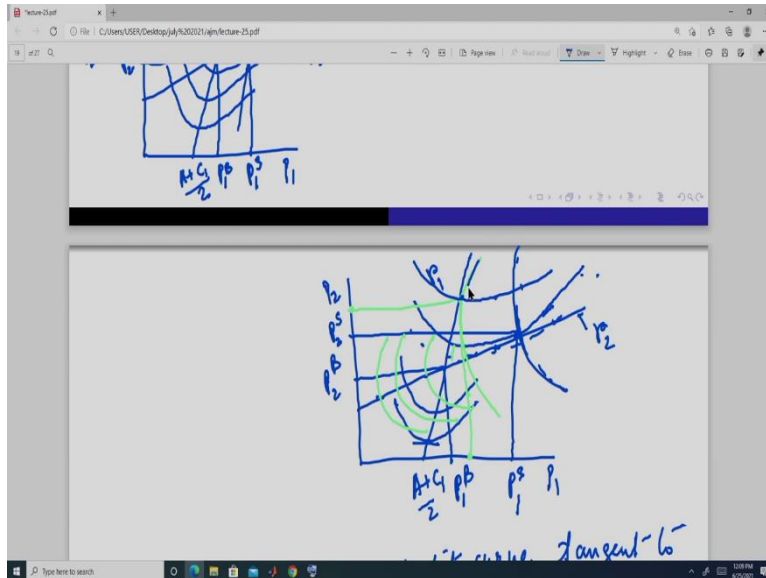
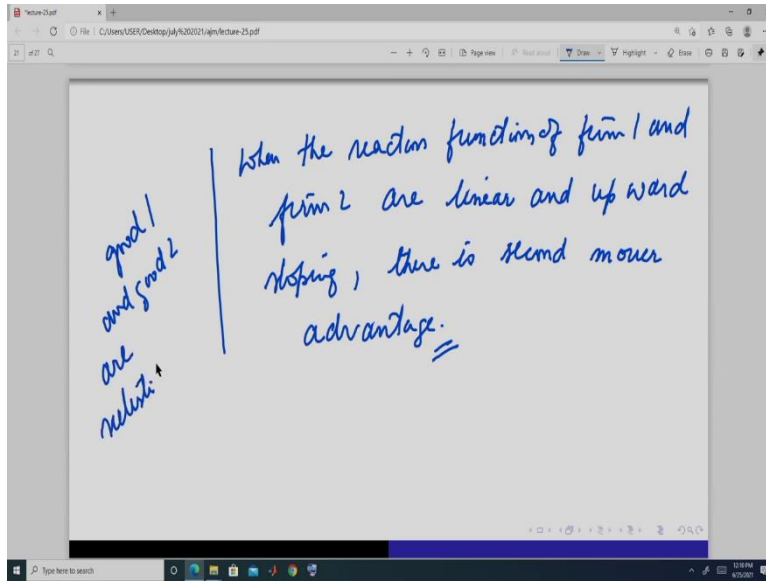


So, what do we get? When firm 1 moves first and firm 2 moves second, so, profit of firm 2 here when it moves second, this is going to be greater of firm 2, when it moves first when, firm 2 moves first and firm 1 move second. So, this is the case, we have got it from this. So, what is happening? This is the iso profit of firm 2. But if it moves first, its iso profit is going to be somewhere here.

It will be based on this reaction function, right. Because it will choose that point in this reaction function of firm 1, which will give you the maximum a . So, it will definitely going to be lie below this. So, because this a is never going to b, because this is tangent that this a. So, that is why, firm we will we get there is always a second mover advantage. So, what we, do we get?

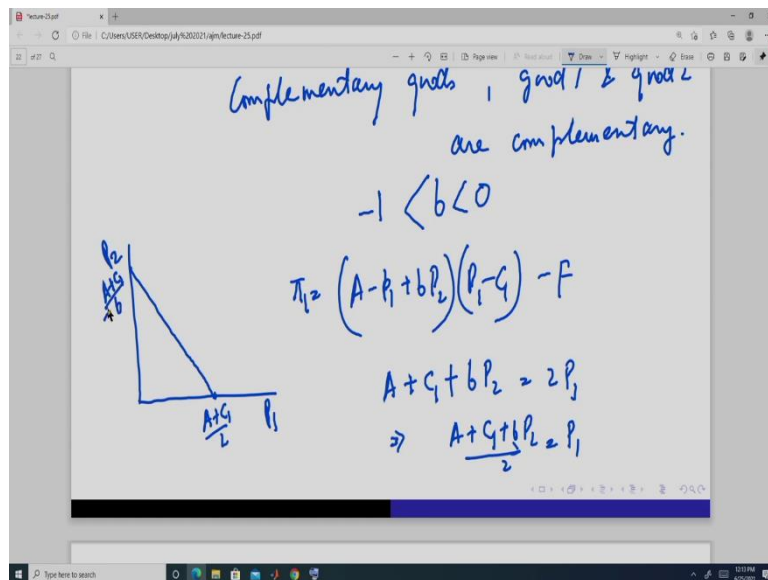
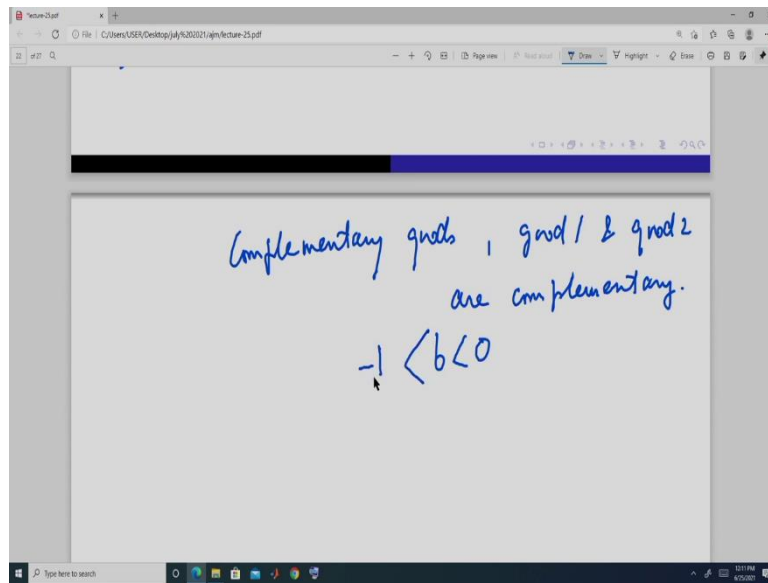
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We get that whenever the reaction functions, so, this result we write it in this form. When the reaction function of firm 1 and firm 2 are linear and upward sloping, there is a second mover advantages. So, this means when they are upward sloping, this means good 1 and good 2 are substitute, okay. If it is not substitute, then the reaction functions would not have been upward sloping, we have got it from this.

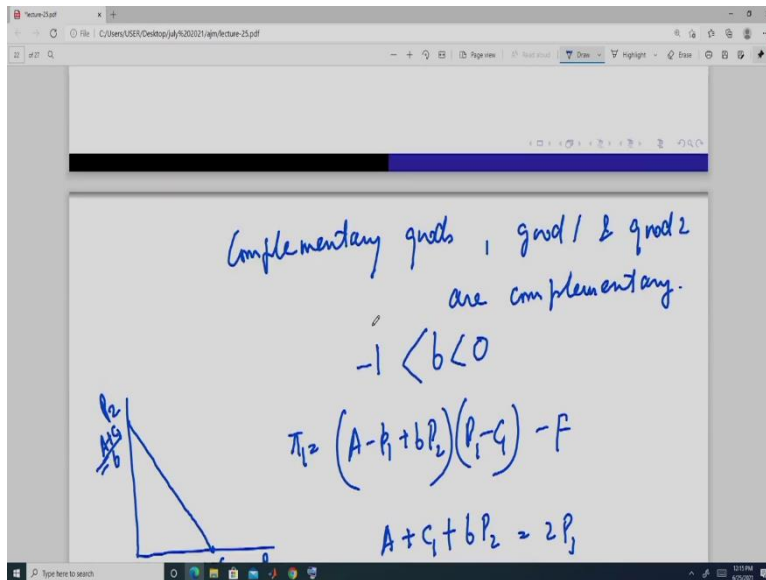
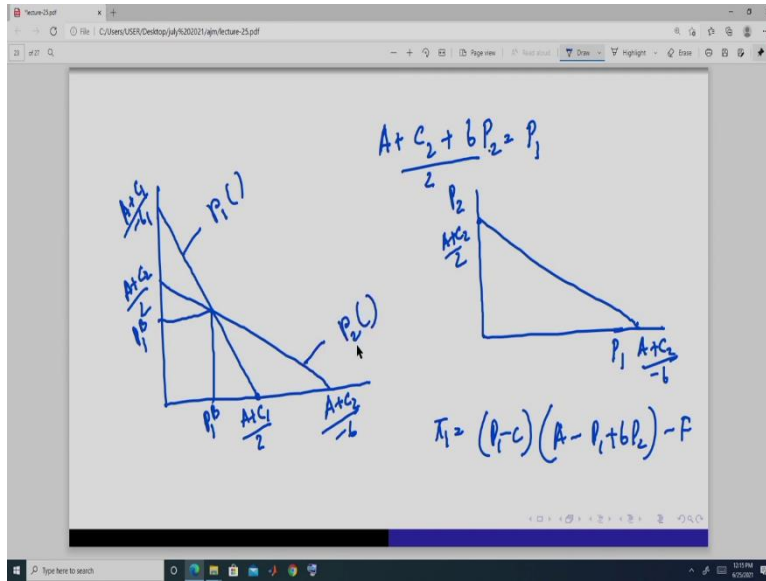
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Now, in case of complimentary goods we know, in case of complimentary goods, that is good 1 and good 2 are complimentary. So, that means b is negative and b is greater than minus 1, okay. So, this so, reaction function is profit of firm 1 is this. We know the reaction function of firm 1. This is going to be a negative portion, part. So, going to be like this. So, this reaction function, if we take price of good 1 here, price of good 2, our price of firm here.

So, this this is negative, this portion because b is negative. So what it, what it will be? So, if this is 0, it is going to be A plus. And if this is 0, this is going to be divided by, is going to be divided by minus b. So, it is positive number, we will get like this.

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Similarly, the reaction function of firm 2, it is going to be  $A + c_2$ , it is going to be like this-  $\frac{A+c_2+bP_2}{2} = P_1$ , sorry, this. And since this is going to be negative so, and this is going to be like this. And this is so, this is positive like this. So, it is going to be like this. This is going to be the price when they are choosing the price simultaneously. So, this is Bertrand. Now in this case, what we have done?

We have assumed that the goods are complementary. And firm 1 chooses price first, firm 2 chooses price second. And the profit function, the iso profit curve is this-  $\pi_1 = (P_1 - c)(A - P_1 + bP_2) - F$ . Now, when  $P_2$  increases, since  $b$  is negative this is going to go down. So, this is so, profit in this direction it is going to go down. Reaction function of firm 1, reaction function of firm 2.

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$$\frac{dP_2}{dP_1} = - \frac{(A + C + bP_2 - 2P_1)}{b(P_1 - C)}$$

First mover advantage when reaction functions are downward sloping and linear.

$b < 0, P_1 - C > 0$   
 $b(P_1 - C) < 0$

$\rightarrow$  good 1 & good 2 are complementary.

$\pi_1 = (P_1 - C)(A - P_1 + bP_2) - F$

$\frac{dP_2}{dP_1} = - \frac{(A + C + bP_2 - 2P_1)}{b(P_1 - C)}$

$\frac{dP_2}{dP_1} = - \frac{(A + C + bP_2 - 2P_1)}{b(P_1 - C)}$

like curve tangent to

So, again by simply looking at this, we get this to be  $-\frac{dP_2}{dP_1} = -\frac{A+c_1+bP_2-2P_1}{b(P_1-c_1)}$ . Now, b is negative so, since b is negative so, since b is negative,  $p_1 - c_1$  is positive. So, this is positive so, this whole term  $-b(P_1 - c_1)$  is negative. So, this is, whole term is positive. So, this when prices are here, this portion reaction function this takes negative value. And when it, positive value and when it is here it is taking negative value. So, the iso profit curves are going to be like this, okay. And profit is going to be higher here.

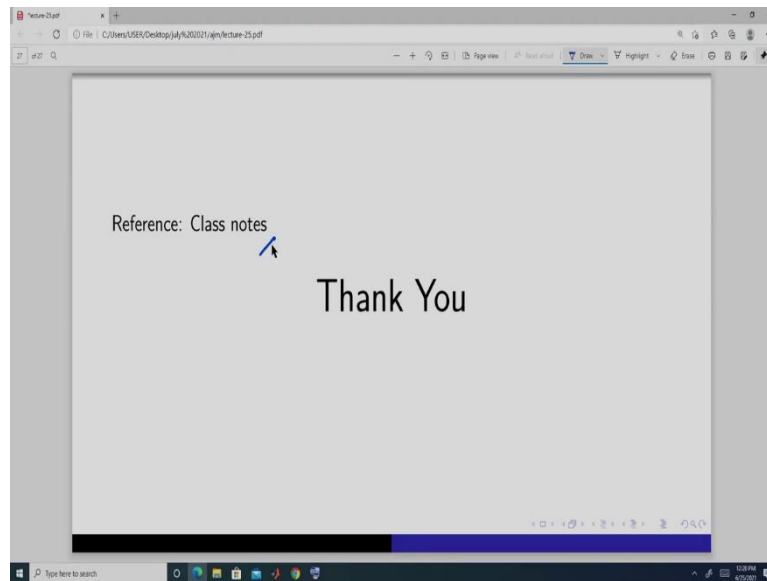
So, again it will be this point is going to be some stackelberg price. And this is going to be stackelberg price, right? Now, here if you look at this here so this is when they move. Now, in firm 1, if you look at firm 2, it will be at, so the of firm 2, this is the reaction function of firm 2. So, it will choose that it is tangent to this one. So, this is the stackelberg. Now, whether there is a first mover or second mover advantage.

Now, in this case you will see that this point is should give firm 2 the maximum profit and this is when they move simultaneously. And this is when firm 2 moves second and firm 1 moves first. So, here this is going to give it a higher profit, than this. So, firm 2 has a disincentive to move, second. So, we get that again there is a first mover, first mover advantage when reaction functions are downward sloping and linear.

So, this is the case when good 1 and good 2 are complimentary in nature. And they are complimentary then we get the result is same as the Cournot, as the quantity competition and when there is no homogeneous product. But when the goods are substitute, we get that there is a second mover advantage and we show that second mover advantage based on this, that if firm 2 move second, its will be in this iso profit curve.

And the iso profit curve of firm 1 is this because it is tangent to this. But if it and this is always going to lie above any iso profit curve that is tangent to this, because this is the reaction function of firm 1. So, that is why firm 2 here is going to gain by moving second rather than moving first or moving simultaneously along with firm 1, okay. So, with this, we end the stackelberg competition.

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And stack it for this portion price competition this class note is sufficient. And it is more or less straightforward, it is not very difficult. Only problem is when you have to find out the, whether there is a first mover advantage or the second mover advantage and it is better if you do it based on the diagrams, rather than comparing those algebraic expressions. Because these algebraic expressions are going to be very messy in this situation.

Because we have got the and because this is the price of firm 1 and, and when we plug in this, in this reaction function we get the price of firm 2 in this form. And we have to plug in this, in this function. So, this is going to be a very messy thing. But, we can find out whether, we can find out a way to compare the situations whether there is first mover advantage or second mover advantage based on the diagrammatic analysis or based on the analysis of the graphs, okay. So, this portion you can look at from these notes is sufficient.

And if you compare it with the quantity competition, you will see if in the case of complimentary goods it is same as the quantity competition. Even if we are doing product differentiation, okay. So, with this I end this portion. So, thank you.