Introduction to Market Structures

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Lecture 34

Tutorial on Bertrand Competition and Stackelberg Quantity Competition

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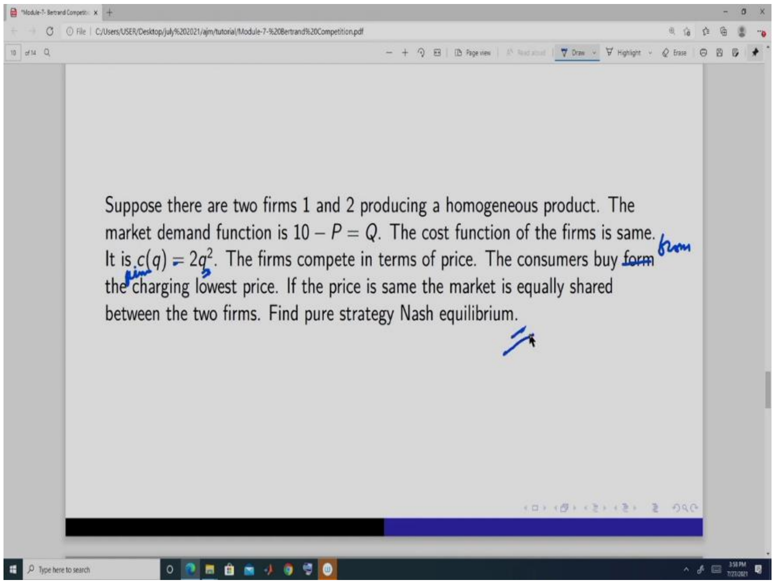

We will now use the decreasing returns scale. Suppose there are firms, firm 1 and firm 2, and the market demand function is this- 10–P=Q. Cost function of firm is same. It is this- $c(q) = 2q^2$, both the firm. And firm competes in terms of price, that is Bertrand competition and a consumer buys from the firm charging lowest price and if the price is same, the market is equally shared between the two firms. So, find a pure strategy Nash equilibrium. We have to solve the pure strategy.

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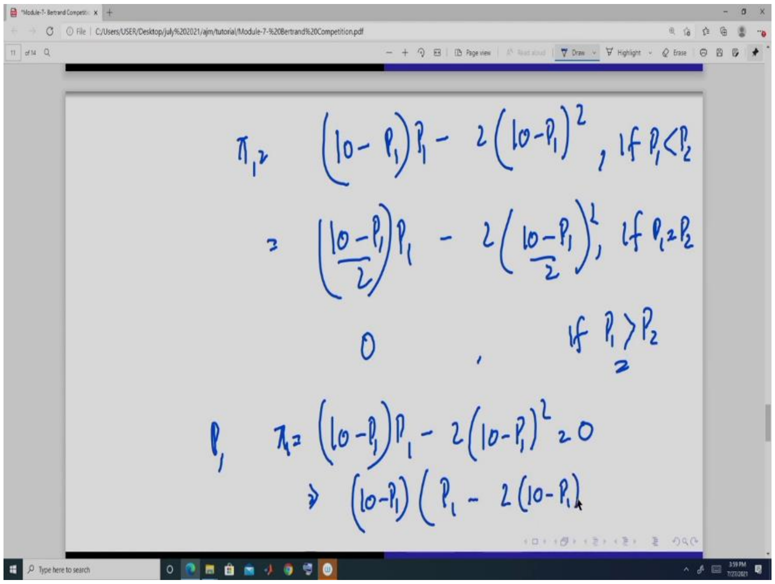

So, here again, we get the profit of firm 1 is this-$(10-P_1)P_1-2(10-P_1)^2$, if P 1 is less than P 2. And it is equal to this-$\frac{(10-P_1)}{2}P_1-2\left(\frac{10-P_1}{2}\right)^2$, if P 1 is equal to 2. And it is equal to 0 if P 1 is greater than P 2. We will get this. So, we find out P such that profit, this is-$(10-P_1)P_1-2(10-P_1)^2$ equal to 0. We get this at two price.

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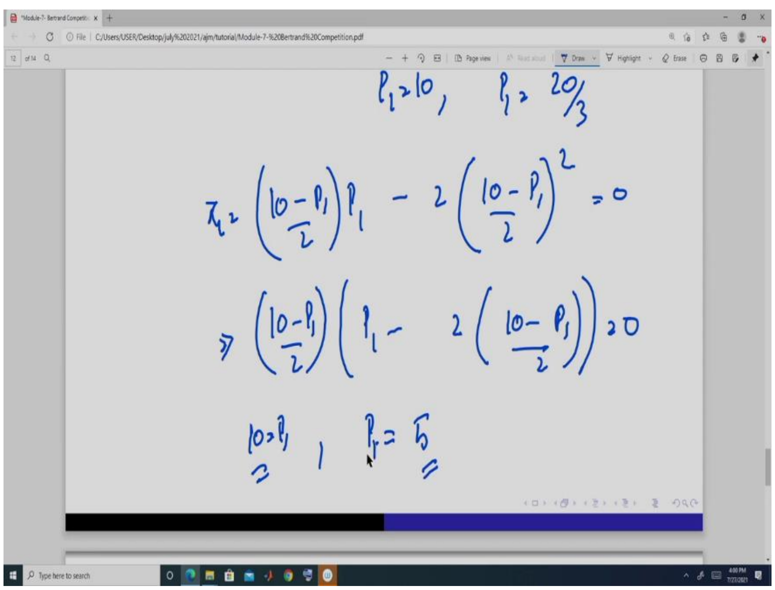

So, we get this is equal to 0. So, at P 1 is equal to 10 and P 1 is equal to 20 by 3, we get this. Again, we find out that this-$\frac{(10-P_1)}{2}P_1-2\left(\frac{10-P_1}{2}\right)^2$, and this is equal to 0. So, this is, this is equal to 0-($\frac{(10-P_1)}{2}$)$(P_1 - 2\left(\frac{(10-P_1)}{2}\right) = 0$. So, we get that P is equal to this, and P is equal to, P is equal to 5. We get this. P 1. So, these prices are going to be same for firm 1 and firm 2.

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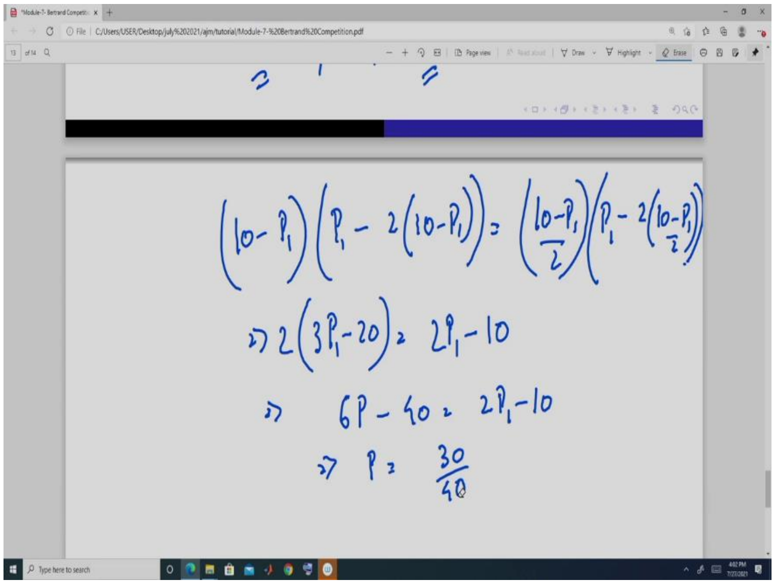

And we get another price when this-$(10 - P_1)(P_1 - 2(10 - P_1))$ is equal to this-($\frac{(10-P_1)}{2}$)$(P_1 - 2\left(\frac{(10-P_1)}{2}\right)$. So, this implies this-$2(3P_1 - 20) = 2P_1 - 10$. So, it is 30 by 4.

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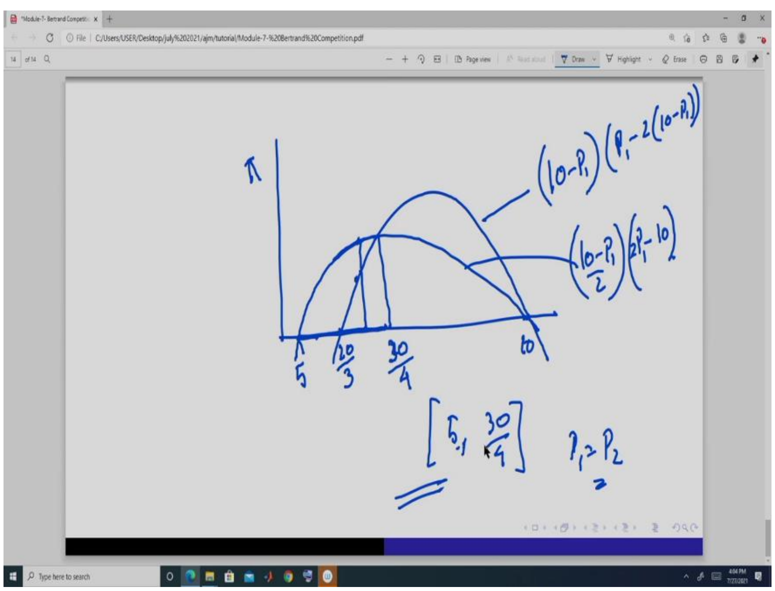

Now, if we plot this function, this function is this. This point is 10, this point is 20 by 3. And this function is, is this. This point is 5 and this is 10. This point is, one second, 30 by 4, okay. So, and here will be profit. So, we will get the same prices for firm 1 and firm 2. And here we know that this range, because if firm 1 sets a price, suppose here, firm 2, if it undercuts, it will get this, but if it sets the same price, it is going to get this. So, if we look at this whole range of prices, the Nash equilibrium lies in this range-[5, 30/4]. So, each price in this range, price in this range constitute a Nash equilibrium. And P 1 is always equal to P 2 and they lie within this range, okay.

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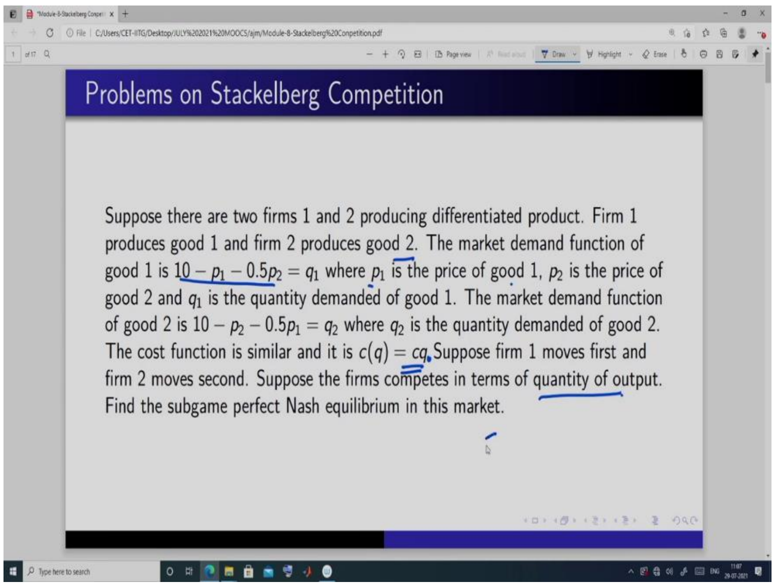

Let us solve some problem on Stackelberg competition. So, in this problem, suppose there are two firms 1 and 2, producing differentiating products. Firm 1 produces good 1, and firm 2 produces good 2. And our market demand function is this-$10 - p_1 - 0.5p_2 = q_1$, okay where p 1 is the price of good 1, p 2 is the price of good 2, and q 1 is the quantity demanded of good 1, which I produced by firm 1.

And market demand function for good 2 is this-$10 - p_2 - 0.5p_1 = q_2$. 10 p 2 minus point, 0.5 p 1 equal to, so here q 2 is the quantity of good 2, which is produced by firm 2. And the cost function for both the firms are same and it is given by c q, okay. Suppose firm 1 moves first and firm 2 moves second. So, it is a sequential gain and that is why it is a Stackelberg. And firms competes in terms of quantity. So, this is again Stackelberg quantity competition with differentiated product. And we have to find the subgame perfect Nash equilibrium.

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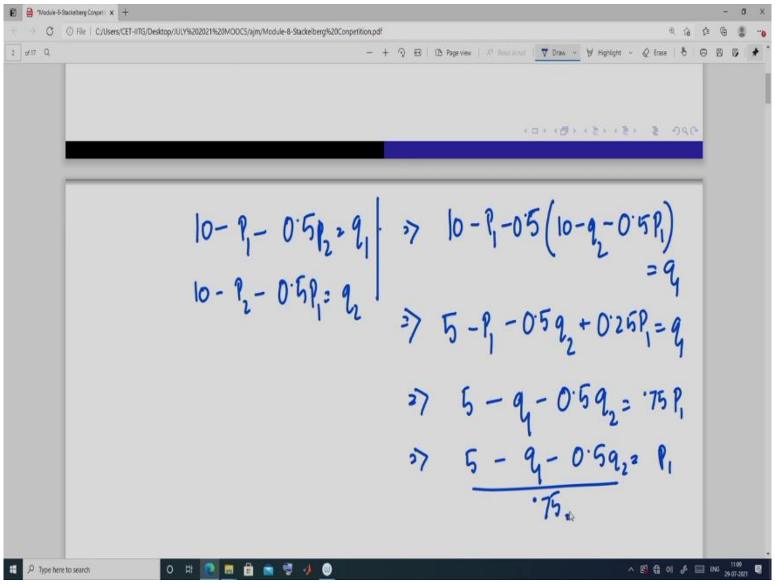

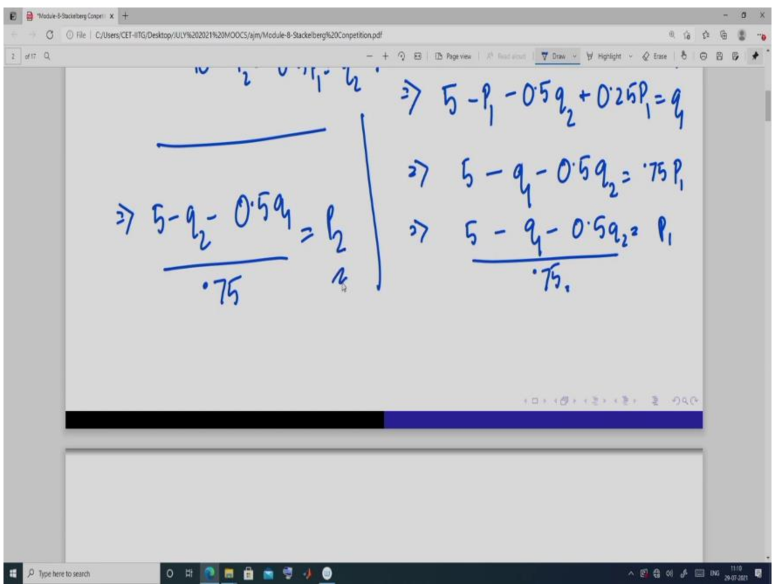

So, from this, first what we will do, we have to convert this-$10 - p_1 - 0.5p_2 = q_1$, this demand equation. So, this, you can write simply because the demand for good 2 is this-$10 - p_2 - 0.5p_1 = q_2$, right? So, from here, you can, you can write this-$10 - p_1 - 0.5(10 - p_2 - 0.5p_1)$. Substitute here, this, you take this here, we will get q 2. And this you can write as q 1. So, this is 5, q 2, which we can write in this. So, this is the inverse demand function of good 1, this -$5 - q_1 - 0.5q_2 = P_1$. Similarly, since the demand functions are more or less same, inverse demand function for good 2, we can write q 1, 0.75 equal to q 1, equal to p 2. This is the price of good 2-$\frac{5-q_2-0.5q_1}{0.75} = p_2$.

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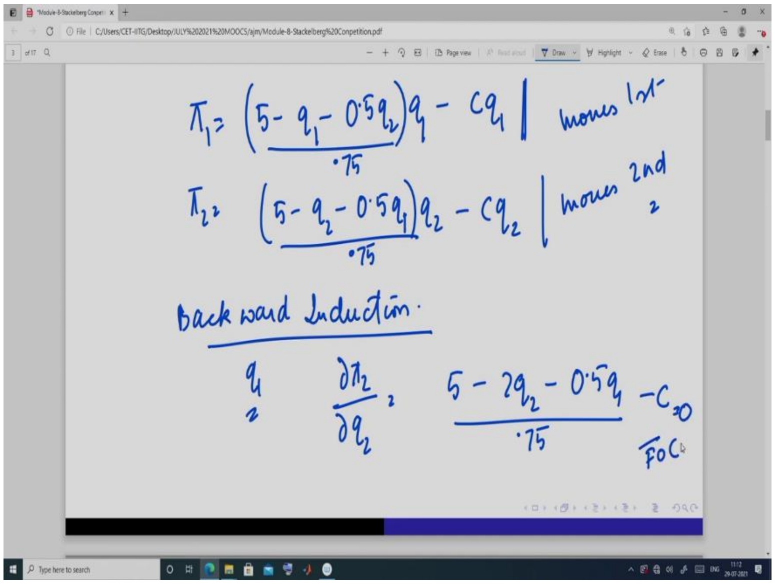

So, from this, we can write the profit function of firm 1. Profit function of firm 1 is 5 minus q 1, 0.5 q 2, q 1, cost is c, marginal cost is constant, c, and it is q 1 divided by. This is the profit function of firm 1-$\pi_1 = \left(\frac{5-q_1-0.5q_2}{.75}\right) q_1 - cq_1$. And profit function of firm 2 is, it is this-$\pi_2 = \left(\frac{5-q_2-0.5q_1}{.75}\right) q_2 - cq_2$.. Now here, this moves first and this moves second, okay. So, we will use backward induction to solve this problem., right? So, suppose q 1 is the output in stage 1. Then, we will find the reaction function of firm 2. It is, it is this minus c, which is equal to 0, which is the first order condition-$\frac{5-2q_2-0.5q_1}{.75} - c = 0$.

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$$\Rightarrow 5 - .75c + 0.5q_1 = 2q_2.$$

$$\Rightarrow \frac{5 - .75c + 0.5q_1}{2} = q_2$$ reaction f^n of firm 2

$$\pi_1 = \left[5 - q_1 + 0.5\left(\frac{5 - .75c + 0.5q_1}{2}\right)\right]q_1 - cq_1$$

$$\pi_1 = \left(\frac{10 - 2q_1 + 2.5 - .37c + .25q_1}{2}\right)q_1 - cq_1$$

$$10 - p_1 - 0.5p_2 = q_1$$

$$10 - p_2 - 0.5p_1 = q_2$$

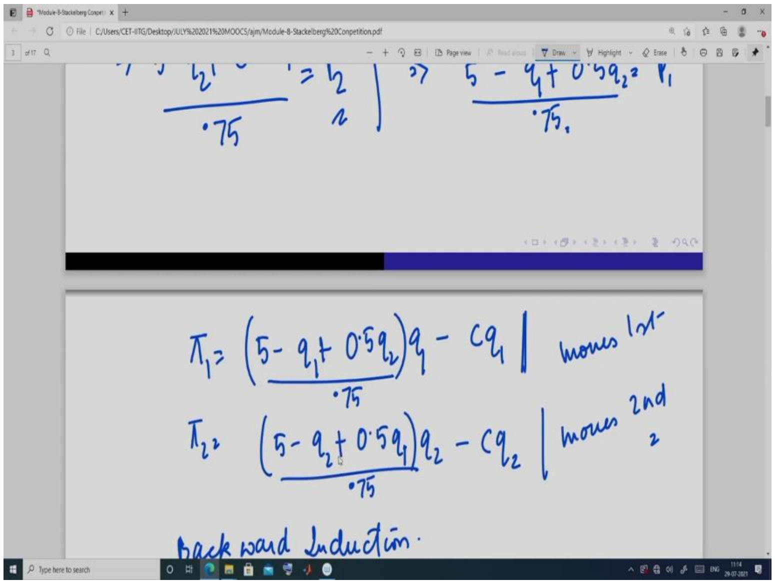
$$\Rightarrow 10 - p_1 - 0.5(10 - q_2 - 0.5p_1) = q_1$$

$$\Rightarrow 5 - p_1 + 0.5q_2 + 0.25p_1 = q_1$$

$$\Rightarrow 5 - q_1 + 0.5q_2 = .75p_1$$

$$\Rightarrow \frac{5 - q_1 + 0.5q_2}{.75} = p_1$$

$$\Rightarrow \frac{5 - q_2 + 0.5q_1}{.75} = p_2$$



So, from this, the reaction function of firm 2 is 5 minus, and this is the reaction function of firm 2- $5 - .75 + 0.5q_1 = 2q_2$. So, given any output of firm 1 in stage 1, how much output firm 2 is going to produce? It is based on this, this is the reaction function of firm 2- $\frac{5-.75+0.5q_1}{2} = q_2$. So, now what do we do? We plug in this in the profit function. Just wait a minute.

This is, okay, this should be plus. I have made a mistake, see. This is plus. So, this is plus. So, it will be plus here, it will be plus here. So, here, it will be plus here, it will be plus here, plus here. So, this reaction function is actually q 1 q 2, this is, if q 1 is equal to 0, it is taking a positive value. This q 2 is taking a positive value. That is something like this. This point is, you can say, this, okay. Now, let us substitute this here, and this is going to be, so we are substituting here, so it is this, 0.5. It is this-$\pi_1 = \left(\frac{10-2q_1+2.5-.37c+.25q_1}{2}\right) q_1 - cq_1$.

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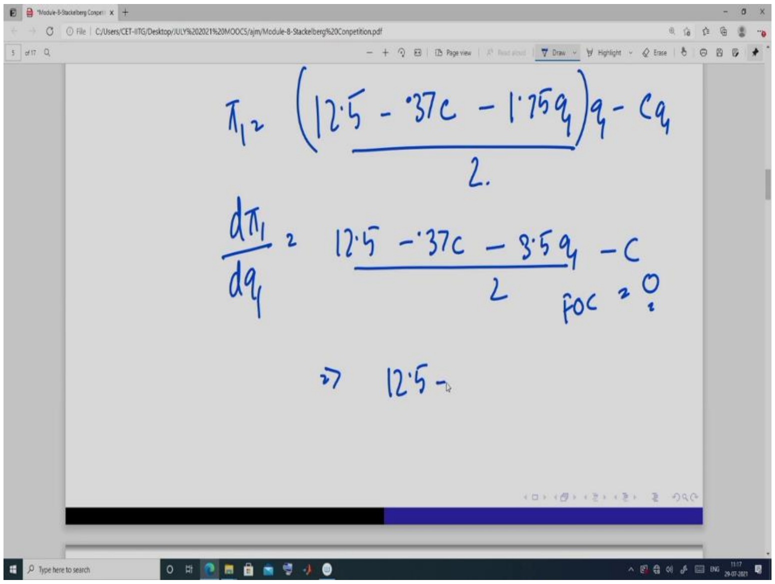

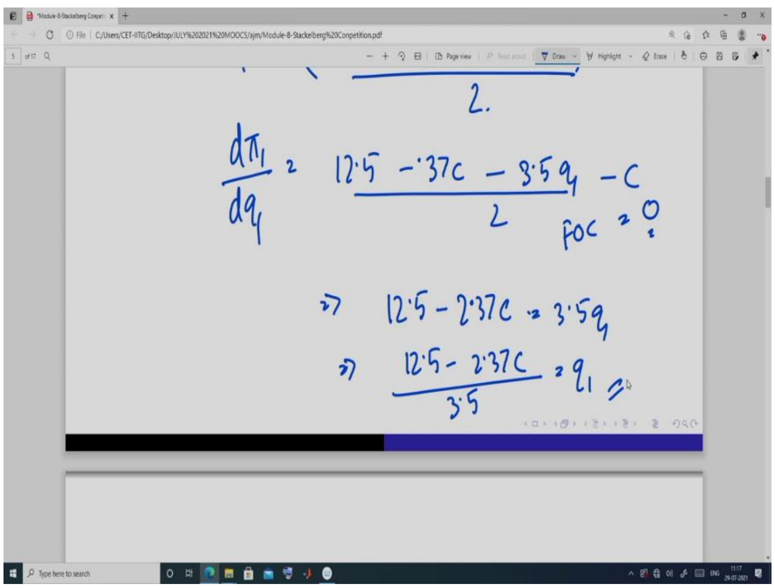

Which, we can write as 12.5. This is 1.75 q 1. We are getting this-$\pi_1 = \left(\frac{12.5-.37c-1.75q_1}{2}\right) q_1 - cq_1$.. Now, we will take this with respect to q 1, and we will get. So, first order condition, we will make it 0, so we get the output of firm, wait, this must be, okay so it is, it will be 2, so it is 2.37 c. So, it is. So, q 1 is this-$\frac{12.5-2.37c}{3.5} = q_1$.

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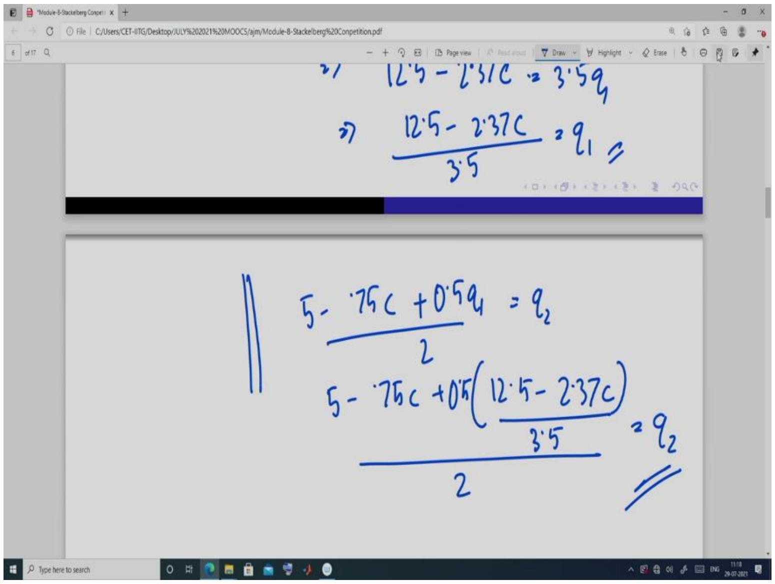

And we plug in this q 1 in the reaction function. Reaction function **is** 5 minus this, 0.75 c plus point, point q 1. This is q 2. So, plug in this. This is going to be, so we have, this is the subgame perfect Nash equilibrium in this case, okay. So, q 1 is this and q 2 is this, right. By plugging in the reaction function, the value of q 1, we will get the q 2, okay.

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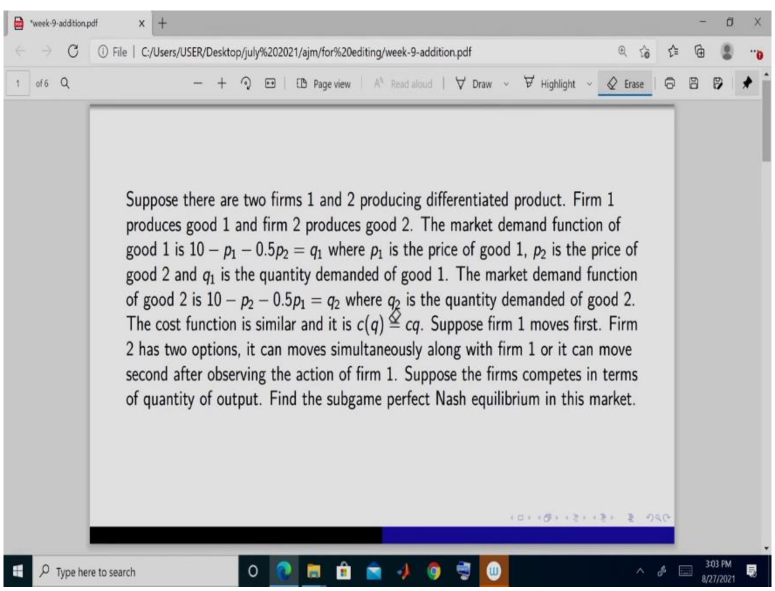

So, we are going to solve this second problem. And this is actually an extension of the first problem.

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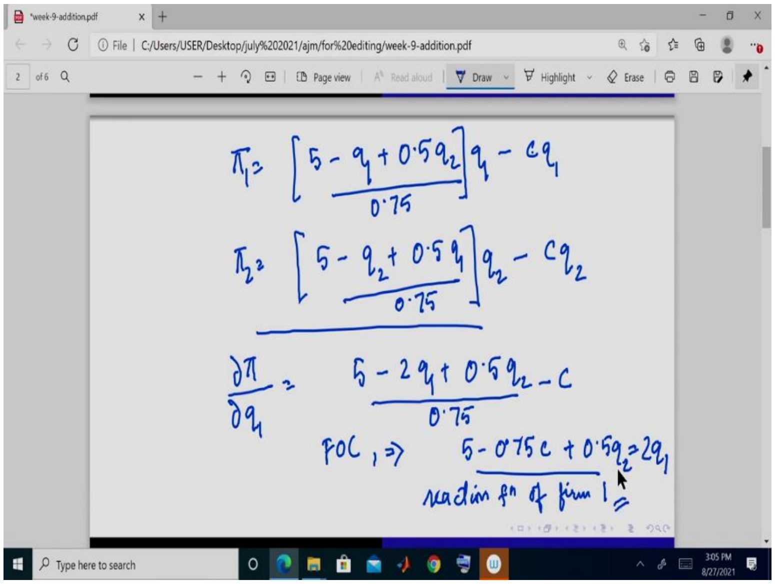

So, in the first problem, we have got the profit function of firm 1, of this nature. This$-\pi_1 = \left[\frac{5-q_1+0.5q_2}{0.75}\right] q_1 - cq_1$. And the profit function of firm 2, this-$\pi_2 = \left[\frac{5-q_2+0.5q_1}{0.75}\right] q_2 - cq_2$. So, here we will find the Cournot reaction function from this profit function. So, if we take this, we will get, then first order condition is going to give us. So, this is the reaction function of firm 1. Given output of firm 2, q 2, what is the optimal output of firm 1? This-$5 - .75 + 0.5q_2 = 2q_1$.

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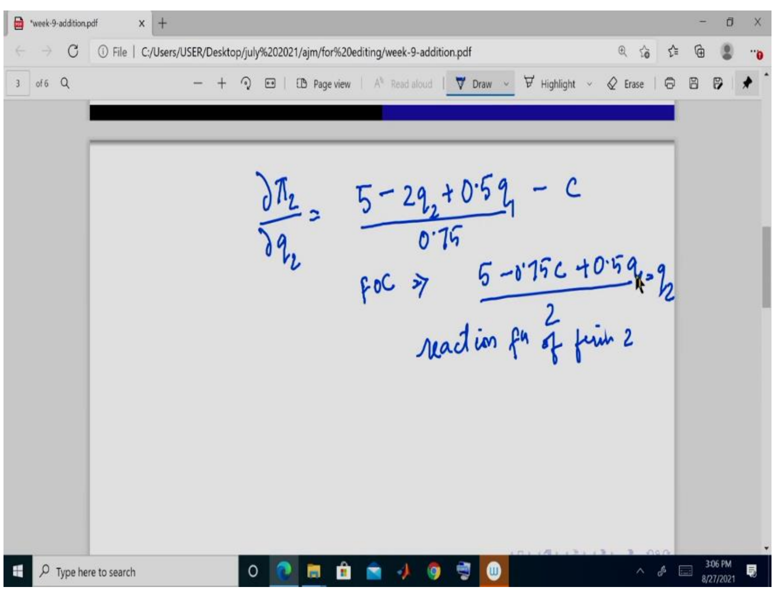

Now, we find out the reaction function of firm 2. And first order condition is going to imply this, so this is the reaction function of firm 2-$\frac{5-.75c+0.5q_1}{2} = q_2$.. So, given any output of firm 1, q 1, what is the optimal output of firm 2 q 2, we have got this. So, if we plot them, this is q 1, this is q 2. This function, if suppose q 2 is 0, then this is going to take a positive value. So, it is something like this. And this point is, this. And, this, if q 1, takes a value 0, it will be somewhere here. And, slope is going to be less than this slope. So, this point is 50 minus, and this is the Cournot outcome. Now, we will study the Stackelberg outcome based on these reaction functions.

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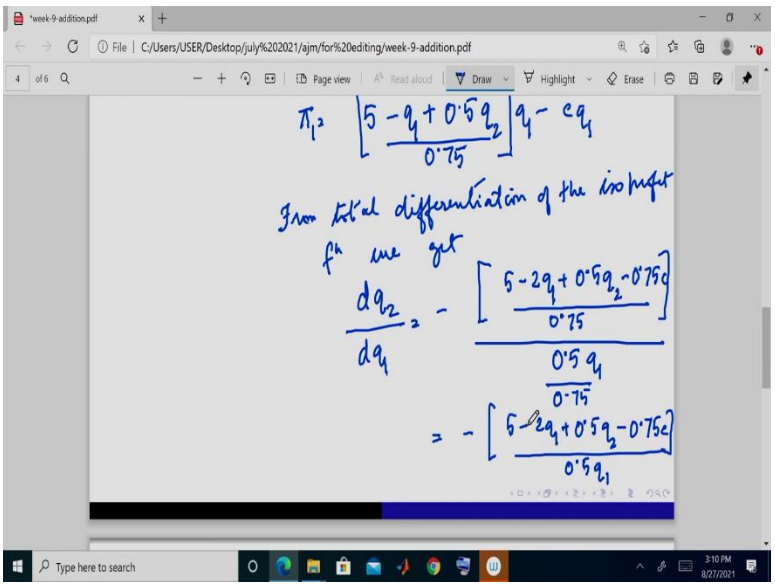

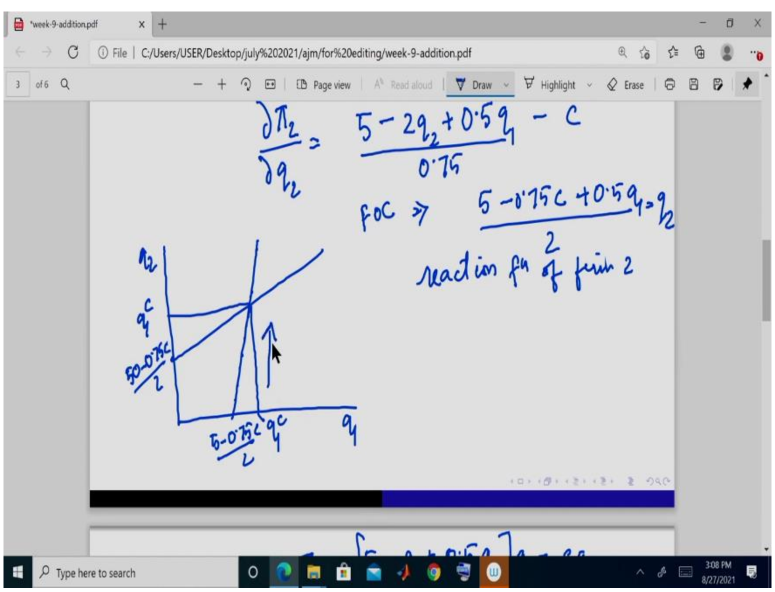

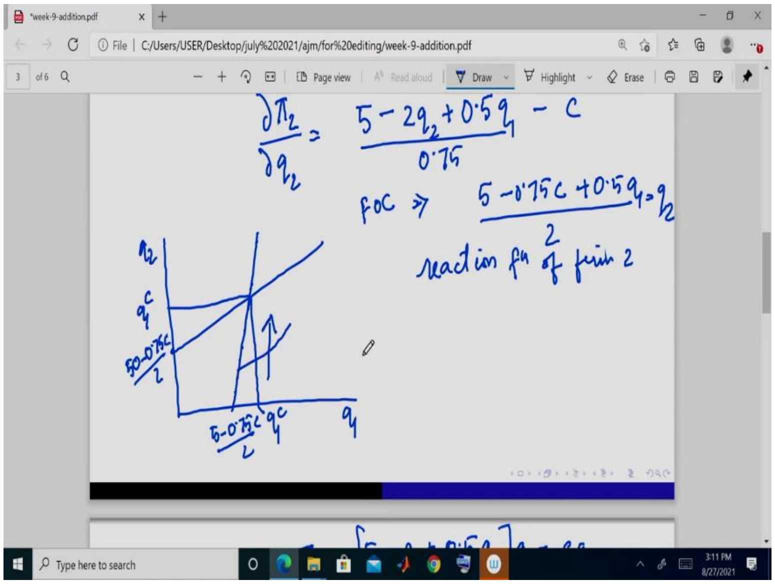

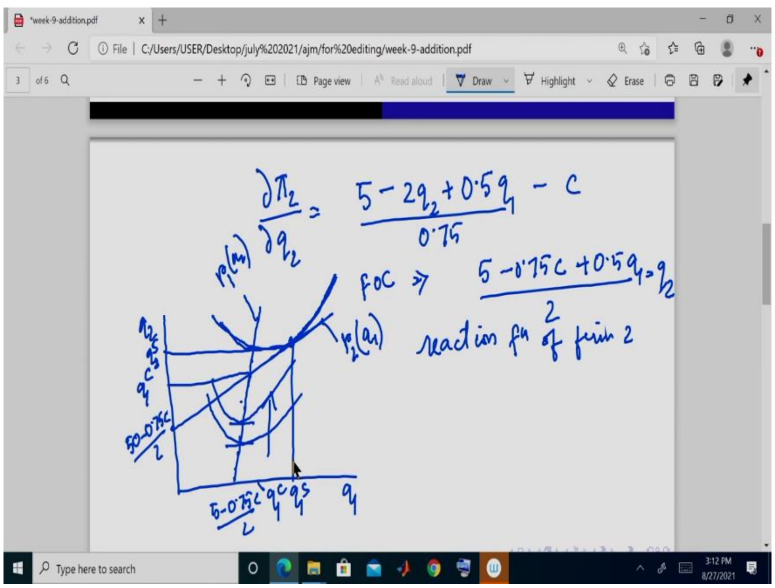

So, we know, the profit function of firm 1 is, it is this. So, we will look at the iso-profit curve of this, generated from this. Now, if you look at this, here, consider this reaction function. So, here if we move in this way, output of firm 2 is increasing, q 2 is increasing. Now, here, if we keep on increasing q 2, keeping q 1 fixed, see the profit is increasing. So, here if we fix this, profit is increasing in this direction. So, profit increases in case of the firm 1. And if we take the, from total differentiation of the iso-profit function, we get this- $\frac{dq_2}{dq_1} = -\frac{\left[\frac{5-2q_1+0.5q_2-0.75c}{0.75}\right]}{\frac{0.5q_1}{0.75}}$. And here, this portion will be simply, it will be this. So, this is going to be this.

Now here, look this portion is actually the reaction function of firm 1. So, we fix q 2, and if we keep on increasing q 1, here, if we fix q 2 and keep on increasing q 1, this is going to take a negative value. It takes a 0 value, when it is in this reaction. So, it will be something like this.

And, if we keep on decreasing q 1, keeping q 2 fixed, then what is happening? This is going to take a negative value, sorry, this is going to take a positive value and this is negative. So, it is going to be a negative. So, it will be something like this. It will be like this. So, it is 0 at this point. So, these are the iso-profit of firm 1.

And the Stackelberg outcome here, it is going to be some point here and it is given by this point, q 1 s, q 2 s because Stackelberg outcome is such that the iso-profit curve of firm 1 is going to be tangent to the reaction function of firm 2. This is the reaction function of firm 2, and this is the reaction function of firm 1. This. So, this is the Stackelberg. Now, we have to find out whether firm 2 is going to choose along with firm 1, in stage 1, or it is going to choose after firm 1 has chosen is output and that is in stage 2.

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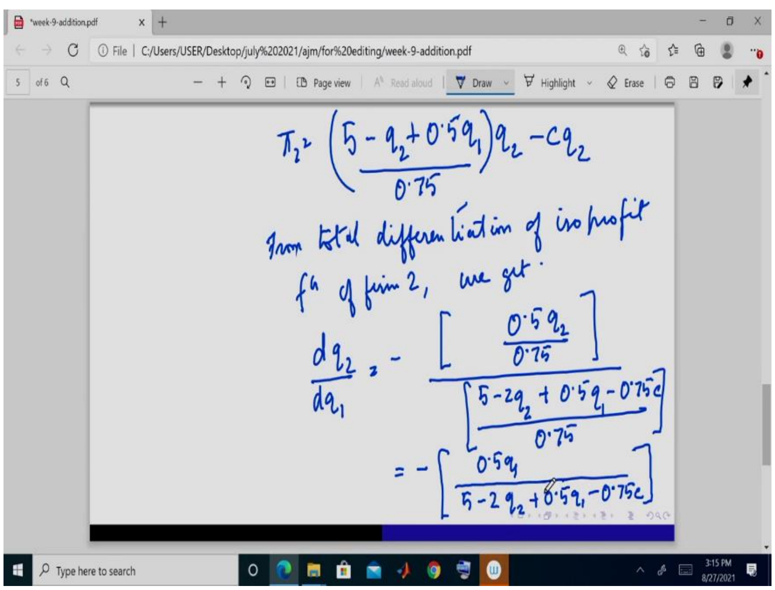

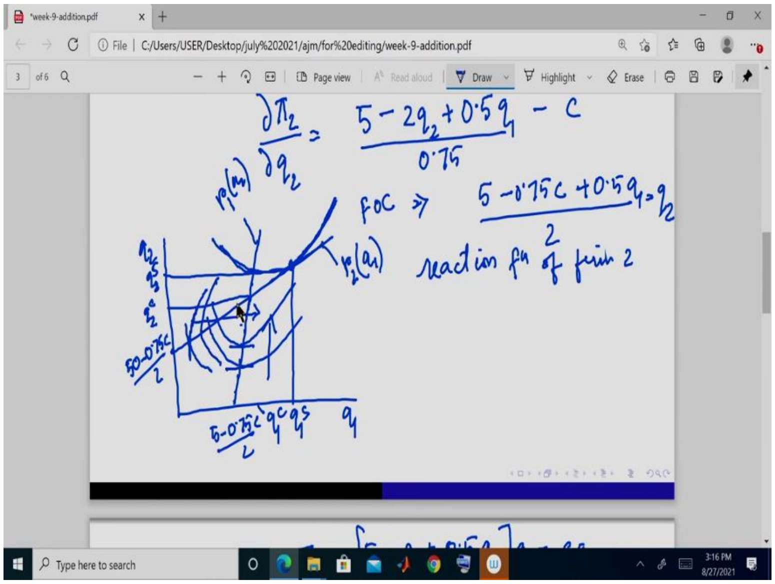

So, we again find the iso-profit curve of firm 2. So, the profit function of firm 2 is this. And then here, if we fix q 2, and keep on increasing q 1, if we fix q 2 here, q 2 here, and keep on increasing the output of firm 1, q 1, then the profit of firm 2 increases. And we got it from this here.

So, it is same in this case also, in this direction profit increases. So, we get from total differentiation of iso-profit function of firm 2, we get this, which is equal to, which is equal to this. So, this is the reaction function of firm 2-$\frac{dq_2}{dq_1} = -\left[\frac{0.5q_1}{5-2q_2+0.5q_1-0.75c}\right]$. So, in this, when we are at the reaction function of firm 2, this takes 0. So, this is a 90 degree.

So, iso-profit curves are like this, this because if we keep fixed q 2, and keep on increasing, q 1 here it will be, this will be negative. And if we keep fixed q 1, and keep on increasing q 2, this will be negative. And this is negative, so it will be positive. So, if we keep this fixed at the reaction function and then keep on (in) decreasing this, then this is going to be positive. So, this is negative. So, that is why this portion is negative and this portion is positive. So, these are the iso-profit curves of firm 2.

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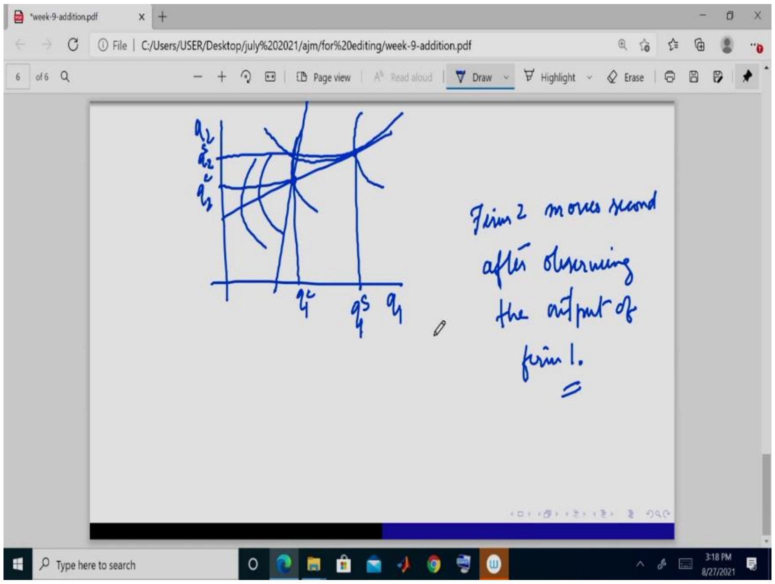

Now, we derive the result. This is q 1, q 2. This is the reaction function of firm 1, this is the, yes, this is Cournot thing, and suppose this is the Stackelberg. And these are the iso-profit of firm 2. So, if it moves simultaneously, along with firm 1, then this is the outcome, Cournot outcome. But if it moves second, then this is the outcome. So, profit here, is more than the profit this, at this point.

So, that is why firm 2 moves second after observing the output of firm 1. So, but in the homogenous good case, we have found that firm 1, firm 2 will always try to move along with, simultaneously along with firm 1. But here, in this case we have found that firm 2 is preferring to move second, that is, in stage 2, after observing the output of firm 1. So, here, we have, there is a second mover advantage.