Introduction to Market Structures
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Lecture – 33
Stackelberg Quantity Competition

Hello, welcome to my course, Introduction to Market Structures. Today, we are going to do Stackelberg Quantity Competition. So, till now we have done Cournot competition and a different versions of Bertrand competition. And today, we will do Stackelberg quantity competition. In Stackelberg quantity competition, we first assume that there are two firms, firm 1 and firm 2.

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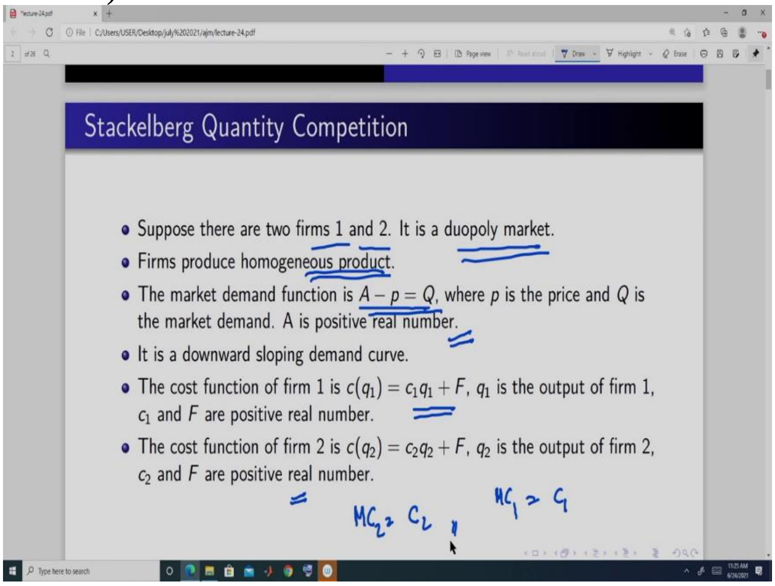

So, we consider a simple market and that is a duopoly market, okay. Now, firms produce homogeneous product. So, that means, whatever output are produced by firms 1 and firm 2, they are similar whether you buy from firm 1 or you buy from firm 2, it does not matter the products are same. And the market demand function is this- A–p=Q. So, it is again A minus p is equal to capital Q. So, p is the market price and big Q is the market demand and A is a positive real number.

So, this is our standard (market) downward sloping demand curve that we are using it for some time. Now, we specify the cost function on each firm. Cost function of firm 1 is this-$c(q_1) = c_1q_1 + F$. So, its marginal cost is constant. So, if we take this as a a cost function, and then the marginal cost of firms 1 is c1. So, this is the variable component and this is the fixed cost and we assume for a simplicity only this version of the cost function.

Again, the cost function of firm 2 is- $c(q_2) = c_2q_2 + F$ c2 into q2 plus F2, where q2 is the output of firm 2 and c2 and F are some positive real number. So, again marginal cost of firm 2 is constant and it is c2, okay. So, this is the specification of the market or specification of the agents who are operating in the market. So, the firms they have a cost function of this nature and the market demand is this. Now, we define the interaction.

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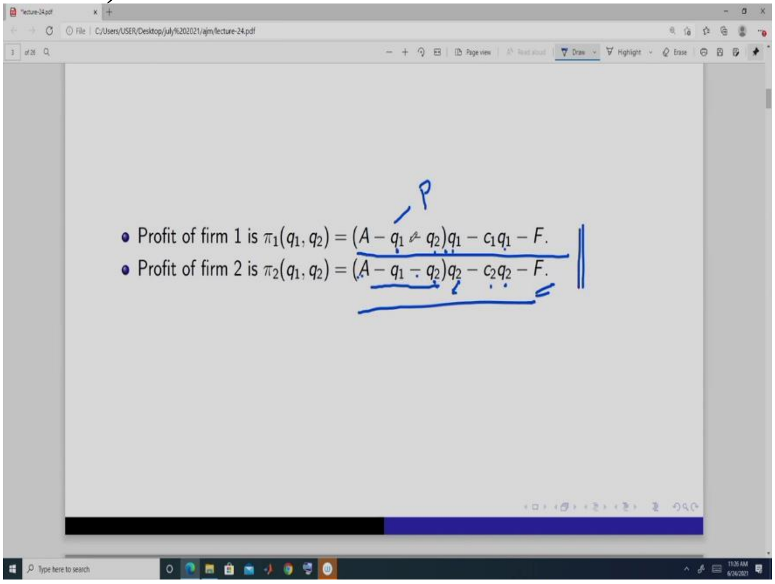

So, here based on this cost function and this market demand, we get the profit function of firm 1 is of this nature-$\pi_1\ (q_1, q_2) = (A - q_1 - q_2)q_1 - c_1q_1 - F$. This is Q1 output of firm 1, this is Q2 output of firm 2. So, this is the aggregate so, this is the market price and this is the output of firm 1. So, this is the total revenue minus total cost of firm 1, so, this is the profit.

And here again, this is A minus q1 minus q2. So, this is the price and into q2. So, this is the total revenue for firm 2 and c2 into q2, this is the variable cost, total variable cost minus F that is the fixed cost. So, this is the profit function of firm 2-$\pi_2\ (q_1, q_2) = (A - q_1 - q_2)q_2 - c_2q_2 - F$ So, we take this as the profit function, okay or you can say the payoff. And now we will define the game.

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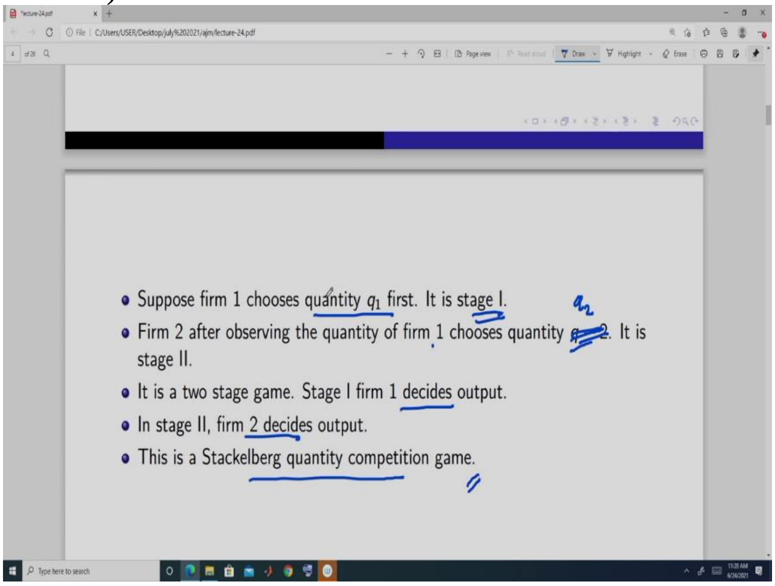

So, here firm 1 first chooses quantity q1, okay and it is called as stage 1. So, firm 1 first chooses this quantity. After observing the quantity of firm 1, firm 2 chooses this quantity and this is q2. So, this should be q2, this is not minus, a, q2. So, it is called the stage 2. So, we get it is an extensive form game or a dynamic game, which has two stages, stage 1, firm 1 decides output. And in stage 2, firms 2 decides output, okay.

So, this form of a market interaction is called a stackelberg quantity competition. Why compete, quantity competition? Because each firm chooses output. And why Stackelberg? Here, it is the decisions are taken sequentially. So, first firms 1 decides the output and then firm 2 after observing the output of firm 1, it chooses its output, that is q2, okay. So, this is the stackelberg. Now, we have already done dynamic games.

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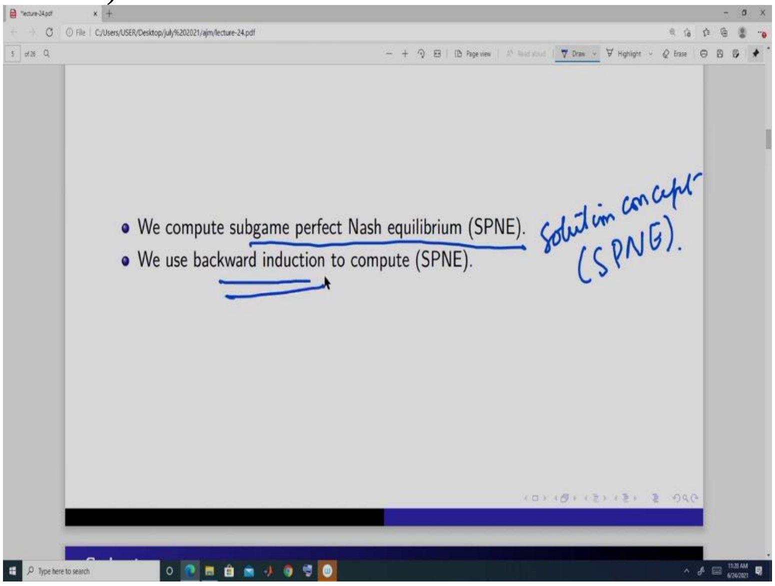

So, what do we do in dynamic games? We look for sub game perfect Nash equilibrium as the solution concept. So, solution concept we are going to use is SPNE, sub game perfect Nash equilibrium. And how do we find the sub game perfect Nash equilibrium? We use something called a backward induction, and we will use that in this market, okay. So, we have two firms, firm 1 decides its output in stage 1. First, I can say it in the first and after observing the output of firm 1, firms 2 decides the output it is going to produce, okay.

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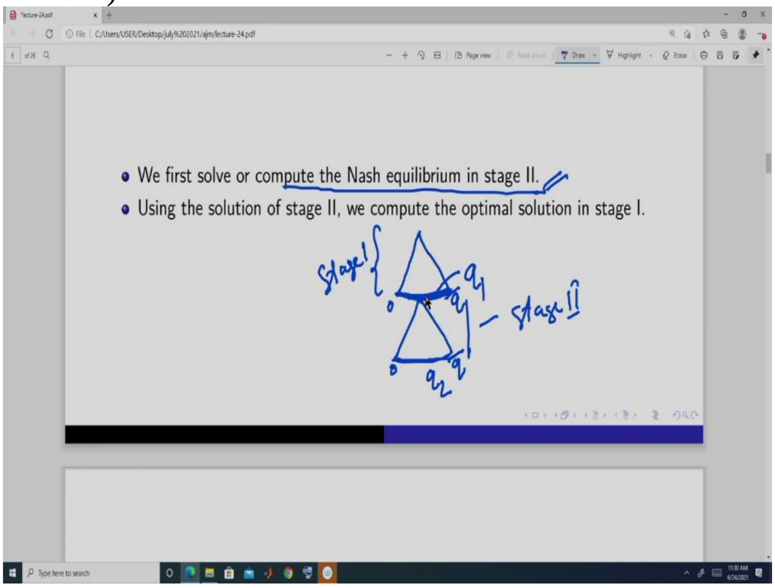

So, that is the stage 2. So, how do we solve this? So, when we are using backward induction, what we will do? We will first solve compute a Nash equilibrium in stage 2. So, it is going to be a game like this. If you look at the game structure, it is something like this where this is, you can say q1 and this is stage 1 and then this is stage 2. And this is q2, q2 again line because it is continuous.

So, I have drawn it line this so, it is from 0 and suppose some sufficiently big number q upper bar. And this is from 0 to sufficiently big number q upper bar, okay. So, this is stage 1 and this, this is stage 2 and this is stage 1. So, now how do we solve this? So, we use backward induction. So, then it means that we assume that suppose firm 1 has produced something, q1. And based on that, we find the optimal output of firm 2.

So, we first solve the stage 2, assuming some output, some outcome in stage 1. And then using that solution of stage 2, we compute the optimal solution in stage 1. So, this is the backward induction. So, first we take some, some value of q1 and then solve this. Because this is 1 sub game. And then we solve this whole game, using this outcome, okay.

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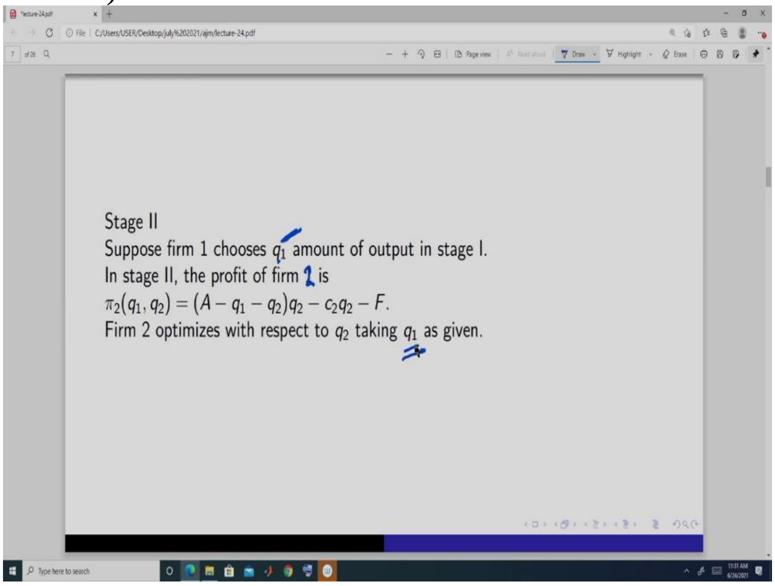

So, what we do? Suppose, the output of firm 1 is some amount q1, okay. So, in stage 2 profit of firm 2 is this, A minus q1 minus q2, q2 minus c2 q2 minus F. So, this is the total revenue and this is the total cost, okay. So, and firm 2, maximizes or or optimizes its profit with respect to q2. Taking q1 as given, because it cannot decide the output, it just simply observe, and it knows that okay firm 1 is already produced this much or it is committed, that it is met.

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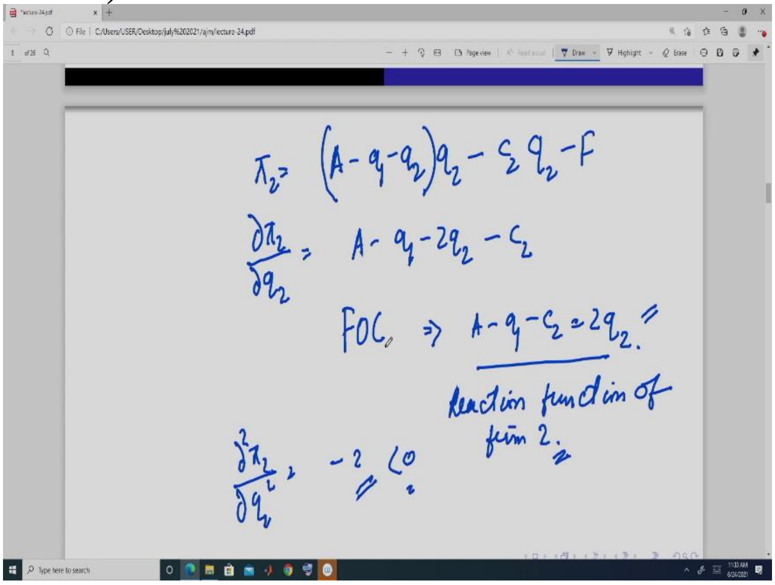

So, based on this we get, this is a profit- $\pi_2\ (q_1, q_2) = (A - q_1 - q_2)q_2 - c_2 q_2 - F$ and when we are and this is differentiable. Because this function is differentiable actually. So, we get, we take the partial derivative with respect to q2 and that is again, first order condition for maximization gives me, this is the reaction function of firm 2- $A - q_1 - c - 2 = 2q_2$. We have derived such reaction function while doing Cournot competition, right? So, it is same as the cournot reaction function of firm 2, okay.

And if you take the second derivative, we will see that with respect to q2 it is negative. So, and it is always negative-$\frac{d^2\pi_2}{dq_2^2} = -2 < 0$. So, we do not have to bother whether it is a maximum point or not, the solution of this is a maximum point or not, is it going to maximize the profit or not, okay. So, we do not need to worry. So, based on this, we get this reaction.

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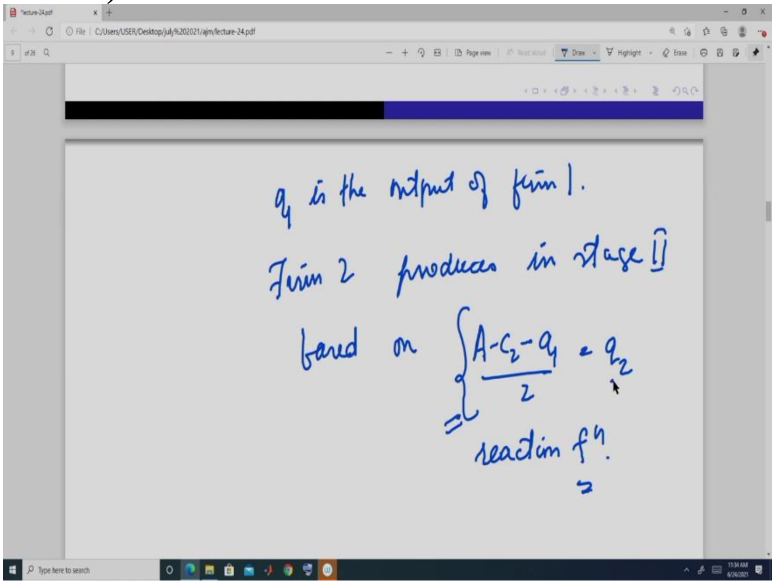

So, now what does this mean? That means, that whenever q1 is the output of firm 1, firm 2 produces in stage 2 based on this function, which is the reaction function-$\frac{A-c_2-q_1}{2} = q_2$. So, it produces based on this function, right? So, this is the outcome in stage 2. So, this is the optimal output. So, when firms 2 decides its output, it always decides based on this reaction function, okay.

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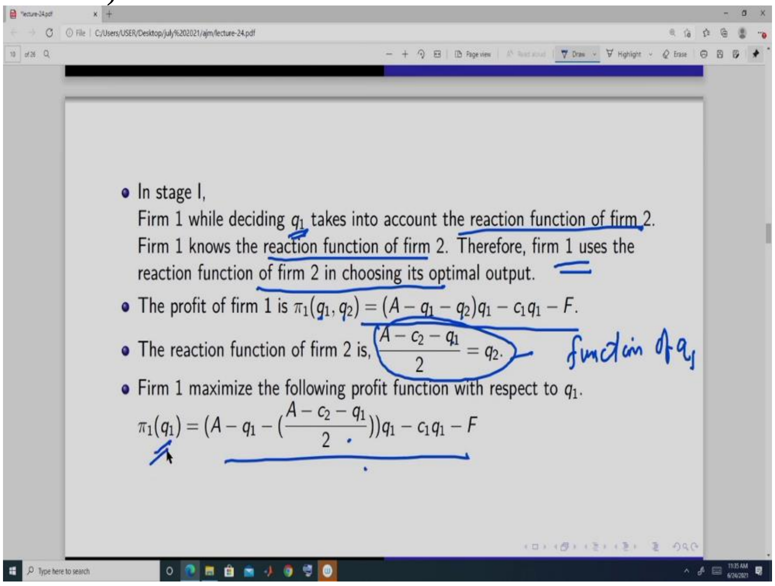

Now, we move to stage 1, what do. In stage 1, firm 2 is deciding q1, and it will take into account the reaction function of firm 2. Because it knows that if I produce some q1 then how firm 2 is going to react? It is going to react based on these reaction function. So, and firm 2 knows the reaction function of firm 2. So, firm 1 knows the reaction function of firm 2, how?

Because all these information's are a common knowledge cost function of firm 1 and firm 2, its common knowledge demand firms, demand mark, market demand is known to both the firms. So, so firm 1 can compute the reaction function of firm 2. So, firm 2 is going to use the reaction function of firm 1, firm 2 in choosing its optimal output, what it is going to do? So, firm 1 profit function is this, we have seen that.

Now, we know the reaction function and we have taken here profit function as a function of q1 and q2. But this q2, firm 1 knows is actually given by these reaction function. It is actually a function of q1. So, we plug in this here in place of this and we get this as the profit function. So, now the profit function of firm 1 is only a function of q1, its own output-$\pi_1(q_1) = \left(A - q_1 - \left(\frac{A-c_2-q_1}{2}\right)\right) q_1 - c - 1\, q_1 - F$. Because it knows that how the firm 2 is going to react, based on its own output.

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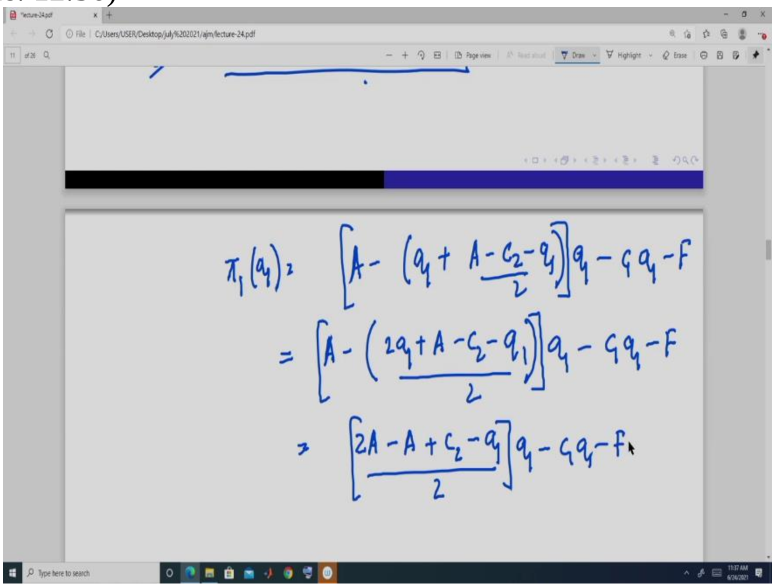


$$= \left[A - \left(\frac{2q_1 + A - c_2 - q_1}{2}\right)\right] q_1 - c_1 q_1 - F$$

$$= \left[\frac{2A - A + c_2 - q_1}{2}\right] q_1 - c_1 q_1 - F$$

$$\pi_1 = \left[\frac{A + c_2 - q_1}{2}\right] q_1 - c_1 q_1 - F$$

$$\frac{\partial \pi_1}{\partial q_1} = \frac{A + c_2}{2} - q_1 - c_1$$

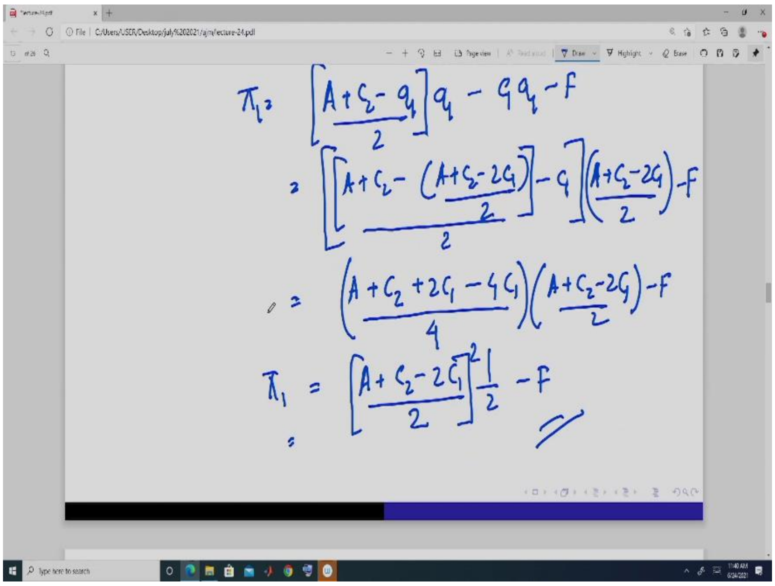
$$\text{FOC} \Rightarrow \frac{\partial \pi_1}{\partial q_1} = 0$$

$$\Rightarrow \frac{A + c_2 - 2c_1}{2} = q_1$$

Stage I, $q_1 = \dfrac{A + c_2 - 2c_1}{2}$

$$\pi_1 = \left[\frac{A + c_2 - q_1}{2}\right] q_1 - c_1 q_1 - F$$

$$= \left[\frac{A + c_2 - \left(\frac{A + c_2 - 2c_1}{2}\right)}{2} - c_1\right]\left(\frac{A + c_2 - 2c_1}{2}\right) - F$$



So, we now find the optimal q1. So, it is, it is this, this-$\pi_1(q_1) = \left[A - \left(q_1 + \frac{A - c_2 - q_1}{2}\right)\right] q_1 - c_1 q_1 - F$. So, profit of firm 1 is actually, it is this-$\pi_1 = \left[\frac{A - c_2 - q_1}{2}\right] q_1 - c_1 q - 1 - F$. Now we maximize this with respect to q1. So, we get, this first order condition gives me, so this equal to. So, this is the output of firm 1, in stage 1-$\frac{A - c_2 - 2c_1}{2}$. So, the stage 1 outcome, is q1 equal to this. So, the profit of firm 1 in stage 1, we have is this, it is this-$\left[[A + c_2 - \left(\frac{A - c_2 - 2c_1}{2}\right)\right] - c_1] \left(\frac{A - c_2 - 2c_1}{2}\right) - F$. Because there is this 2, so, this whole thing is going to be. So, this is the outcome in stage 1-$\left(\frac{A - c_2 - 2c_1}{2}\right)^2 \frac{1}{2} - F$. So, in stage 1 profit of firm 1 is this, and output of firm 1 is this. Now, based on this we can find out the outcome in stage 2.

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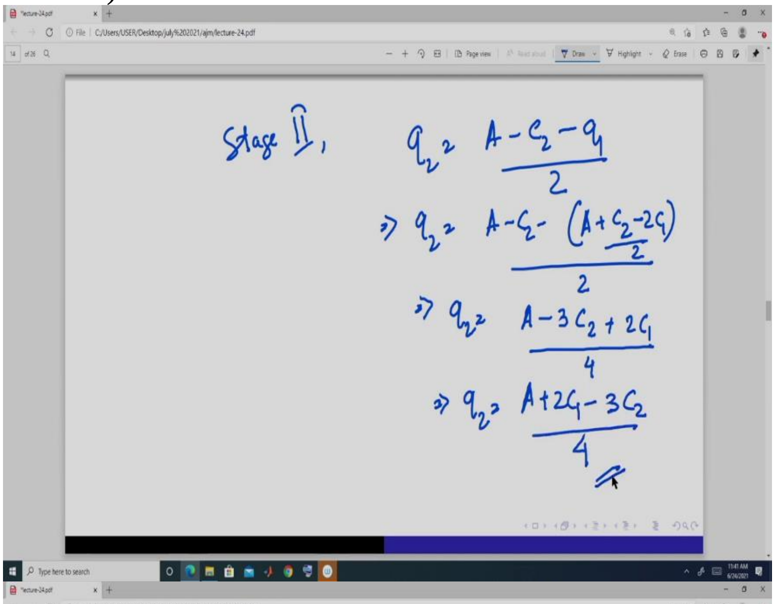

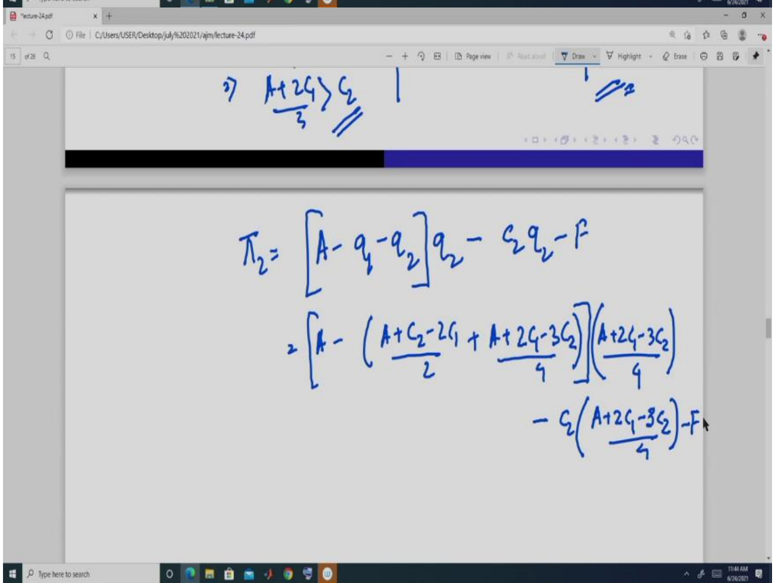

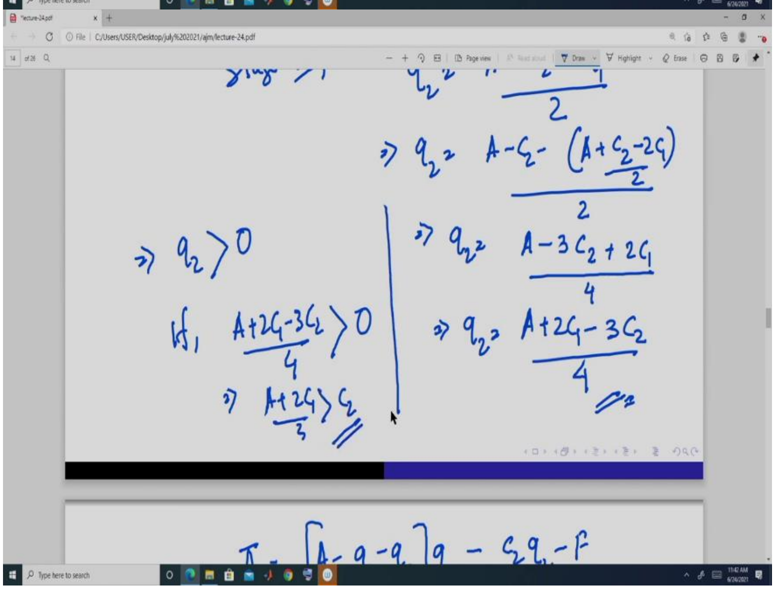

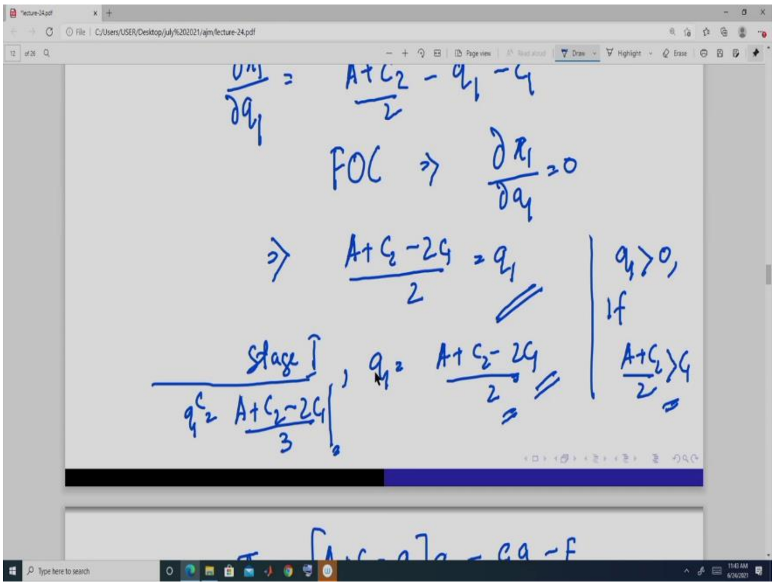

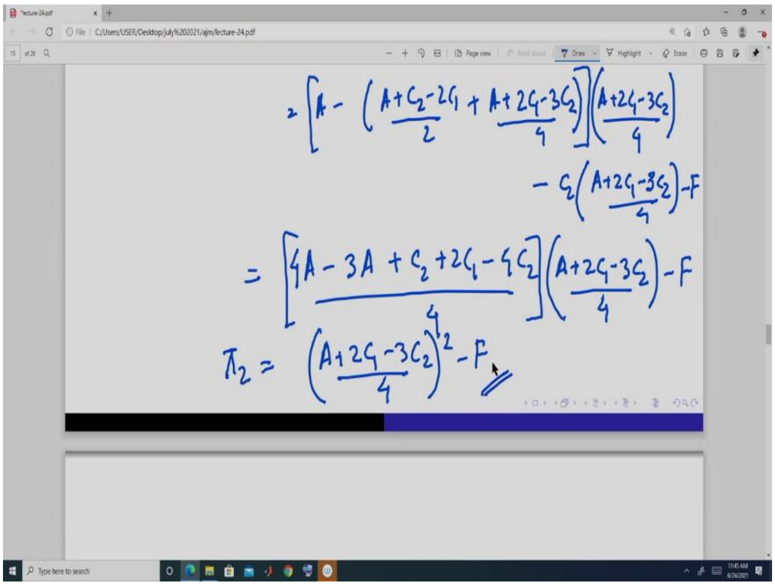

So, in stage 2, what happens? Q2 is A c2 minus q sorry, it is the Cournot reaction function-$q_2 = \frac{A-c_2-q_1}{2}$. So, we get this and so, q2 is, it is this-$q_2 = \frac{A-3c_2+2c_1}{4}$. So, we write q2 is equal to this-$\frac{A+2c_1-3c_2}{4}$. And so, now the profit of firm 2 in stage 2 is, is this-$(A - q_1 - q_2)q_2 - c_2q_2 - F$. So, now here remember, this from here we have to put a condition that is q2 is positive, q2 is positive if, if this is. So, this implies c2 must be this-$\frac{A+2c_1}{3}$.

And here, this condition for this to be positive, q1 is positive if A plus c2. So, this is always going to be positive. Because, c, it is this output is actually if you look at this, this is greater than the Cournot output. Because in Cournot, you remember Cournot output is this-$q_1^c = \frac{A+c_2-2c_1}{3}$, right? We will show again this and we get this.

So, this is greater than Cournot and we know in Cournot it is positive. So, it is positive, but this is less than the Cournot output and this is so, we need this condition-$\frac{A+2c_1}{3} > c_2$. Now, let us compute the profit of firm 2. Profit of firm 2 now simply substitute the values, we get this. So, profit of firm 2 is this-$\pi_2 = \left(\frac{A+2c_1-3c_2}{2}\right)62 - F$ and in stage 2.

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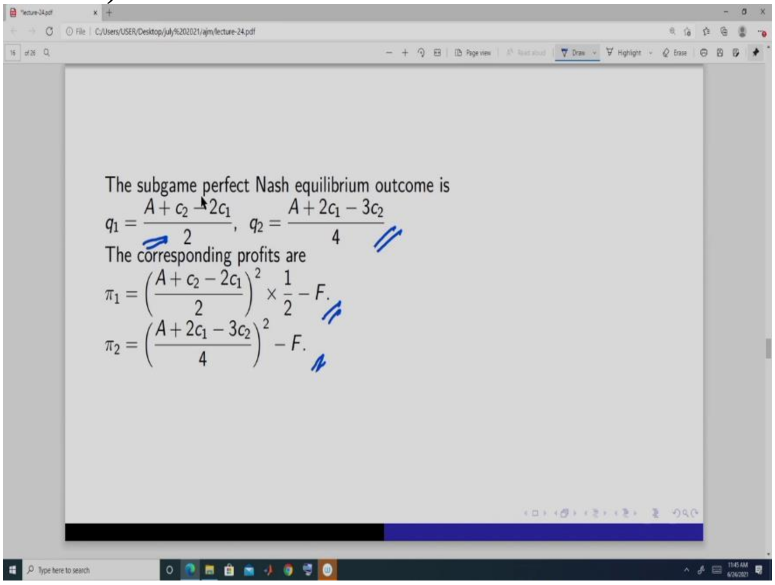

So, we get, so, here the sub game perfect Nash equilibrium outcome is this for, for firm 1, this is the output- $q_1 = \frac{A+c_2-2c_1}{2}$ and this is the output for firm 2-$q_2 = \frac{A+2c_1-3c_2}{4}$. And the profits are this. This will be decided in stage 1 and this will be decided in stage 2 and these enter profits, right? Now we can compare the profits.

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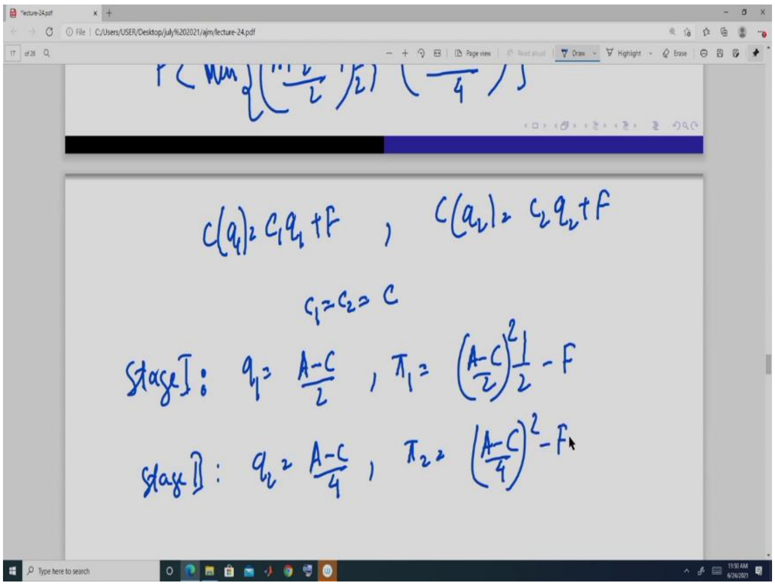

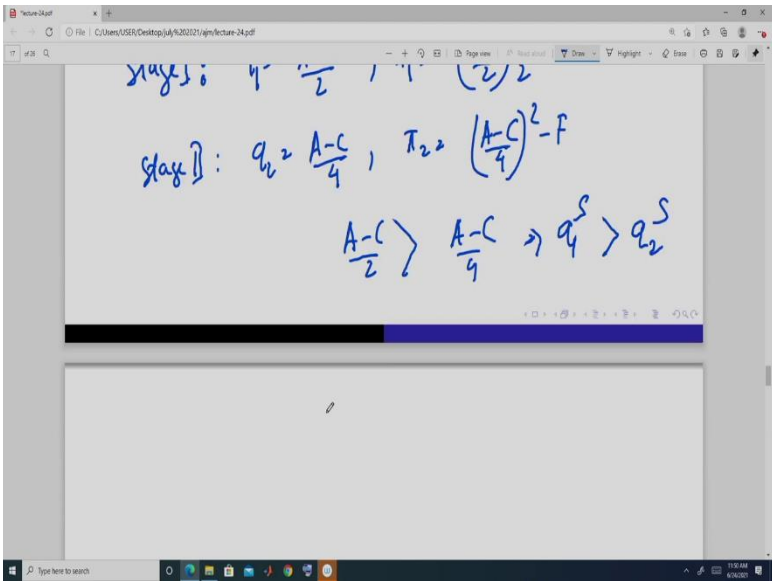

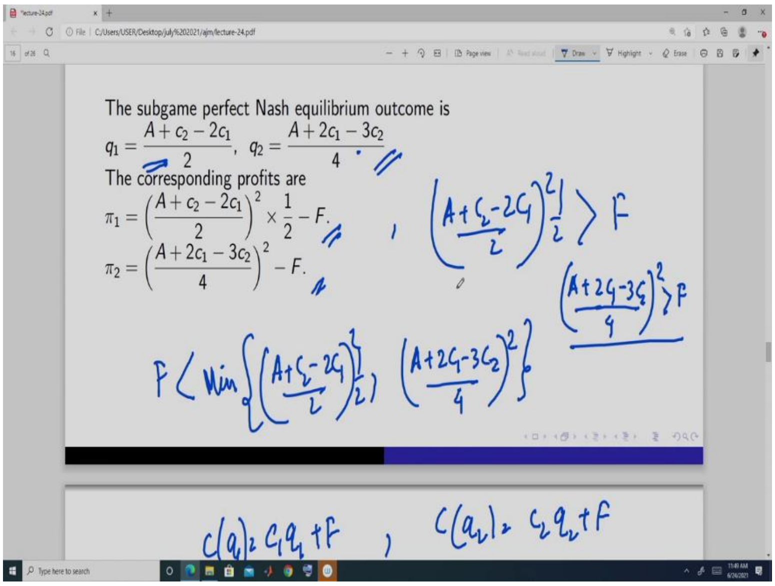

So, whether the profit of firm 1 is greater than profit of firm 2. But now here, there is a firm 1 and firm 2 are not similar. Firm 1's cost function is this-$c(q_1) = c_1q_1 + F$ and firms 2's cost function is this- $c(q_2) = c_2q_2 + F$. So, whether they are making higher profit or lower profit it also depends on this marginal cost, this, this marginal cost, see? So, it will be not possible to compute. So, we have to take that, okay if marginal cost is lower for firm 1 or it is greater for firm 1 like that, different conditions.

So, for simplicity suppose we do this assumption, c1 is equal to c2 which is equal to c, okay. But this, this kind of by taking this kind of output we will compare the profit and that we will do through diagram, not algebraically. Because algebraically it will be messy and it will not give us very clear picture, it will be a conditional statement. So, let us take this. So, then q1 is what? q1 is this-$\frac{A-c}{2}$ in stage 1, q1 is this and profit is this-$\pi_1 = \left[\frac{A+c_2-2c_1}{2}\right]^2 \frac{1}{2} - F$, okay.

So, here, here we have to make further an assumption. And that is the F should be less than this amount. So, F should be, this-$\left[\frac{A+c_2-2c_1}{2}\right]^2 \frac{1}{2} > F$. And so, based on this, F should be less than minimum of this. So, f should be less than min of, we get this. And F should be minimum of this two- $\min\{\left[\frac{A+c_2-2c_1}{2}\right]^2 \frac{1}{2}, \left[\frac{A+2c_1-3c_2}{4}\right]^2$. Otherwise, it will not be profitable to produce at all, okay. So, this, we should not overlook these kind of conditions.

So, now here profit of this. So, in stage 2, q2 is going to be here it is 2 c1 minus 3 c2. So, again c divided by 4, it is this. Now, if we compare these two outputs- Stage i: $q_1 = \frac{A-c}{2}, \pi_1 = \left(\frac{A-c}{2}\right)^2 \frac{1}{2} - F$, Stage ii: $q_2 = \frac{A-c}{2}, \pi_2 = \left(\frac{A-c}{4}\right)^2 - F$, we get this is definitely greater than this. So, this implies q1 stackelberg is greater than q2 stackelberg where, firm 1 moves first and firm 2 moves second.

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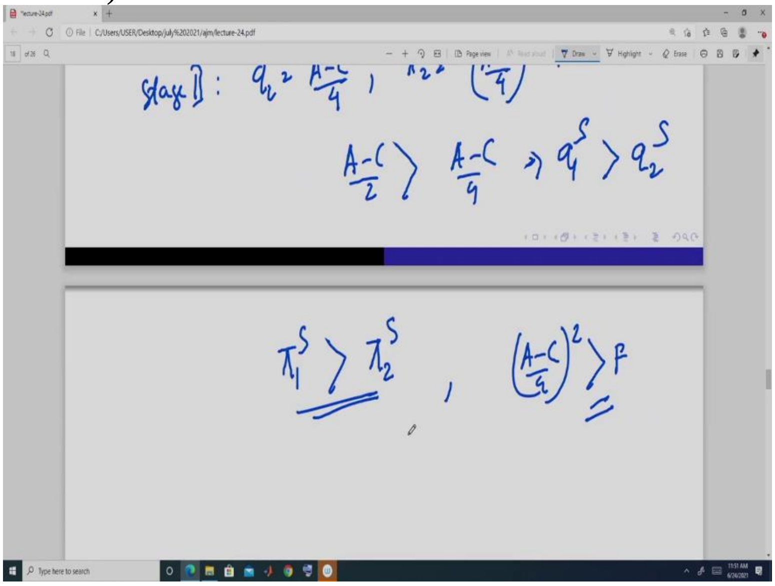

And profit, so, from this we get the profit also. Profit of firm 1 in stackelberg is greater than profit of firm 2 in stackelberg. Because, this is definitely less than this, obvious. And here, we have to make an assumption that this$\left(\frac{A-c}{4}\right)^2$is greater than F, okay. So, we get this.

Now, the question is so, we get that if the firms are similar in terms of cost, then the firm which moves first or the firm which decides output first, it is going to on higher profit then the firms which is going to decide or choose output later or it is in second's stage, okay. So, this immediately gives that if I move first then I have an advantage. So, this is the first, here we can say that it has a first mover advantage.

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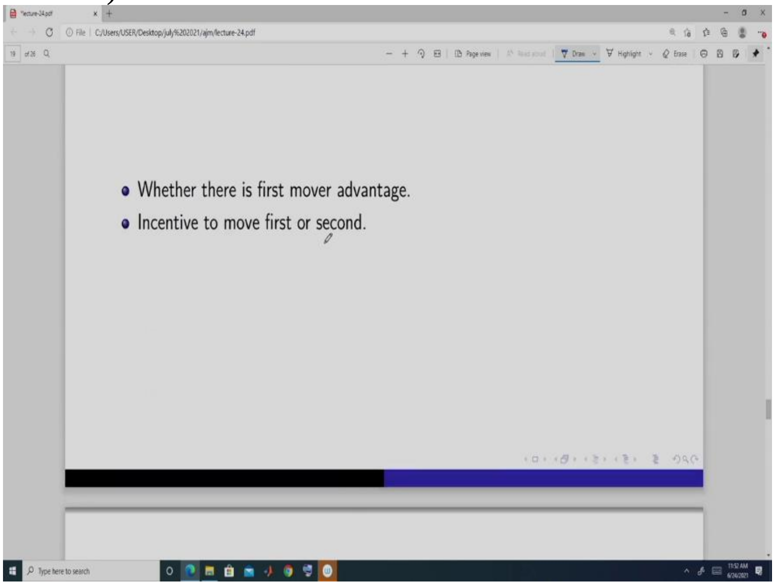

So, whether there is a first mover advantage? Yes, there is a first mover advantage, we have found it when the cost functions are similar. Now, we will see what is, what happens when costs are not similar in this. And we will do it diagrammatically and related to this topic we have a thing like whether there is an incentive to move first or move second, this we will consider later.

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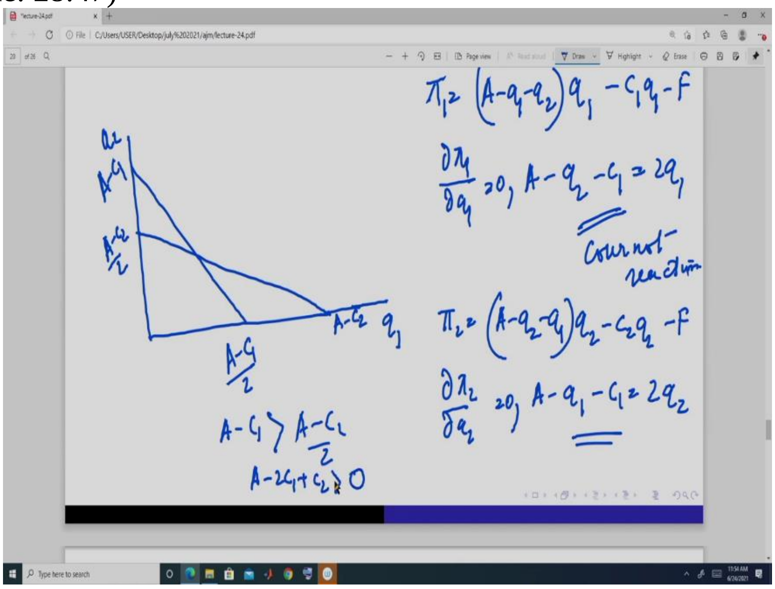

Now, see our profit function is of firm 1, is this-$(A - q_1 - q_2)q_1 - c_1q_1 - F$. So, if we simply, this is the cournot reaction function-$A - q_2 - c_1 = 2q_1$. And for firm 2, this- $(A - q_2 - q_1)q_2 - c_2q_2 - F$ we get this- $A - q_1 - c_1 = 2q_2$. So, this is again the firm 2's reaction function. If we plot them, we get this and this is A minus c1. So, here we assume there is an implicit assumption that this- $A - c_1$ is greater than A minus c2 half-$\frac{A-c_2}{2}$. So, this means that A minus twice c1 plus c2 is greater. Again, we have this less than this.

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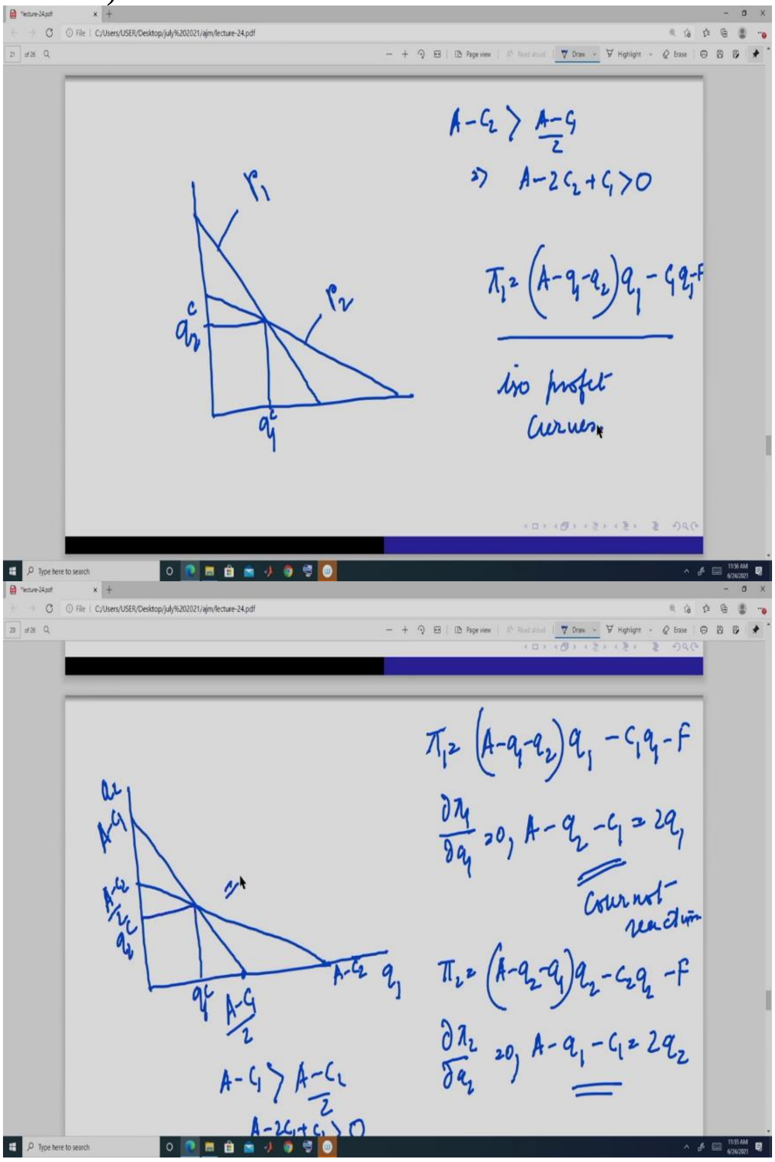

So, so, this implies, is greater than this and we know that we assumed this thing in the cournot thing. So, this is cournot output, this is cournot output, we have got this. Now, we will use this diagram, how? See, so, the if we take this, this is the reaction function of firm 1 and this is the reaction function of firm 2, this is the cournot output of firm 1 and this is a cournot output of firm 2, okay. And profit function of firm 1 which is a function of q1 and q2, this- $(A - q_1 - q_2)q_1 - c_1q_1 - F$. So, now we draw something called iso profit curves. What are iso profit curves? Iso profit curves are actually level curves.

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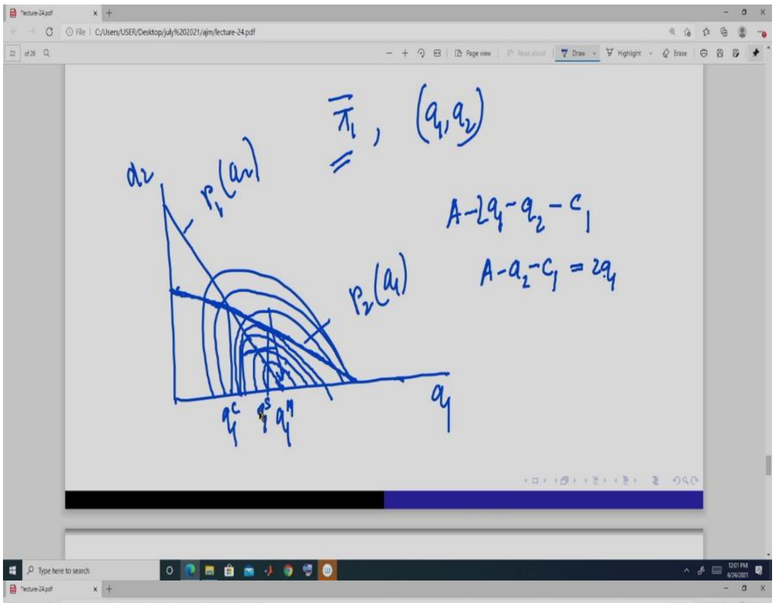

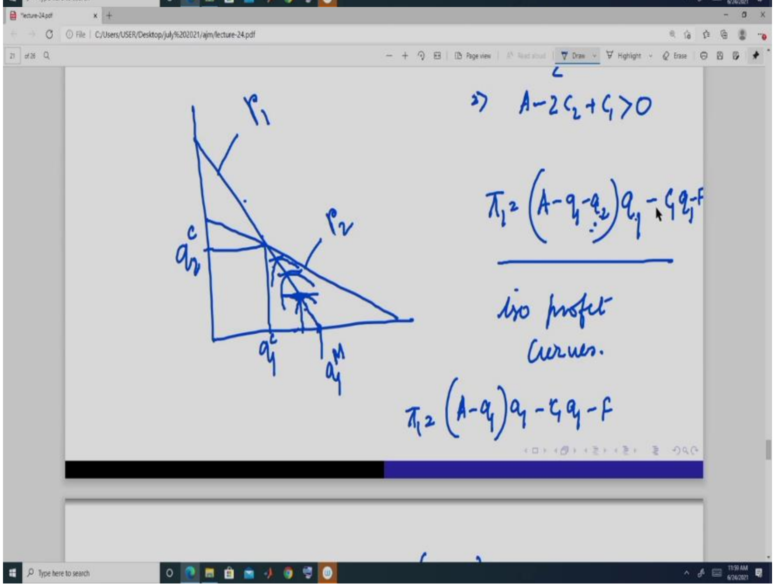

So, what we do? We fix the profit of firm 1 this and look at the combination of q1 and q2 such that it gives me the same level of profit. Now, here in this reaction function, if we fix the level of q2, some level and then find the optimal A, so, that is given by this reaction function. So, this function-$\pi_1 = (A - q_1 - q_2)q_1 - c_1q_1 - F$, if we plot this is going to be you can look at this. So, it is what? If we take the derivative, right?

Now, A this- $A - q_2 - c_1 = 2q_1$is equal to this line, but if this is q is greater than this, so, you fix the level. Suppose, I fixed the level here of q2. So, if q1 is less than this amount, then this is positive increasing. And if q is greater than this amount, then it is negative. So, we get a like this. So, this is like this, that is why it is a maximum point, given a fixed q2. Similarly, we will have a like, like this. Here it is increasing and then it is decreasing.

Again, it is like this. So, for each point we fix some q2 and we will get the, how it looks this. Then and when q2 is 0, this is what? It is this again, it is a, again it is what? It is a quadratic function-$(A - q - 1)q_1 - c_1 q_1 - F$ or you can see it is also a function of the monopoly profit function, monopoly profit function, right? and this is maximum at this. So, this is the q1 when it is a monopoly. And when q2 increases, we get this.

We shift from this function to this function. And so, the, we know the profit is maximum and the monopolist. So, as we move here, so q2 is increasing, this is increasing. So, that means this portion is decreasing this, if we even if we keep constant what happens? Profit goes down. So, if we take these as the reaction functions of firm 1 and firm 2, cournot reaction function, so, this is the output when firm 1 is monopoly and this is the output when firm 2 is Cournot.

And here we get curves like this, where this is 10 means, derivative is 0 here of the iso profit curve. Iso profit curve, means this curve. So, the combination of q1 and q2 such that profit is same. So, it is this combination of q1 and q2 such this profit is constant. And if we go up like this, profit goes down. Because this is the monopoly profit and as q2 increases, you look at this function q2 increases, this portion is decreasing keeping q1 constant so, it is going profit is going down.

So, profit increases in this direction, so, this is the maximum point. So, now what firm 1 does? Firm 1 knows that firm 2 is going to be produced output based on this reaction 1. So, firm 1 in stage 1 will decide that point of this reaction function, which is going to give me the maximum profit. So, we will have this kind of reaction function, iso profit curve. And we will choose that one which is going to be the, giving me the maximum.

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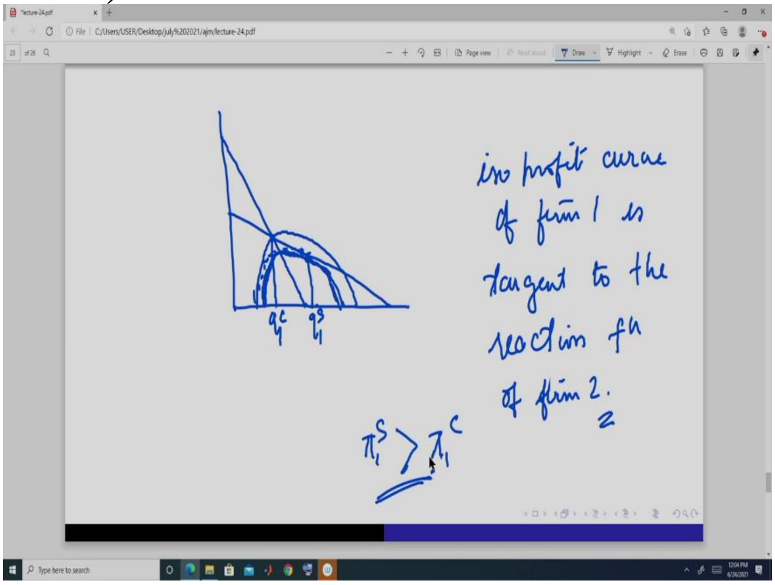

Now suppose, so, from here we get it has to be some tangent like this. So, it is suppose this. This is going to be the Stackelberg output, why it is going to be tangent? Because if a firm is suppose it is not tangent, so, let us draw again. Suppose, this line is tangent at this point. So, this is stackelberg output of firm 1, this is the Cournot, okay.

Now, suppose this is not the output, somewhere different. It has produced something less, then it is producing here, firm 2 is produced based on this. If it, if firm 1 produces here, firm 2 is going to produce based on this reaction function, so, it will be here. And that will be lying somewhere in a reaction function, iso profit curve, which lies below this. If it is more, here because this line is only tangent.

So, it will always all these iso profit curves are going to lie below this. So, profit of firm 1 is not maximized, if the iso profit curve lie above this iso profit. Because if this is tangent to this from below, it is like this. So, all the iso profit which lies above it are going to give less profit. So, any output here it means output of firm 2 is here. So, so there must be a (tan) iso profit curve which passes through this, but this will lie above this iso profit, which is tangent. So, that is why profit is going to be less.

So, that is why this is going to be the stackelberg, wherever it is tangent. So, iso profit curve of firm 1, is tangent to the reaction function of firm 2, right? And this is this Cournot output, right? So, this passes through this iso profit, and this is the stackelberg. So, what do we get? So, the profit in stackelberg is greater than profit in Cournot, whenever the iso reaction functions are like this, downward sloping we get the profit of firm 1 which moves first is higher in Stackelberg than the Cournot.

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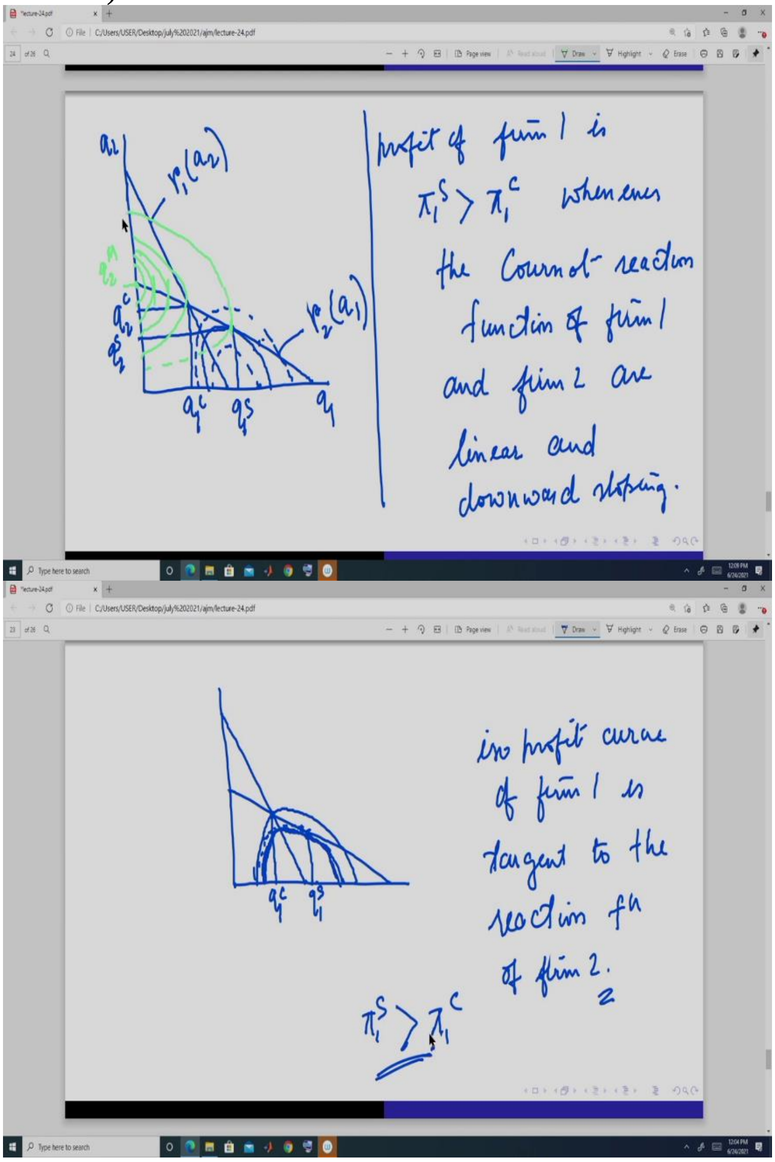

So, we write it in this form, that profit of firm 1 is this-$\pi_1^s > \pi_1^c$ whenever Cournot reaction function of firm 1 and firm 2 are, are you can say linear and downward sloping, or as given in this figure, right? Now, from here we again try to find out whether there is a first, so, from this we get to see what is happening. So, profit of firm is this. Now, let us look at from the context of firm 2, okay. This is the reaction function of, okay and suppose this is the reaction, okay.

This is q1 c cournot output, this is Cournot output c, right. We know whenever the cournot reaction function intersects, we get the cournot output. And we have seen this to be the stackelberg output of firm 1 and this is the stackelberg output of firm 2. Because in stackelberg competition what happens? Firm 1 takes the reaction function of firm 2 as given and based on that rather than the output. So, given this reaction function, firm 1 chooses that combination of that q1 which maximizes its profit, okay.

So, these are the iso profit curves of firm 1 and this is 1. Because this iso profit curves gives higher profit to firm 1 than this. But if you choose anything based on this suppose this output or this, if you choose this output firm 2 is going to produce here. If it produce here, so, the iso profit that passes through this point lies above this. So, profit is less.

Now, here let us draw the similar iso profit curves of firm 2. So, firm 2’s monopoly output is this, this is firm 2's monopoly output. Its reaction function is going to be some, something like this, somewhere like this where it is like this way and it will be like here. And this is the stackelberg, this is the cournot. So, from here, what do we get?

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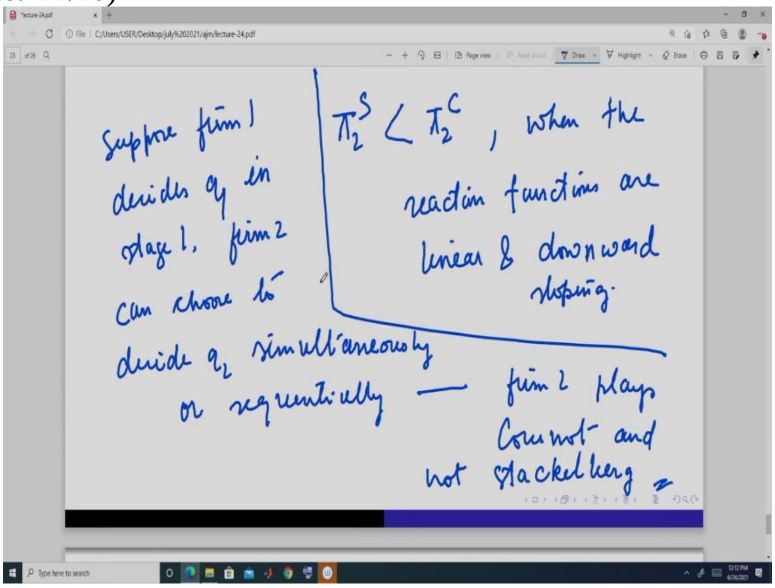

We get that the profit of firm 2 in stackelberg is always going to be less than the Cournot, right less than the cournot, when reaction functions are linear and downward sloping, we have got this. So, from here what do we get? We can conclude that if firm 1 is moving in stage 1, and if firm 2 is given a choice whether to move in stage 1 or in stage 2 like, a firm 2 has a choice to play cournot or to stackelberg firms 2 is always going to play Cournot from this.

So, that is why we see that firm 2 will never want to be the second mover in this case. Because then it has a first mover advantage. So, if we write the problem in this way that suppose firm 1 decides q1 in stage 1 and firm 2 can choose to decides q2 simultaneously or sequentially then we get in this scenario, firm 2 will play Cournot and not Stackelberg, okay.

So, in this context whether there is a first mover advantage or not we get that the firm 1 has a first mover advantage in this cournot in stackelberg quantity competition. And firm 2 has an incentive to move along with firm 1, rather than move later after observing the output of firm 1, okay. So, from this graphical analysis, we get this, okay.

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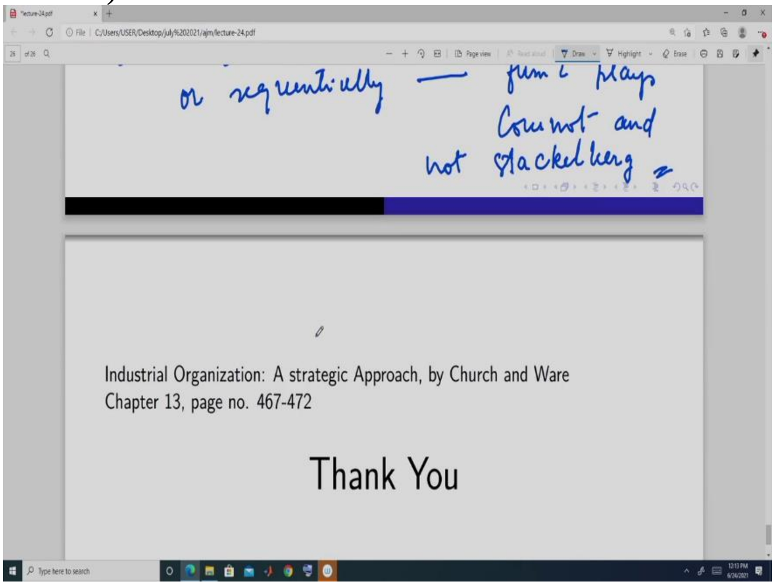

So, you can read this portion from this book by Church and Ware, A Strategic Approach, Industrial Organizational or strategies approach chapter 13-page number this, or these class notes are sufficient enough. And in the next class we will do the price competition in this kind of setup. Thank you.