# Introduction to Market Structure
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**Lecture – 32**
## Bertrand Competition with Decreasing Returns to Scale

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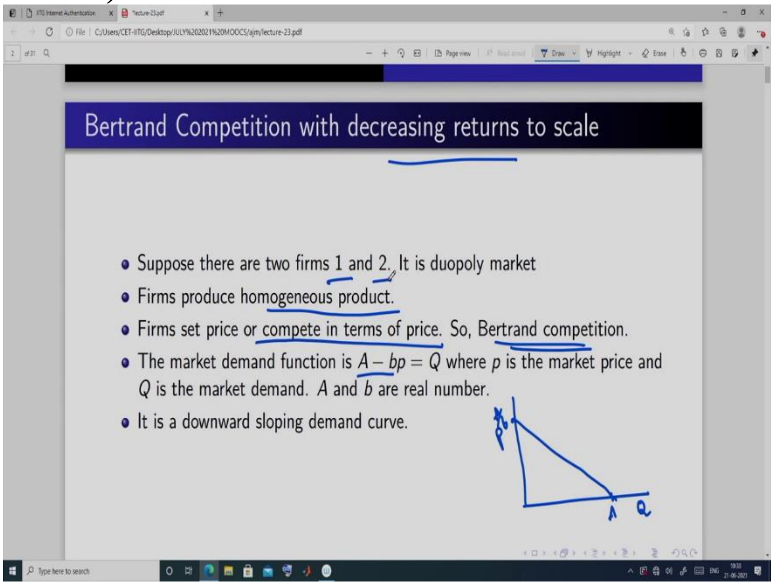


Hello, welcome to my course, Introduction to Market Structures. Today, we are going to do Bertrand Competition with Decreasing Returns to Scale. We are all familiar with decreasing returns to scale, so, we will do that. Suppose, we again assume that there is a duopoly market, that there are two firms, firm 1 and firm 2. So, since there are 2 firms so, it is a duopoly market.

And firms produce homogeneous product, homogeneous products means that the goods produced by firm 1 and firm 2 are completely substitutable. So, whether I buy from firm 1 or I buy from firm 2, it does not matter. And firms compete in terms of price. So, there is Bertrand competition. And the market demand is this, A minus bp equal to Q- A–bp=Q. Q is the here market demand and p is the market price, that is the lowest price in the market, okay.

And A and b are some positive real number. And so, since it is this, so our demand curve if we take p here and quantity here, it is A by p, and this is A. So, it is a downward sloping demand curve. And throughout this course, till now we have only done downward sloping demand curve, okay. So, we continue with that type of demand curve.

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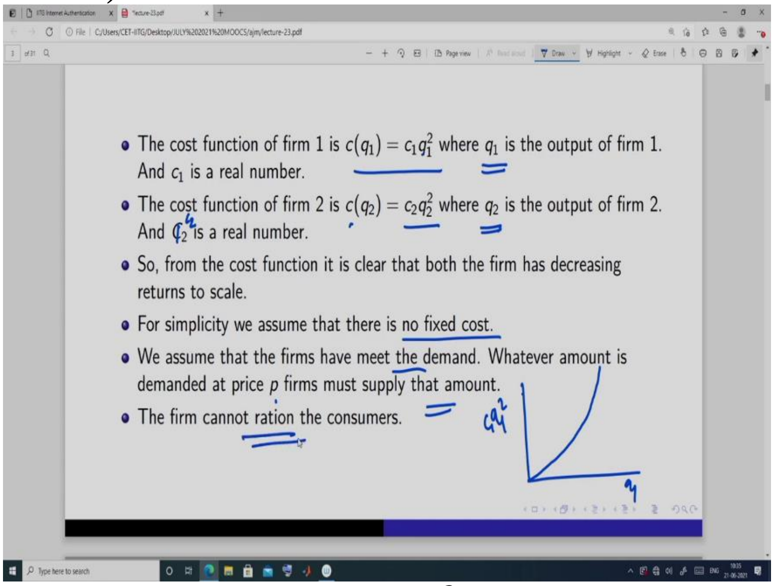


Now, the cost function of firm 1 is this-$c(q_1) = c_1 q_1^2$. So, it is c1 q1 square. So, q1 is the output of firm 1, okay. So, it is square. So, this means the cost function is going to this kind of form, right. If I take q1 here and so, it is this, it is this, right?. So, and if we have this kind of cost function, then we know it is decreasing returns to scale. So, that is if we go on increasing output, so, the marginal cost is increasing, okay. And similarly, the cost function of firm 2 is c2 into q2 square, where q2 is the output of firm 2.

And here c1, this should have been small c. These two are positive real numbers, okay. So, it is from the cost function of both the firms, it is clear that they are facing decreasing returns to scale. And for simplicity in this case, we will assume that there is no fixed cost, okay. Again, further we assume that the firms meet the demand. It means that suppose, firm 1 is the lowest, is has set the lowest price.

So, there will be some quantity demanded at that price? So, firm 1 has to supply that whole amount, okay. So, whatever amount is demanded at price p, firms must supply that amount, okay. So, it mainly, so suppose there is a 100 units demand for the output, the firm cannot say that it is only going to supply 80, it has to supply the 100 units, okay. So, therefore the firms cannot ration the consumers, okay. So, it has to meet the demand.

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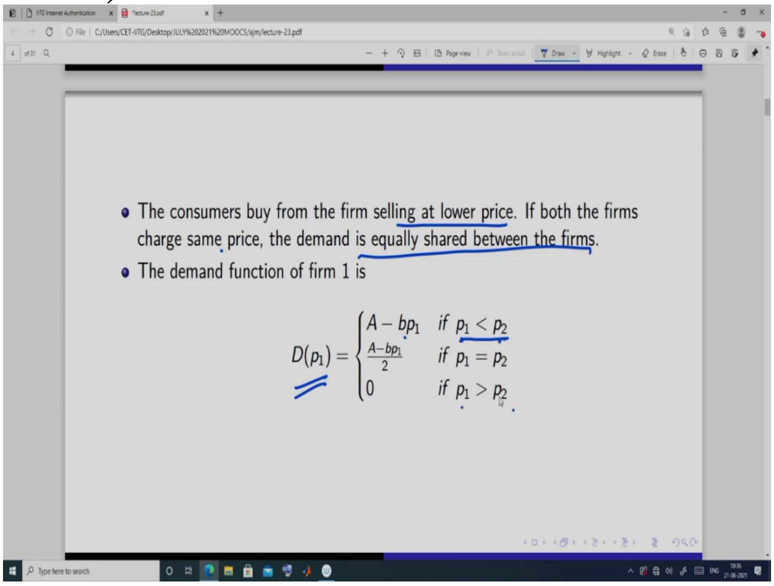


And further, we assume that the consumer, consumers buy from the firm selling at a lower price, okay. And if both the firms charge same price, demand is equally shared between the firms, okay. So, the prices if they set, same price, the demand is equally set. If a firm 1, suppose sets a price lower than firm 2, then everyone is going to buy from firm 1. So based on that, assumption, the demand curve faced by each firm, this is of this nature.

So, if price of firm 1 is less than price of firm 2, is p1 which is less than price of firm 2, p2 then demand is A every, the total demand or the aggregate, the market demand is this much-$A - bp_1$ and this is going to be supplied by firm 1. If the price of firm 1 and firm 2 is same, then at this price suppose this is the demand so, it is equally shared-$\frac{A-bp_1}{2}$. And if the price of firm 1 is greater than the price of firm 2, so, its demand is 0. So, no one is going to buy from this firm.

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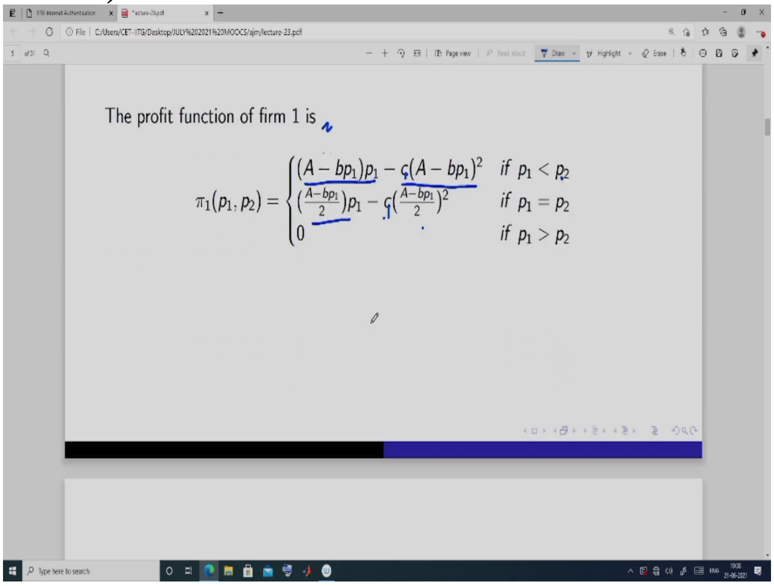


So, based on this demand curve now, we get the profit function of firm 1. So, profit function of firm 1 it is, it is this- $(A - bp_1)p_1 - c(A - bp_1)^2$. So, this is the total quantity supplied by firm 1 or produced by firm 1, since the price of firm 1 is lower than the price of firm 2 into price. So, this is the total revenue of firm 1. And this is the total cost, this is the output it produces or it sells and that is square of it. And here, what we do to keep the things simple, okay.

So, since it is A, we, there is c1 and, okay so, this c1. Again, if the prices are same firm 1 and firm 2 sets the same price, then the amount supplied or produced by firm 1 is this much,- $\frac{(A-bp_1)}{2}p_1 - c\frac{(A-bp_1)^2}{2}$ A minus bp 1 divided by 2. Because it is going to get half of the market demand into price. So, this is the total revenue. So, this is the output produced by firm 1. So, this is the total cost. So, this is the total profit of firm 2, if it shares the market, okay. So, this is c1 and this is c2, okay.

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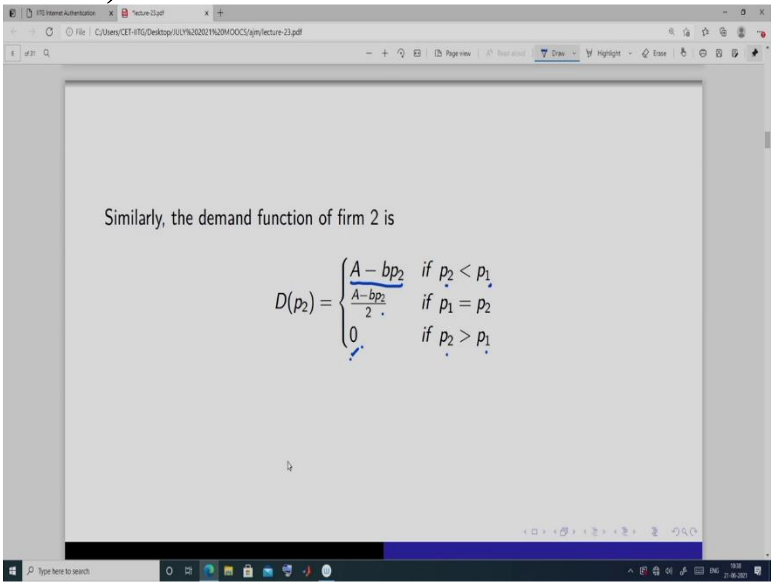


Similarly, the demand for firm 2 is this-$(A - bp_2)$. If the price of firm 2 is less than the price of firm 1, then the whole market is supplied by the firm 2. And so, this is the total demand that is faced by a firm 2. But if the price of firm 1 is same as the price of firm 2, then the total demand that firm 2 faces is half of this half-$\frac{(A-bp_2)}{2}$, So, that is why it is this, okay. And if the price of firm 2 is greater than the price of firm 1 so it gets 0 demand.

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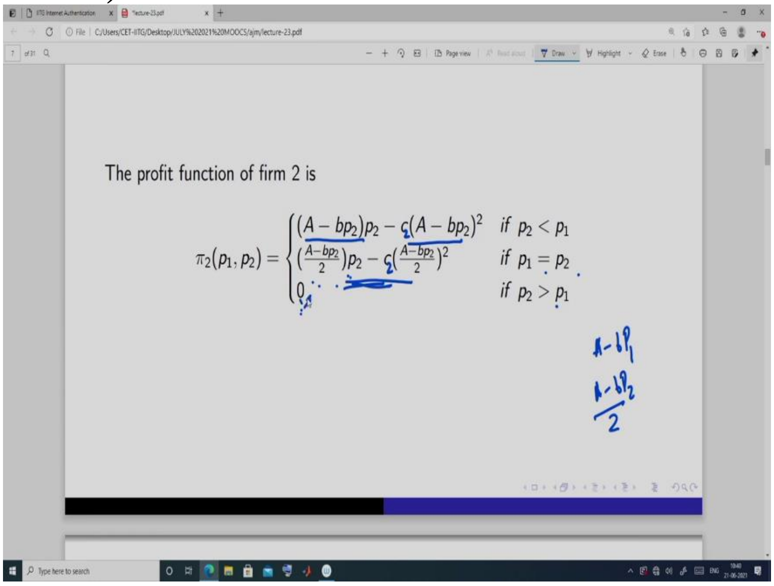


Based on this, we get the profit function of firm 2, this nature. So, here this is c2 and this is c2, okay. So, it is A minus b, this is the total demand into the price. So, this is the total revenue and this is the cost, okay. If firm 2 is only supplying everyone or it is meeting the demand. Because the price of firm 2 is less than price of firm 1. This is the profit when price of firm 1 and prize of firm 2 is same.

So, this is the, this is what? This is the total market demand at this price is this, or you can say this, p1 and is equal to p2. And half of this is this. So, this is the total supply, amount supplied by firm 2 into or produced by firm 2 into price. So, this is the total revenue. And this is the total cost. So, total revenue minus total cost gives you the profit-$\frac{(A-bp_2)}{2}$ $p_1 - c\frac{(A-bp_2)^2}{2}$. And if the price of firm 2 is greater than the price of firm 1, then no one buys from firm 2, so price is 0, okay. So, this is the profit function so or the payoff of firm 2.

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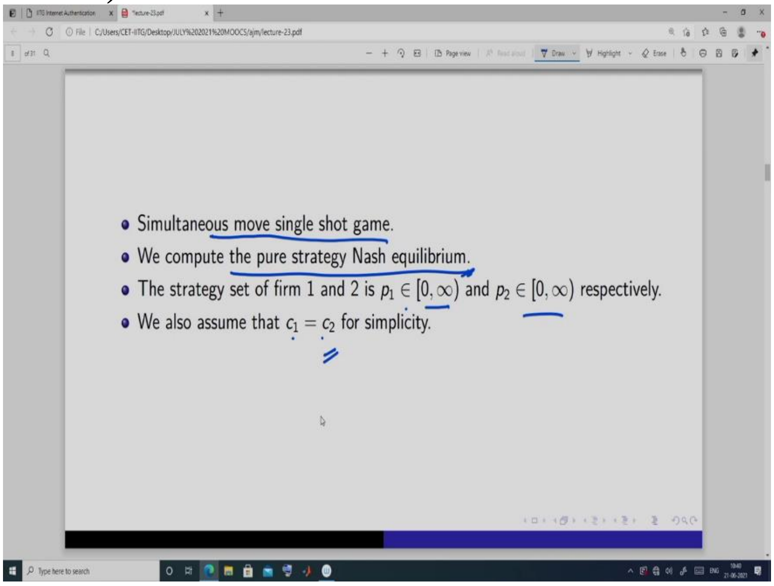


Now, this game it is a Bertrand competition. So, in a Bertrand competition we know the firm 1 and firm 2 both selects or chooses price at the same time. So, it is a simultaneous move single shot game. So, here we find the all compute the pure strategy Nash equilibrium. And the strategy set of firm 1 and firm 2 is this, for firm 1 and this for firm 2 respectively. We further assume, that c1 and c2 is same, okay. We will relax this assumption later on, okay. So, now we the find the pure strategy Nash equilibrium in this setup or in this market game, okay.

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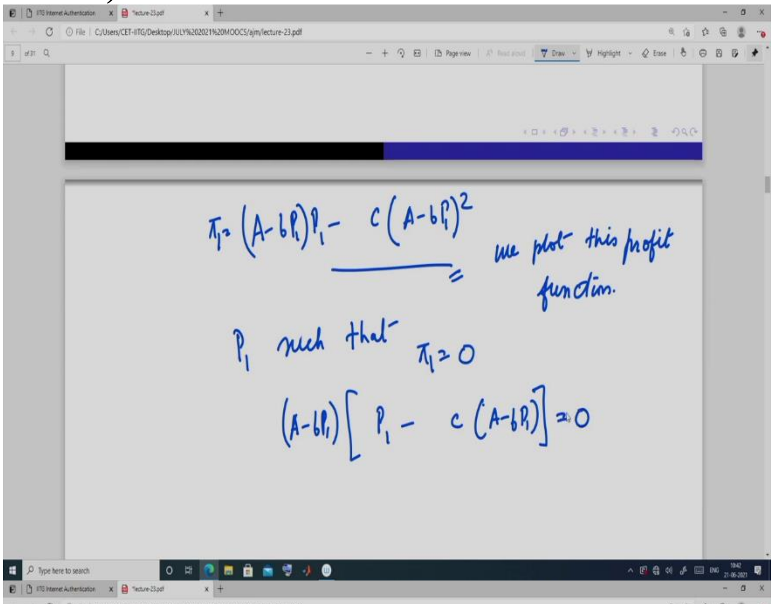

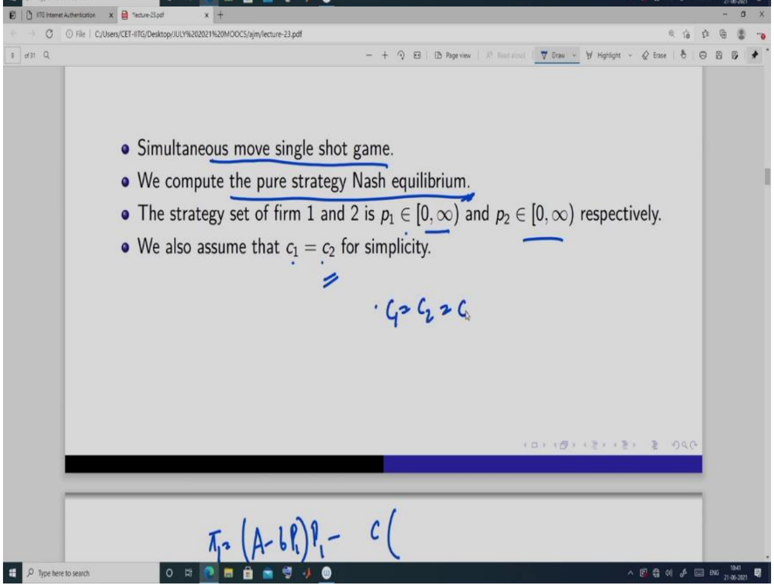


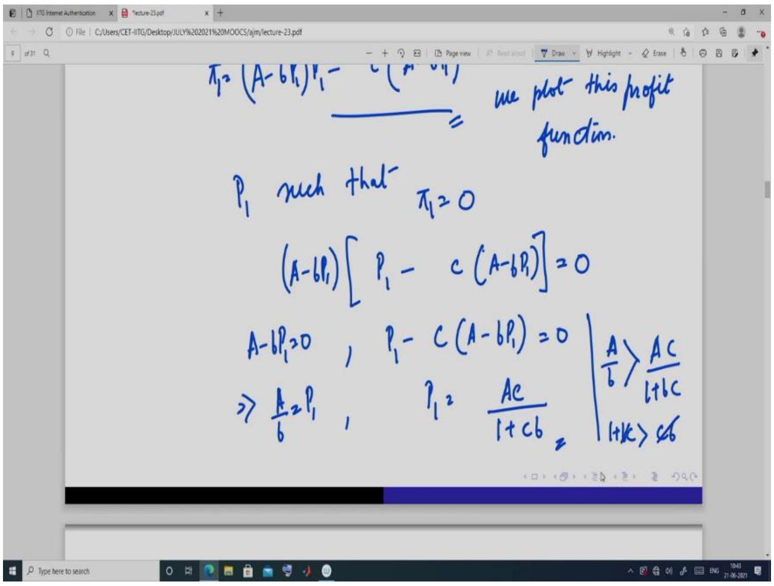

So first, we denote this, we do calculate it for profit of firm 1. So, it is this, right. Now c1 is equal to c2 and suppose that is, here we make it like this, c1 is equal to c2 is equal to sum c, okay. And so, it is this-$\pi_1 = (A - bP_1)P_1 - c(A - bP_1)^2$. Now, we plot this function, okay. What do we do here? So, we first find out the p's such that, this is equal to 0. So, let us write it in a better way.

So, p1 such that this is equal to 0. So, how do we find it? We simply take here in because of this simple functional form, we can, we get this-$(A - bP_1)[P_1 - c(A - bP_1)]$. So, from here we get. So, this gives me A by b and this gives me and from here, it is easy to see that A, this is greater than, this is greater than this. So, this we denote this.

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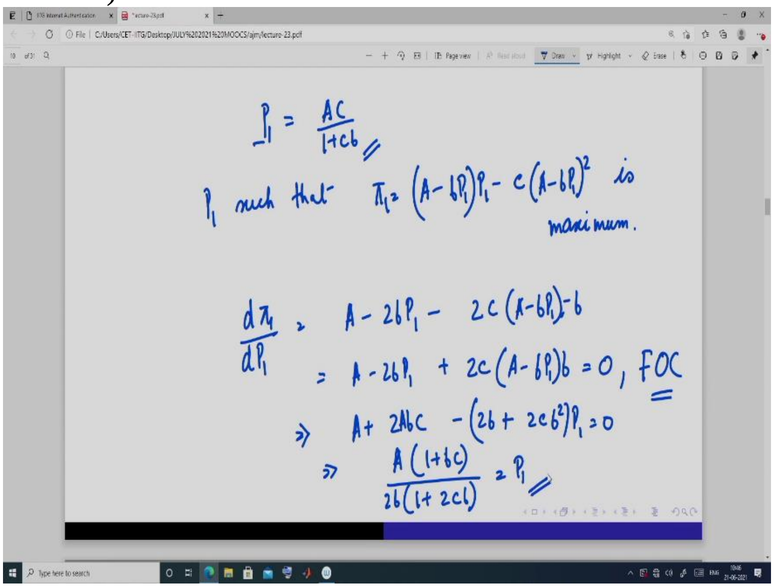


So, we denote that p1 under bar is equal to AC, remember we will use this price and we have 1 price like this- $\frac{AC}{1+cb}$, okay. Now, again let us find the a price such that, so p1 such that this-$\pi_1 = (A - bP_1)P_1 - c(A - bP_1)^2$is maximum, okay. How do we find out that price, that p? So, we find out that p by simply finding the monopoly price of this. And this is equal to, and this is equal to 0 is given by first order condition-$A - 2bP_1 + 2c(A - bP_1)b = 0$. So, first derivative is equal to 0. So, this gives what? This is 2Ab from here. So, we get this price, okay. So, this is one monopoly price-$P_1 = \frac{A(1+bc)}{2b(1+2cl)}$.

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$$\hat{\pi}_1 = \left(\frac{A-bP_1}{2}\right)P_1 - c\left(\frac{A-bP_1}{2}\right)^2$$


$P_1$ such that $\hat{\pi}_1 = 0$

$$\Rightarrow \left(\frac{A-bP_1}{2}\right)\left[P_1 - c\left(\frac{A-bP_2}{2}\right)\right] = 0$$

$$A - bP_1 = 0, \quad P_1 - c\left(\frac{A-bP_2}{2}\right) = 0$$

$$\Rightarrow \frac{A}{b} = P_1, \quad P_1(2+cb) = Ac$$

$$P_1 = \frac{Ac}{2+cb}$$

maximum.

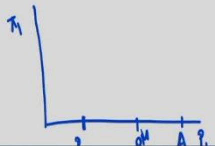
$$\frac{d\pi_1}{dP_1} = A - 2bP_1 - 2c(A-bP_1)\cdot(-b)$$

$$= A - 2bP_1 + 2c(A-bP_1)b = 0, \quad \text{FOC}$$

$$\Rightarrow A + 2Abc - (2b + 2cb^2)P_1 = 0$$

$$\Rightarrow \frac{A(1+bc)}{2b(1+2cb)} = P_1$$

$$\Rightarrow P_1^M = \frac{A(1+bc)}{2b(1+2cb)}$$



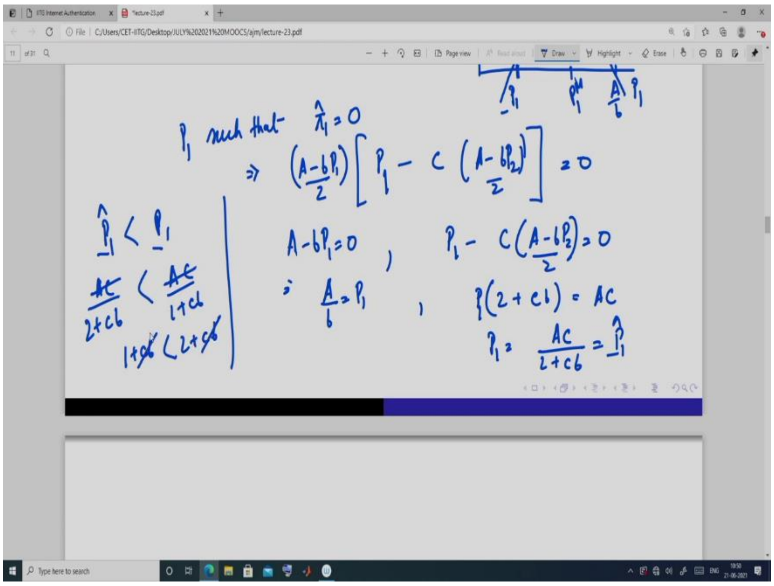

$$(A-bP_1)\left[P_1 - c(A-bP_1)\right] = 0$$

$$A - bP_1 = 0, \quad P_1 - c(A-bP_1) = 0 \qquad \frac{A}{b} > \frac{Ac}{1+bc}$$

$$\Rightarrow \frac{A}{b} = P_1, \quad P_1 = \frac{Ac}{1+cb} \qquad 1+bc > cb$$

$$\underline{P}_1 = \frac{Ac}{1+cb}$$

$P_1$ such that $\pi_1 = (A-bP_1)P_1 - c(A-bP_1)^2$ is maximum.



So, based on this three price, what do we get? If we take here and profit of this, this is p1 lower bar. This is A by p and this is the monopoly price of firm 1. So, we denote this price as p1 M which is the monopoly price. And this is A, and this is, you will have something like this, this is the profit function.

Now, consider this, this function-$\pi_1 = \frac{(A-bP_1)}{2} P_1 - c\left(\frac{A-bP_1}{2}\right)^2$. And suppose, denote this by pi 1 hat, okay what do we get? So, this again, we want to find out p1, such that this is equal to 0. So, this is given by, so, this is equal to 0, 0. So, this gives me, or this gives me, okay. And this is same as the price we have got here and this. But this price, which we denote as p1 lower bar is not same as this p1. And we denote this as p1 hat lower bar, okay. And it is easy to see, that p1 hat lower bar is less than p1 bar. Because A minus C 2 plus cb is less than A-$\frac{AC}{2+cb} < \frac{Ac}{1+cb}$, okay.

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$P_1$ such that $\hat{\pi}_1 = \left(\frac{A-bP_1}{2}\right)P_1 - c\left(\frac{A-bP_1}{2}\right)^2$ is maximum.

$$\frac{d\hat{\pi}_1}{dP_1} = \frac{A-2bP_1}{2} - 2c\left(\frac{A-bP_1}{4}\right)\cdot -b$$

$$\Rightarrow \frac{A}{2} - bP_1 + \frac{Acb}{2} - \frac{cb^2P_1}{2} = 0, \text{ FOC}$$

$\hat{P}_1^M < P_1^M$

$\frac{A(1+cb)}{b(2+cb)}$

$$\Rightarrow \hat{P}_1^M = \frac{A(1+cb)}{b(2+cb)}$$

$$\frac{d\pi_1}{dP_1} = A - 2bP_1 - 2c(A-bP_1)\cdot -b$$

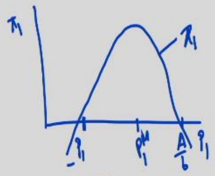
$$= A - 2bP_1 + 2c(A-bP_1)b = 0, \text{ FOC}$$

$$\Rightarrow A + 2Abc - (2b + 2cb^2)P_1 = 0$$

$$\Rightarrow \frac{A(1+2bc)}{2b(1+cb)} = P_1$$

$\Rightarrow P_1^M = \frac{A(1+2bc)}{2b(1+cb)}$

$$\hat{\pi}_1 = \left(\frac{A-bP_1}{2}\right)P_1 - c\left(\frac{A-bP_1}{2}\right)^2$$




$P_1$ such that $\hat{\pi}_1 = 0$

$$\Rightarrow \left(\frac{A-bP_1}{2}\right)\left[P_1 - c\left(\frac{A-bP_2}{2}\right)\right] = 0$$

$$\frac{d\hat{\pi}_1}{dP_1} = \frac{A-2bP_1}{2} - 2c\left(\frac{A-bP_1}{4}\right)\cdot -b$$

$$\Rightarrow \frac{A}{2} - bP_1 + \frac{Acb}{2} - \frac{cb^2P_1}{2} = 0, \text{ FOC}$$

$\hat{P}_1^M < P_1^M$

$$\frac{A(1+cb)}{b(2+cb)} < \frac{A(1+2cb)}{2b(1+cb)}$$

$$\Rightarrow \hat{P}_1^M = \frac{A(1+cb)}{b(2+cb)}$$

$$2(1+cb)(1+cb) < (2+cb)(1+2cb)$$

Now again, here we also find the p1 such that p1 hat, which is equal to A minus bp 1, that is when the market is equally shared between firm 1 and firm 2, such that this- $\pi_1 = \frac{(A-bP_1)}{2} P_1 - c\left(\frac{A-bP_1}{2}\right)^2$ is maximum. So, we get this, this is equal to 0, that is the first order condition- $\frac{A-2bp_1}{2} - 2c\left(\frac{A-2bp_1}{2}\right) - b => \frac{A}{2} - bp_1 + \frac{\frac{Acb}{2} \quad cb^2P_1}{2} = 0$. And we get from here, p1 is equal to A sorry, right? And we call this the, the monopoly price-$P_1^m = \frac{A(1+cb)}{b(2+cb)}$. if we have this. And it is easy to see, we will also mark it by hat, that p1 hat M is less than p1 M because it is A 1 plus cb divided by sorry, here I have made a mistake and it is, okay. So, we compare these two prices and we get.

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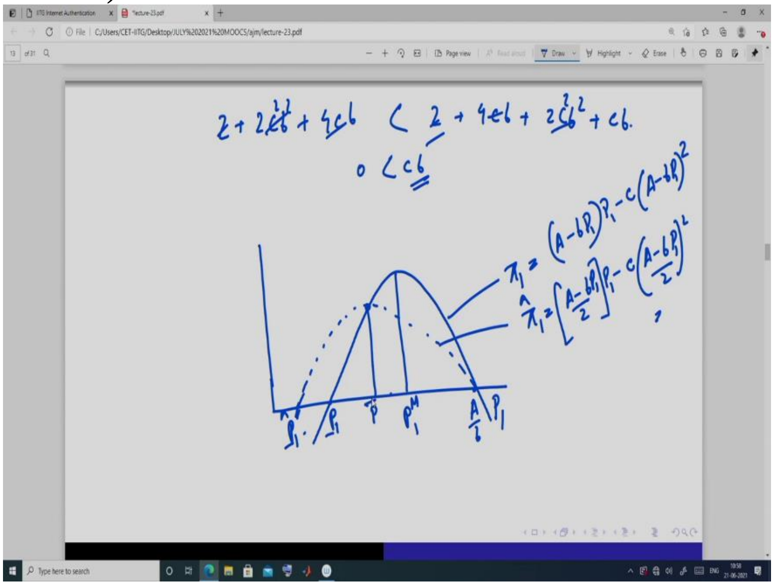

So, it is, just wait a minute let me, we get this and we get this to be this-$2 + 2c^2b^2 + 4cb < 2 + 4cb + 2c^2b^2 + cb$, and then will be, and so, this whole term get cancels and there is. And since both of them are positive so, we get this- 0<cb. So, this means that, okay let me draw this, because, everything will be now based on this diagram. This is A by b, this is suppose this is pM, this is p and see here this curve is, this is p1 lower bar hat. And p1 is somewhere here or it may be here. So, we will get something like this, this curve is this hat, where this is, and this is, this is this. So, we have a price like this, which we called p bar.

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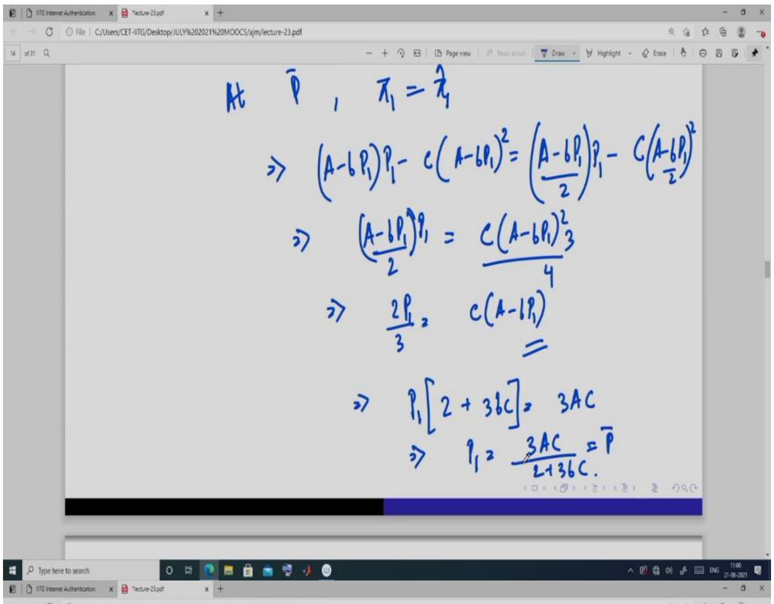

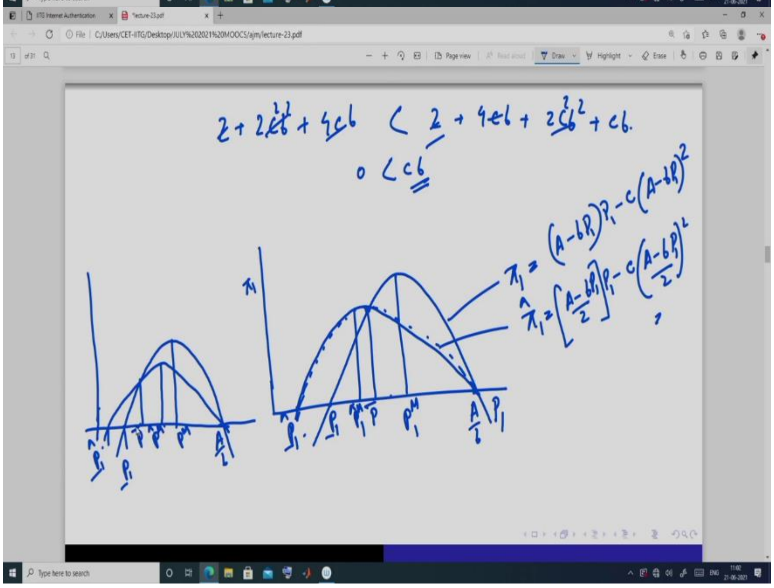

And this price is such that at p bar, we have p, this is equal to this-$\pi_1 = \pi\hat{}_1$. So, this means we have a situation where, whether the market is shared or whether the market is supplied solely by firm 1 profit is same, this-$(A - bP_1)P_1 - c(A - bP_1) = \left(\frac{A-bP_1}{2}\right)P_1 - c\left(\frac{A-bP_1}{2}\right)^2$. So, if we this, we get this to be, we get this-$\frac{2P_1}{3} = c(A - bP_1)$. And from here, we get the price-$P_1 = \frac{3AC}{2+3bC}$. Now, this, so this is equal to p upper bar, this price.

And this price is this. And we can have a graph like this, where this is the pM bar monopoly price, okay. Or we may have a situation something like this, okay. And this is p upper bar and here this is the p upper bar, okay. So, here monopoly output when or the profit price that maximizes this profit, where market is shared is below this and here it is above this price, okay.

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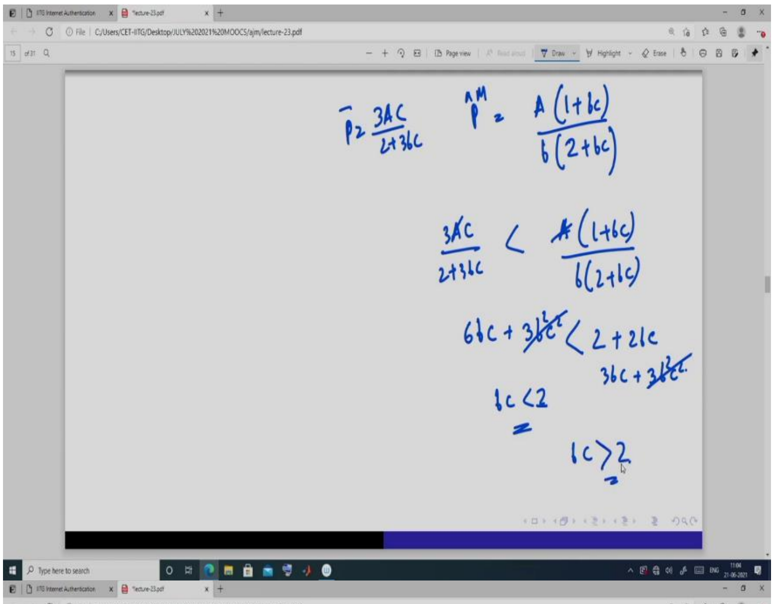

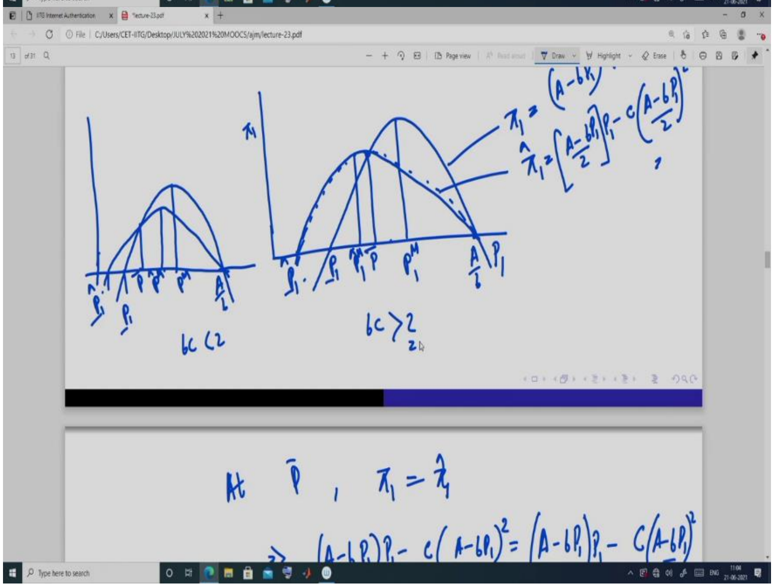

So, and we can see that under what condition we will get this. We will simply compare this, c this-$\frac{3AC}{2+3bC}$ and compare this price which is and which is this, this price-$\frac{A(1+bc)}{b(2+bc)}$. So, we get that if we compare 3AC, this is less than this. If we have this- bc<2, then this price is less than this or if this is greater than 2, i.e. bc>2, then this. So, here this, this case is b into c less than 2, here this is b into c is greater than 2, okay. So, we get this.

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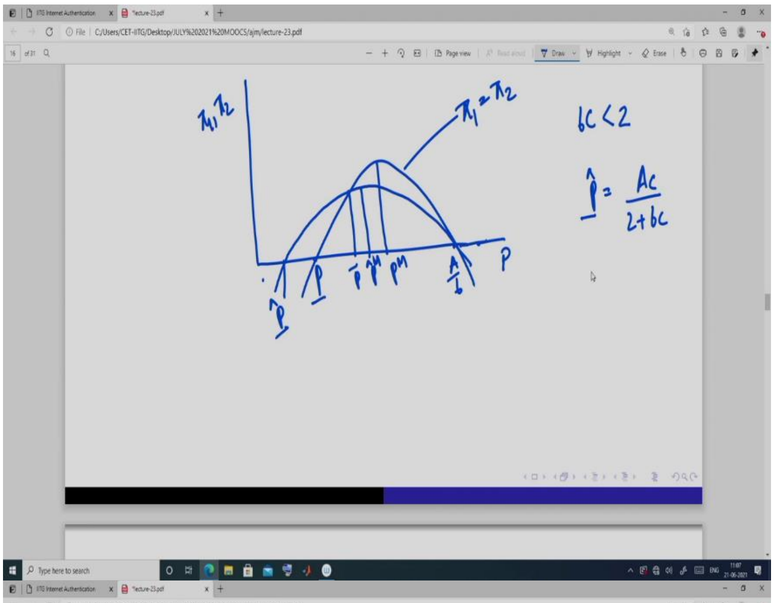

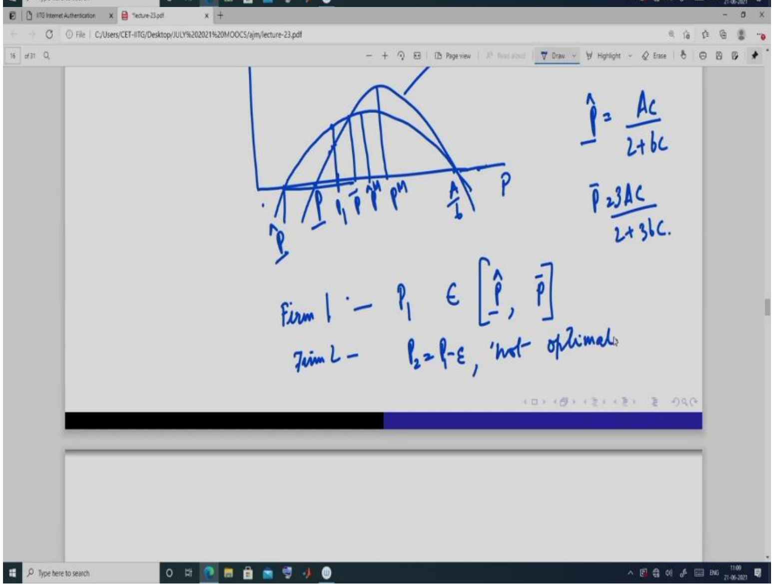


Now, here if you look at this profit function of firm 1 and firm 2, you will find that they are same. So, if we take this, this is suppose. So, we can write it like this, this is A by b, this is the monopoly price and this is p, where this is again you can write something like this. And this is, this is p bar and suppose this is p hat M, okay. Here we have profit of firm 1 and firm 2 here it is price, okay. We get this situation. Now, what we will do? We will try to find a pure strategy Nash equilibrium, in this case where bc is less than 2, okay.

Now see we have derived that here and that price is this, and another price is this. This-$\frac{\text{Ac}}{2+\text{bc}}$ p A is 3AC 2 plus 3bc, this-$\frac{3\text{AC}}{2+3\text{bc}}$, right? So, we will require these two price only. Now here, how to find the pure strategy Nash equilibrium? Now suppose, firm 1 sets a price p1, okay which lies between in this range, okay this range, anywhere. Then firm 2, if it sets a price p 2 which is p 1 minus epsilon amount, then it will get the whole market and suppose this is p1. And if it shares, they are going to get this profit if firm 1, firm 2 undercut its profit is going to be here. So, this is not optimal.

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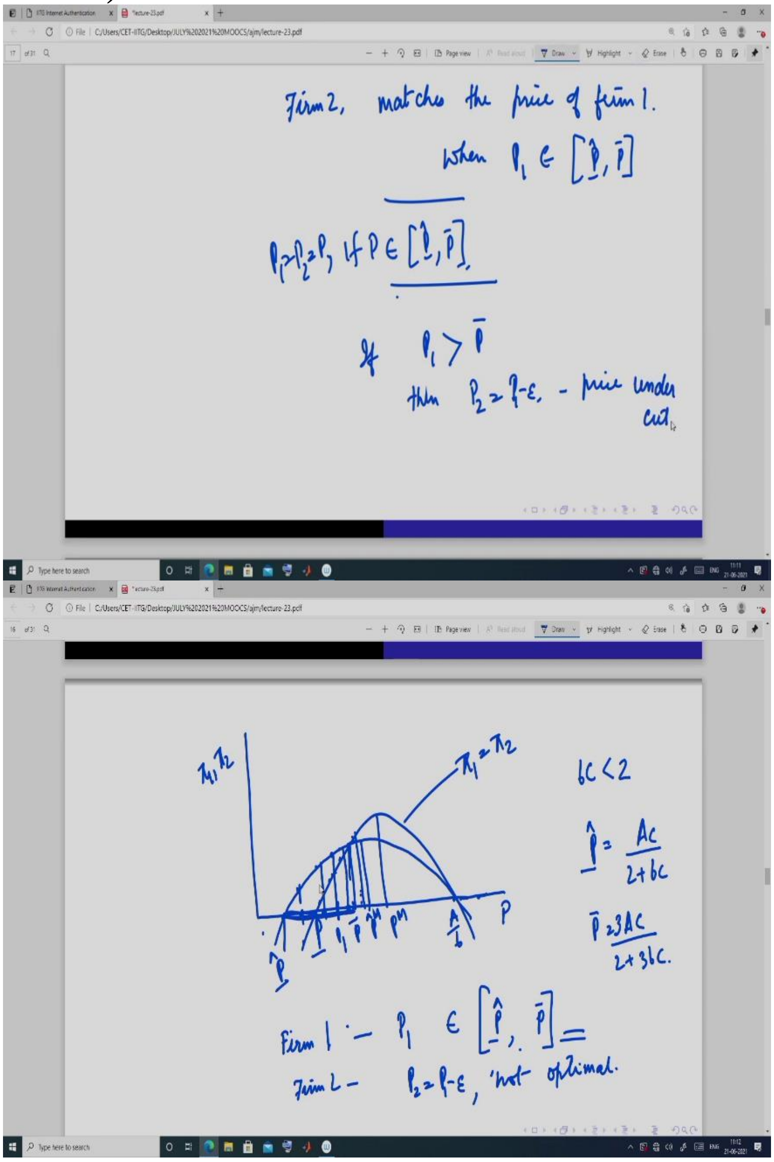


So, firm 2, what it does? Firm 2 matches the price of firm 1. Similarly, when p1 belongs to, because you take any price here, you take a price here. If the price is here, then if it undercuts its profit is going to be negative. But if it shares, market equally it is going to be positive. If it is price is here, then if it firm 1 is at this price, then firm 2 if it undercuts profit will be here, if it shares profit will be here.

Take this price, if firm 1 sets this price, if firm 2 undercuts profit will be here, but if it shares the price, it will remain here. So, at this price, both the profits are same. So, there is no tendency to undercut. So, in this range of price, that is in this range we see both the firm has a tendency to match the price. So, we get that p1 belonging to this, then if so, we get p1 is equal to p2 is equal to p, if p belongs in this range, okay.

Now, the same and if suppose p1 is here, here in this way. Then if it undercuts, firm 2 undercuts it is going to be at a higher curve, higher profit, then if it shares. So, if p1 is greater than p1 then p2 is p1 epsilon amount. So, there is a price, undercut. So, here any price there is going to be continuous price wars. But in this range, price are going to be matched.

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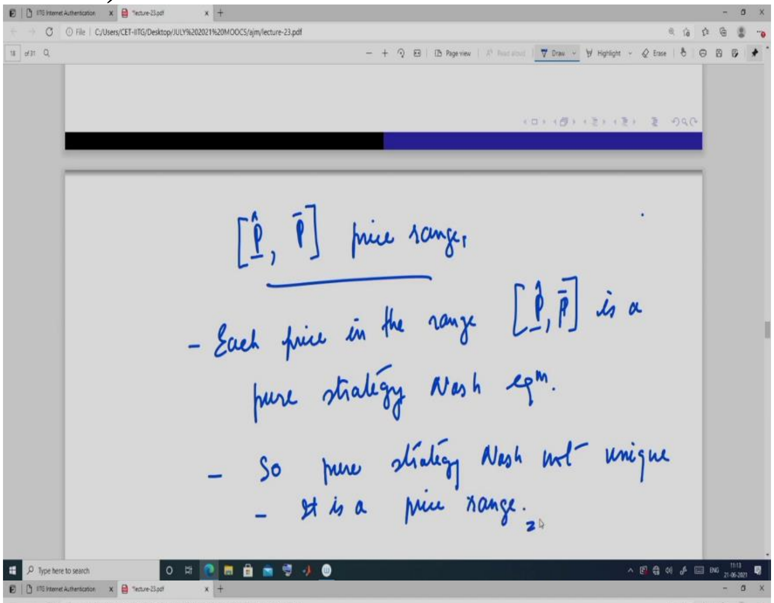


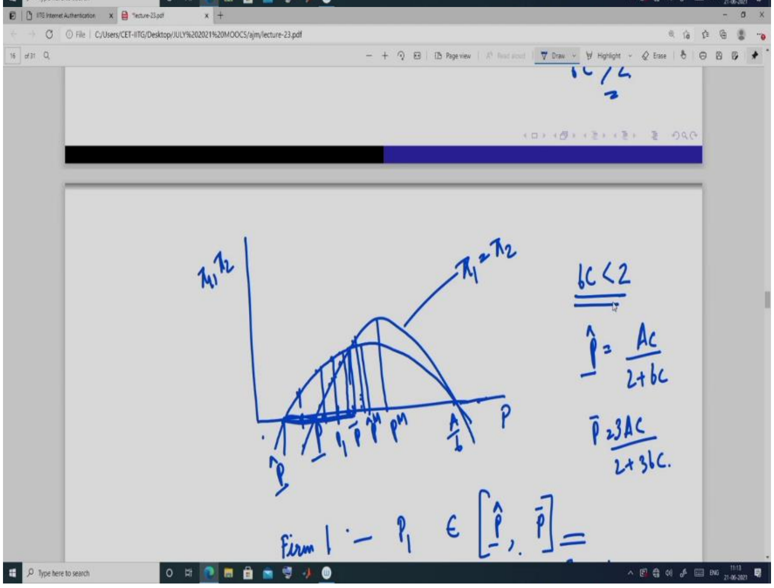


So, based on this a, we get that this range, this price range. So, so, we get this range. So, each price in the range. This is a pure strategy Nash equilibrium. So, so, pure Nash equilibrium is not unique. It is a price range, okay this whole range. So, what do we get? So, this is the case when we have done this and we get this situation.

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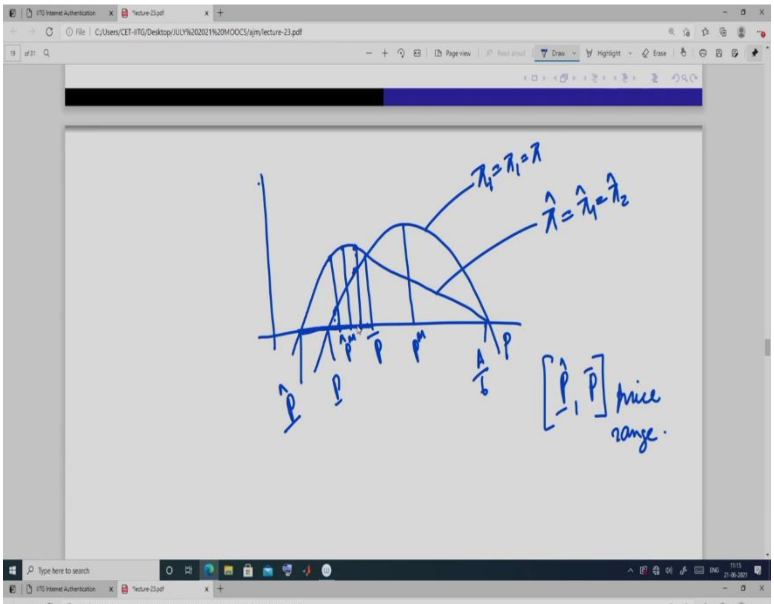


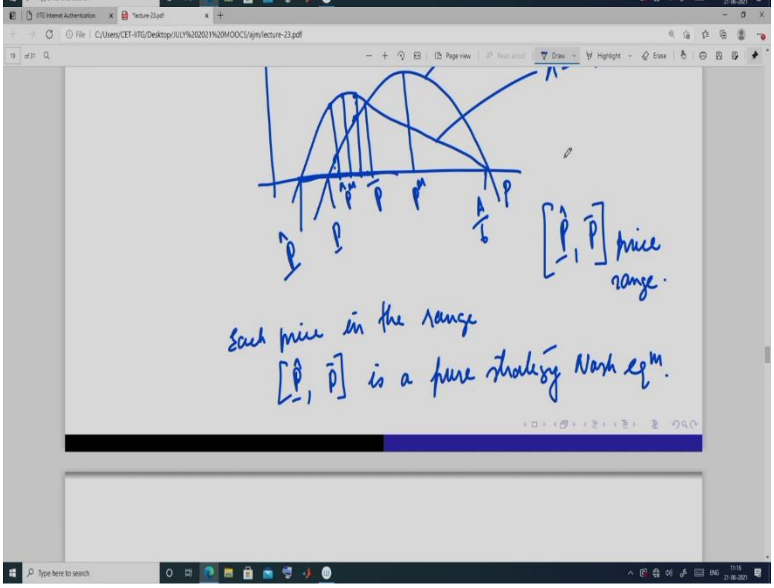


Now, if we simply take the other case, that is this one. This is hat, this is p. Here again, this whole range is the same thing. This same argument, this price range, each price in this range is a pure strategy Nash equilibrium. Same argument you take any price here, there is no tendency to undercut. Because if it undercuts, it will be in this curve and if it shares, it will be here. You take any price here, if it undercuts it will be here, here.

And if it shares it will be here. So, there is no tendency to undercut. So, in this whole region the tendency is to match the price of the other firm rather than to undercut it. So, but if price is greater than this, they will undercut. So, for this region we get that again, each price in the range p is a pure strategy Nash equilibrium, okay. And it is necessarily not unique, okay. So, this is the case we get, when we take the c1 and c2 to be same.

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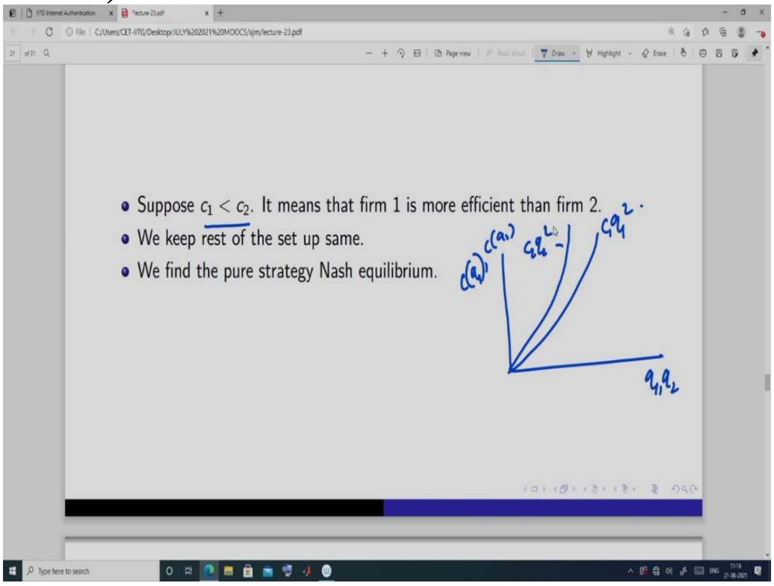


Now, we take suppose this, c1 and c2 are not same. And for simplicity, we take and for suppose one case, there is c1 is less than c2. It means, firm 1 is more efficient than firm 2. Because if we look at the cost curve, here c1 and c2 and here this if this, this is c2, because c2 is greater than c1, okay. We keep the rest of the setup same and we find the pure strategy Nash equilibrium in this case. What we will do?

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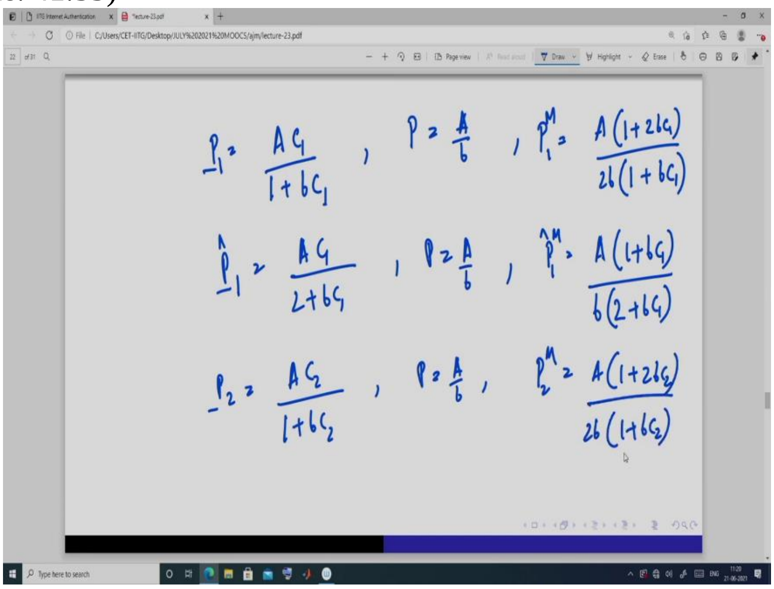

Here, we will simply find this is this price, sorry. So, first let us find this price. This price is going to be AC 1 plus this-$P_1 = \frac{AC_1}{1+bC_1}$. Similarly, we will have one price that is this-$P = \frac{A}{b}$ at which profit is 0. And this price is going to be, and this is going to be A C1 2 plus b c1, this I think it is this, it is this-$P_1^M = \frac{A(1+bc_1)}{b(2+bc_1)}$, okay. So, we have got this prices and similarly we will get this prices-$P_2 = \frac{AC_2}{1+bC_2}$, $P = \frac{A}{b}$, $P_2^M = \frac{A(1+2bc_2)}{2b(2+bc_2)}$. We will evaluate or we will find it in the same way. Only c1 will be replaced by c2 and c1 is less than c2.

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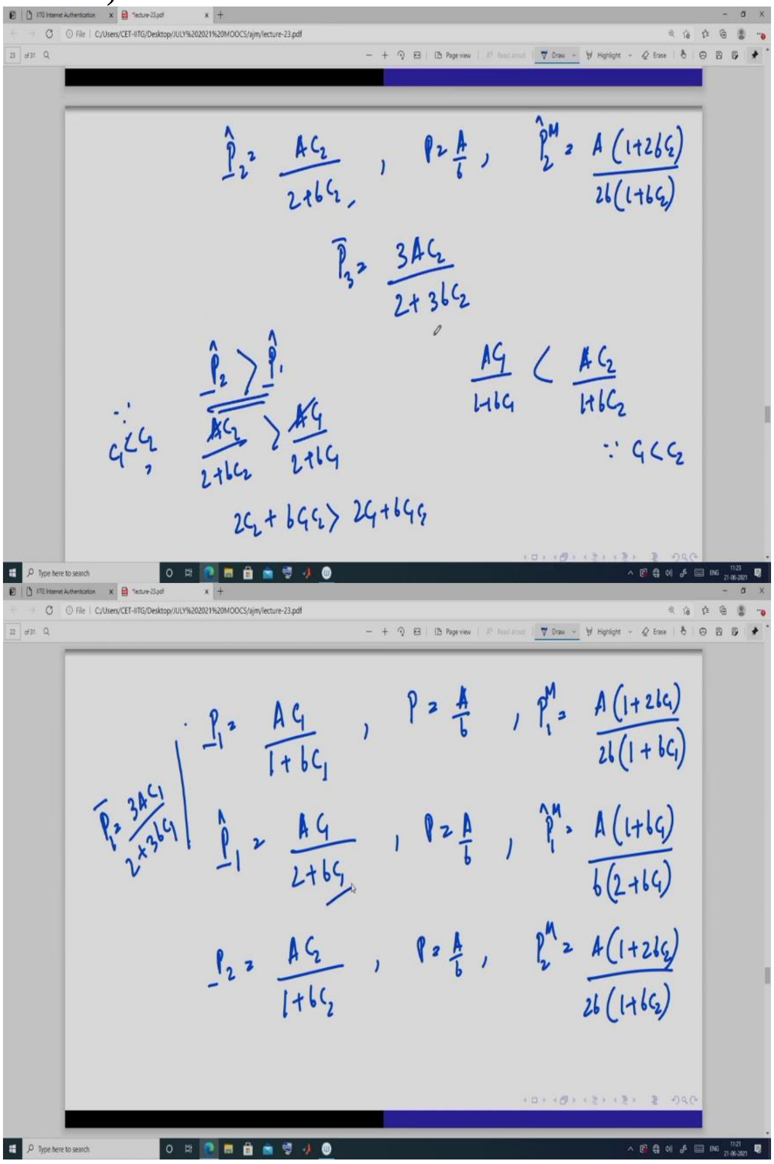

Now, here we will further, what we will have p bar 1 which is 3AC by 2 plus 3 bc1-$\bar{P_1} = \frac{3AC_1}{2+3bC_1}$ . And here again we will have this, which is 3AC 2 which is 2 plus 3 bc2, okay, this-$\frac{3AC_2}{2+3bC_2}$. Now we plot this, first do the comparison. Now here, if we compare this-$\frac{AC_1}{2+bC_1}$ and compare this-$\frac{AC_2}{2+bC_2}$, what do we get is that let see, p this is greater than this. Because c2 is greater than c1, AC 2, 2 bc 2. This is greater than A C1 2b c1, easy to see-$\frac{AC_2}{2+bC_2} > \frac{AC_1}{2+bC_1}$.

So, we cancel out, so, we get this-$2c_2 + bc_1c_2 > 2c_1 + bc_1c_2$. Now because c1 is, since c1 is less than c2. Now similarly we compare this, we also compare these two prices A. So, if this is the A C1 this is also going to be less than A C2 1 plus bc 2-$\frac{AC_1}{1+bC_1} < \frac{AC_2}{1+bC_2}$. Because you are simply, since c1 is less than c2, okay.

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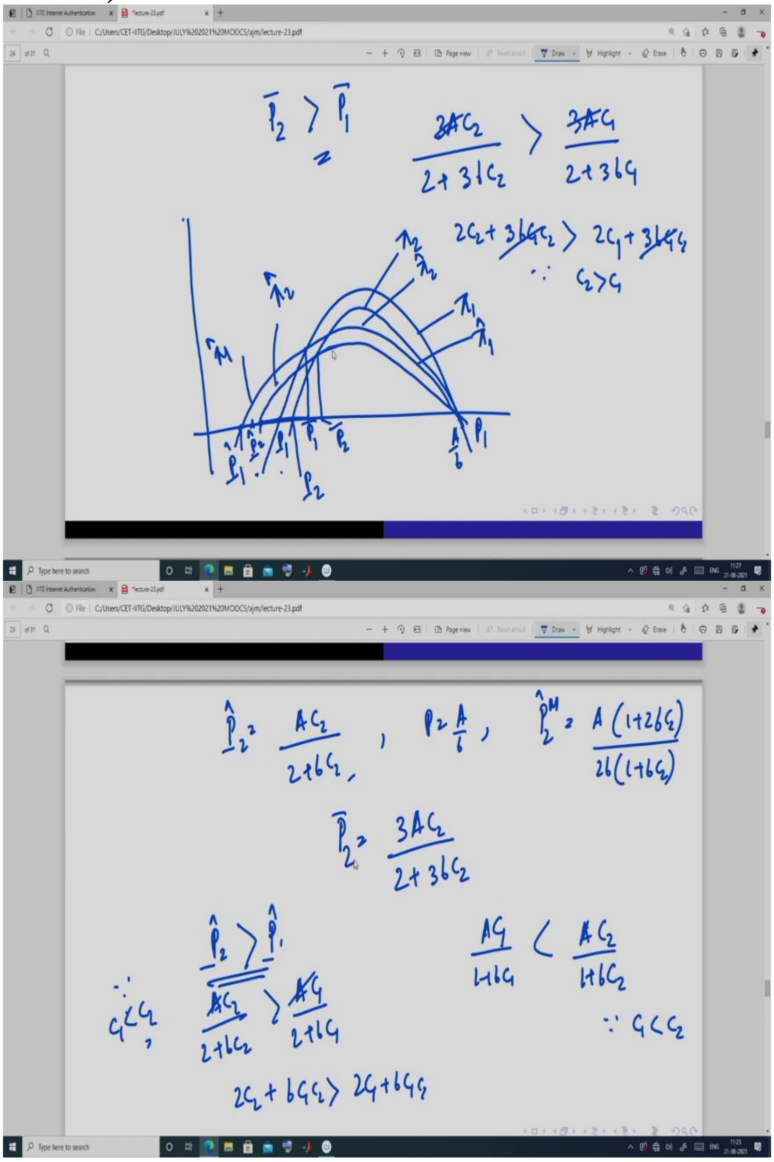

Again, we get that p1, p1 this is going to be greater than this-$P_2 > P_1$. How? Same argument, so since c2 is greater than c1, we get this. So, based on these and we can compare the monopoly prices also and we will get the same thing, okay. So, what we do? We get this, plot this, okay. This is for this and this is p1 bar, this is. So, in the last case, when these two prices of both the firms are seen, we got this as the pure strategy Nash equilibrium, okay.

But right now, we will get a slightly different. So, this is p1bar and we will have a price like this, this is and this price is p bar 2, we have a price like this. This curve is p2 hat when the firm 2 is sharing the market. And this price is p2 hat lower bar, okay and this price is p upper bar 2 and this price is, this price, these two curves are nothing. So, this is p1, okay this is p pi 2 hat, this is pi 1 hat, okay. This curve should always lie actually, because this, this point I have made a mistake.

And this price is this, okay. Now, look here firm 1 suppose, sets a price here, okay. Then if it is not matched by firm 2, then its profit is going to be here. Because this is, when it serves the firm, market alone and this is when it shares. But here if firm 2 shares the market, then here also its profit is going to be negative, because this is the profit curve of firm 2 when it shares.

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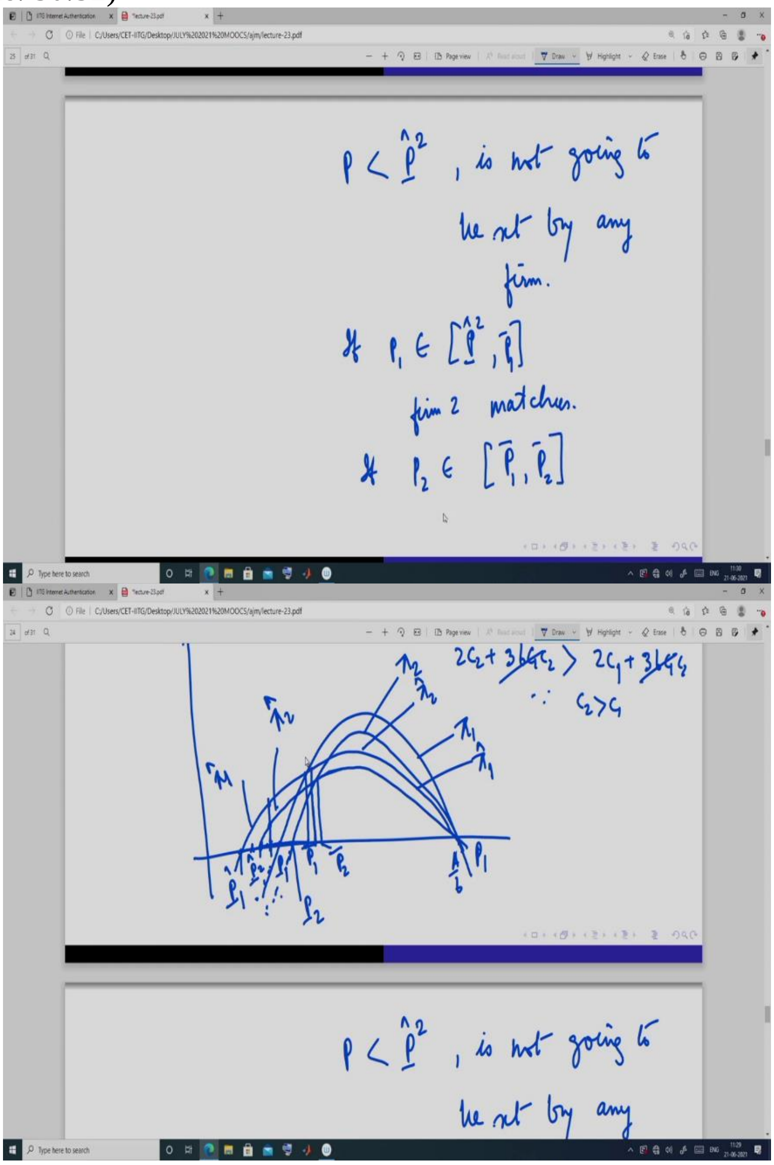

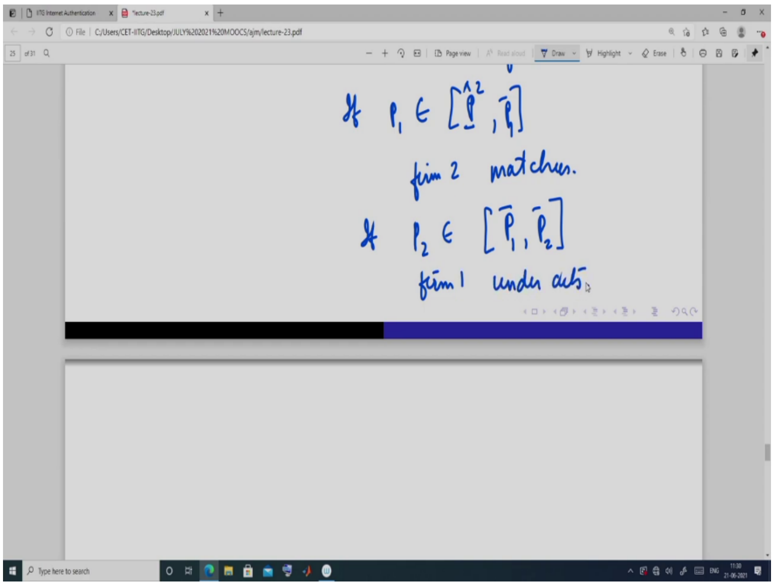


So, any price so, p below this is not going to be set by any firm, okay. This is now, so, we get this as the low. Here, if we take this price and if firm 1 sets this price, firm 2 if it undercuts, it is given by here, it will be negative. But if it shares, it will get this. And if firm 2 also undercuts, it will, since it will not undercut because it has same that price. And its profit is this, if it shares and profit of firm 2, if it shares is this. But if it undercuts, it will be here in this curve.

So, it will be negative so, that is why it will be if p1 belongs to this. And it is less than this, right? this is lower than this. So, firm 2 matches, we get this. Now, take a price, which is here in between these two, firm 1 sets this price. So, if firm 2 matches this price, profit of firm 1 is here, matches. And if firm 2 set, set same the same price it is here. But if firm 1 slightly reduces, its profit is at the higher level. So, if p2 suppose belongs to a price, which is firm 1 undercuts.

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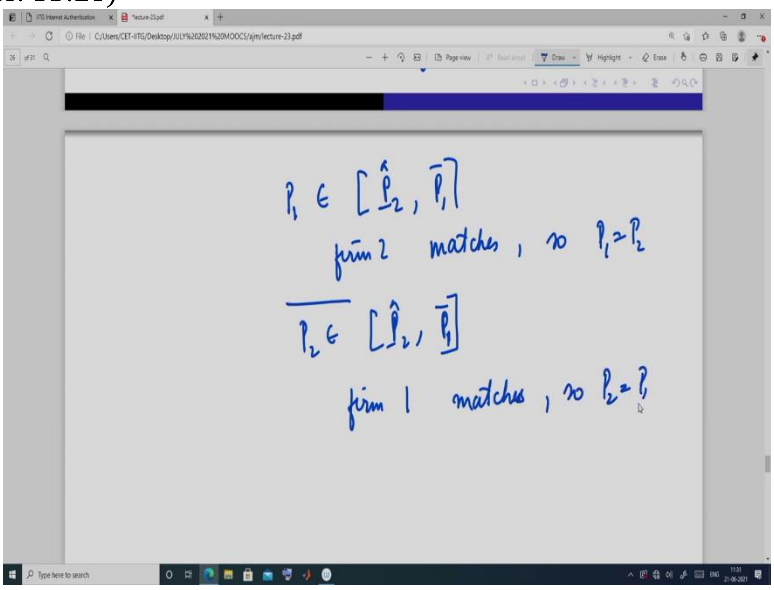
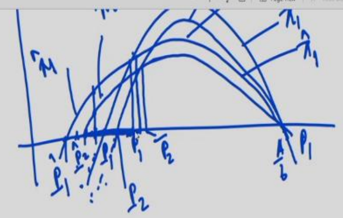


firm 2 matches, so $p_1 = p_2$

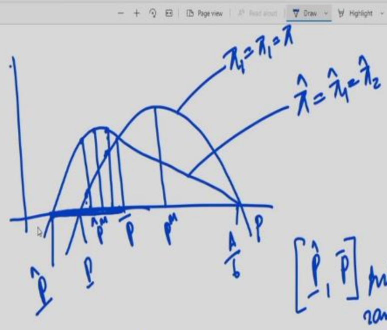
$p_2 \in [\hat{p}_2, \bar{p}_1]$

firm 1 matches, so $p_2 = p_1$

pure strategy Nash eqm, each price in the range $[\hat{p}_2, \bar{p}_1]$




$p < \hat{p}^2$, is not going to be set by any firm.




$[\hat{p}, \bar{p}]$ price range.

Each price in the range $[\underline{p}, \bar{p}]$ is a pure strategy Nash eqm.

So, so when they are going to match? So, when p1 lies between this firm 2 matches. And so, p1 is equal to p2. Again, if p2 lies in this range, this range firm 1 matches. So, p2 is equal to p1. So, we get that the pure strategy, pure strategy Nash equilibrium, pure strategy each price in the range, this. So, each price in this range is a pure strategy. So, again here it is not necessarily unique, but it is a range.

So, it will be from this range, region to this. So, this is when we have asymmetry in cost function but there is a decreasing returns to scale, we get a range and that range is this to this. I will, but in when there is no asymmetry, when the cost function is same, then we do not get this, we do not get instead we get what? We get a this kind of, this whole range this, this range from p hat lower bar to p upper bar.

But here, we have two p hat lower bar and two p upper bar. And from that, we get the lower p upper bar and the higher p hat, okay. So, that is the and so, with this we conclude the Bertrand competition. So, Bertrand competition we have done in different forms. We have first assumed that when the cost function, marginal cost function is constant, it may be different. So, then we have found out the pure strategy Nash equilibrium.

In certain cases, we have got pure strategy Nash, Nash equilibrium in certain cases we have not got. Then we have introduced capacity constraint and in capacity constraint we have shown that under certain combination of capacities, we get pure strategy Nash equilibrium, under certain we do not get any pure strategy Nash equilibrium. And in this case, when we have decreasing returns to scale, we have shown that there is a range of pure strategy Nash equilibrium.

Or you can say it is a continuum of pure strategy Nash equilibrium. So, we get different varieties of Nash equilibrium in this Bertrand competition. So, that is why Bertrand competition is very interesting, okay. So, with this I end this portion, for this decreasing returns to scale, this class notes are sufficient. Because it is not there in any these, textbooks that I have mentioned, okay. Thank you.