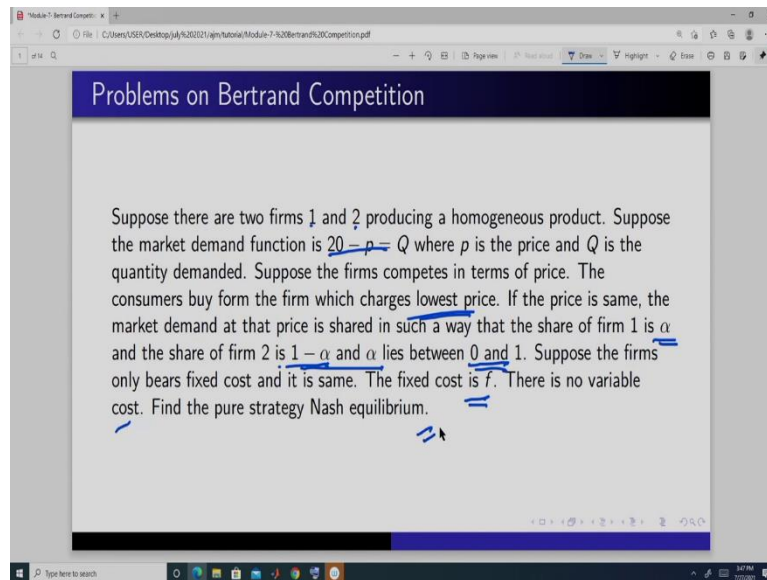


Introduction to Market Structures
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Lecture No. 31
Tutorial on Bertrand Competition

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The screenshot shows a presentation slide with the following text:

Problems on Bertrand Competition

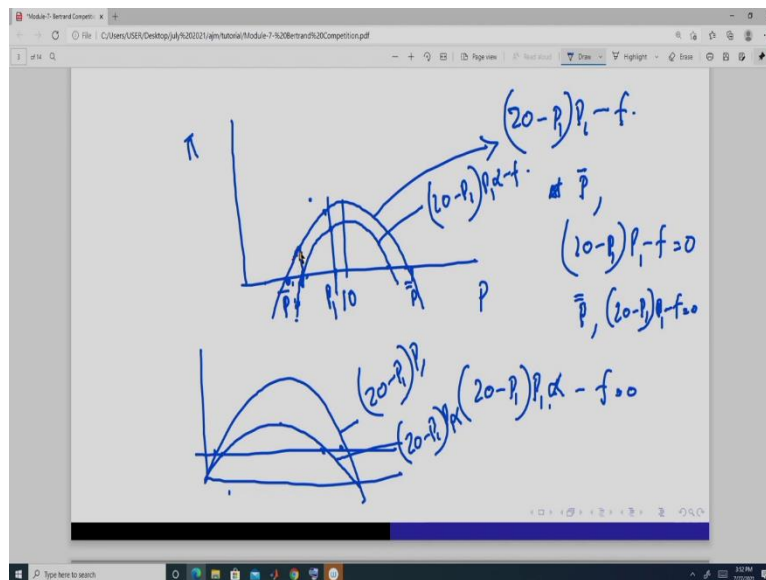
Suppose there are two firms 1 and 2 producing a homogeneous product. Suppose the market demand function is $20 - p = Q$ where p is the price and Q is the quantity demanded. Suppose the firms compete in terms of price. The consumers buy from the firm which charges lowest price. If the price is same, the market demand at that price is shared in such a way that the share of firm 1 is α and the share of firm 2 is $1 - \alpha$ and α lies between 0 and 1. Suppose the firms only bears fixed cost and it is same. The fixed cost is f . There is no variable cost. Find the pure strategy Nash equilibrium.

Let us do some problem on Bertrand competition. So, in the first problem, let us suppose assume that there are two firm, firm 1 and firm 2 producing homogeneous product, market demand function is this- $20 - p = Q$ and they compete in terms of price and buyers or the consumers buy from their lowest price and if the market is, if the market price is such that the price is same set by firm 1 and firm 2, then the market demand is share in this ratio- $1 - \alpha$.

That is firm 1 will get α and firm 2 will get $1 - \alpha$ and α lies between 0 and 1. So, we have done that it is half, half, but not it is suppose α and $1 - \alpha$. And suppose the firms only bears fixed cost and it is same and the fixed cost is f and there is no variable cost, find the pure strategy Nash equilibrium. We have solved one version of this in the class.

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$$\begin{aligned} \pi_1 &= (20 - p_1)p_1 - f && \text{If } p_1 < p_2 \\ &= (20 - p_1)p_1 - f && \text{If } p_1 = p_2 \\ &= -f && \text{If } p_1 > p_2 \end{aligned}$$



No pure strategy Nash eqn

So, in the tutorial we will. So, the profit function of suppose firm 1 you can write it in, demand function is this, $\pi_1 = (20 - P_1)P_1 - f$ if p_1 is less than p_2 or it is this $-(20 - P_1)P_1\alpha - f$ if p_1 is equal to p_2 and it is equal to 0 if p_1 is less than. Now, here we can have two situation, either we can assume this. So, it means that we incur fixed cost only when we produce or since we want to produce but no one is buying from us, so we may have a situation like this also. We have already incurred fixed cost. So, it is something like this- $-f$, okay. So, we get this kind of things. So, you can do this.

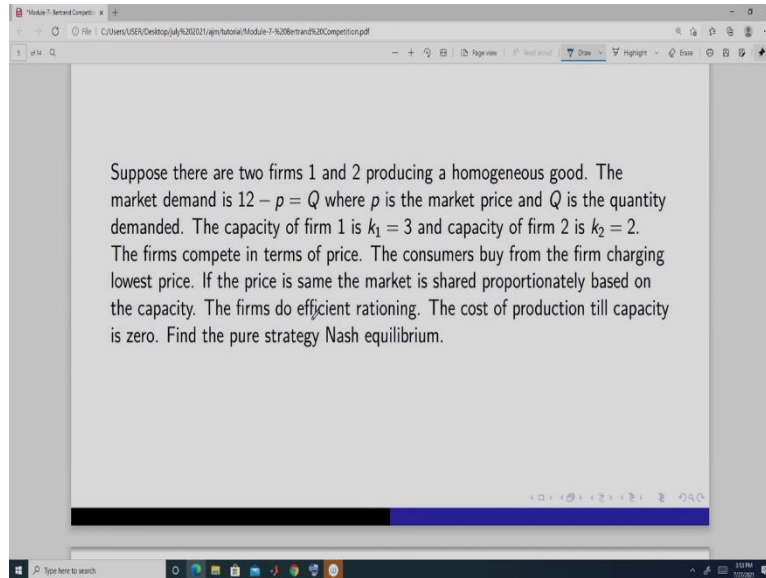
So, I am assuming this or you can take 0 also. It will not make much difference. But it is better to take this, because we have assumed that the fixed cost is independent of the output. So, if we plot this function, what do we get? If we take profit here and let us first plot this function, this. Now, this we will have one p , this p , suppose this is \bar{p} or, at \bar{p} this is equal to, and again this is, again we have another \bar{p} at which, this is 0, so it is somewhere here, \bar{p} . And we will get the profit function here. This point is 10, right. This is the profit function here.

Now, if we plot this when they share it, this will be somewhere here when we equate it to 0, because this is now multiplied with this. This is lying between 0 and 1. So, this we can say f divided by now α , α lies between 0 and 1. So, this term is now more than f . So, this is going to be lying here, sorry this is going to lie here, because it has to be greater. And this point again this which is higher, when we have equate to 0, it will be lying here. So, it will be some curve like this. This is, it is like this. Or you can take like this, sorry. This is α , α is lying between 0 and 1. So, it is like this. f is something here. So, we get these points.

So, these points are these points here. So, similarly we will get this for firm 2 also, because everything is same. Now, if firm 1 suppose sets a price like this, then firm 2, if this is suppose firm, firm 2 if it sets this price, it will get this profit. But if it sets slightly less, it will get this profit. So, it will do this. So, they are going to compete like this and they will finally reach this price. At this price, what is happening? If this slightly reduce, it gets this.

But if it again matches, then it gets some price which is below this. So, again it will not do one thing. Firm 1 will not match. Then firm 2 it is not optimal to, it will go on increasing the price, then it will again reach this point, again this. So, we will get a cycle like this and we will have no pure strategy in this case, no pure strategy Nash equilibrium, okay. So, this is same thing. Only thing here I have only changed the sharing method.

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Now, let us do one another problem. Suppose there are two firms, firm 1 and firm 2 and they produces homogeneous product and the market demand function is this- $12-p=Q$. Capacity of firm 1 is k_1 , capacity of firm 2 is k_2 , k_1 is 3, capacity of firm 2 is k_2 and it is 2. And the usual Bertrand thing applies that is the consumers buys from the lowest firm price, firm that sets the lowest price. And if the price is same then the market is shared proportionally based on their capacity. Firms do efficient rationing. And the cost of production till capacity is 0. We have to find the pure strategy Nash equilibrium here.

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Handwritten equations on a whiteboard:

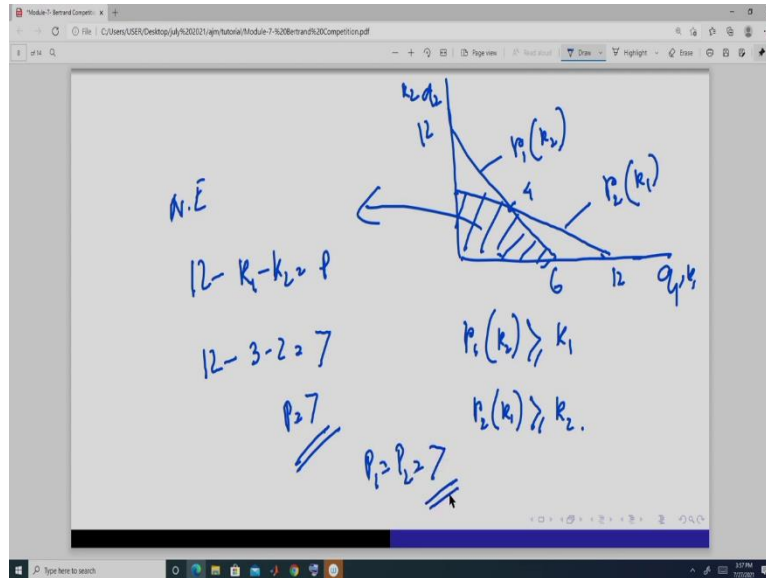
$$10 - p > Q.$$
$$\pi_2 = (12 - q - q_2)q_1, \quad \frac{12 - q_2}{2} = q_1$$
$$p_1(k_2) = \frac{12 - k_2}{2}$$

Handwritten equations on a whiteboard:

$$\pi_2 = (12 - q - q_2)q_1, \quad \left\| \frac{12 - q_2}{2} = q_1 \right.$$
$$p_1(k_2) = \frac{12 - k_2}{2}$$
$$\pi_2 = (12 - q - q_2)q_2, \quad \frac{12 - q_1}{2} = q_2$$
$$p_2(k_1) = \frac{12 - k_1}{2}$$

Handwritten equations on a whiteboard:

$$p_1(k_2=2) = \frac{12-2}{2} = 5, \quad k_1=3$$
$$5 = p_1(k_2=2) > 3$$
$$p_2(k_1=3) = \frac{12-3}{2} = \frac{9}{2}, \quad k_2=2$$
$$\frac{9}{2} = p_2(k_1=3) > 2$$



So, this demand function is this, right. So, if we try to find the Cournot this, Cournot reaction function it is I think 12, sorry. It is this $\frac{12 - q_2}{2} = q_1$. So, this we can write, this is the Cournot reaction function. You can write Cournot reaction function is, so again, now here plug in the, so if we take this, capacity of firm 2 is 2. So, this we get what, it is 5. And capacity of firm 1 is k_1 is 3. So, this is this $p_1(K_2 = 2) > 3$ which is equal to 5. Again, it is 3 and capacity of firm 2 is 2. So, in this situation we have got this.

So, the, if, reaction function is, if this is q_1, q_2, k_2, k_1 , okay we have got this and point is actually 4. And for in this situation we know for this situation when the is Nash equilibrium, pure strategy Nash equilibrium is 12 minus $k_1 k_2$, this price. And here we have got this, because this is greater than k_1 and we have got this. And this is, so price is equal to 7. Both the firm set the same price p_1 is equal to p_2 and it is equal to 7. So, this is the pure strategy Nash equilibrium $p_1 = p_2 = 7$. We have shown this and we are simply using this, okay.