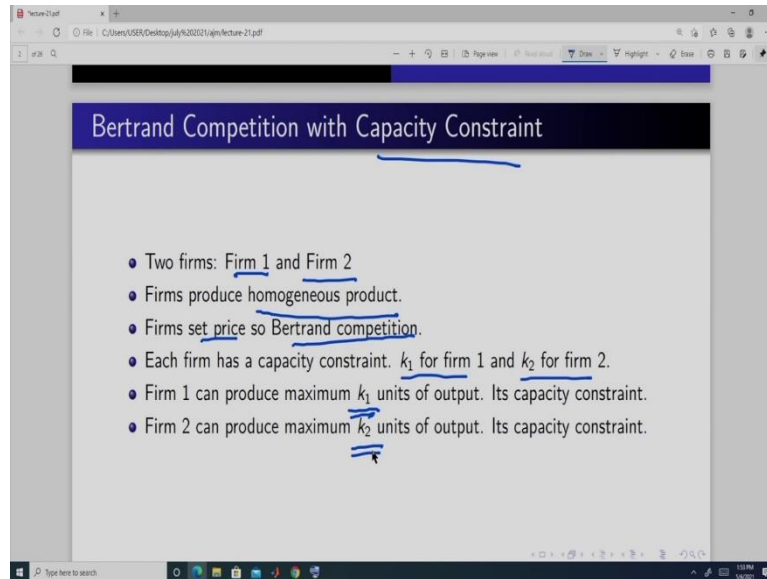


**Introduction to Market Structures**  
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**Lecture No. 29**  
**Bertrand Competition with Capacity Constraints**

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Hello everyone, welcome to my course introduction to market structures. So, today we are going to do Bertrand competition with capacity constraint. Now, we have already done Bertrand competition what is generally done in Bertrand competition. In Bertrand competition we have 2 firms and each firm chooses a price and the firm which sets the lowest price everyone or all the consumer buys from that firm.

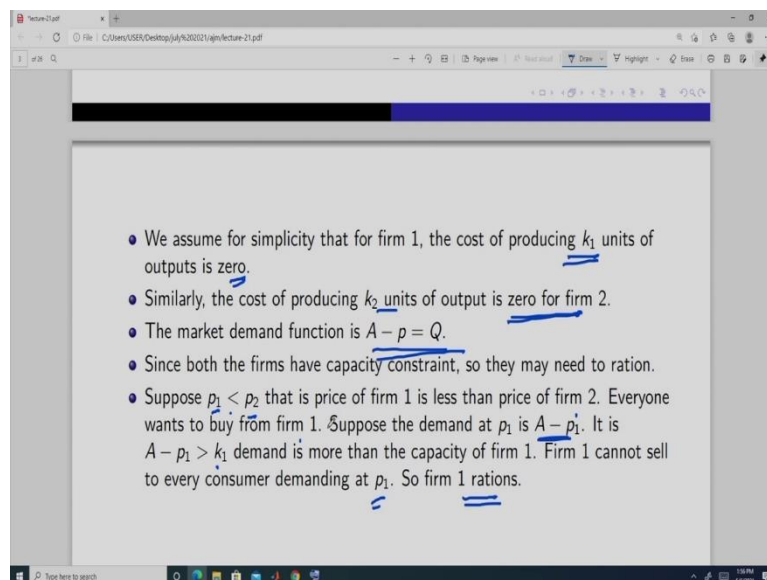
So, there is a competition to set the price and we have seen that when there is no fixed cost and the marginal cost are same the both the firm sets the prices at price is equal to marginal cost and that is Bertrand paradox. Next when there are fixed cost we have seen that there is no pure strategy Nash equilibrium and when we have different marginal cost and 0 fixed cost at that in this situation also we see that there is no pure strategy Nash equilibrium.

Now, we introduce a new thing in this Bertrand competition and that is capacity constraint. What do we mean by capacity constraint? Capacity constraint means that the firms cannot produce whatever amount of output it wants to produce. It has some constraints on the output it can produce. So, to study this we have taken a duopoly market that is there are two firms firm 1 and firm 2. Both the firm produces homogeneous product that means the output whether

I buy from a consumer buys from a firm 1 or firm 2, it does not matter what the firm produces same type of good, okay.

And since each firm sets price so it is a Bertrand competition both the firms set a price. And now we specify the capacity. So, each firm has a capacity constraint that is  $k_1$  for firm 1 and  $k_2$  for firm 2. What do we mean by this capacity constraint? That means that firm 1 can produce maximum  $k_1$  units of output. So, it is given by its capacity and firm 2 can only produce maximum  $k_2$  units of output it cannot produce more than firm 1 cannot produce more than  $k_1$  units of output and firm 2 cannot produce more than  $k_2$  units of output. So, that is, this is given us capacity. Now, we specify the cost.

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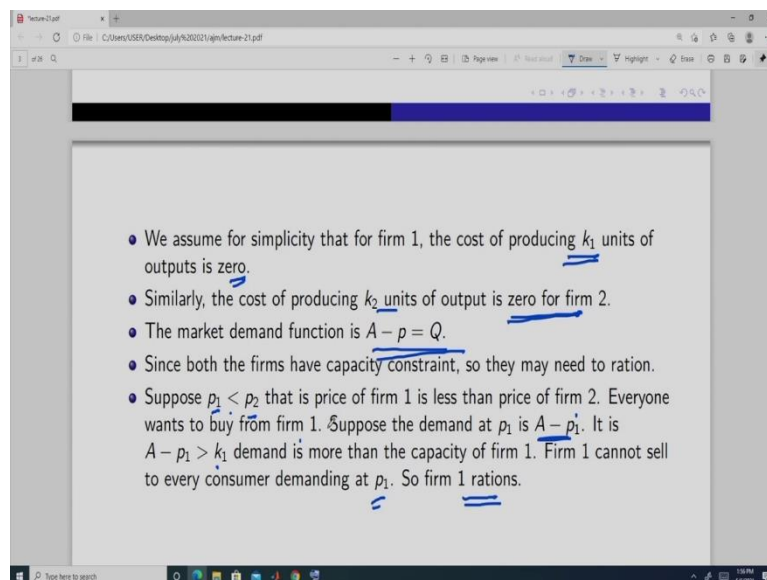
So, here we assume for simplicity that the cost is 0 for firm 1 for producing  $k_1$  units of output, its cost is 0. So, it does not incur any cost to produce this output. This is actually a simplest to simplify the game a computation that we are going to do later on to find Nash equilibrium we make this assumption. Similarly firm 2 to produce  $k_2$  units of output it does not incur any cost, okay. And the market demand is a linear downward sloping demand curve and that is this-  $A - p = Q$  and more or less we have assumed this demand curve till now.

So, this is our demand curve. Now here since each firm has a capacity constraint. So, they may have to do something called rationing and what do we mean by that? So, it is suppose price of firm 1 is  $p_1$  and price of firm 2 is  $p_2$ . And price of firm 1 is less than the price of firm 2, okay so this. So, what will happen everyone will want to buy from firm 1 because the price of firm 1 is less than the price of firm 2.

Suppose the aggregate demand a market demand at price  $p_1$  is this-  $A - p_1^*$  and it is such that the demand is more than the capacity of firm 1. So, firm 1 cannot satisfy the demand that is generated in the market. So, firm 1 cannot sell to every consumer that is demanding at this price. So, it can only satisfy the demand of only a fraction of that or some portion of that some will be left untouched or we will not be served by firm 1. So, firm 1 has to do something called rationing. So, it is something like this suppose you have 4 chocolates and you have to distribute it among suppose 7 children.

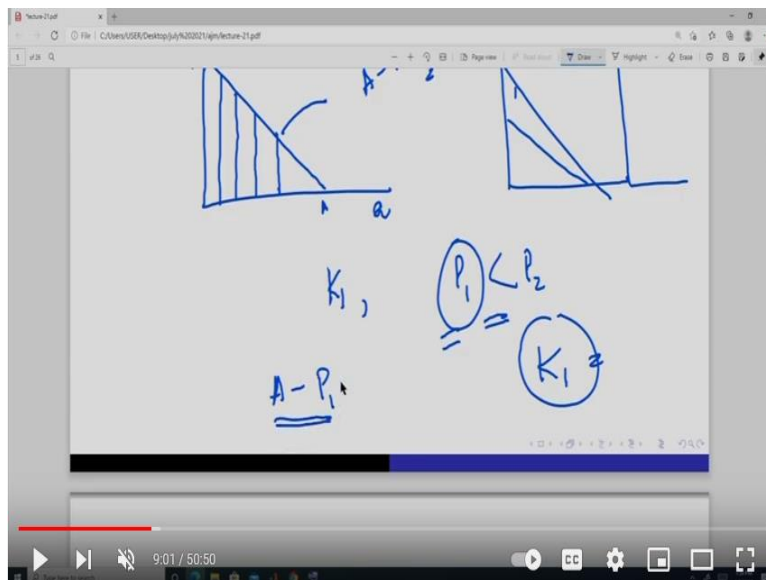
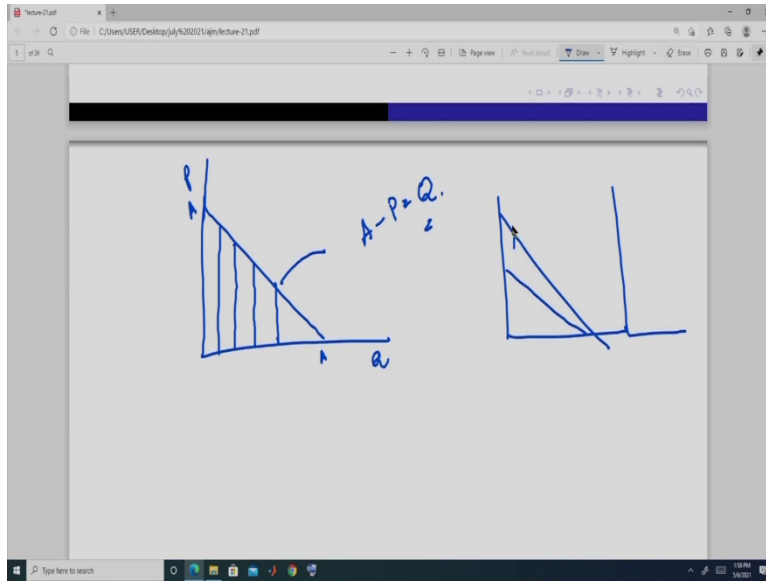
Now, you cannot divide this chocolate. So, these 5 chocolates how you have to you have to choose 5 out of this 7 children how are you going to do it? You will use some rule. So, here also we will specify certain rules to ration how to select which consumers are going to get this product from firm 1, okay.

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So, we have to specify a rule and one rule is this efficient rationing. So, efficient rationing says that those who value the good most they are going to get the that good from the low price firm. So, in this case firm 1 is going to serve or going to serve the demand of those consumers whose demand or whose valuation is highest or maximum and what do we mean by evaluation it means the willingness to pay. So, it means those whose willingness to pay is maximum or is highest they are going to be served first.

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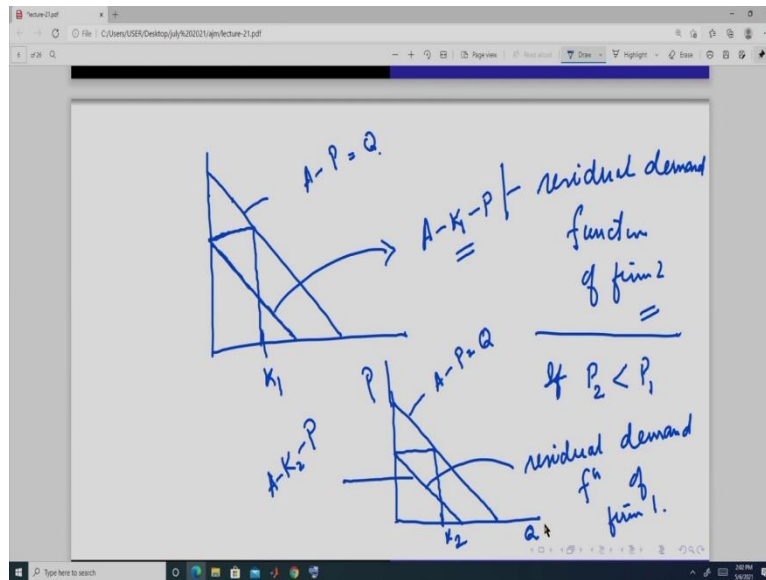


So, in terms of suppose in this case, our demand curve is this. Suppose this is  $q$  this is  $p$ , in this axis  $p$  this is  $A$  this is  $A$ . So, demand curve is this, right? This heights are giving the maximum willingness to pay by a consumer, okay. Now, this is a market demand curve. So, this demand curve we have got from horizon till summation of individual demand curves, okay. So, at so if you want this quantity what is the maximum it is willing to pay it is generally if we take individual demand curve of individual person then it is given by this height.

But here for simplicity we will assume that suppose this only gives you that willingness to pay. So, there may be some consumers who are willing to pay so, it may be something like this there is this one demand curve this is another demand curve and we have got the market demand curve firm horizontal summation of these two demand curves. So, that these demand curves everyone so, each individual whose demand curve is this their willingness to pay is higher than this something like this we will get, okay.

So, here so, capacity of firm one is this and we have assumed that this is the situation suppose  $P_1 < P_2$ . So, everyone wants to buy from firm 1 since the price is less. But firm 1 can only sell this much amount only satisfy this much quantity demanded- $K_1$ . And what is quantity demanded at this it is total demand is this- $A - P_1$ .

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So, from this demand curve, what do we get is? So, first  $K$  units are going to be sold to those who value the goods maximum. So, this demand curve is something like this-  $A - K_1 - P_1$ . This is the residual demand curve of which is faced by firm 2. So, firm 1 is always going to sell this much at each prices. So, if the price is below this price then only it can sell its full capacity that is  $k_1$  otherwise if the price is here then it will be able to sell only this much amount of quantity if here it is only this much.

So, for prices less than this it is going to first sell this much so, firm 2 whose price is higher than the price of firm 1 is going to get this demand curve. So, from this demand curve we remove this much quantity and what is left is this that is being served by firm 2, okay. So, this is how the effect of rationing. So, what we do that when price of suppose price of firm 2 is less than the price of firm 1. So, in that case here it will be something like this. So, if price of firm 2 is less than price of firm 1, so in that case so firm 2 is going to sell the first  $k_2$  units to those who value it maximum.

This is the residual demand function of firm 1 in this situation this. So, this is  $A$  minus  $k_2$  minus  $p$ -  $A - K_2 - P_1$ , okay. So,  $P$  is here  $Q$  is here, okay. So, now I hope you have understood what do we mean by the rationing So, rationing here when the moment a firm rations it means it will

only serve those consumers whose valuation is high. Now how they are going to identify that we are not bothered about it. So, we are not modeling.

So, that is a very valid question and it is a very difficult question also to set to address. But we are not addressing it here we have address such kind of thing when we were doing price discrimination in the case of Monopoly market. But here we are assuming that suppose firm 1, if it sets a price lower than the price of firm 2, it can identify the buyers who is valuing more and who is valuing it less, okay.

And if firm 2 sets a price lower than the price of firm 1, then it can identify the buyers in terms of their valuations, okay. We assumed that. So, now we come to the demand function of each firm.

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$A - P_2 Q$

Demand function of firm 1 is

$$q_1 = \begin{cases} \text{Min} \left\{ K_1, (A - P_1) \right\}, & \text{if } P_1 < P_2 \\ \text{Min} \left\{ K_1, \frac{(A - P) K_1}{K_1 + K_2} \right\}, & \text{if } P_1 = P_2 = P \\ \text{Min} \left\{ K_1, A - K_2 - P_1 \right\}, & \text{if } P_1 > P_2 \end{cases}$$

So, demand function of firm 1 that it faces is so its quantity demanded is  $q_1$  it is equal to minimum of  $k_1$  if  $p_1$  is less than  $p_2$ -  $q_1 = \text{Min} \{K_1, (A - P_1)\}$ , if  $P_1 < P_2$ . If firm 1 sets a price less than the price of firm 2 can either. So, total demand is  $A$  minus  $p_1$  since the demand curve is  $A$  minus is this, right? now its capacity is  $k_1$  so it is can satisfy the demand of  $k_1$  or whichever is less than this, okay. And it is mean  $k_1$   $A$  minus  $P$   $k_1$   $K_1$  plus  $k_2$  if  $p_1$  is equal to  $p_2$  is equal to  $p$ -  $q_1 = \text{Min} \left\{ K_1, \frac{(A - P_1) K_1}{K_1 + K_2} \right\}$ , if  $P_1 = P_2 = P$ .

So, if both the firms set a same price that is  $p$  then either its firm sell its capacity given if the total demand is less than its capacity, then it is shared proportionally based on their capacity it is this. So, this is the demand at price this and if so, this  $\frac{(A - P_1) K_1}{K_1 + K_2}$  is proportionally shared. So,

out of these  $k_1$  fraction of this whole capacity is going to be served by firm 1. And the next is  $k_1$  if firm sets a price which is greater than the price of firm 2, then either its sales is full capacity that is  $k_1$  or it serves only the residual whichever is less-  $\text{Min}\{K_1, A - K_2 - P_1\}$ , if  $P_1 > P_2$ .

If the residual demand is less than its capacity it is only sells is residual. So, this is the quantity demand that the firm faces in this kind of Bertrand competition with capacity constraint, okay. So, we have consider all the possible cases here.

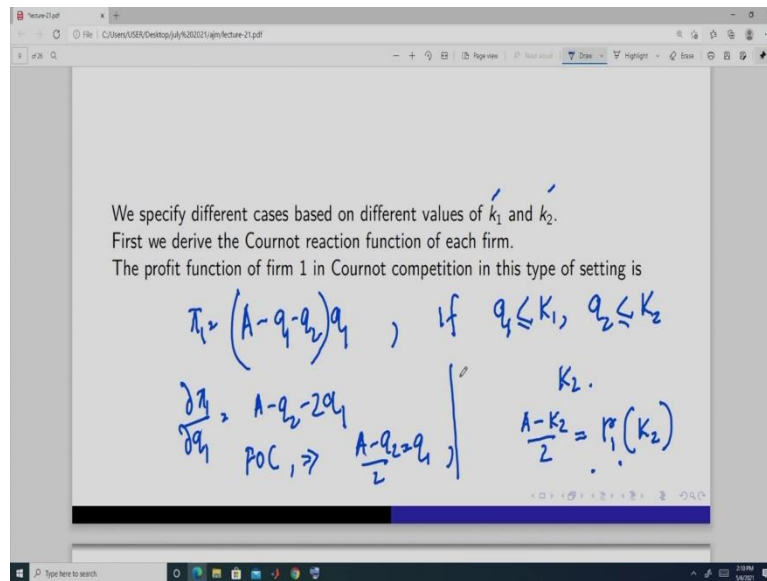
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Demand function of firm 2 is

$$q_2 = \begin{cases} \text{Min}\{K_2, A - P_2\}, & \text{if } P_2 < P_1 \\ \text{Min}\left\{K_2, \frac{(A-P)K_2}{K_1+K_2}\right\}, & \text{if } P_2 = P_1 = P \\ \text{Min}\{K_2, A - K_1 - P_2\}, & \text{if } P_2 > P_1 \end{cases}$$

Next, similarly, for firm 2, we get  $q_2$  is equal to  $\text{min } k_2$  or  $A$  minus  $p_2$  if  $p_2$  is less than  $p_1$ -  $\text{Min}\{K_2, A - P_2\}$ , if  $P_1 > P_2$ . So, if  $p_2$  is less than  $p_1$  then it serves if the whole demand at  $p_2$  is this, so, either it can sell this much amount or its capacity that is  $k_2$ . Or it sells  $k_2$  if price of firm 1 and firm 2 are same and that is suppose  $p$  then either firm 2 sales up to its capacity or if the aggregate demand at that price is less than the aggregate capacity then its capacity then it sells this fraction-  $\text{Min}\left\{K_2, \frac{(A-P)K_2}{K_1+K_2}\right\}$ , if  $P_1 = P_2 = P$ , okay. And when it sets the higher price either it sells its capacity or it sells up to sells its residual demand if  $p_2$  is greater than  $p_1$ , okay. So, this is the demand curve faced by firm 2-  $\text{Min}\{K_2, A - K_1 - P_2\}$ , if  $P_1 < P_2$ .

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Now, we will specify different values of  $k_1$  and  $k_2$  different way and based on that we will specify different combinations of  $k_1$  and  $k_2$  to solve the, or to find the pure strategy Nash equilibrium. So, before doing that let us first look at the Cournot reaction function of this in this case. So, profit of firm 1 can be written in this way if  $k_1$  is like this-  $\pi_1 = (A - q_1 - q_2)q_1$ , if  $q_1 \leq K_1, q_2 \leq K_2$ . Now, so, this is the Cournot profit function and provided this are satisfied then optimizing with respect to  $q_1$  we get.

So, first order condition gives us. So, this is the Cournot reaction function-  $\frac{A - q_2}{2} = q_1$ . From this reaction function we can derive another reaction function and it is of this nature. Suppose capacity of firm 2 is  $k_2$ , okay. Then what is the best response suppose firm 2 is producing at its capacity  $k_2$ , what is the best response of firm 1? Best response of firm 1 based on this is this-  $\frac{A - K_2}{2}$ . So, this we can write in this way as the reaction function of firm 1 when firm 2 is producing at its capacity.

So, now plug in different values of  $k_2$  different capacity of firm 2 and you will get the best response of firm 1 what is the best capacity it wants this or it wants to produce it is this from the Cournot thing, right? So, we will require this remember.



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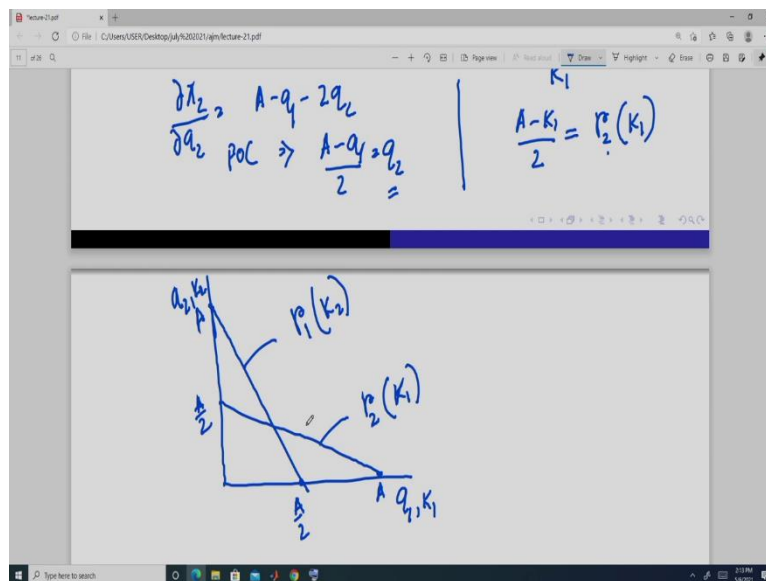
The profit function of firm 2 in Cournot competition in this type of setting is

$$\pi_2 = (A - q_1 - q_2)q_2, \quad q_1 \leq K_1, \quad q_2 \leq K_2.$$
$$\frac{\partial \pi_2}{\partial q_2} = A - q_1 - 2q_2 \quad \left| \quad \begin{array}{l} K_1 \\ \frac{A - K_1}{2} = P_2(K_1) \end{array} \right.$$
$$\text{FOC} \Rightarrow \frac{A - q_1}{2} = q_2 =$$

Next profit of function of firm 2 in this Cournot if we take this is the aggregate price this is the market price and this is the output given this since 0 cost so, cost portion is not there, i.e.  $\pi_2 = (A - q_1 - q_2)q_2$ . So, first order condition gives us this-  $\frac{A - q_1}{2} = q_2$ . So, this is the reaction function of firm 2 given output of firm 1. So, now suppose the capacity of firm 1 is  $k_1$  you plug in the capacity here of firm 1 suppose it is producing at its capacity then the firm 2 the best output a base response of firm 2 is given by this function.

And this is so, plug in values of capacity of firm 1 and you will get the best response of firm 2 this-  $\frac{A - K_1}{2} = P_2(K_1)$ , right? So, these are the so, now we will define the capacities based on these two reaction functions, okay.

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So, if we simply plot these reaction functions this. So  $q_1, q_1$  comma  $k_1, q_2$  comma  $k_2$  this is  $A$  and this point is  $A$  by  $2$  and this you can say this from this here when this-  $P_2(K_1)$ , is equal to  $0$  that means  $k_2$  is  $A$   $k_2$  is here for each  $k_2$  we get and when  $k_2$  is  $0$  this is  $A$  by  $2$ . So, we get this; similarly, the reaction function of this is  $A$  and this is  $A$  by  $2$  and this is react, plug in the value of  $k_1$  we get the best response of firm  $2$  when  $k_1$  is  $0$  this  $k_1$  is  $0$  this is equal to  $A$  by  $2$  this is this and when this is equal to  $0$ , so, it is  $A$ , right?

So, when  $k_1$  is  $A$  this is equal to  $0$  you can say. Now, we so, this hope you are following what I have done. Now, we specify the case.

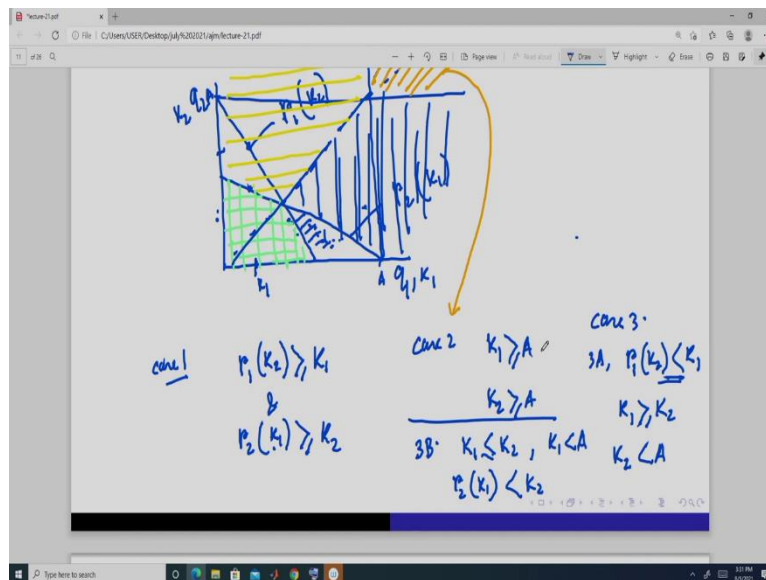
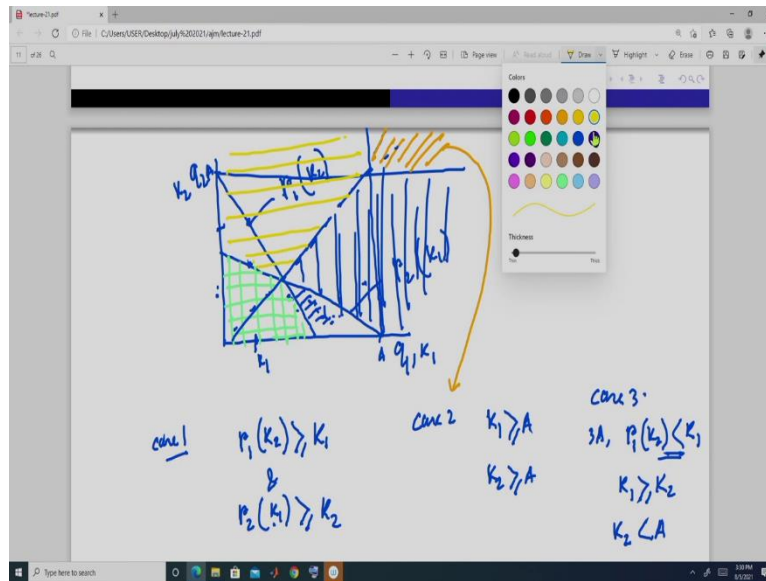
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The screenshot shows a PDF viewer window displaying a slide with handwritten blue ink. The text on the slide reads: "Case 1:  $k_1 \leq r_1(k_2)$  and  $k_2 \leq r_2(k_1)$ .  
If firm 2 is producing at capacity, the capacity of firm 1 is such that it cannot be more than its best response. So  $k_1 \leq r_1(k_2)$ .  
If firm 1 is producing at capacity, the capacity of firm 2 is such that it cannot be more than its best response. So  $k_2 \leq r_2(k_1)$ ." There are two graphs. The top graph shows a downward-sloping line with a vertical line at  $k_2$  and a horizontal line at  $k_1$ . The bottom graph shows a downward-sloping line with a vertical line at  $k_1$  and a horizontal line at  $k_2$ . The graphs illustrate the relationship between capacity and best response.

So, first case is this that capacity of firm 1 is always less than equal to this function this reaction function of firm 1. And capacity of firm 2 is always less than equal to this reaction function. That means, here if firm 2 is producing at its capacity that is  $k_2$  the capacity of firm 1 is such that it cannot be more than its best response. So, that is why it is this. So, in this situation if this is A and this is A by 2 you plug in any value here  $k_2$  you will the  $k_1$  is here if it is  $k_2$  is this  $k_1$  is this if  $k_2$  is this  $k_1$  is this  $k_2$  is this  $k_1$  is this if  $k_2$  is 0  $k_1$  is this A by 2, okay.

Similarly this if firm 1 is producing at capacity, then capacity of firm 2 is such that it cannot be more than its best response. So,  $k_2$  is less than  $k_1$ , i.e  $k_2 \leq r_2(k_1)$  so, it is something like this. So, plug in here suppose this is the capacity of firm 1 if it is producing here firm 2 is capacity is should not be greater than this, if capacity of firm 1 is this capacity of firm 2 should not be greater than this. So, it is from here, okay.

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Case 3: 3A;  $k_1 > r_1(k_2)$ ,  $k_1 \geq k_2$  and  $k_2 < A$ . The capacity of firm 1 is greater than the capacity of firm 2. If firm 2 is producing at capacity, the capacity of firm 1 is such that it is more than its best response. So  $k_1 > r_1(k_2)$ . Firm 2 cannot serve the whole market when price is zero.

Case 3: 3B;  $k_2 > r_2(k_1)$ ,  $k_2 \geq k_1$  and  $k_1 < A$ . The capacity of firm 2 is greater than the capacity of firm 1. If firm 1 is producing at capacity, the capacity of firm 2 is such that it is more than its best response. So  $k_2 > r_2(k_1)$ . Firm 1 cannot serve the whole market when price is zero.

So, output of firm 1 here output of firm 2 our capacity of firm 1 capacity of firm 2 this we get as the reaction function of firm 1, this we get reaction function of firm 2. Now and in case 1 we have got this whole region and this whole region but when we combine these so, what is the case 1 saying? Case 1 says that  $r_1(k_2) \geq K_1$  is greater than equal to  $k_1$  and this is  $k_2 - r_2(K_1) \geq K_2$ . So, if I fix this  $k_1$  then the reaction function from this we will get it should be here from this. So,  $k_2$  should be less than this.

So, here in this region and here if I fix suppose in here  $k_2$  then the reaction from this it should be in this line so, it should be less than this here. Now, if I specify  $k_2$  here then if we only have this then it should be here. But take this  $k_1$   $k_2$  is should be less than this portion, so, that is why we will not include this portion and not include this portion only this portion is in case 1. So, suppose let us mark it with a different color this green color this region is actually the case 1 this, this green color, okay.

Now, in case 2 we know that case 2 is this  $k_1$ ,  $k_1$  is greater than equal to  $A$  and  $k_2$  is greater than equal to  $A$ . So, here this point is  $A$  and this point is  $A$  so, this region is actually giving us is giving us case 2. So, case 2 is this- $K_1 \geq A, K_2 \geq A$ . So, the market size is sufficiently big so, that both the firm can satisfy the or meet the whole market, okay. And this portion is given by this case this  $k_1$ . So, suppose the capacity of firm 1 is greater than the capacity of firm 2 and if firm 2 is producing at capacity that is  $k_2$  the capacity of firm 1 is such that it is more than its best response.

It is like this- $k_1 > r_1(k_2)$  and firm 2 cannot serve the whole market when price is 0. So, case 3 is so, this line we will get this this is the 45 degree line. Now, here case 3 is we have defined that there are two part case 3A and case 3B. So, 3A is  $r_1(k_2)$  this is less than  $k_1 - k_1 > r_1(k_2)$ ,  $k_1$  is greater than equal to  $k_2$  and  $k_2$  is less than  $A$ . So, this portion is obviously and this portion is also part of it because this  $r_1$  if you fix  $k$  here it has to be greater than.

So, these points is satisfy this points. So, we get this, okay. This is case 3A and 3B is given by this this region. This region is 3B and 3B is it is this I am writing 3B here. So, it is  $k_1$  less than equal to  $k_2$ ,  $k_1$  is less than  $A$  and  $r_2(k_1)$  this is less than  $k_2 - k_2 > r_2(k_1)$ . So, this portion. So this portion also comes. So, these are the three cases and in case 3 we have two sub cases.

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We solve for pure strategy Nash equilibrium in case 1.  
 $p_1 \in [0, \infty)$  and  $p_2 \in [0, \infty)$ .

$A - K_1 - K_2 = P^*$   
 $K_2 \leq P_2(K_1) = \frac{A - K_1}{2}$   
 At  $P^*$ ,  $Q = K_1 + K_2$   
 Suppose  $P_2 = P^* = P_1$   
 $\pi_1 = P^* K_1 = (A - K_1 - K_2) K_1$

$\pi_2 = P^* K_2 = (A - K_1 - K_2) K_2$   
 Let  $P_1 > P^* = P_2$ .  
 then, firm 1 runs the individual market  
 So,  $A - K_2 - P_1 = Q_1$ ,  $Q_1 \leq K_1$   
 $\pi_2 = (A - K_2 - P_1) P_1$

So, based on this we have got what we know the all the possible combinations of cases of  $k_1$  and  $k_2$  we have specified. Now, we have to find a pure strategy Nash equilibrium. So, that is what is going to be the  $p_1$  and what is going to be the  $p_2$  in this market  $p_1$  and  $p_2$ .  $P_1$  strategy set is this 0 to infinity,  $p_2$  strategy set is 0 to infinity. Suppose, we have this situation now, let us assume that there is a price and that is  $A$  minus this is  $P^*$  we have a price like this- $A - K_1 - K_2 = P^*$ . In this case we will definitely have a positive price like this.

Because in case 1 the capacity this is less than equal to this which is equal to this- $K_1 \leq P_1(K_2) = \frac{A - K_2}{2}$  and  $k_2$  is less than this- $K_2 \leq P_2(K_1) = \frac{A - K_1}{2}$ . So, we will have a price like this. Now, we will show that this is the pure strategy Nash equilibrium in this case. Now, if suppose  $P_2$  is equal to  $P^*$ , okay and suppose  $p_1$  is also equal to their price then the profit of firm 1 because it is selling  $k_1$  and price is this- $P_2 = P^* = P_1$ . So, it is selling at this price and it is

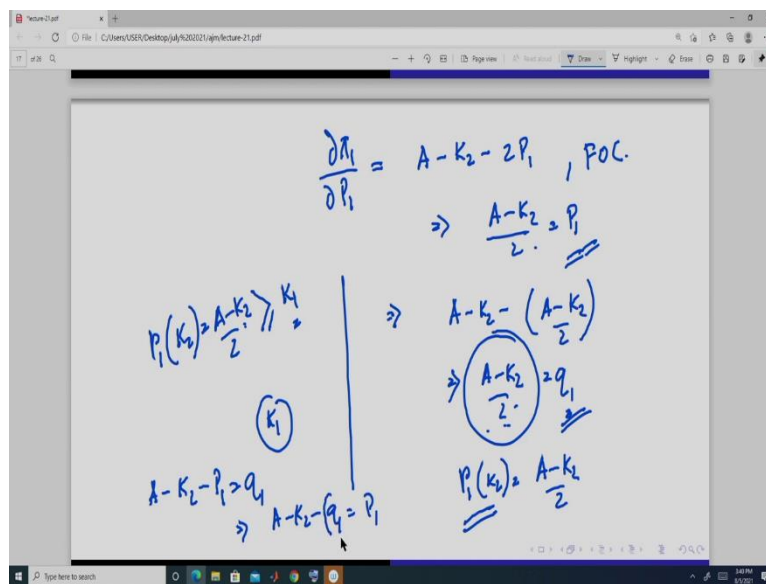
selling up to its capacity  $k_1$  and this is same as this, this  $\pi_1 = P \cdot K_1 = (A - K_1 - K_2)K_1$  and here mind this we will have a unique price here, why we will have a unique price?

Because our demand function is something like this it is  $A - P = Q$  it is  $A$  and this is  $A$ . So, it is a downward sloping straight line. So, if we fixed any fixed quantity here you will get a unique price if you fixed any quantity here we will get a unique price and at this price at  $P^*$  demand  $Q$  is, equal to  $k_1 + k_2$ . So, if this is  $P^*$  then this  $Q$  is equal to  $k_1 + k_2$ . So, it is such that both the firms can supply up to its capacity it is not less than that neither it is more than that, okay. And here we see that the profit of firm 1 is this-  $\pi_1 = P \cdot K_1 = (A - K_1 - K_2)K_1$  if it is.

And the profit of firm 2 profit of firm 2 is this and this is equal to this-  $\pi_2 = P \cdot K_2 = (A - K_1 - K_2)K_2$ . Now we have to show whether this we have to show that this is a pure strategy Nash equilibrium, okay. So, how do we proceed? Here let assume, let's suppose  $p_1$  is greater than  $P^*$  and it is equal  $P_2$ . So, if firm 2 sets a price  $P^*$  then suppose firm 1, set a price higher than this, then this implies what? Then it will firm 1 serves the residual market so the demand curve of firm 1. So, the demand curve of firm 1 is-  $A - K_2 - P_1 = q_1$ , if  $q_1 \leq K_1$   $k_2 \leq q_1$  provided  $q_1$  is less than equal to, okay.

So, if this is the case then what is the profit of firm 1. Profit of firm 1 is this-  $\pi_1 = (A - K_2 - P_1)P_1$ . Now, so, there can be many such prices which satisfy this we have to find that price which is the maximizing this.

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So, what we do let us we are taking partial because in this case there is a strategic interaction. So, price this profit is also a function of  $p_2$ , okay although it is not appearing explicitly but it is appearing based on the condition, right? So, that is why because this is the condition, right? This- $P_1 > P^* = P_2$ . So, that is why I am taking a partial. So, this is this first order condition will imply that it is this- $\frac{d\pi_1}{dP_1} = A - K_2 - 2P_1, \frac{A-K_2}{2} = P_1$ . So, if you plug in this price in the demand function of firm 1 that is the residual demand function if this.

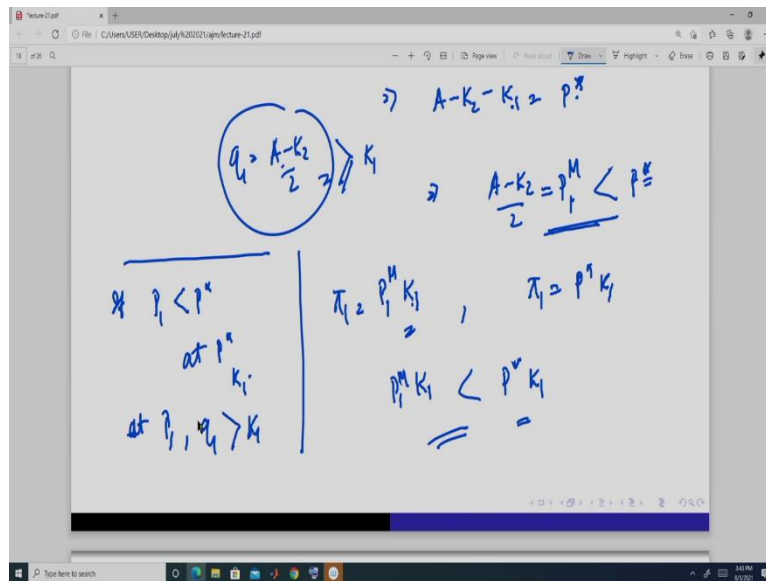
Now, this- $\frac{A-K_2}{2}$  is equal to  $q_1$  now, we know that this is  $r_1$  is given by this form. So, if firm 2 is supplying up to its capacity then what is the best response of firm 1 it is given by this- $r_1(K_2) = \frac{A-K_2}{2}$  and if firm 1 suppose charges a higher price and it is charges a higher price so, it is serving the residual market in that case what is the optimal price? it will set optimal price is this- $\frac{A-K_2}{2} = P_1$  an optimal quantity to sell is this much, right?

So, this is when it is acting as a monopolist in the residual market and this when it is given a capacity of firm 2, what is the optimal output of firm 1 it is given by this- $r_1(K_2) = \frac{A-K_2}{2}$ . Now, in case 1 we know that this which is equal to this this is greater than or equal to  $k_1$ - $r_1(K_2) = \frac{A-K_2}{2} \geq K_1$ . So, in this case what is happening if our firm 1 sets a price think initially thinking that his price is going to be higher than the price set by firm 2 that is  $P^*$  then it is acting as a monopolist in the residual market.

In that case, if it wants to maximize or it wants to set the monopoly price in the residual market, it sets a price this- $\frac{A-K_2}{2} = P_1$  and the output itself is this- $\frac{A-K_2}{2}$ . Now, this output is actually greater than equal to its capacity. So, it means what, even if it wants to sell this much amount of output, it cannot sell this much. So, it will sell  $k_1$ . Now, remember this the demand function is of firm 2 firm 1 is this one-  $A - K_2 - P_1$  and this you can get this-  $A - K_2 - q_1 = p_1$ .

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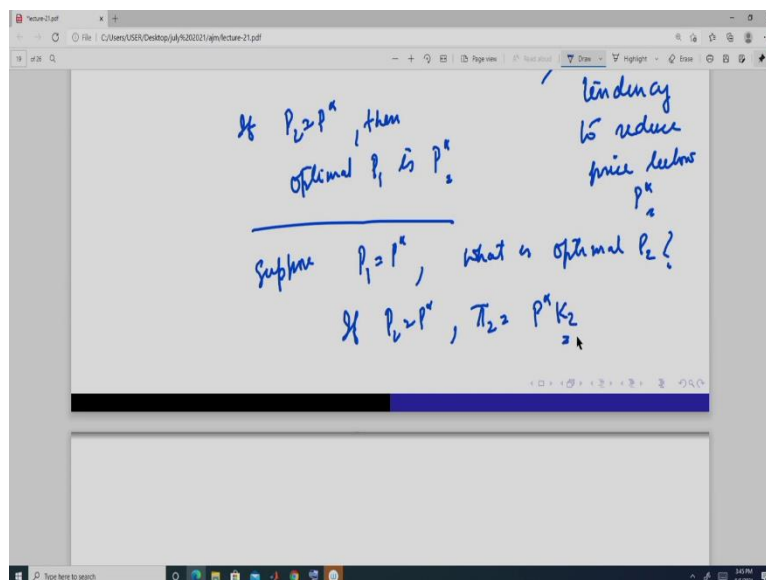
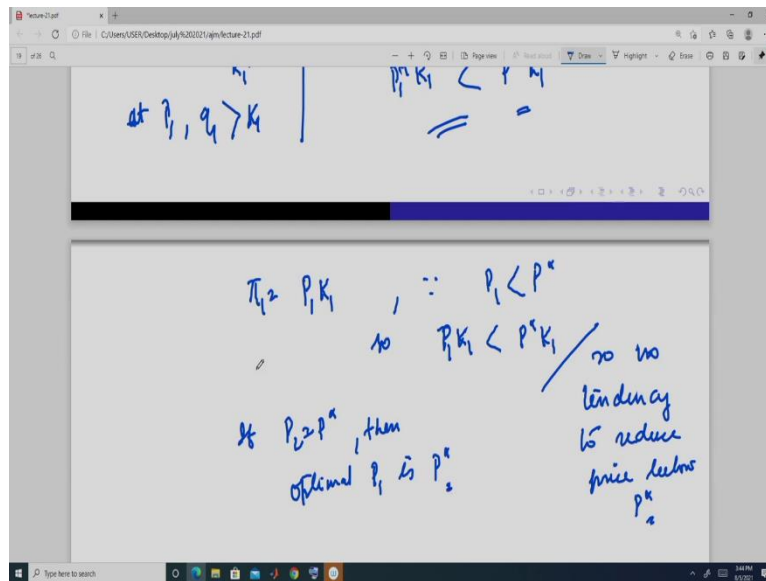


Now, here, when we plug in  $k_1$  here, when we plug in  $k_1$  we get the price  $P^*$ . So, this means what and here the monopoly in the residual market if this is equal to this firm should be selling this much amount should be producing this. So, and this is actually greater than  $k_1$ . Now if we plug in this here, we will get a price which is less than this. So, this means this means which is the this which is the you can I am writing this  $P^M$ .

This is supposed to  $P_1^M$ , okay this is less than  $\text{star} - \frac{A - K_2}{2} = P_1^M < P^*$ . Now, so, here profit because it will sell if it is acting as a monopolist in the residual market this, this is the profit of firm 1  $- P_1^M K_1$  because it cannot sell this much. So, this is less than greater than equal to this here. So, it will be able to sell only this and if it sets  $P^*$  its profit is this  $- P^* K_1$  and we have shown that this is less than this. So, that is why we get this  $- P_1^M K_1 < P^* K_1$ .

So, this means that profit up firm 1 is higher if it sets a sets the price  $P^*$  rather than trying to act as a monopolist in the residual market. Because if it wants to act as a monopolist in the residual market, it will end up charging a price which is less than this  $P^*$ . So, that is why this is not optimal. So, this strategy of firm 1 is not optimal  $- P_1 > P^* = P_2$ . Now, if suppose firm 1 sets a price which is less than  $q$  if  $P_1$  is less than  $P^*$ . So, at  $P^*$  it sells  $k_1$ . So, at  $P_1$   $q_1$  this will be greater than this will be greater than,  $k_1$ .

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So, if this is the case then profit of firm 1 is going to be  $P_1$  into  $k_1$  and since  $P_1$  is less than  $P^*$ . So, is less than  $P^* k_1$ . So, to know tendency to reduce price below  $P^*$ . So, we get that if  $P_2$  is equal to  $P^*$  then optimal  $P_1$  is  $P^*$ , right? Now, we have to show whether it is a Nash equilibrium or not. So, again suppose  $P_1$  is equal to  $P^*$ , okay when  $P_1$  is equal to  $P^*$  we have to find what is optimal  $P_2$ . So, we know if  $P_2$  is also equal to  $P^*$  then profit of firm 2 is  $P^*$  into  $k_2$ .

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If  $P_2 > P^*$ , then demand of firm 2 is

$$A - K_1 - P_2 = q_2, \quad q_2 \leq K_2$$

$$\pi_2 = (A - P_2 - K_1)P_2$$

$$\frac{\partial \pi_2}{\partial P_2} = A - 2P_2 - K_1, \text{ FOC}$$

$$\Rightarrow \frac{A - K_1}{2} = P_2$$

$A - K_1 - P_2 = q_2, \quad q_2 \leq K_2$

$$\pi_2 = (A - P_2 - K_1)P_2$$

$$\frac{\partial \pi_2}{\partial P_2} = A - 2P_2 - K_1, \text{ FOC}$$

$$\Rightarrow \frac{A - K_1}{2} = P_2^M$$

$$\Rightarrow A - K_1 - \frac{A - K_1}{2} = q_2$$

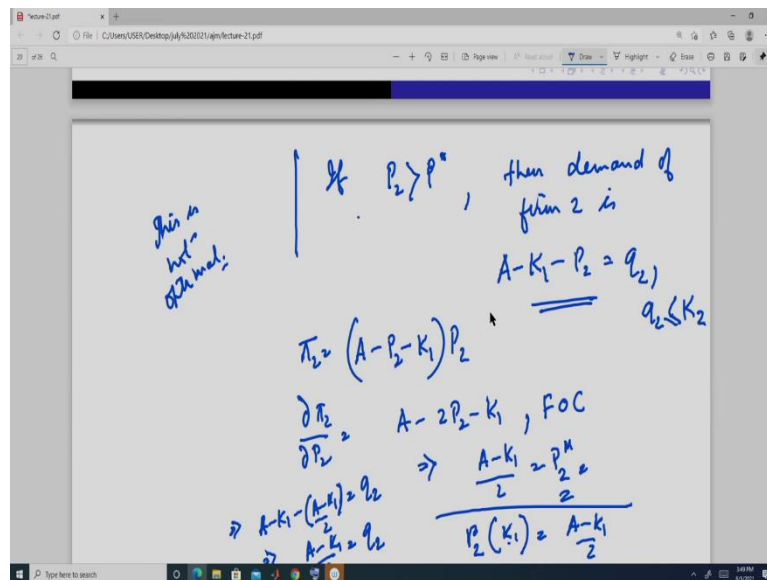
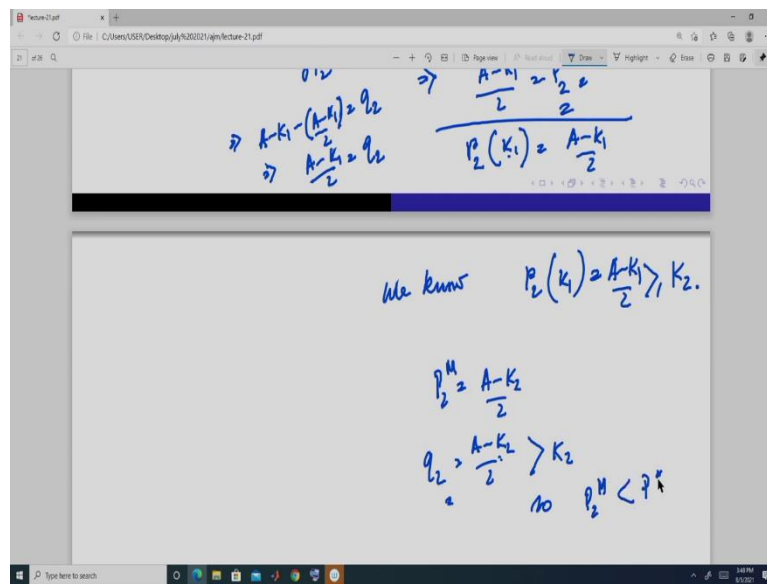
$$\Rightarrow \frac{A - K_1}{2} = q_2$$

$$P_2(K_1) = \frac{A - K_1}{2}$$

Now, if  $P_2$  is suppose greater than  $P^*$  then of firm 2 is  $A - K_1 - P_2 = q_2$  it will start the residual market provided  $k_2$  is less than equal to  $K_2$ . So, we have to find a price  $p_2$  such that it is greater than  $P^*$  and also it is maximizing its profit. Because there can be many such  $p_2$ s. So,  $P_2$  profit function of firm 2 now is this  $\pi_2 = (A - P_2 - K_1)P_2$ . So, again optimizing with respect to  $P_2$ , we get this first order condition gives this  $\frac{d\pi_2}{dP_2} = A - 2P_2 - K_1$ .

So, this is what? So, this is again the Cournot reaction output. So, if  $r_1$  is producing up to its capacity that is  $k_1$  what is the optimal output of firm 2 it is given by this  $a$  which is equal to this  $P_2(K_1) = \frac{A - K_1}{2}$ . So, let us this is  $M$ , so we get this. And from here we get so, we get this, it is this  $A - K_1 - \frac{A - K_1}{2} = q_2$ .

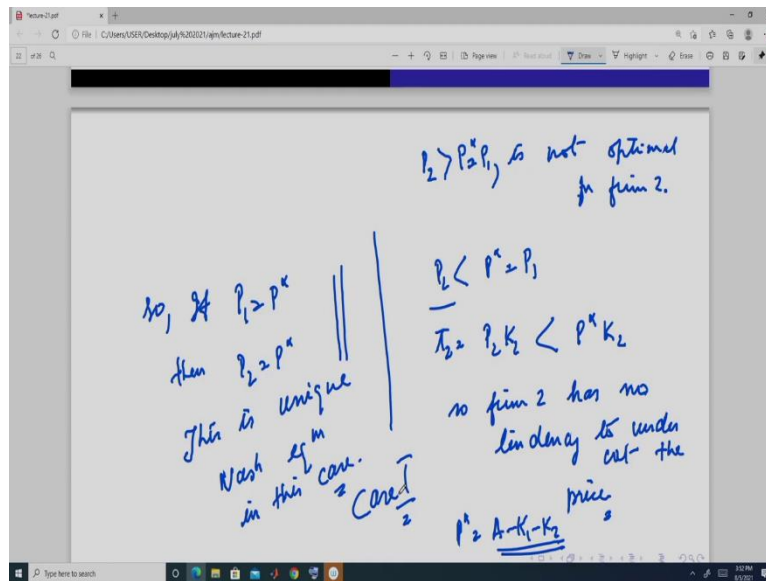
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So, and we know that  $r_2 k_1$  which is equal to  $A$  minus this is greater than equal to  $k_2$ , i.e.  $p_2(K_1) = \frac{A - K_1}{2} \geq K_2$ . So, even if firm 2 wants to sell this it will not be able to sell it will only be able to sell  $k_2$ . Now, if it wants to sell this it sets a price which is  $p_2^M$ . And  $p_2^M$  is equal to  $\frac{A - K_2}{2}$ . So, at this price quantity is it sales is it can it wants to sell is this-  $\frac{A - K_2}{2}$  since it is a downward sloping demand curve the residual demand curve is also downward sloping.

So, these since this is greater than  $k_2$ , so,  $p_2^M$  is actually less than  $P^*$  here. So, if that is the case then profit of firm 2 when it sets  $p_2^M$  and  $k_2$  this is going to be less than  $P^* k_2$ . So, this is not optimal  $P_2 > P^*$ .

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So, we get that  $P_2$  greater than  $P^*$  is not optimal for firm 2. Now, if  $p_2$  is less than  $P^*$  which is equal to  $P_1$  say firm 1 sets that price in this case profit of firm 2 is  $P_2$  into  $k_2$  this is definitely less than  $P^* k_2$  -  $P_2 k_2 < P^* k_2$ . So, firm 2 has no tendency to undercut the price. So, from here we get that. So, if  $p_1$  is equal to  $P^*$  then  $P_2$  is also equal to  $P^*$ . So, therefore, this is unique Nash equilibrium in this case and this is case 1.

So, in case 1 we get that there is going to be a unique Nash equilibrium and that Nash equilibrium is given by this case this here. So, it is such that the price is equal to prices such that it is the firms are producing at their capacity. So,  $P^*$  is equal to this -  $P^* = A - K_1 - K_2$ . So, this is  $P^*$ . So, when both the firms are going to set a common price that is  $P^*$  and at that  $P^*$  the demand is such that both the firms produce up to its capacity.

And this is the unique thing because we have seen that there is no tendency of any firm to set a price higher than  $P^*$  neither there is a tendency to lower the price, okay. And for the other cases that is in case 2 and case 3 we will do later in the next class. Thank you.