Introduction to Market Structures
Professor. Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati
Lecture No. 28
Bertrand Competition with and without fixed cost

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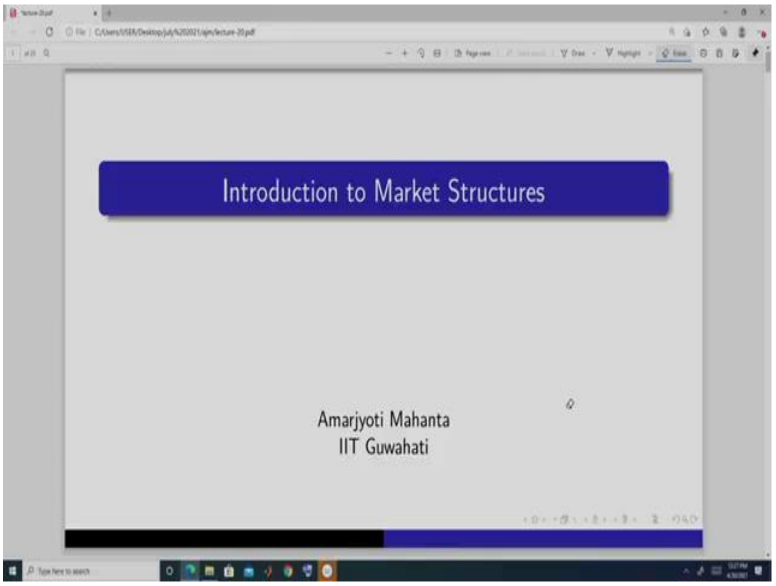

Hello, everyone. Welcome to my course Introduction to Market Structures. We have completed the Cournot model. We have done both Cournot duopoly market, Cournot oligopoly market. In Cournot competition what we have seen that the firms decide the output. And the aggregate, and based on the aggregate output the market price is determined, okay. And while deciding the output they play a strategy game.

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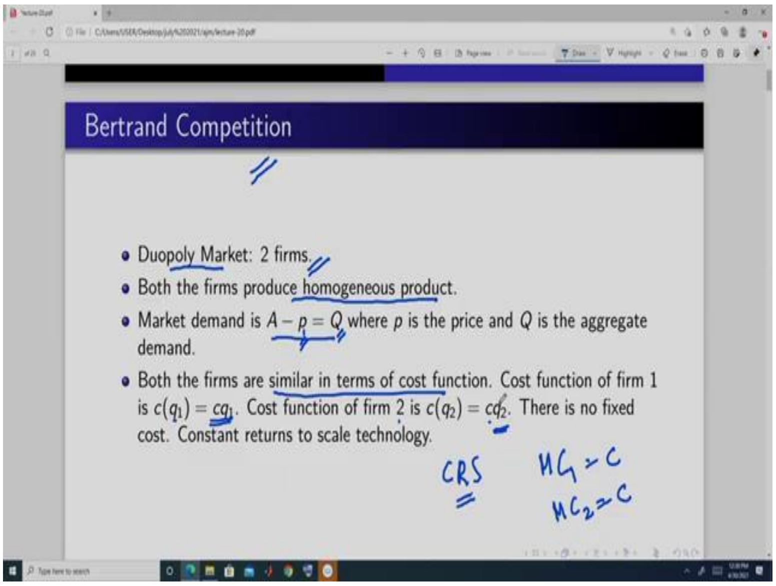

Today, we are going to do Bertrand competition. In Bertrand competition what happened instead of quantity they will choose price. So, how, now let us specify the market. So, for simplicity we will assume that there are two firms. So, it is a duopoly market, okay. Next, both firms produces homogeneous product. What do we mean by a homogeneous product? Homogeneous product means that whether I buy from firm 1 or buy from firm 2 buy being the same price it does not matter. The type of good or the nature of the good is going to be same. So, they are perfectly substitutable, okay.

So, market demand is this-A-p=Q. It is same as what we have considered in the Cournot. So, this p is the market price. We will come to this. But now in a market we know there is only one price and it is this price and this is the aggregate quantity demanded at that price, okay. Again, for simplicity we assume that the, we will see in this Bertrand competition we will take different types of costs.

So, first both the firms are similar in terms of cost function. So, cost of firm 1 is c into q1, q1 is the output of firm 1. And cost function of firm 2 is c into q2. So, this is a CRS technology, constant returns to scale and there is no fixed cost, okay. And both the firms are same. So, here marginal cost of firm 1 is c, marginal cost of firm 2 is again c, okay. So, they are same.

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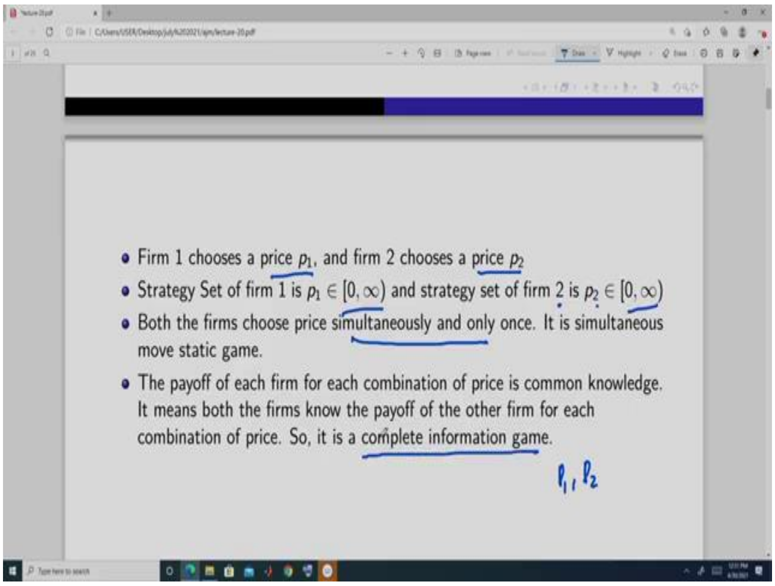

Now, let us specify the game, the strategic interaction in this. So, firm 1 chooses a price that is p1 and firm 2 chooses a price p2, okay. Now, here just previously I have said that in market share is only one price. So, the market price is actually the minimum of these two price, okay. So, okay so strategy set of firm 1 is this p1 and which lies between 0 to infinity and strategy set of firm 2 is again this which lies between 0 to infinity, okay. And both firm, both the firms choose price simultaneously and only once. So, this is a static game of, static game which is played simultaneously, okay.

Next, firm 1 knows the payoff of firm 2 for all the combinations of price p1 and p2. Firm 2 knows all the payoffs of, payoffs for all the combinations of p1 and p2 of firm 1. So, therefore, it is a complete information static game okay and it is played only once.

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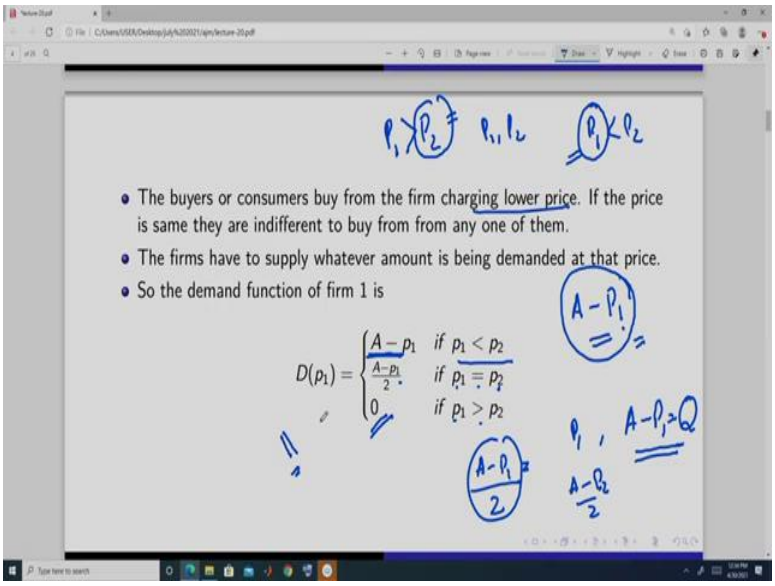

Now, we specify some further details, okay. Like buyers, they will buy from the firm that is charging the lower prices. So, if we have two price p1 and p2 and if p1 is less than p2, then everyone is going to buy from firm 1. And if p1 is greater than p2, then everyone is going to buy from firm 2, okay. So, this will change the demand curve faced by each firm, okay. We will come to it. And further if firm 1 sets a price p1 and suppose p1 is less than p2, then there must be a demand and that demand is going to be this much- $A - P_1$. So, this demand has to be satisfied by firm 1, whatever be the quantity, okay.

So, this firm has to supply this much amount of quantity, okay. So, based on these two assumptions, we specify the demand curve of firm 1. So, firm 1 if suppose sets a price p1 and it is less than p2, then it has to serve the whole market. And at p1 market demand is A minus p1. So, this whole amount is to be satisfied or is to be supplied by firm 1. So, demand curve of firm 1 is this- $A - P_1$.

And if firm 1's price is same as the price of firm 2, p1 is equal to price p2, then the market is equally shared. It is this-$\frac{A-P_1}{2}$. So, at this, this is the amount of quantity demanded and this is the demand faced by firm 1. Similarly, firm 2 will face this if the prices are same. And if price of, set by firm 1, p1 is greater than p2, then the amount is 0. It does not, nobody buys from that firm, okay. So, based on this demand function, we will get the payoff function.

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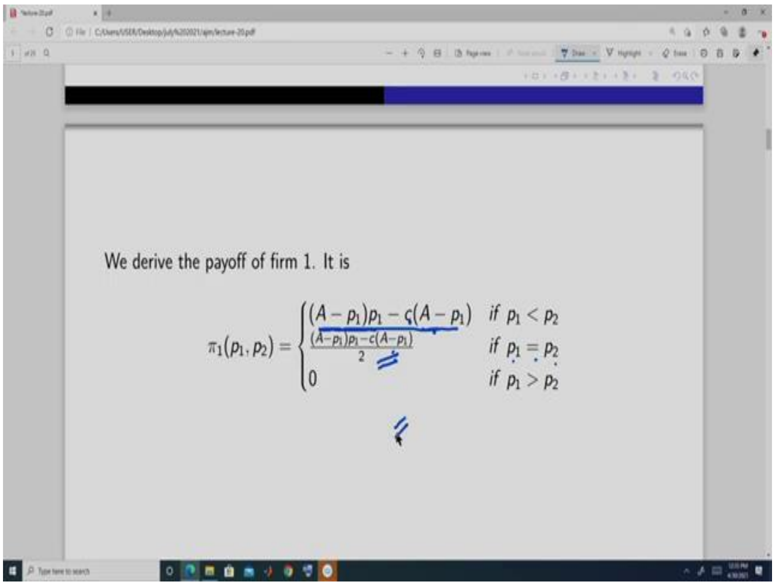

So, the payoff of firm 1, if price of firm 1 is less than the price of firm 2, then it has to serve the whole market. So, the demand is this much, so demand into price. So, this is the total revenue and this is the marginal cost c into the amount it is going to produce A minus p1. So, this is the total cost. So, this is total revenue minus total cost. So, this is the profit-$(A - p_1)p_1 - c(A - p_1)$.

So, profit of firm 1 when it sets a price p1 and firm 2 sets a price p2 and suppose p1 is less than p2 it is this- $(A - p_1)p_1 - c(A - p_1)$. And suppose price are such that p1 is equal to p2, then the profit is this-$\frac{(A-p_1)p_1-c(A-p_1)}{2}$, because this is the total revenue minus total cost divided by 2, because the this is going to be shared half, this is again going to be shared half. So, this is the payoff. So, this is the payoff function of firm 1.

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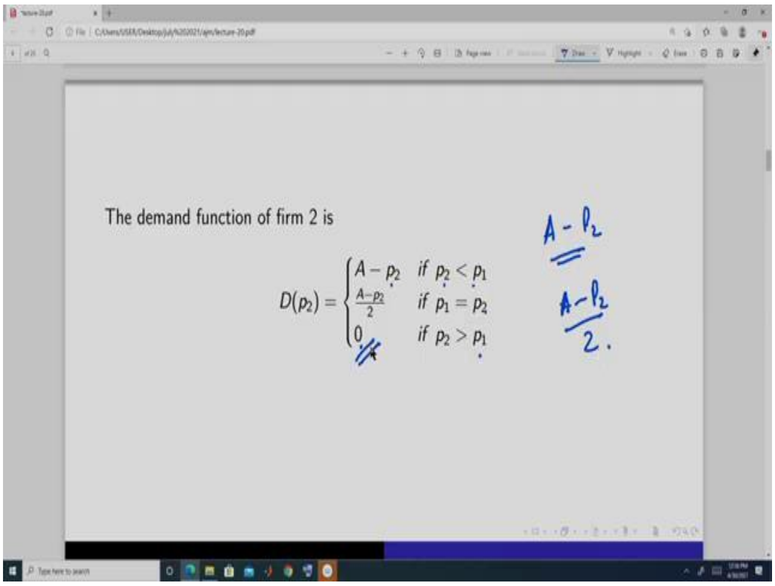

Similarly, demand function of firm 2 is A minus p2, if p2 is less than p1 that is if firm 2 sets a price less than the price of firm 1, it has to serve the whole market and the demand at p2 is this- $A - p_2$. And if it sets a price which is same as the price set by firm that is p1 is equal to p2, then the market is shared equally. So, the demand is this much- $\frac{A-p_2}{2}$. And if firm 2 sets a price which is greater than the price of firm 1 they need, gets 0 demand that is nobody buys from it. So, its demand is 0.

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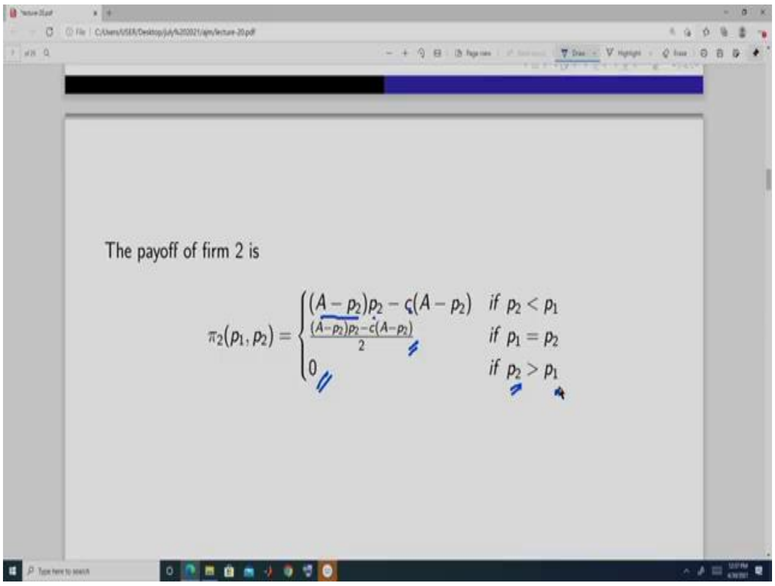

So, from here, from this demand function, we get the payoff of firm 2. Payoff of firm 2 is this- $(A - p_2)p_2 - c(A - p_2)$. So, this is the demand that the firm 1, firm 2 faces when its price is less than the price of firm 1. So, demand into price. So, this is total revenue, marginal cost and it is same as the average cost. So, this is into the amount it has produced, so total cost. So, this is the profit. And when prices are same p1 is equal to p2, then this is A minus p2 into p2 divided by 2.

Again, c into A minus p2 divided by p2. This is the total cost and this is the total revenue. So, the profit is this-$\frac{(A-p_2)p_2-c(A-p_2)}{2}$. So, it is simply half of this if the prices are same, but if they match and if in one case it does not match, okay. And profit is 0 if price is, of firm 2 is more than the price of firm 1. Now, given this specification we have to find the pure strategy Nash equilibrium of this game, okay.

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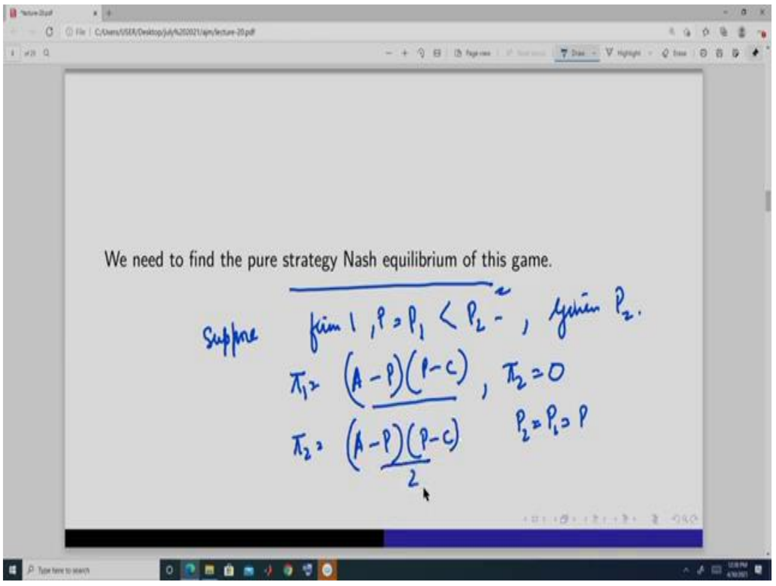

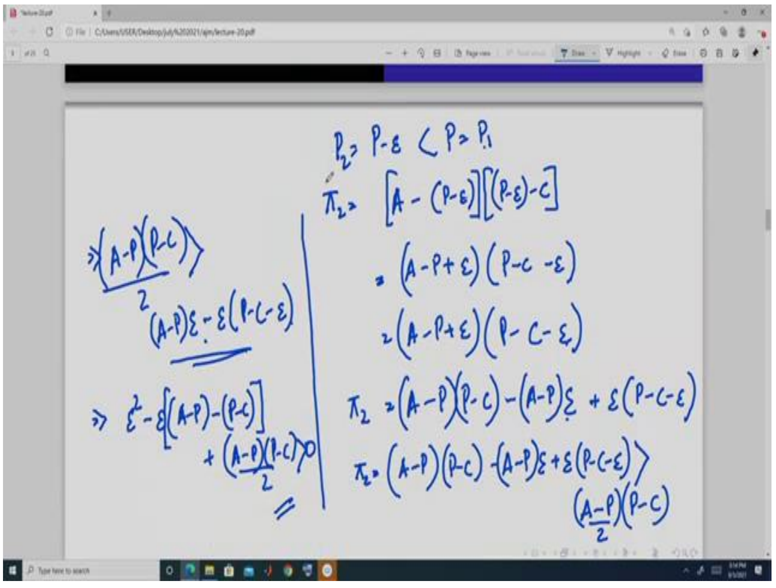

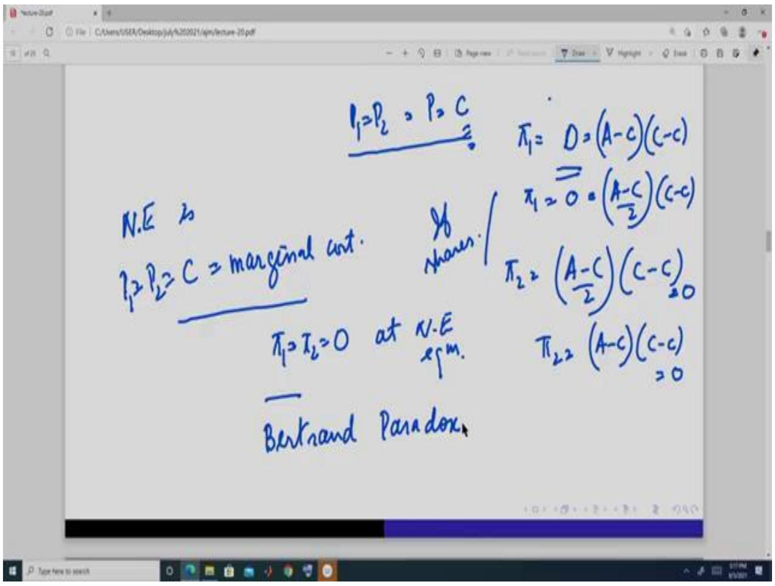

So, first I will show you algebraically and then we will do it geometrically that is through diagram and through diagram it will become more clear. Suppose firm 1 sets a price p1, okay and p1 is suppose less than p2, already there is a p2, given p2, So, the profit of, and suppose this is equal to p, i.e $P = P_1$ is A minus p, p minus c. This is the profit- $\pi_1 = (A - P)(P - c)$. And profit of firm 2 is 0. So, firm 2 if it sets a price p2 is equal to this, p1 is equal to p1, then the profit of firm 2 is, it is this-$\pi_{1=2} = \frac{(A-P)(P-c)}{2}$, right? And instead if firm 2 sets a price which is p2 suppose is equal to p minus some epsilon which is this- $P_2 = P - \varepsilon < P = P_1$, then the profit of firm 2 is, it is this-$\pi_2 = [A - (P - \varepsilon)][(P - \varepsilon) - \varepsilon]$, right?

Now, if we compare this profit and this profit, we see that, this is c, right? So, we have, we can get this- $(A - P + \varepsilon)(P - c - \varepsilon)$. And then if we, okay solve this we have got this. This can be written this way- $(A - P)(P - c) - (A - P)\varepsilon$. This part is multiplied with this part and then we are left with e p this-$(A - P)(P - c) - (A - P)\varepsilon + \varepsilon(P - c - \varepsilon)$, okay. So, if firm 1 sets a price p1 and firm 2 sets a price epsilon less than that price, then the profit of firm 2 is going to be this-$(A - P)(P - c) - (A - P)\varepsilon + \varepsilon(P - c - \varepsilon)$. Now, if this epsilon is very small, if it is very small, then this part we can show that this which is, this- $(A - P)(P - c) - (A - P)\varepsilon + \varepsilon(P - c - \varepsilon)$ is actually greater than this- $\frac{(A-P)(P-c)}{2}$when firm 1 sets this and it shared a market.

So, when the firm 1, firm 2 set the price same as the price of firm 2 and shared the market equally, because from here we get that, so if epsilon is sufficiently small we can construct an epsilon like this which is, which satisfies this- $\frac{(A-P)(P-c)}{2} > (A - P)\varepsilon - \varepsilon(P - c - \varepsilon)$. So, if this is because then actually we get a quadratic form and this quadratic form is of this nature that is, I should have taken it in this form, this, so we can have epsilon which satisfies this-

$\varepsilon^2 - \varepsilon[(A - P) - (P - c)] + \frac{(A-P)(P-c)}{2} > 0$. And this will give us that since there exists an epsilon, so we will, it is optimal for firm 2 to reduce the price or undercut the price if firm 1 sets a price p, then the firm 2 will set a price p minus epsilon.

So, like this it will go on, because we have taken this price to be some arbitrary price. So, like this, this will continue, and then finally, we will get that the price is equal to c. And when the price is equal to c, so profit of firm 1 is 0, because it is A minus c if firm 1 sets this price. And this is, will also be equal to 0 if it shares the market at that price and profit of firm 2 it is going to be the same and it is going to be shares and it is going to be, so this is equal to 0- $\pi_2 = \frac{A-c}{2}.(c - c) = 0$, this is equal to 0-$\pi_2 = (A - c)(c - c) = 0$. So, price of firm 1 and firm 2 is going to be such that it is going to be equal to marginal cost.

And in this case, we have only one pure strategy Nash equilibrium. So, Nash equilibrium is p1 is equal to p2 is equal to c which is equal to marginal cost. We get this. So, when the cost function are same and marginal cost function are such that the marginal cost is constant and they do not have any fixed cost in that case in Bertrand competition or when the firm set the price or compete in terms of price, we get that the pure strategy Nash equilibrium is to set a price is equal to marginal cost. And this will lead to the profit of firm 1 and profit of firm 2 is 0 at Nash equilibrium. So, this is called something called a Bertrand paradox.

And why it is a paradox, because there are only two firms. And two firms are sufficient to generate 0 profit. Generate 0 profit here means that the through the price competition, the price is reduced to marginal cost. And when the price is equal to marginal cost, the firms are not earning any supernormal profit. So, they are earning same as what they get in a competitive market. But in competitive market, we need many firms. But here with only two firms, we can generate this outcome. So, that is why it is called a Bertrand paradox.

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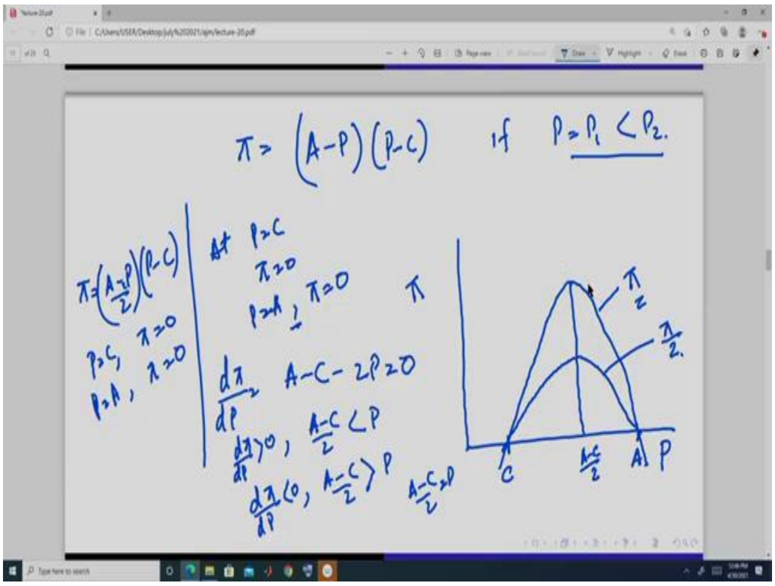

Now, let us do it diagrammatically. So, profit function, we can write it this way. If suppose p is equal, if it is this-$\pi = (A - P)(P - c)$, if $P = P_1 < P_2$. Now, if we try to plot this function here, in this axis we take price and here we take profit. If we take this, let us, this is c and this is A, okay. At p is equal to c, profit is equal to 0. At p is equal to A, profit is equal to 0. And when we differentiate with respect A, we get A minus c, this. So, this is, slope is increasing as long as p is less than this, i.e $P < \frac{A-c}{2}$ and this is this, right. So, it is maximum at price is equal to A minus c divided by 2.

So, we get this curve something like this. So, this is the profit, right. Now, if we take this curve, so again here when p is equal to c, profit is equal to 0 and p is equal to A, profit is equal to 0. And this is same as this curve only it is at the half distance. So, this point is the, you can say the monopoly price, this curve, okay, half the market share. Now, so we get the function, the plot of the payoff function in this form, okay.

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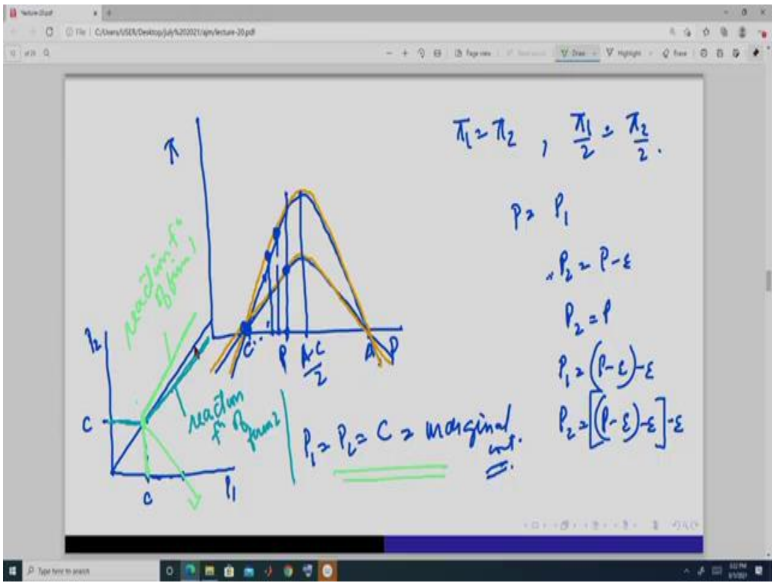

Now, let us look here, right. And since the payoff functions are same, so suppose this is for firm 2, because if we look at the payoff function it is same, right? Now, this is, now set the price like this. So, this is suppose p. P is equal to, firm 1 sets a surprise p. If firm 2 sets a price slightly less than this that is p2 is p minus epsilon something close to it, its market share is this. But if it sets a price which is p2 is equal to p, then it is here. Both the firm gets here.

So, firm 2 is going to set this price and moment it sets this price, its payoffs from here to here, right? So, it is like this. Now, if firm 2 sets this price, now firm 1 is going to set a price which is p slightly less than this, slightly less than here. So, if it sets here, its market is this. Now, firm 2 will know that if it sets a slightly less than this, then if it sets the same pA it will get this profit, but it is still higher. So, p2 is going to be this- $P_2 = [(P - \varepsilon) - \varepsilon] - \varepsilon$. Like this it will go on. So, this undercutting will go on till this point, because for any price which is greater than c this curve lies below this curve.

So, there is the moment you undercut the price, you get a bigger market share and your profit is also higher. So, that is why price is should be always equal to this that is equal to marginal cost-$P_1 = P_2 = C = MC$. So, the reaction functions of firm 1 and firm 2 in this can be represented in this form. Suppose this is the 45 degree line, and this is the marginal cost, so marginal costs are constant. And it is same.

Now, if firm 1 set a price suppose it is this, firm 2 will set a price which is less than this, we have seen this from here, from this diagram. So, any price of firm 2 greater than this. So, how it is going to respond, what it is A. So, it will be less than A. So, it will be lying here, slightly less than. And when it is c, it will be c. So, this is you can say c and then this. And if firm, price of firm 1 goes below this, it is not going to reduce the price, so it is this. So, this is the reaction function of firm 2. And the reaction function of firm 1 we will get in this way.

Firm 2 if it charges a price like this, it will set the price such that it will be slightly less than the price of firm 2. From this diagram we know this. So, it will be like this. So, it will above lie above the 45 degree line. And at price, when firm 2 sets the price c, it is going to set the price c like this. And if it moves below this, it will not charge any price below this, so it will be like this. So, this green line is the reaction function of, firm 1 and these two reaction function intersects at this point. So, that is why this is the Nash equilibrium point, okay. So, we get this as the outcome based on this reaction function.

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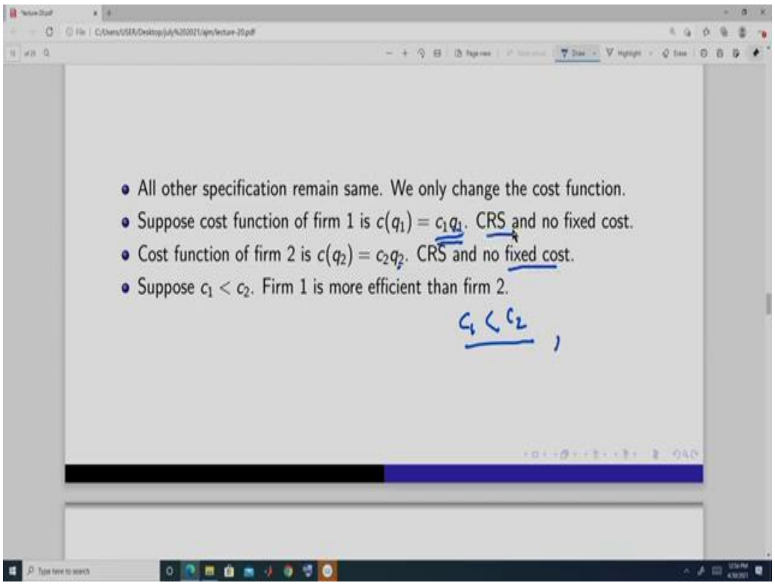

Another case is suppose we now take different function, different cost function. So, what do we take? We take the cost function of firm 1 to be this- $c(q_1) = c_1q_1$, okay. So, it is c1q1. Cost function of firm 2 is c2q2, q2 is the output, here c1 is the output. And further we assume c1 is less than c2. So, this means that firm 1 is more efficient than firm 2 and there is no fixed cost and there is CRS, okay. So, this is the specification of the cost. So, we know that firm 1 can produce output at a much lower cost than the firm 2 and because of this thing and they do not have any fixed cost. All the cost is only the variable cost and the demand curve function is same as earlier.

So, firm 1 if it sets a price lower than the price of firm 2 it gets the whole demand. This-$A - p_1$ if it sets the price same as the price of firm 2 it has to share the market equally. And if it sets a price higher than the firm 2 it gets 0 demand, okay. Similarly, for firm 2 if it sets a price less than the price of firm 1, then it gets the whole market demand. If it sets a price which is same as the price of firm 1, then it gates or it has to share the market equally and it gets 0 demand if the price is higher than the price of firm 2, okay firm 1.

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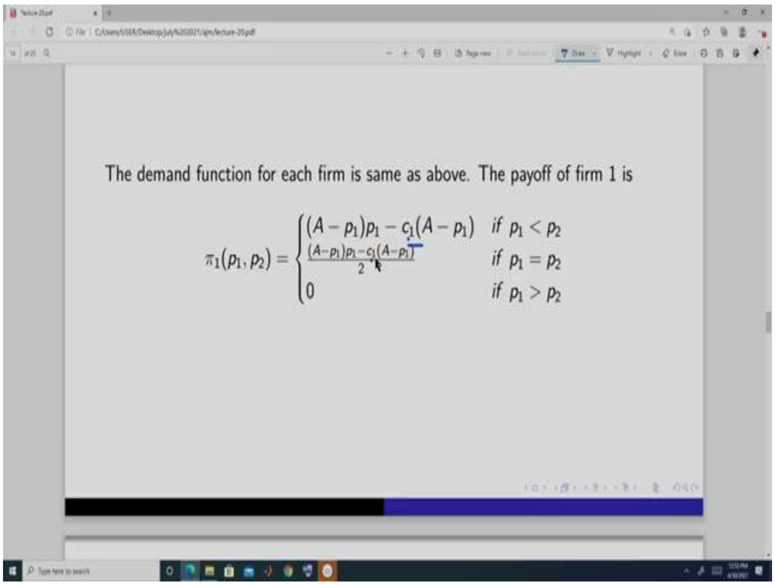

Now, given this we know the payoff function of firm 1 is going to be like this- $(A - p_1)p_1 - c_1(A - p_1)$. Here instead of c earlier it is going to be c1 and here it will be c1. Here in this case firm 1 is getting the whole market, in this case the firm 1 is getting half of the market or half of the total quantity demanded at that price, okay.

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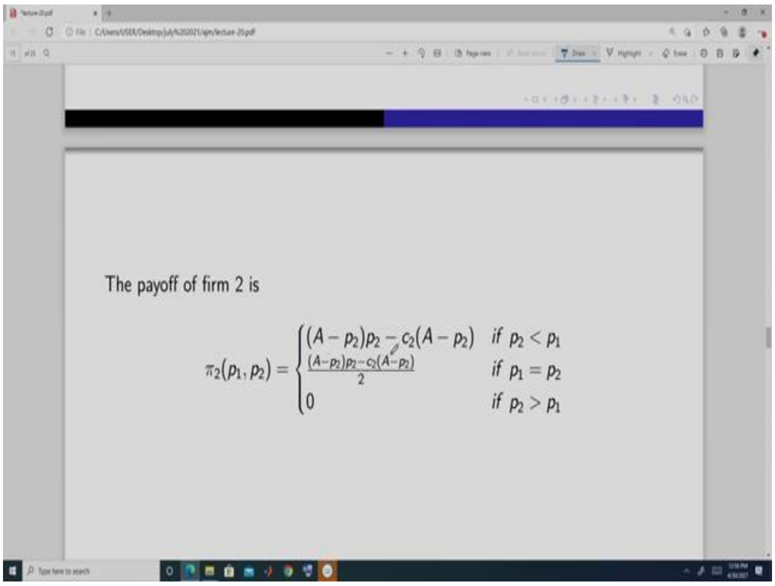

And for firm 2 at p2 when p2 is less than p1 it is getting the whole market that is at p2 whatever is being demanded firm 2 is serving or selling. It is getting this payoff- $(A - p_2)p_2 - c_2(A - p_2)$. This is the quantity and this is the price, this is the revenue, this is the total cost. So, this is total revenue minus total cost gives you the total, gives you the profit. And if the price is same as the price set by firm 1, then it shares the market equally and profit is this- $\frac{(A-p_2)p_2-c_2(A-p_2)}{2}$, okay. Now, let us solve this. And in this case we will do it through diagram, okay.

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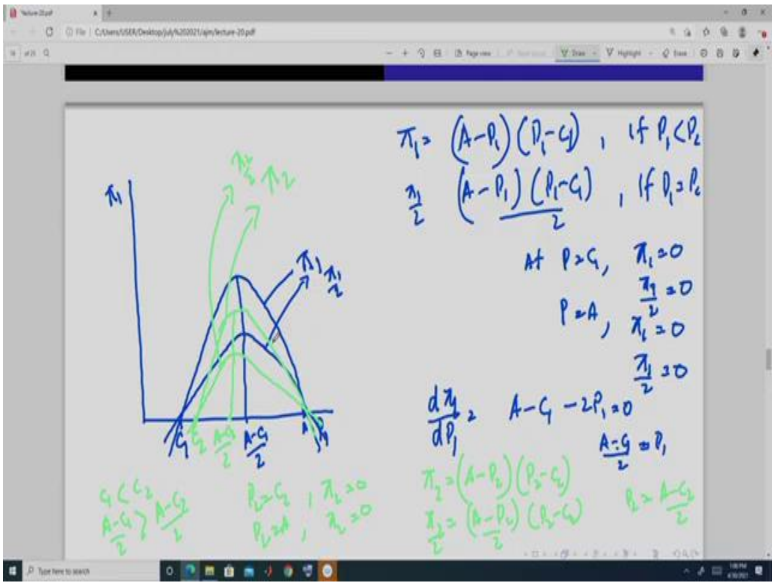

So, first let us look at the profit function of firm 1. Firm 1, profit function is this- $\pi_1 = (A - p_1)(p_1 - c)$, right? if p1 is less than p2 and this- $\pi_1 = \frac{(A-p_1)(p_1-c)}{2}$ if p1 is equal to p2, right.

So, I plug here c1. It is 0. At p is equal to c1 profit is 0, p is equal to A profit is 0, half of this is also, because this is simply half of this. And this is if we differentiate it with respect to p1, we get it first order condition- $\frac{d\pi_1}{dP_1} = A - c_1 - 2P_1 = 0$. So, this gives if, so this is the optimal point. If p1 is less than this, then it is increasing. If p1 is greater than this, then it is decreasing. So, it is something like this. So, this is the, and if market is shared, is you have got half of the market then this, this is for firm 1, okay.

Now, here for firm 2 it is this-$\pi_2 = (A - p_2)(p_2 - c)$, right? So, it is same except at p2 is equal to c2, profit is 0 and p2 is equal to A, c2 is 0. And if we differentiate, we get it to be if p2 is equal to A minus c2 and from here we know since we are given that c1 is less than c2, so A minus c1 divided by 2 this is greater than A minus c2- $\frac{A-c_1}{2} > \frac{A-c_2}{2}$. So, this curve is going to be somewhere some, so this is the profit of firm 2 and this curve is half of profit, okay and this point is A minus c2, this and this point is c2. So, now, I hope this payoff functions, the diagram of the payoff function is clear. Now, we will derive the Nash equilibrium here, okay.

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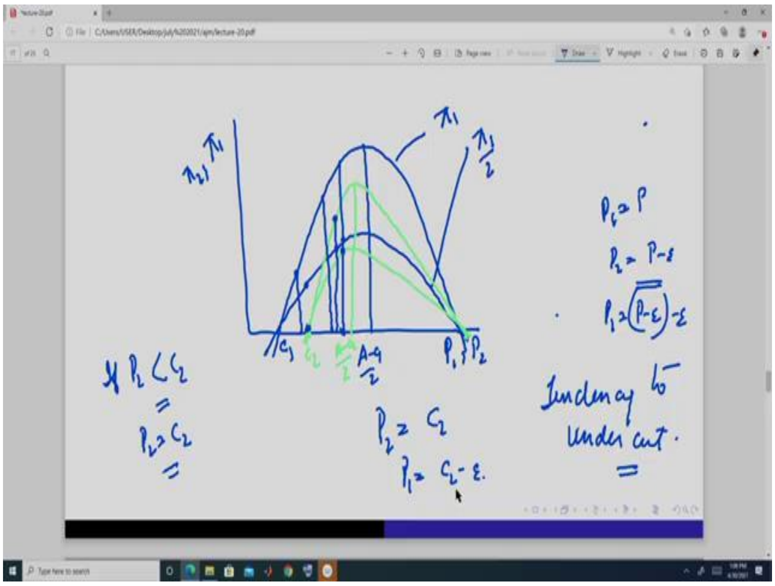

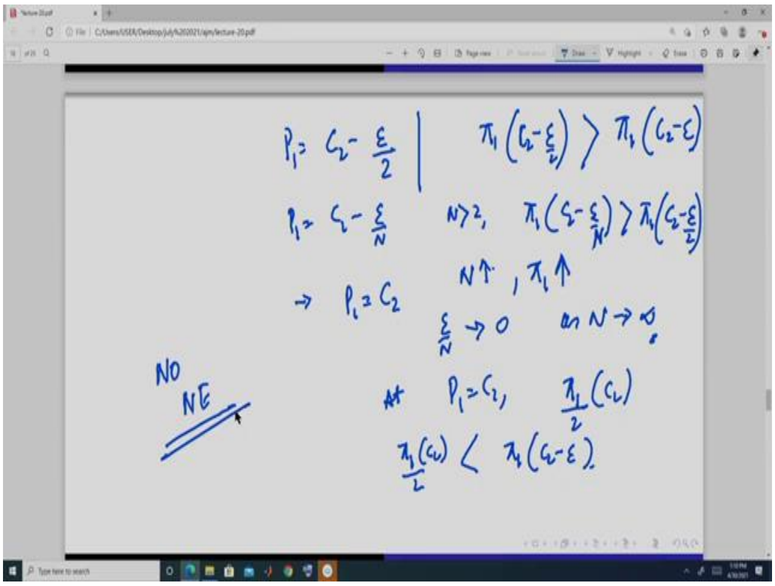

So, you will see that we may not have Nash equilibrium in this. Both are present. okay A minus c2 divided by 2 and this is c2. Now, suppose firm 1 sets a price somewhere here this price. This is p1 equal to p, this. If firm 2 sets a price which is same as this, it is going to get this as a profit and firm 1 is going to get this as a profit. But firm 2 if it sets a price slightly less, its profit is going to be here. So, firm 2 is going to set this price. So, we get this.

Now, if firm 1 sets the price same as this is profit is going to be here, but if it sets the price slightly less than this, so if p1 is this, then it moves profit to here. So, it is going to be like this. So, we see that there is a tendency to undercut and it will go on. So, it will reach like this. So, suppose p1 is equal to c2 firm 2 sets a price, firm 1 set a price which is equal to c2, okay. If it is here then firm 1, see firm 1 has set this. Firm 2 if it sets a price, firm 2, wait, let us, so like this it is going on. And suppose firm 2 sets a price p2 is equal to c as it is going goes on decreasing, it reaches a price c2 and suppose firm 2 that is p2 is equal to c2.

Then if firm 1 matches this price, then its profit is going to be here. But if it sets a price slightly less than this is, its profit is going to be here, right? So, p1 is c2. The moment it sets this price firm 1, firm 2 is not going to set any price, because if it sets a price less than this, if p2 is less than c2 its profit is going to be negative. So, it will set the price is going to be like this only.

Now, here see instead of this price, if firm 1 had set a price which is p1 c2, this- $P_1 = C_2 - \frac{\varepsilon}{2}$, half of this, profit here is going to be greater than, when it is this- $\pi_1\left(C_2 - \frac{\varepsilon}{2}\right) > \pi_2(C_2 - \varepsilon)$, because as the price increases profit increases, because it is less than this price- $\frac{A-c}{2}$. This is the monopoly price, the price at which this we have derived it now. This is maximum. So, firm 1 is going to increase again further if profit is this and N greater than 2, profit is going to be greater, epsilon, like this- $\pi_1\left(C_2 - \frac{\varepsilon}{N}\right) > \pi_2(C_2 - \frac{\varepsilon}{2})$, and will go on increasing. So, profit of 1 is going to go on increasing.

So, finally, it will hit p1 is equal to c2, because epsilon 2 tends to 0 as N tends to infinity, right. So, and when the moment p1 is equal to this c2, firm 2 sets a price which is also c2, its profit is going to be here. So, it was going like this. So, it was going, increasing like this. The moment it hits, it comes from here to here. So, again when it is here, so at p1 is equal to c2 is this which is c, so this is less than epsilon which is this amount here. So, like this it will continue.

So, we see that firm 1 will undercut and then it will go on increasing till the price is c2. And moment it reaches c2, it will again undercut and then it will go on increasing till it reaches c2. So, we see that there is no pure strategy, because when it is firm 2's price is c2 optimal response of firm 1 is to set a price which is slightly less than c2.

Now, when it sets a price which is slightly less than c2 that is epsilon less than c2, then it is better off by again taking half of that A. If we take this c2, but then if we take this half of that distance this is epsilon then it is better off. Again it is further better off if we reduce that distance further. So, it will continue like this and as this portion is going on reducing and reducing, it will hit, the price will hit c2. The moment price again hits C2, profit goes down. So, goes down to this level. So, market is shared equally.

Although firm 2 is earning 0 profit, but it is market is shared equally so that is why firm 1 will again reduce the price by some epsilon. Now, if you keep on reducing that epsilon, your profit is again going on increasing. So, you will go on reducing the epsilon and finally again hit c2. So, this process will go on and so there is no pure strategy Nash equilibrium. So, no pure strategy Nash equilibrium in this equation.

So, the moment we have a CRS production function and so our cost function is like this, but they are different. If suppose one of the firm is efficient, another farm is relatively less efficient then so in a Bertrand competition, we do not have any pure strategy Nash equilibrium, okay. So, it is based on this diagram.

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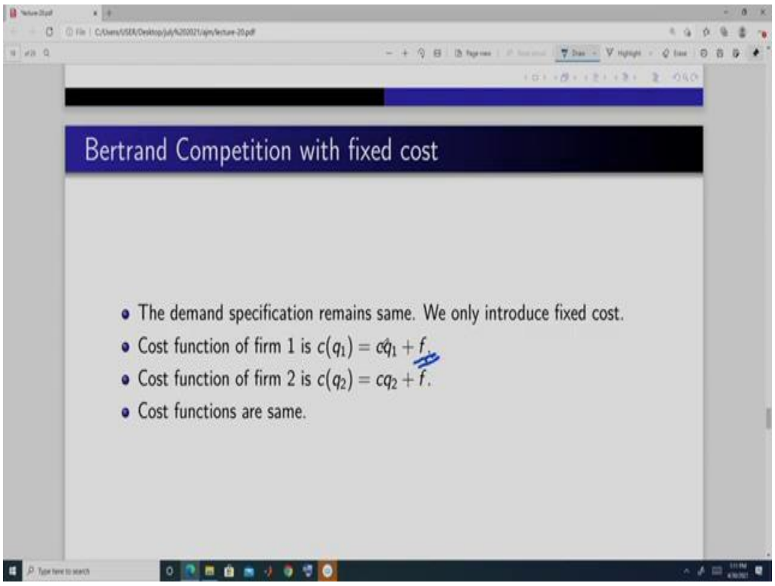

Now, let us introduce fixed cost, okay. So, cost demand specification remains the same as earlier. And the cost function it is also same, but there is a fixed cost component and it is this f.

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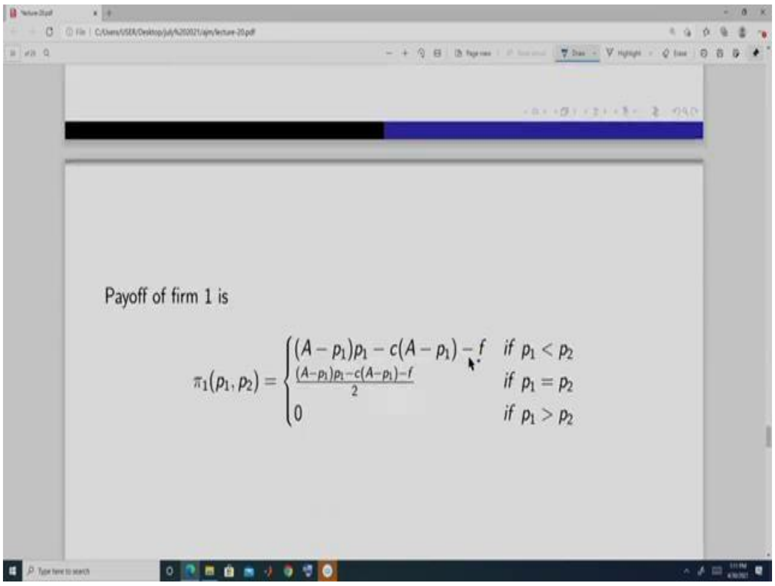

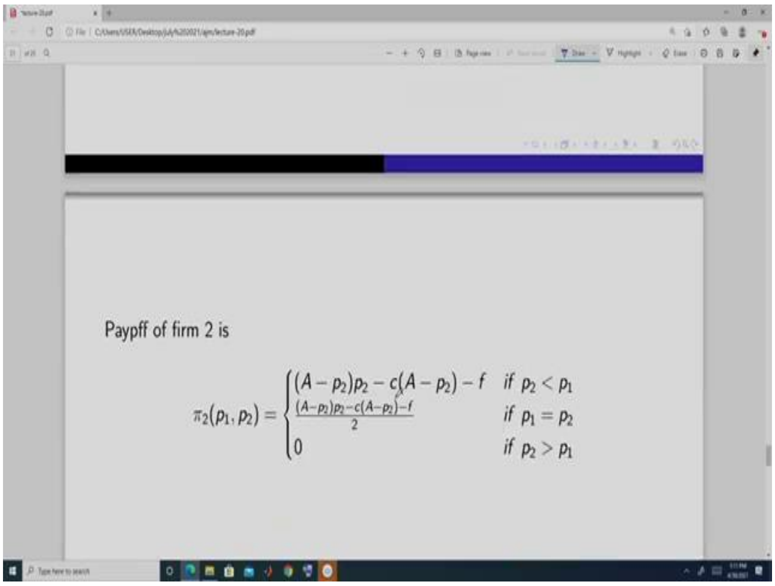

So, we have a component like this - $\pi_1(p_1, p_2) = (A - p_1)p_1 - c(A - p_1) - f$, if $p_1 < p_2$ and we have a component like this here. So, this is the payoff of firm 1 and this is the payoff of firm 2-$(A - p_2)p_2 - c(A - p_2) - f$.

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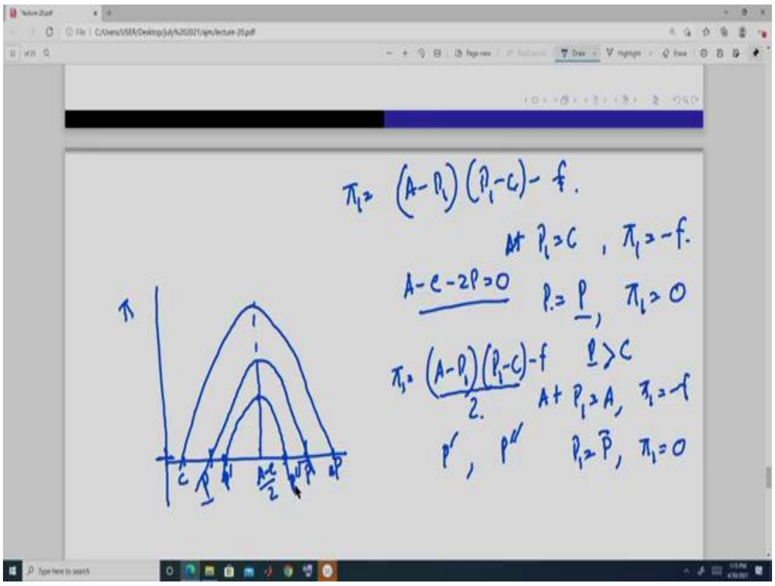
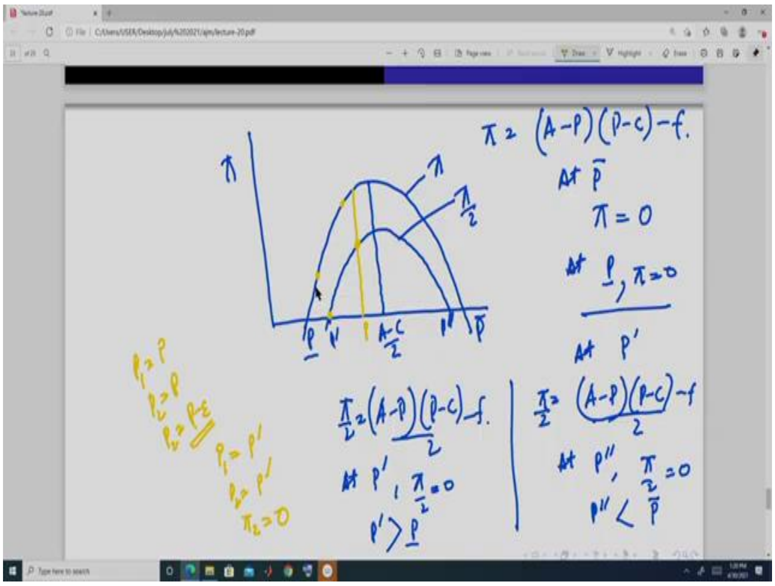
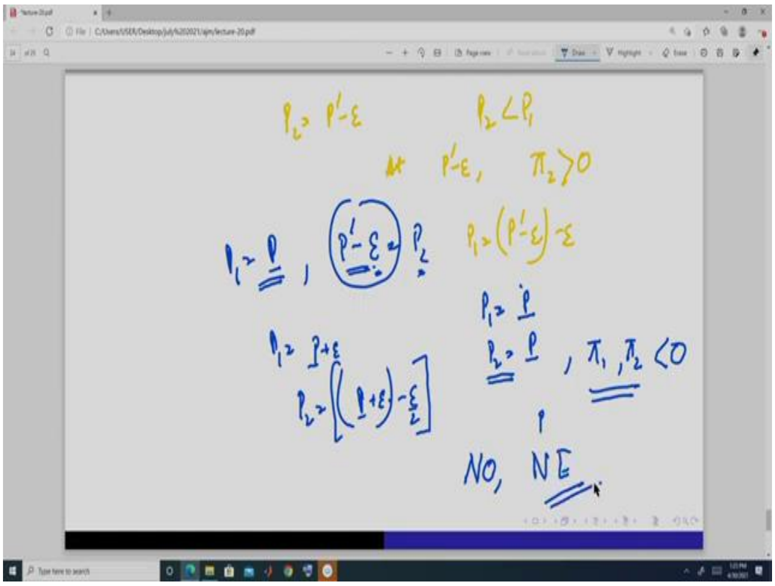
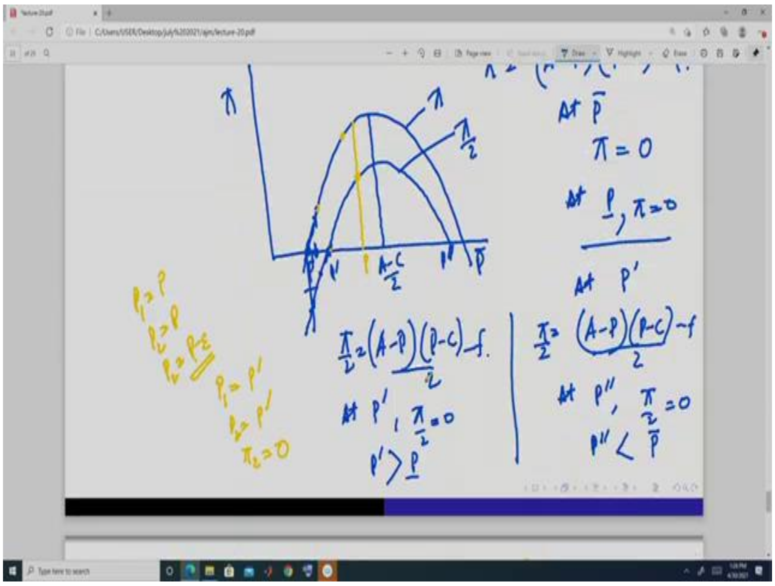



π = (A−P)(P−C)−f.
At P̄
π = 0
At P̲, π = 0
At P′
π/2 = (A−P)(P−C)/2 − f
At P″, π/2 = 0
P″ < P̄
π/2 = (A−P)(P−C)/2 − f.
At P′, π/2 = 0
P′ > P̲
A−C/2
π
π/2
π₂ = 0


P₂ = P′−ε
P₂ < P₁
At P′−ε, π₂ > 0
P₁ = P̲ , P′−ε = P₂
P₁ = (P′−ε)−ε
P₁ = P̲
P₂ = P̲ , π₁, π₂ < 0
P₁ = P̲+ε
P₂ = [(P̲+ε)−ε/2]
NO, NE


At P̄
π = 0
At P̲, π = 0
At P′
π/2 = (A−P)(P−C)/2 − f
At P″, π/2 = 0
P″ < P̄
π/2 = (A−P)(P−C)/2 − f.
At P′, π/2 = 0
P′ > P̲
A−C/2
π
π/2
π₂ = 0

Now, again we explain it, find the pure strategy, whether we have a pure strategy Nash equilibrium or not through the diagrams, okay. So, it is this- $\pi_1 = (A - p_1)(p_1 - c) - f$, right? Now, here you will see that c is same, okay but profit is 0 if you take this, at p1 is equal to c profit is, this portion is 0, so profit is minus f. So, we have a price which is p is equal to lower bar at which p1 is equal to 0, okay. The moment, and p bar is greater than c, marginal cost, okay. And further at p1 is equal to A this profit is, this function is 0, this function is again this is 0, so this is 0. So, we will have negative profit, i.e $\pi_1 = -f$.

So, we will have p is equal to some upper bar at which this is equal to 0. And if we differentiate this, again we get the same thing that is first order conditions this is this- A-c-2P=0. So, if we plot p here, profit here, this is p lower bar and this is p, this is p upper bar and, okay earlier it was, so this was c and this was A and fixed cost was not there, cost profit function as like this as a function of p. But now when we have a fixed costs, so it has become something like this, okay.

So, remember this thing. And this, this thing- $\pi_1 = \frac{(A-P_1)(P_1-c)-f}{2}$, right? So, this will be again further lower. It will be something like this. This portion, but rest it is, this is going to be the maximum. So, this is suppose p dash and this is supposed p double dash. This is p dash, this is p double dash, okay. So, what we have now it is like this. This is p upper bar. At p upper bar this is 0. This is p lower bar. At p lower bar is 0. This is equal to A minus p, p minus c, f, okay.

Now, we have again at p dash suppose we take the and the market is shared between is this. So, when we, earlier this point where they are equal they, it was same, this point, but here it is not like this, because at p upper bar, at p upper bar this is equal to this, but there is half it is, so it has to be lower than p. So, that if it is lower this will take a higher value. So, overall it is. So, that is why it should be at p double dash this is equal to 0 and p double dash is less than p upper bar.

Similarly, earlier again when we take this, at this, this is equal to 0-$\frac{\pi}{2} = 0$ and this p dash is greater than p lower bar. Why, because when we, what is happening, this price is less than, so the maximum for this and this it is same and it is A minus c divided by 2. Now, since this p lower bar is less than this, so as we increase the p here profit increases, right? It is same in this case also. So, at this price only it will make profit, here it will. So, it will be something like this. So, this is the profit, this is when it is shared. Now, these two payoff functions are same for both the firms, right? Now, we have to find the pure strategy Nash equilibrium in this game.

I hope it is clear how we have got this, okay. This is p double dash and this is p dash. This is p lower bar. This is p upper bar, okay. How do we get the Nash equilibrium here? Suppose firm 1 sets a price which is this, firm 1, p1 is equal to p. Now, if firm 2 sets the same, its profit, both firm is going to get this. But if it sets a price which is this epsilon, it will get here. So, this is the best response. So, **so** like this there is a tendency to undercut. And finally price will be here.

Now, see if firm 1 has supposed to reach this price, suppose p1 is equal to p dash. Now, if firm 2 sets the same price, if p2 is equal to p dash, then profit of firm 2 is also 0. It is this. But if instead, if it sets a price which is p2 is epsilon, then what is happening, p2 is less than p1. So, it shifts to this level, which is positive. So, at this is positive. So, firm 2 will not set p2 is equal to p dash, but it will slightly reduce it. So, if it reduces then it is getting some positive amount. So, it will like this. So, firm 2 will again reduce. So, firm 1 is going to like this, right? So, firm 2 is going to go like this. It will continue reducing the prices. So, it will finally reach this point, okay. So, at this point, what is happening? So, it will go on.

So, finally, suppose p1 is equal to p lower bar. Now, at this, if firm 2 sets p is equal to lower bar, then this in this A it is negative, right? So, both the firm p1 and p2 both are getting negative. So, firm 2 will not set a price like this. So, firm 2 is going to set, is not going to set a price which is in this case below p dash. Because what is happening, if we reduce go on you will continue. But moment you are here, if it hits, then you get a negative A. So, here if you want to share the market it is better to share here, right?

Now, what is going to happen? Firm 1, since firm 2 is not reducing the price anymore, because it is less here, but if firm 2, firm 1 sets a price slightly higher than this, it will, firm 2 will set a price less than it and then again it will go here. Moment it is here firm 1, is suppose, suppose firm 1 price is at p lower bar, okay and firm 2 is still it is at some price which is this minus some epsilon, okay. Now, here instead if firm 1 is slightly go on increasing but less than this, then what is happening, is profit is increasing, because you look at this graph as it moves here is profit increases.

So, suppose firm 2 stops here. So, it will increase like this. Moment it sets a price higher than this, then moment it sets a price which is suppose this plus small amount some epsilon, then best response for firm 2 is to set something like this- $P_2 = [(P + \varepsilon) - \frac{\varepsilon}{2}]$, slightly less than this. So, again firm 1 will reduce and then they will go on doing that and it will reach this price. And then the other firm is not going to reduce any further. So, it will stop at some price like this. Then firm 1 is again going to go on increasing. It will, the moment it increases more than p lower bar then firm 2 is going to react and it is. So, in this case also we have no pure strategy, Nash equilibrium.

So, if we introduced fixed cost in Bertrand competition with CRS production function or CRS cost function then we do not have any pure strategy Nash equilibrium. So, what do we get? So, in Bertrand competition where the firm set prices, if the marginal costs are seen and it is constant, that is CRS, and there is no fixed cost, then we have a pure strategy Nash equilibrium and it is such that the prices are equal to marginal cost. So, they do not make any profit and this is called the Bertrand paradox.

Now, if the marginal costs are constant, and but they are different, so one firm is more efficient than the other firm. So, we have shown that in that case also if market is shared equally when the prices are same, in that case, we do not have any pure strategy Nash equilibrium and it was given here. Next, we keep the marginal cost same and constant and also we introduced fixed cost. So, in this case also we see that there is no pure strategy Nash equilibrium in the market, okay.

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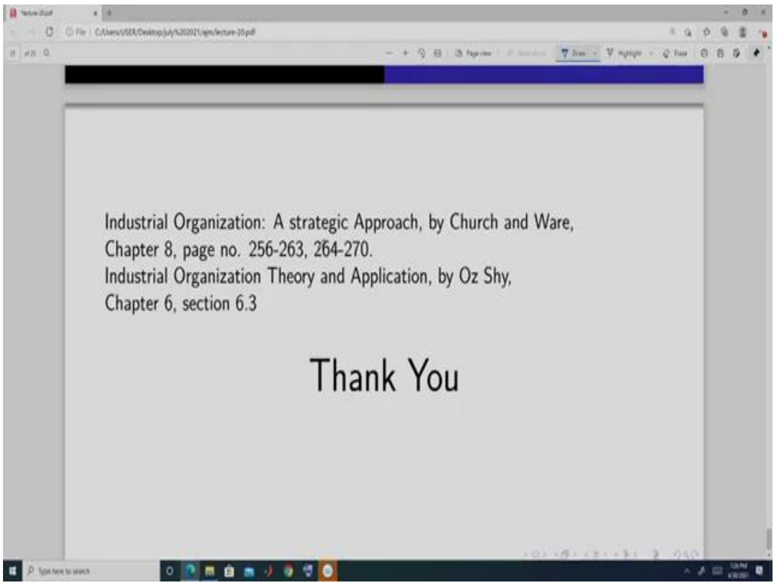

So, with this, we end this portion of Bertrand competition and you can read it from this portion. From Church and Ware you can read from these pages and from Industrial Organization Theory and Application by Oz Shy you can read section 6.3. Thank you.