Introduction to Market Structures
Professor Amarjyoti Mahanta
Department of Humanities and Social Sciences
Indian Institute of Technology Guwahati
Module: 7
Lecture 27: Tutorial

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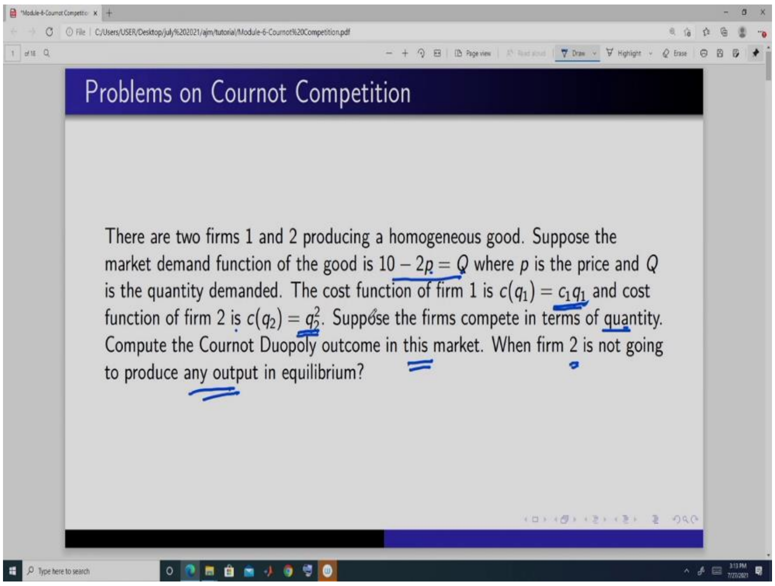

So let us discuss some problems on Cournot competition. So in this example suppose there are 2 firms producing a homogenous product and the market demand function is this- 10–2p=Q where p is the price and Q is the quantity demanded. Cost function of firm 1 is this- $c(q_1) = c_1 q_1$, cost function of firm 2 is this- $c(q_2) = q_2^2$.

So it is CRS and it is decreasing returns to scale and suppose the firms compete in terms of quantity so it is a Cournot competition. Compute the Cournot duopoly outcome in this market and when firm 2 is not going to produce any output in equilibrium. So we have to find out this also, right?

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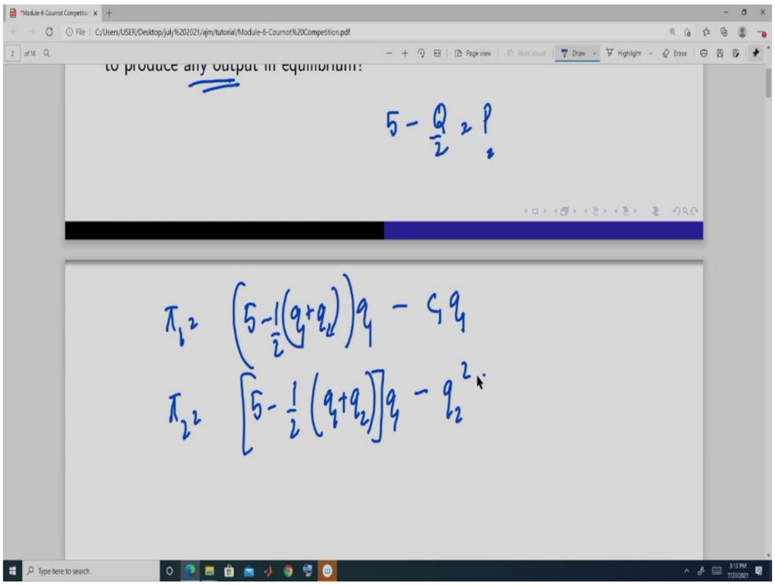

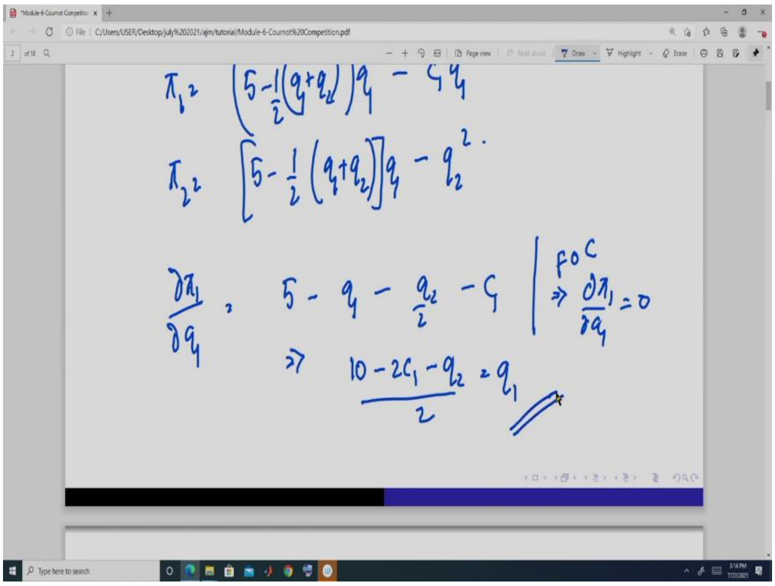

Now so let us, so from this, we get the inverse demand function is, it is this- $5-\frac{Q}{2}=P$. So the profit function of firm 1 is this- $\pi_1=\left(5-\frac{1}{2}(q_1+q_2)\right).q_1-c_1q_1$ profit function of firm 2 is-$\pi_2=\left(5-\frac{1}{2}(q_1+q_2)\right).q_1-q_2^2$ where q1 and q2 are output of firm 1 and firm 2. It is q square I think. It is this, they are differentiable, so we optimize this with respect to their outputs. So this we get- $\frac{d\pi_1}{dq_1}=5-q_1-\frac{q_2}{2}-c_1$, First Order Condition implies. So this, is the reaction function of firm 1- $\frac{10-2c_1-q_2}{2}=q_1$, okay.

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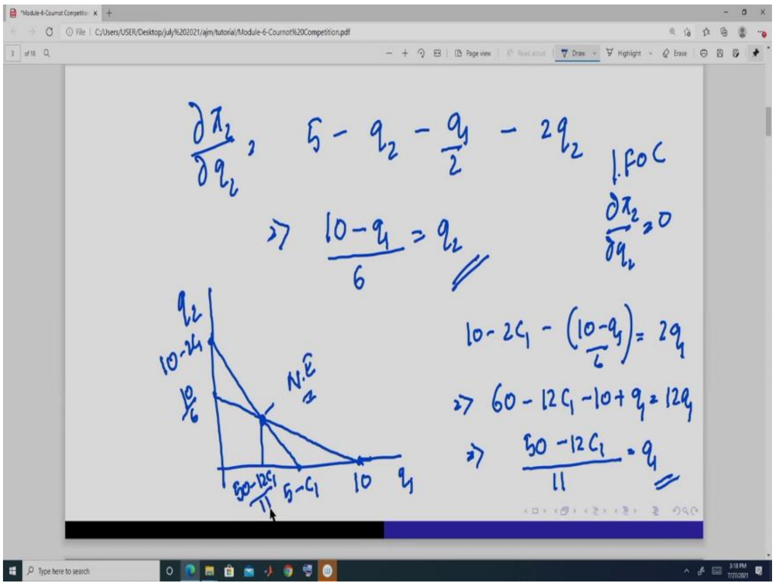

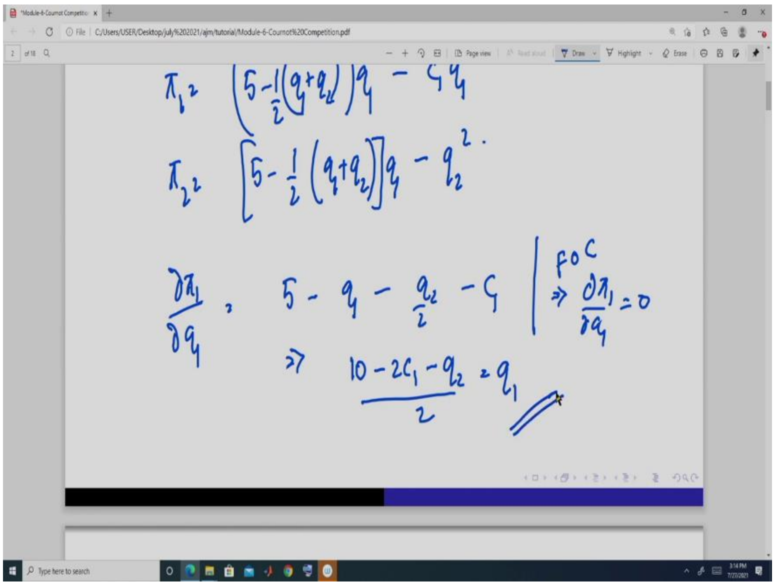

And so here again, you get this first order condition implies is equal to this. So we get this is the reaction function of firm 2- $\frac{10-q_1}{6} = q_2$. So if we plot these reaction functions, okay. If we are plotting these 2 reaction functions, this, so this is and this is equal to 0. This is 5 and when this is equal to 0 then because when q1 is equal to 0, it is going to be this point is 5 minus c1 and when q2 is going to be 0 then it is, this is, so we take common because this is equal to 0. So this is 5, see this 10 minus 2c1 equal to q2 so this point is 10 minus 2c1 and this point is 5 minus c1.

It is this and this reaction is q is equal to, this is 10 and this point is suppose 10 by 6. And these two intersect here. So this is the Nash equilibrium. And we get it by solving these two reaction functions. So it is 10 minus 2c1. If we do this, we get this, this is output of firm 1- $\frac{50-12c_1}{11} = q_1$ so this point is 50 minus 12c1 divided by 11.

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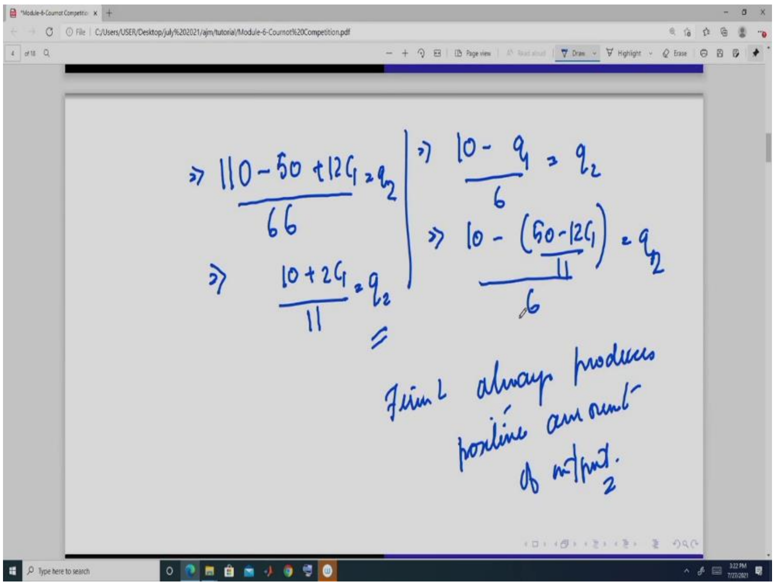

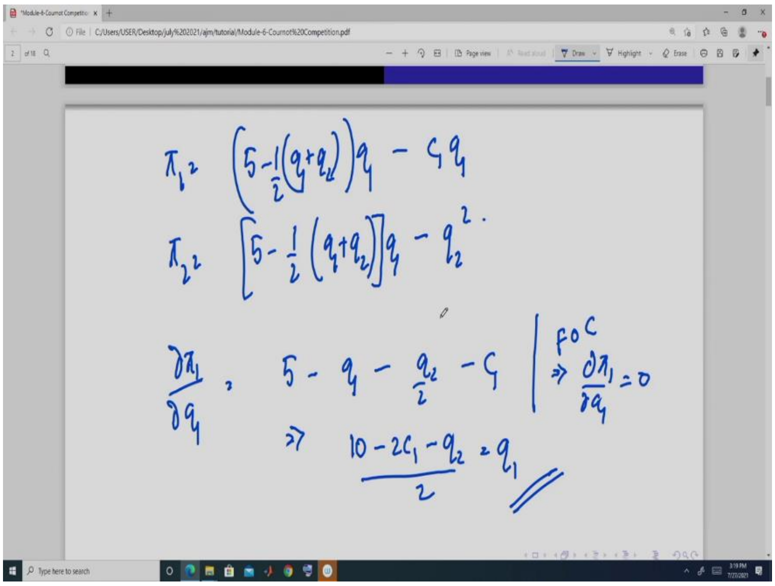

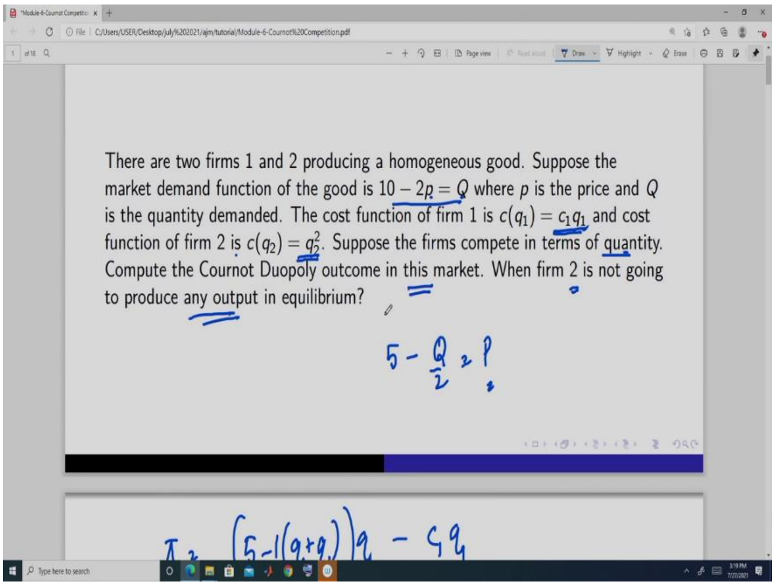

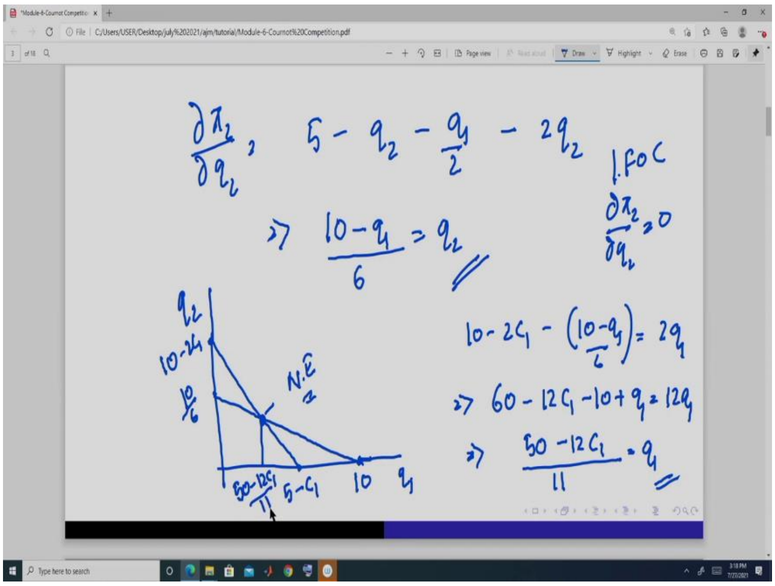

And this height is, it is 10 minus q1, this. So it is 10 minus 50 minus 12 c1 C2. So this is we can again write this, our 10 plus 2c1 divided by 11, this- $\frac{10+2c_1}{11} = q_1$. So these- $\frac{10+2c_1}{11} = q_1, \frac{10-q_1}{6} = q_2$ are the two pure strategy Nash equilibrium or Cournot Nash equilibrium outcome of this game and we can plug in them in the profit function here and we will get the profit.

Now here again we have to find when firm 2 is not going to produce any output in equilibrium. So it is, so this is the reaction function of firm 1, right? Now it is not going to produce any output, when? So when this, this and it is this, suppose this is the reaction function of firm 1 which is given by this point, this point- $\frac{10-2c_1-q_2}{2} = q_1$or okay, the question is this. When firm 2 is not going to produce any output in equilibrium, when firm 2 is not going to produce.

So see here it is this is the Nash equilibrium outcome. When do we get this? When this is less than this and this is less than this, right? Now this is the reaction function of this. So this has to be so much here that it has to go below it. Then only it is not going to produce anything but and it is not this diagram, okay.

So an output for this equal to 0, c has to take some negative here, which is not possible. So that is why firm 2 always produces positive amount of output, always. But firm 1 may produce 0 when c1 is sufficiently high that is when this is or this line is less like this.

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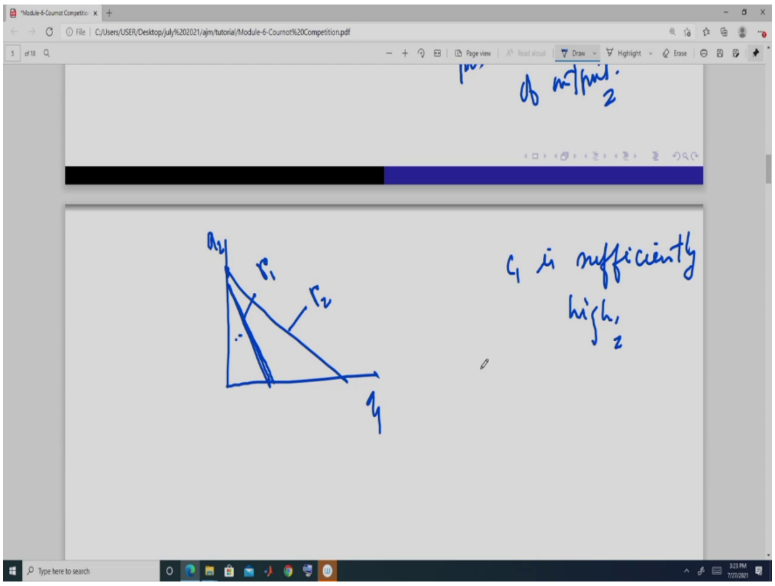

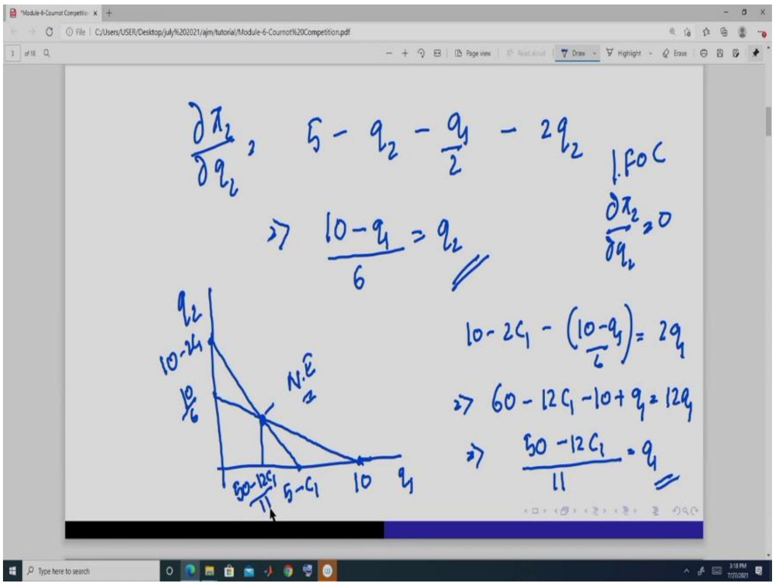

So if we have a reaction function of firm 1 and this is the reaction function of firm 2. So firm 1 will not produce anything, firm 2 is going to produce this much amount of output. But since we cannot reduce this below put this curve below this or we cannot have this curve greater than this, why?

Because if you look at this point, this is 5 minus c1. So it cannot be greater than 10. So that is why we cannot have this curve line above this curve, okay. Reaction function of firm 1 lying above the reaction function of firm 2. So that is why firm 2 is always going to produce some positive amount but firm 1 may not in this situation when c1 is sufficiently high, okay. So we get this.

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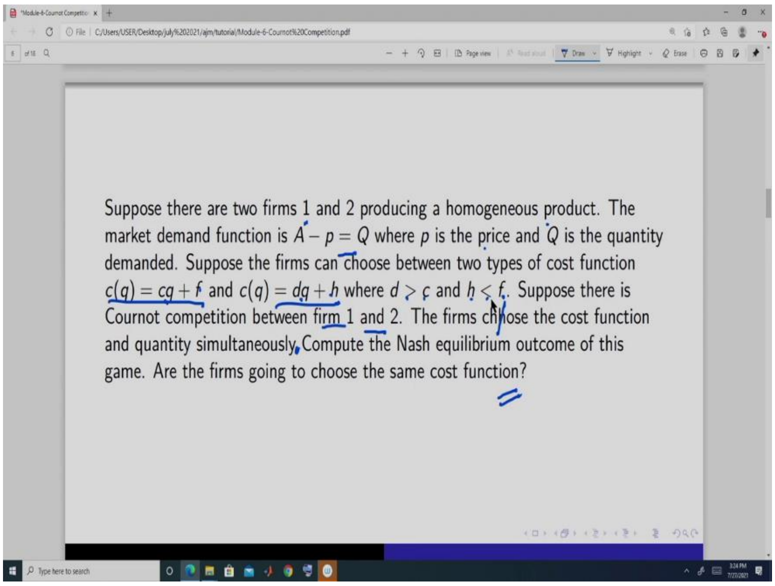

Now next, solve another problem. This is slightly involved problem. Suppose there are two firms; firm 1 and firm 2 and they produce a homogenous product and market demand function is this- A–p=Q. A is a positive number and Q is the quantity demanded, p is the market price. Suppose the firm can choose between 2 types of cost functions. Its cost function can be of this nature- $c(q) = cq + f$ and cost function can be of this nature- $c(q) = dq + h$. Both has this option where d is greater than c and h is less than f.

So here in this cost function marginal cost here is less than the marginal cost this. But here fixed cost is greater in this case than this. And suppose there is Cournot competition between firm 1 and firm 2. And the firms choose the cost function and quantity simultaneously, okay. So these two decisions are taken simultaneously and compute the Nash equilibrium outcome of this game. Are the firms going to choose the same cost function? okay So these are the questions.

Now we have this specification. So choosing between 2 different types of cost functions, it means choosing different type of technique or different combinations of input bundle to produce the same amount of output or to choose a different technology or to choose in such way that the setup cost is also here different because f and d, okay.

So there are many possibilities which may give rise to this situation. It is mainly because of 2 different production functions or it may be two different types of price of inputs or it may be different plant sizes that we have discussed that is one land plot is big, another land plot is small and they have same – they can choose between machines and labor. Both of them are variable. All these are possibilities are there.

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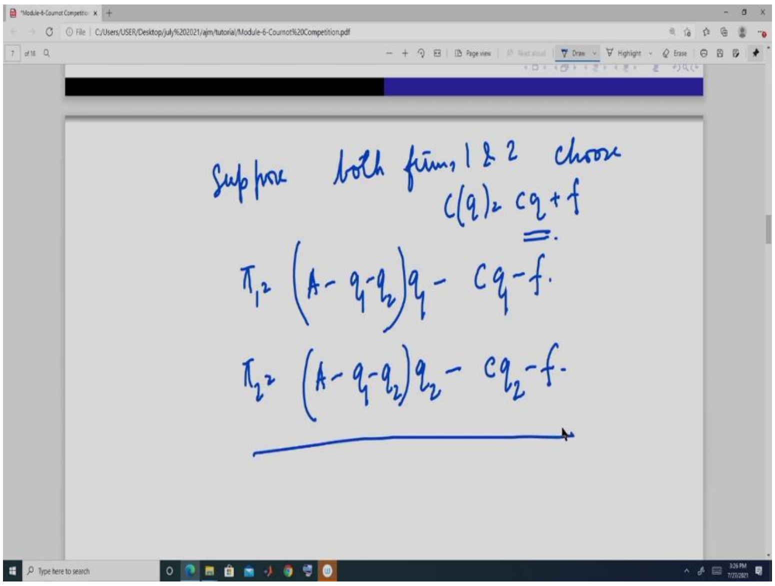

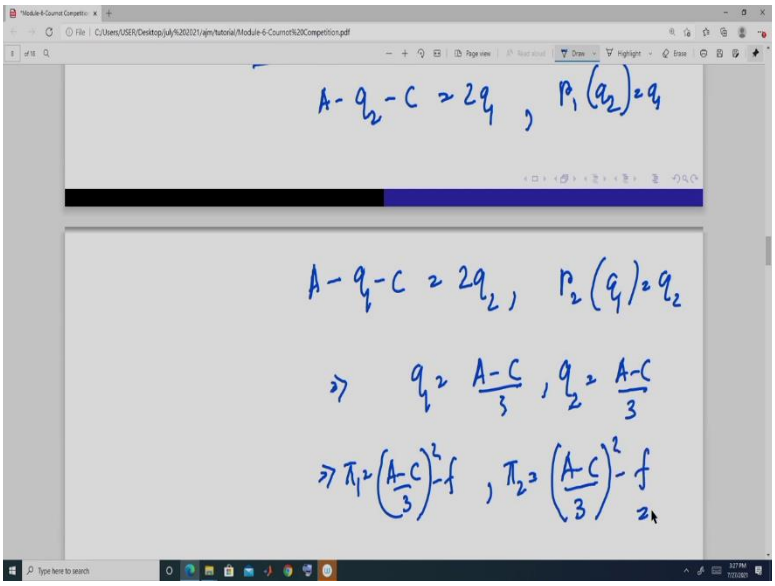

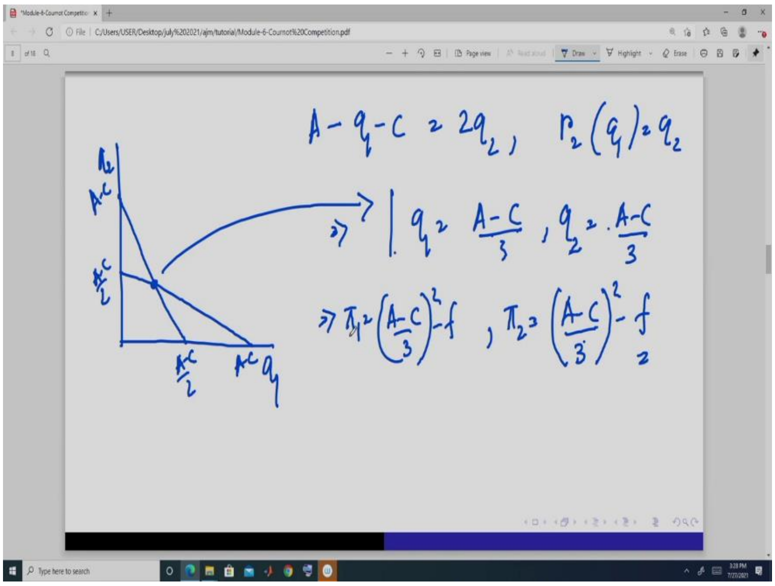

Now in this case, suppose, both firms 1 and 2 choose this production function- c(q)= cq+f. Then profit of firm 1 is this- $\pi_1 = (A - q_1 - q_2)q_1 - cq_1 - f$, profit of firm 2 is this-$\pi_2 = (A - q_1 - q_2)q_2 - cq_2 - f$. So, we do the usual process so the reaction function of firm 1 from this we get A minus this- $A - q_2 - c = 2q_1, p_1(q_2) = q_1$. This is the reaction function and the reaction function of firm 2 is- $A - q_1 - c = 2q_2, p_2(q_1) = q_2$ so if we solve this we get the Cournot outcome as we have already got this- $q_1 = \frac{A-c}{3}, q_2 = \frac{A-c}{3}$ and in this situation we know profit of firm 1 is, it is this-$\pi_1 = \left(\frac{A-c}{3}\right)^2 - f, \pi_2 = \left(\frac{A-c}{3}\right)^2 - f$, right? Now if we look at the reaction functions, and this is the point, this point. This point is this, right?

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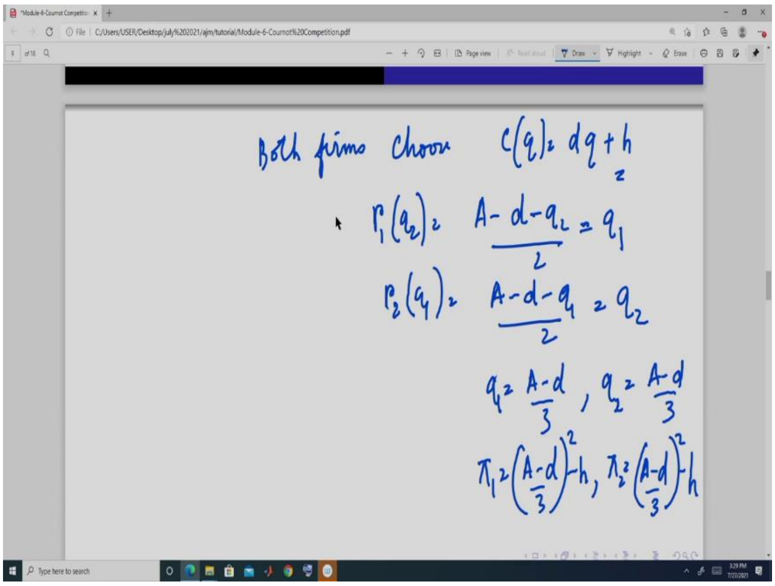

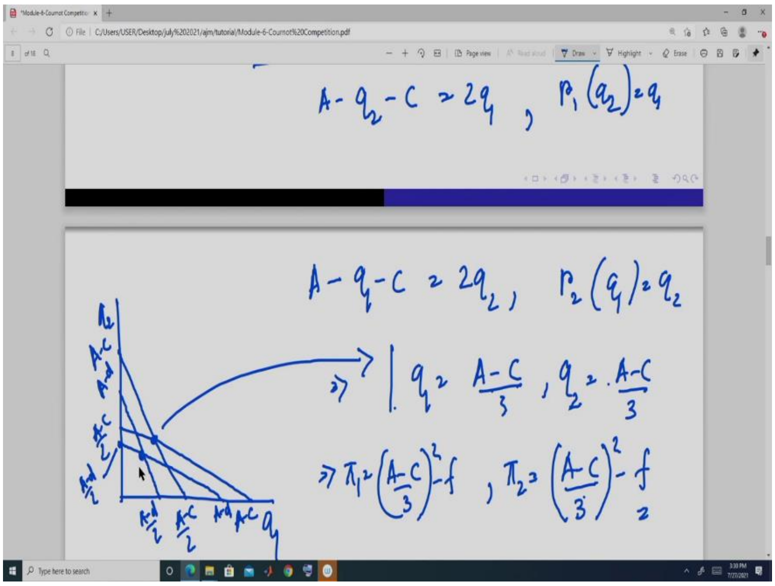

And now suppose both the firms, both firms choose this-c(q)= dq+h then again we know what is going to be the reaction function. It is this-$p_1(q_2) = \frac{A-d-q_2}{2} = q_1, p_2(q_1) = \frac{A-d-q_1}{2} = q_2$ and so the outcome is, so profit is- $\pi_1 = \left(\frac{A-d}{3}\right)^2 - h, \pi_2 = \left(\frac{A-d}{3}\right)^2 - h$. And in this reaction function we will get d is greater than c so we will get line like this A minus d this is A minus and this is this A minus d this point is A minus d by 2 so this is the outcome, right?

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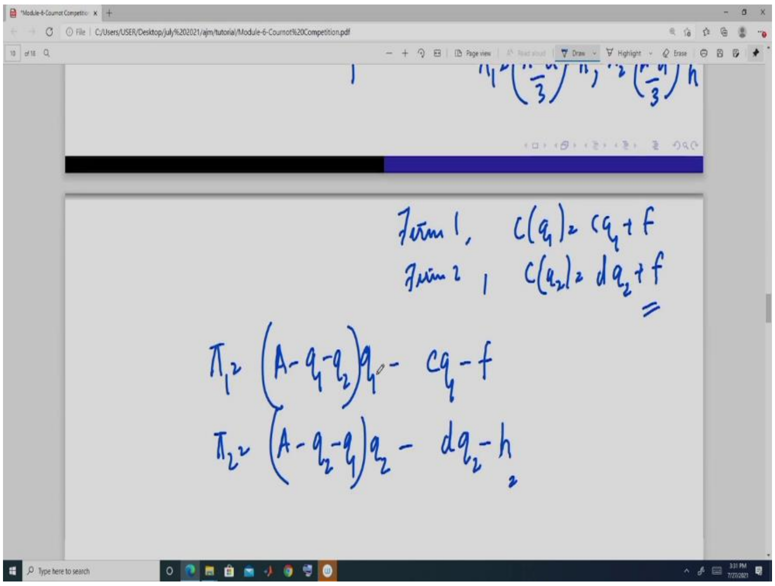

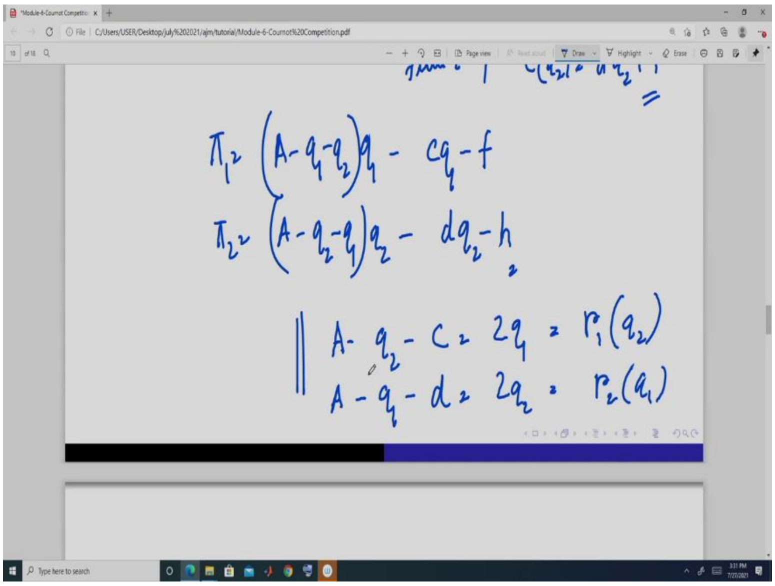

Now if firm 1 if any one of them chooses. Suppose firm 1 chooses this- $c(q_1) = cq_1 + F$ and firm 2 chooses this- $c(q_2) = dq_2 + F$, then what is going to happen? Profit of firm 1 is this- $\pi_1 = (A - q_1 - q_2)q_1 - cq_1 - f$. Profit of firm 2 is this-$\pi_2 = (A - q_2 - q_1)q_2 - dq_2 - h$. And the reaction functions of firm 1 is going to be n reaction function of firm 2.

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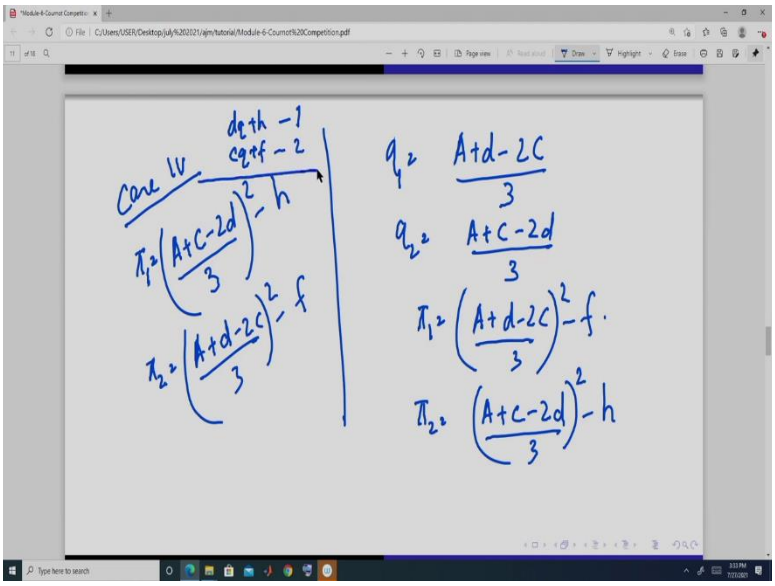

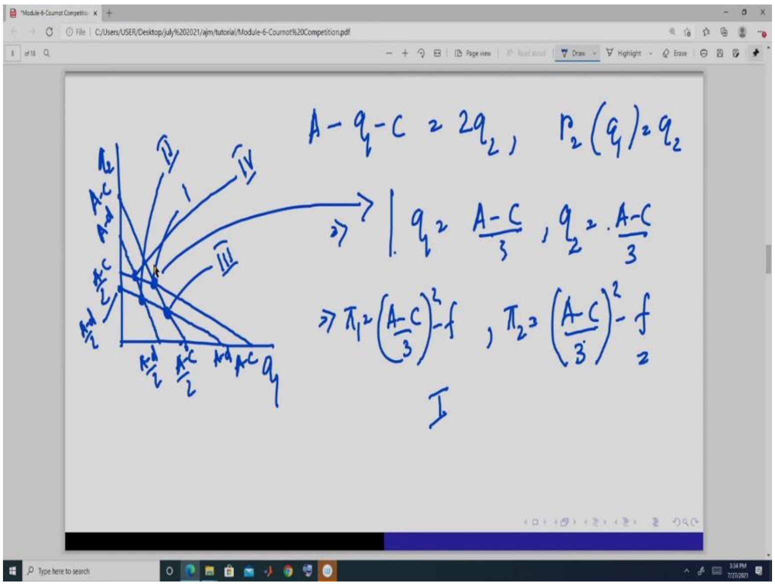

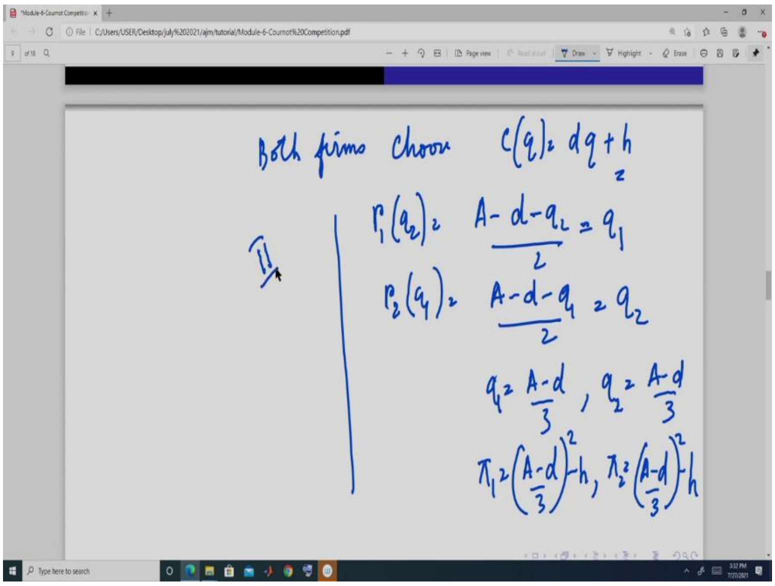

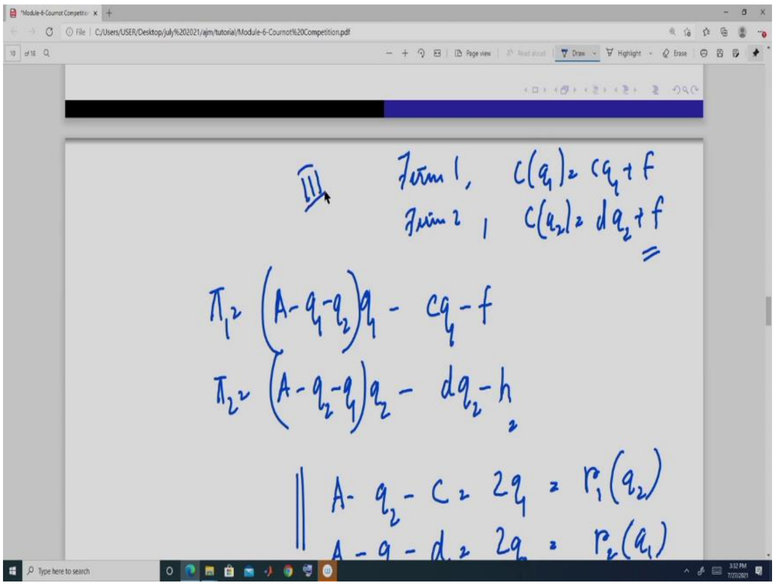

Solving these two, solving these two we get that q1 is A plus d, i.e. $q_1 = \frac{A+d-2C}{3}$ and q2 is A this and this-$q_2 = \frac{A+c-2d}{3}$. So and you can look at profit, profit is also. This is for firm 1- $\pi_1 = \left(\frac{A+d-2c}{3}\right)^2 - f$ and for firm 2-$\pi_2 = \left(\frac{A+c-2d}{3}\right)^2 - h$. It is this and if we look at this diagram, reaction function of firm 1 is this, reaction function of firm 2 is this. So this is the outcome. So this is when we are in case 1. This is case 2 so this is case 1. This is case 2. This is case 3. And case 4 is so we will just get the opposite. It is going to be h because in case 1 firm 1 is choosing the h. This is firm 1 and this is firm 2, okay. So we get this- $\pi_1 = \left(\frac{A+c-2d}{3}\right)^2 - h, \pi_2 = \left(\frac{A+d-2c}{3}\right)^2 - f$. Now we have to compare this profit and these decisions are being taken simultaneously. So it is something like. So this is case 3 and this is case 4.

So now when we can have this case or this case or this case or this case. So when we have this case that means firm 1 has chosen this. Its reaction function is this because its cost is, marginal cost is c and firm 2 has also chosen marginal cost such that it is c and they have got this here. So then firm 2 has 2 options either to choose c or it can choose D these two. If I fix the near firm 1 suppose this is the A, it has 2 choices, either to choose this or to choose this. Now how we get this? Which one is chosen?

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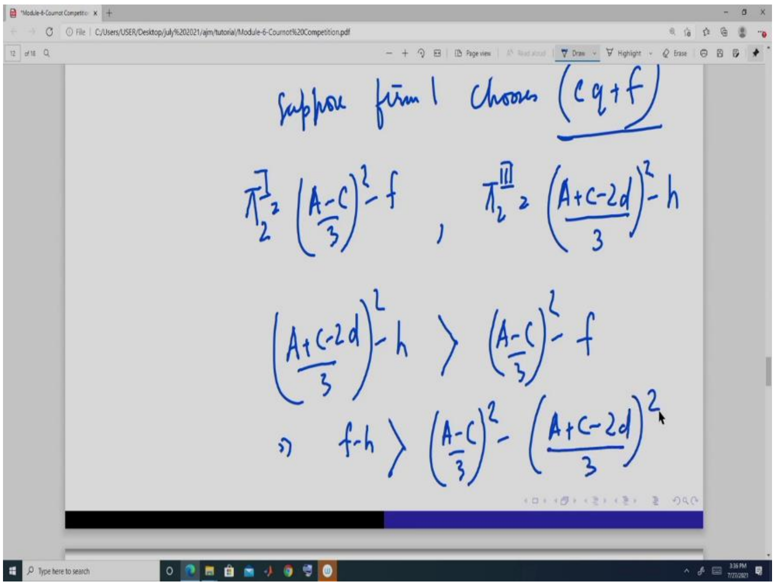

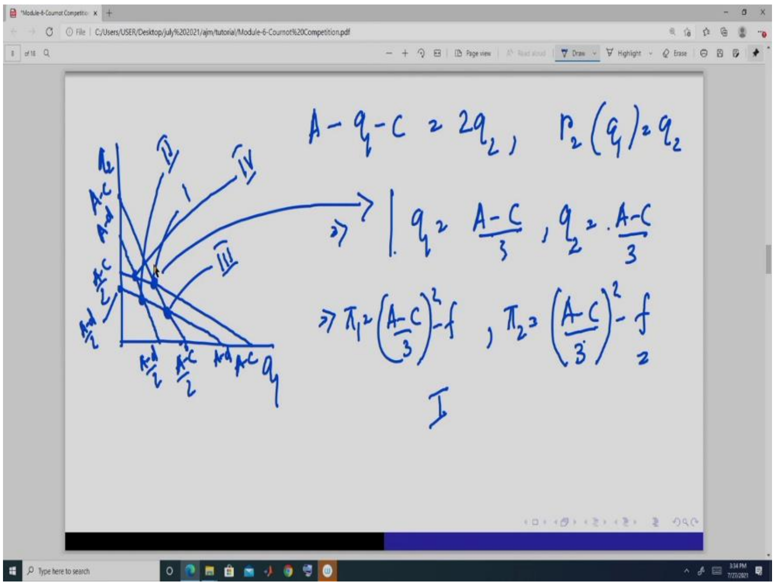

So suppose firm 1 chooses this- cq+f, okay. Then firm 2 profit it has to compare between these 2 cases. Case 3 and case 1, case 3 and case 1. So the profit of firm 1 in case 1 is, of firm 2 is- $\pi_2^1 = \left(\frac{A-c}{3}\right)^2 - f$ and profit of firm 2 in case 3 is it is this-$\pi_2^{iii} = \left(\frac{A+c-2d}{3}\right)^2 - h$, when it is going to choose this. It has to compare between these two when it is possible. When this$\left(\frac{A+c-2d}{3}\right)^2 - h$, is greater than this- $\left(\frac{A-c}{3}\right)^2 - f$ and this implies f minus h should be this.

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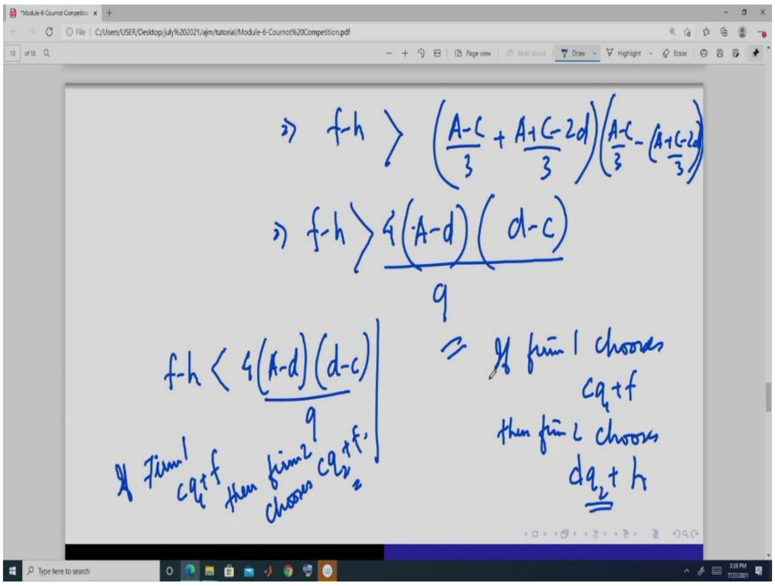

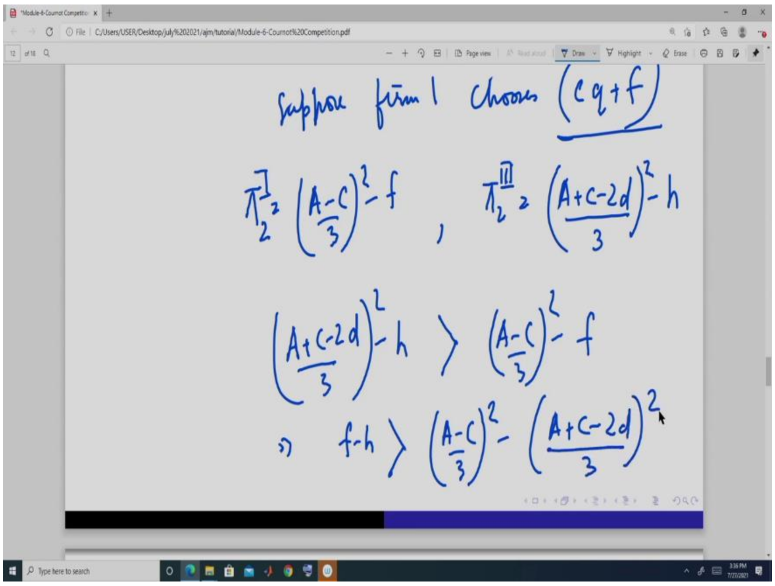

So this implies that f minus h, so this implies f minus h should be greater than this- $\left(\frac{A-c}{3}-\frac{A-c+2d}{3}\right).\left(\frac{A-c}{3}-\left(\frac{A+c-2d}{3}\right)\right)$. So this we know f is greater than h but that difference should be greater than this if this is the case then when firm 1 choses this firm 2 chooses. So if this is the case when then if firm 1 chooses cq this then firm 2 chooses this as the cost function, right? or if this is not satisfied if then firm 2 both will, if firm 1 chooses this then firm 2 chooses this only. If this, then firm, if firm 1 chooses cq plus f then firm 2 chooses cq2 plus f. It should be h.

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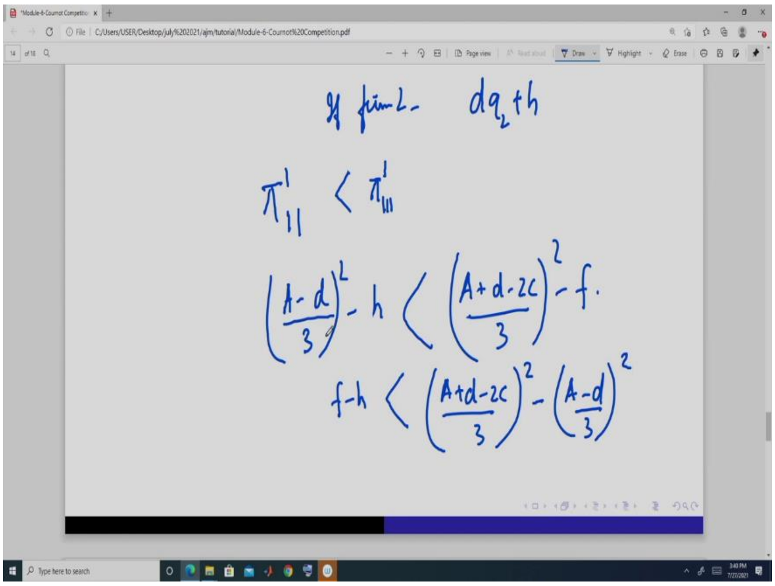

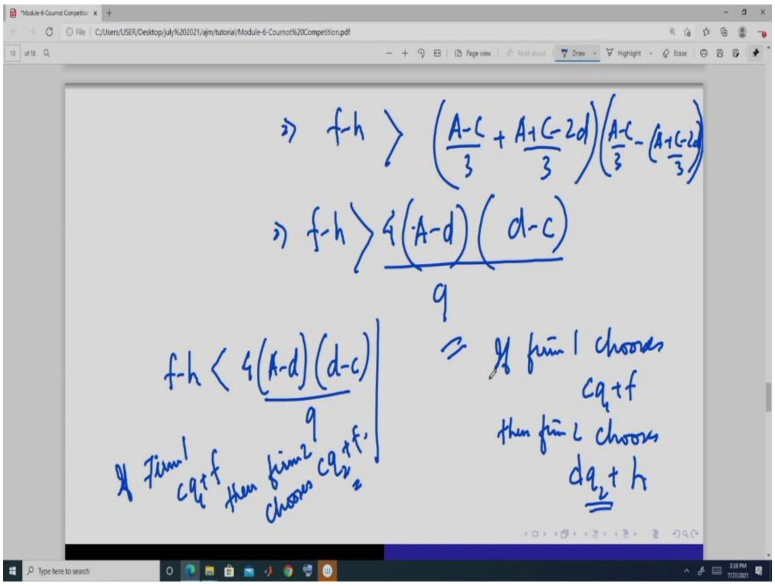

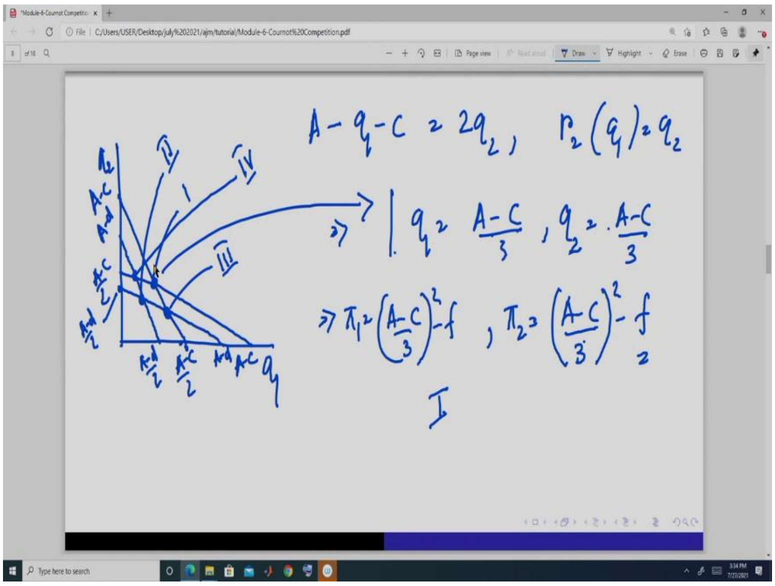

Now if firm 2 is suppose choosing this. If firm 2 is choosing this-$dq_2 + h$ given firm 1 has chosen this so is it optimal for firm 1 to find out whether that it is a Nash equilibrium. So we have to compare this with firm 1 this point this should be. So suppose firm 2 has chosen this then we have to compare firm 1 whether it will choose this point or this point. This point or this point oh not 4, it is 2. It has to be 2. So then this means. It is this-$\left(\frac{A-d}{3}\right)^2 - h < \left(\frac{A+d-2c}{3}\right)^2 - f$, so then it means it is this.

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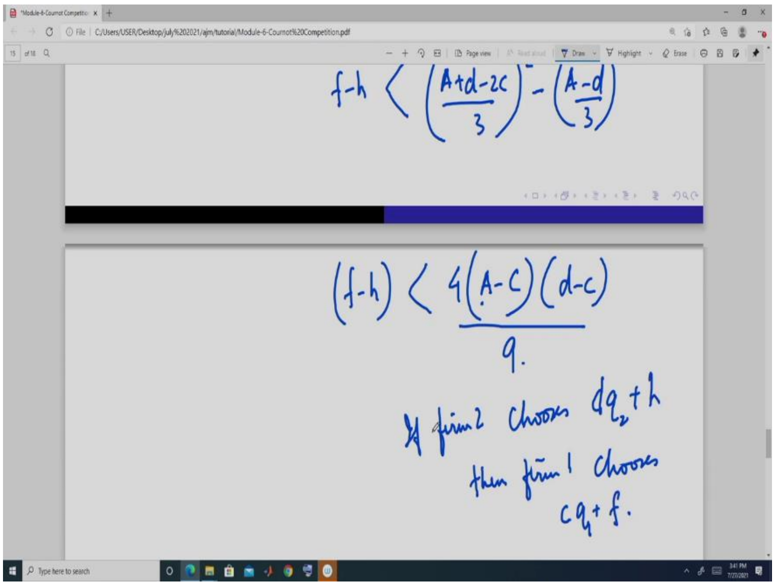

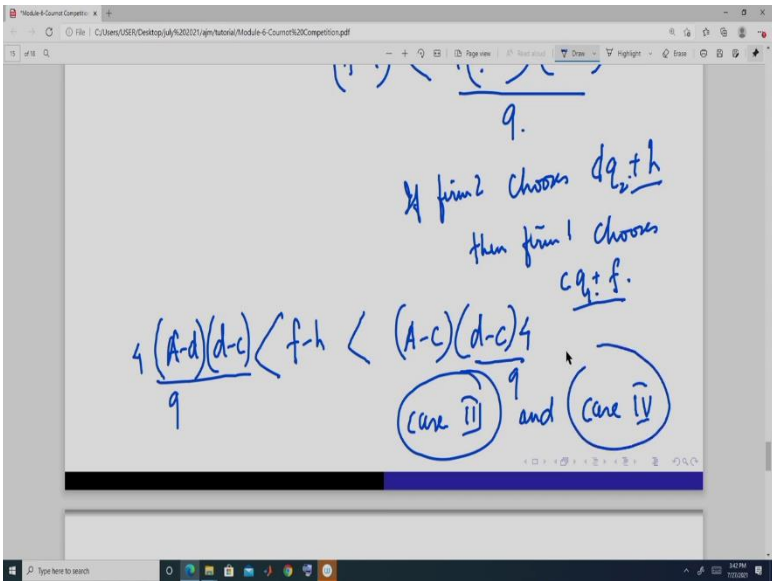

So again this means it should lie here. So if this is the case then if firm 2 chooses this cost function-$dq_2 + h$ then firm 1 chooses- $cq_1 + f$ it has to be h chooses this cost function. So we get that if this is less than A minus c d minus c 4 by 9 and if this is greater than A minus d into d minus c. If it lies here then the outcome is either case 3 and case 4, both can happen. Either firm 1 chooses this cost function-$cq_1 + f$ and firm 2 chooses this cost function- $dq_2 + h$. So this is the case- case iii, or firm 1 chooses this cost function- $dq_2 + h$ and firm 2 chooses this cost function- $cq_1 + f$. So this is the case, i.e case iv.

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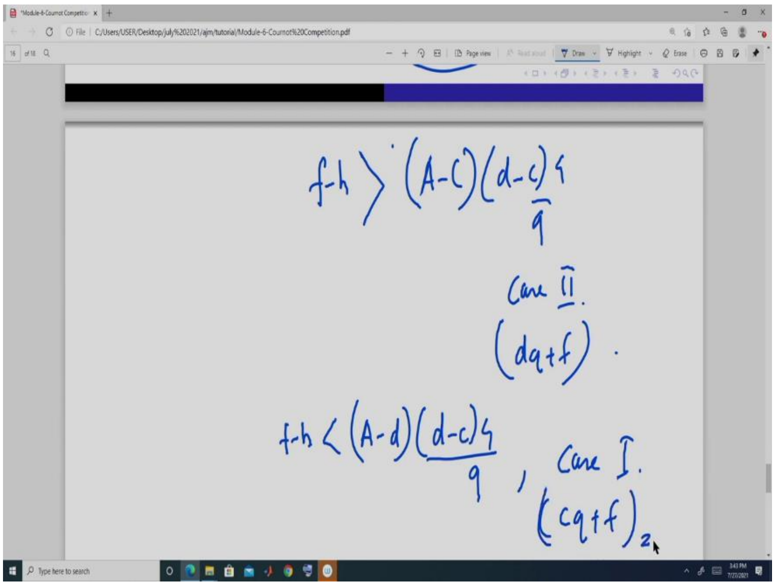

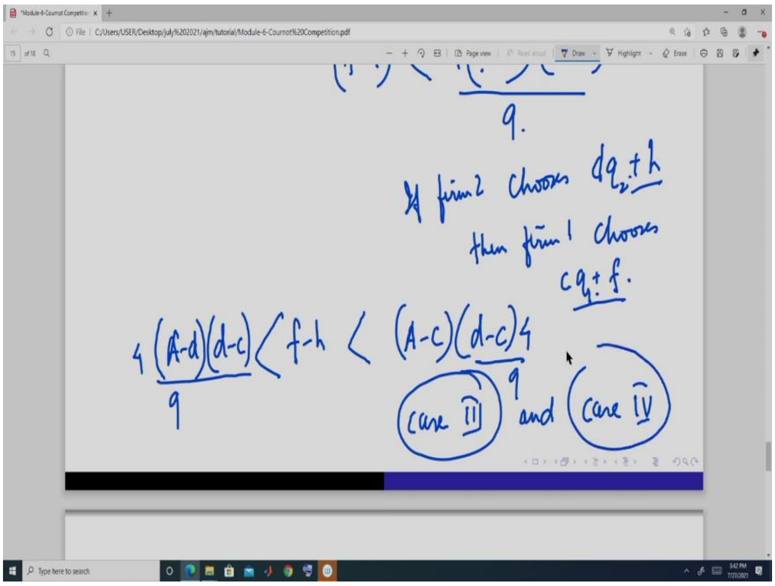

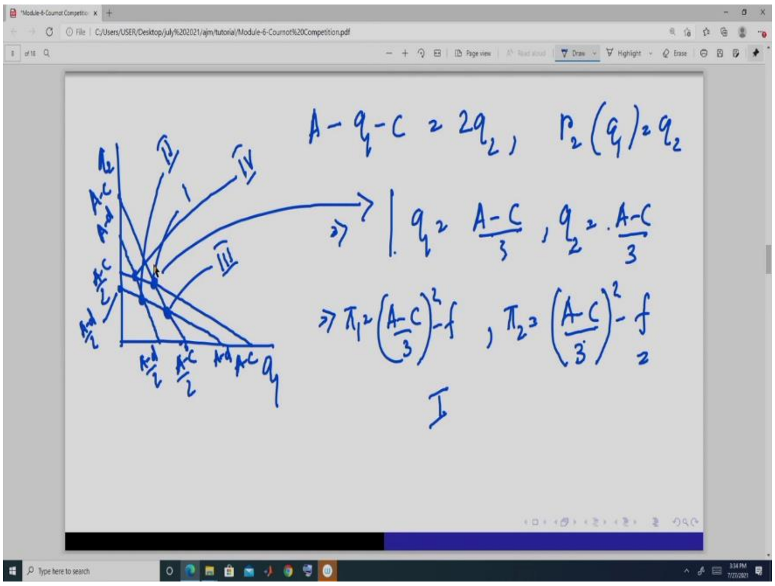

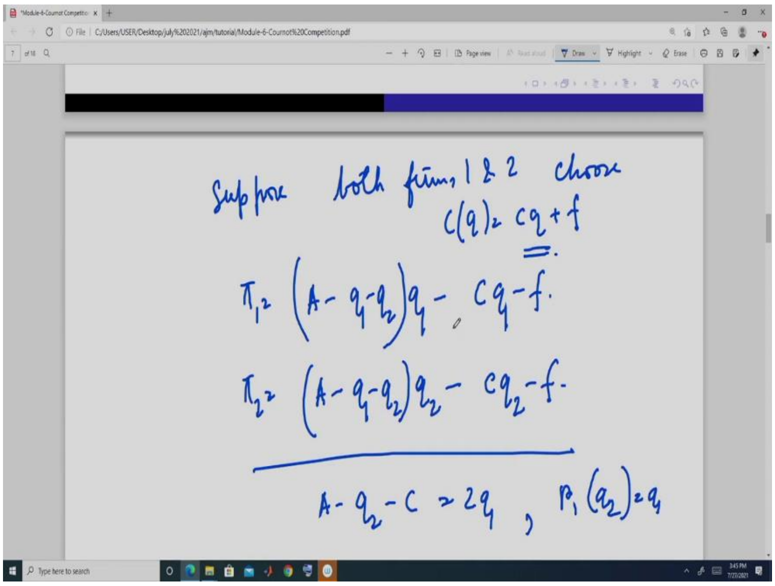

And if this is greater than, this, i.e. f-h> (A-c)(d-c)4/q then we have case 2. Both firms choose this cost function-(dq+f) and if we have this,- f-h< (A-c)(d-c)4/q . This is less than this then we have case 1 and both firms choose this as the cost function. So what do we get? We get many possibilities. So we have one situation where we have a symmetric situation that is case 2 and case 1. We have this when, the difference between the fixed cost is either sufficiently high or the difference is sufficiently low. Case 2 when it is sufficiently high, case 1 when it is sufficiently low.

When it lies between a range then we have an asymmetric situation and it is this. Firm 1 chooses first type of cost function and firm 2 chooses second type of cost function. This is 1 that is case 3 and we may have a this case also. So we have multiple Nash equilibrium here in this equation. See which simply introduction of a choice over the cost function that is if you have choice over cost function that means a choice over different types of technology or different prices of the input then it will, break that unique equilibrium that we get.

Because if you look at this case, so we have these four possible outcomes now, right? depending on different and either we can have this as a unique outcome, either we have this as a unique outcome or we can have these two as a multiple outcome, right? Because when this happens we will also have this, right? But either we can have 1 or we can have this 2, okay this case. So depending on the variety of the cost function and which we get from either production function or from the prices of inputs, okay. Thank you.