Introduction to Market Structures
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Module 7: Cournot Competition
Lecture 26: Cournot Oligopoly

Hello everyone. Welcome to my course Introduction to Market Structures. So we will initially do what we were doing in the last class that is Cournot duopoly and so we assume that there are two firms and the market demand is this- A-p=Q.

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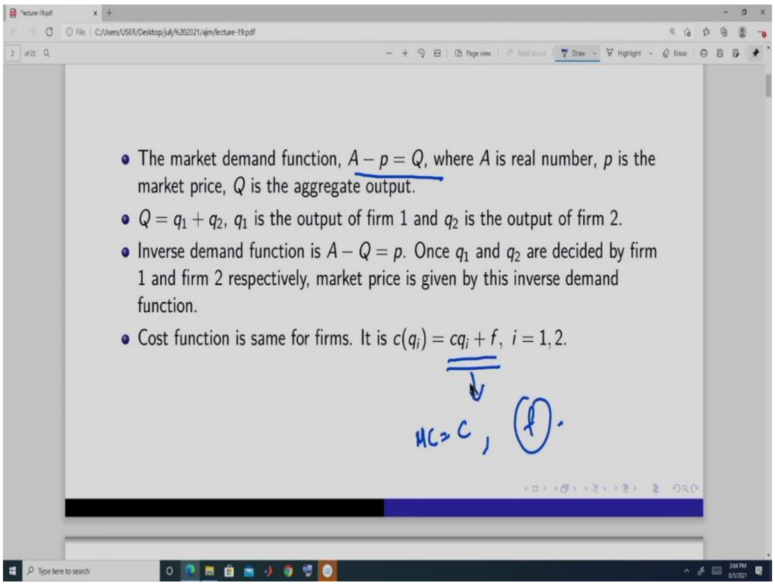

So cost function is of this form- $c(q_i) = cq_i + f, i = 1,2$. So we have assumed that the marginal cost is constant and so here marginal cost is c and we have a fixed cost which comes from here. So this kind of cost function we can see when we have either one factor is fixed and another factor is varying. Then we can have or we can vary most of the factors and suppose we have all the factors and we have paid some license fee so that is giving us this f fixed cost or suppose the rent that is we have paid in land so that may give this kind of fixed cost, okay.

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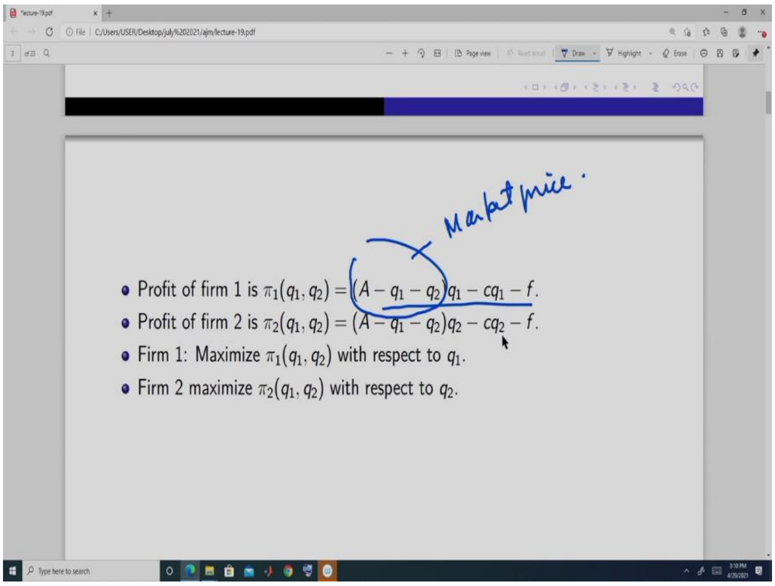

So profit of firm 1 is given by this function. It is this- $\pi_1(q_1, q_2) = (A - q_1 - q_2)q_1 - cq_1 - f$ where this portion$(A - q_1 - q_2)$ is the market price and market price into output of firm 1 is giving its the revenue of firm 1 and this is the total cost. So this is the profit function of firm 1 and similarly this is the profit function of firm 2- $\pi_2(q_1, q_2) = (A - q_1 - q_2)q_2 - cq_2 - f$. And firm 1 what it will do, it will maximize this profit with respect to q1 taking q2 as given and firm 2 will maximize this with respect to q2 taking q1 as given, okay.

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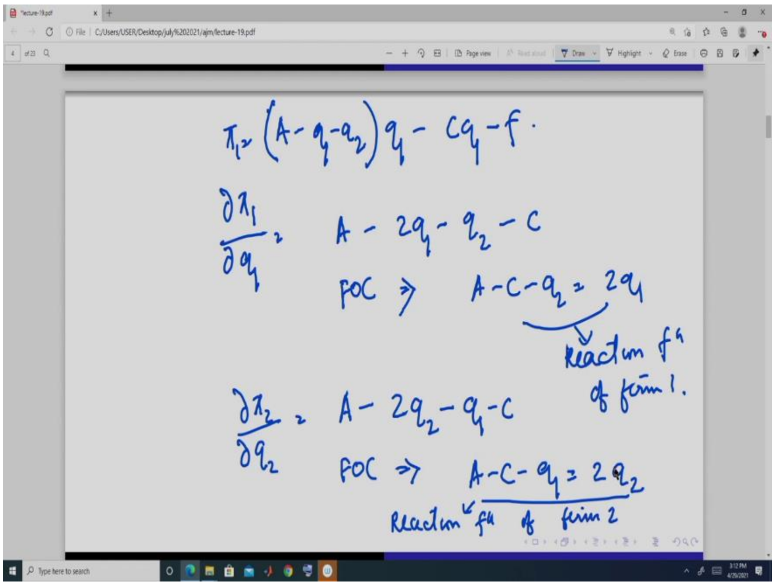

So for firm 1 we see since it is differentiable since the profit function if you look at the profit function it is this, right? and we have assumed that the cost function is same for 2 firms. So we get this is and First Order Condition implies-$\frac{d\pi_1}{dq_1} = A - 2q_1 - q_2 - c$. So this- $\mathrm{A} - \mathrm{q}_2 - \mathrm{c} = 2\mathrm{q}_1$ is the reaction function of firm one.

Similarly, we will find the reaction function of firm 2 and this will be equal to again the First Order Condition will imply- $\mathrm{A} - \mathrm{q}_1 - \mathrm{c} = 2\mathrm{q}_2$. So this is what reaction function of firm 2. So we have got these two reaction functions and we know that the pure-strategy Nash equilibrium is the point given by the intersection point of the reaction functions.

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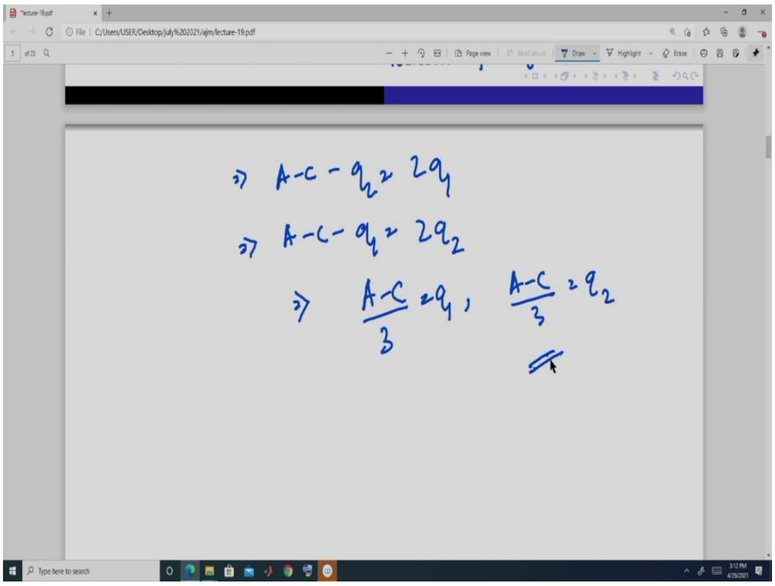

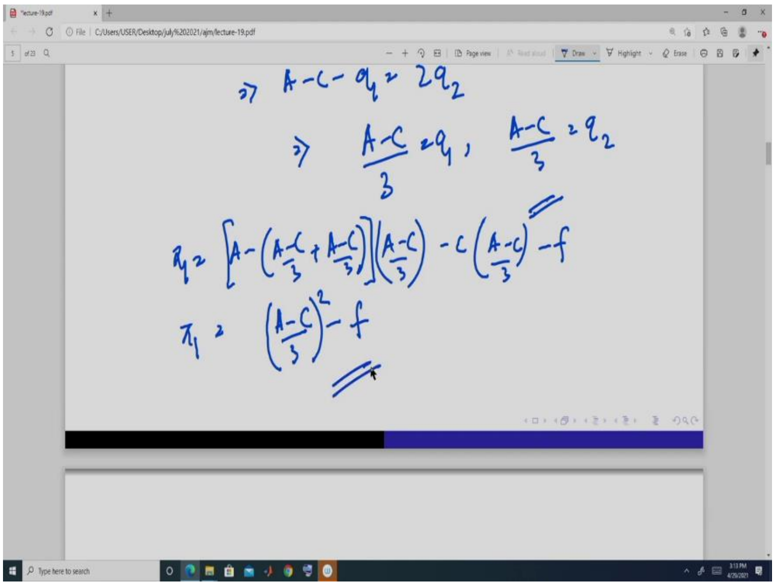

So here in this case the Nash equilibrium is given by solving these two equations- $\mathrm{A} - \mathrm{q}_2 - \mathrm{c} = 2\mathrm{q}_1, \mathrm{A} - \mathrm{q}_1 - \mathrm{c} = 2\mathrm{q}_2$, right? This and we get after solve this- $\frac{\mathrm{A-c}}{3} = \mathrm{q}_1, \frac{\mathrm{A-c}}{3} = \mathrm{q}_2$. So this is the Cournot outcome and when we plug in this in the profit function of firm 1, it is this pi 1 that is the profit of firm 1. It is going to be this- $\pi_1 = \left[A - \left(\frac{A-c}{3} + \frac{A-c}{3}\right)\right] . \left(\frac{A-c}{3}\right) - c\left(\frac{A-c}{3}\right) - f$ and if we, when we solve this we get. So this is the profit of firm 1 in the Cournot competition- $\pi_1 = \left(\frac{A-c}{3}\right)^2 - f$

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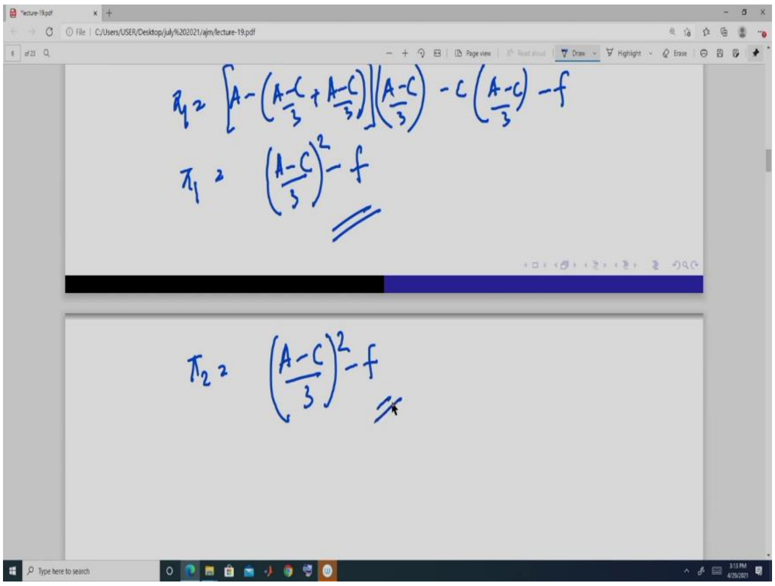

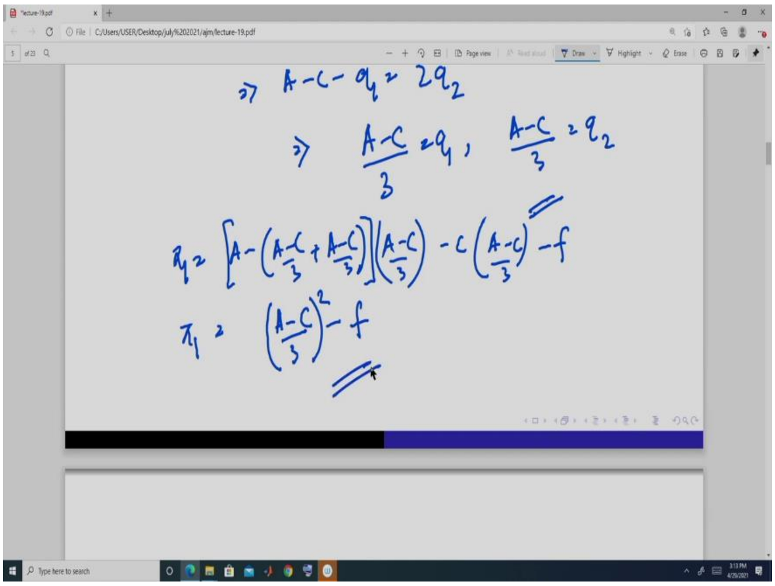

And similarly profit of firm 2 is-$\pi_2 = \left(\frac{A-c}{3}\right)^2 - f$, so whenever both the firms are deciding the output simultaneously and only once and then we get the Cournot outcome and the Cournot outcome outputs are this for firm 1, this is for firm 2. Profit for firm 1 is this and profit for firm 2 is this. So this is the Cournot model.

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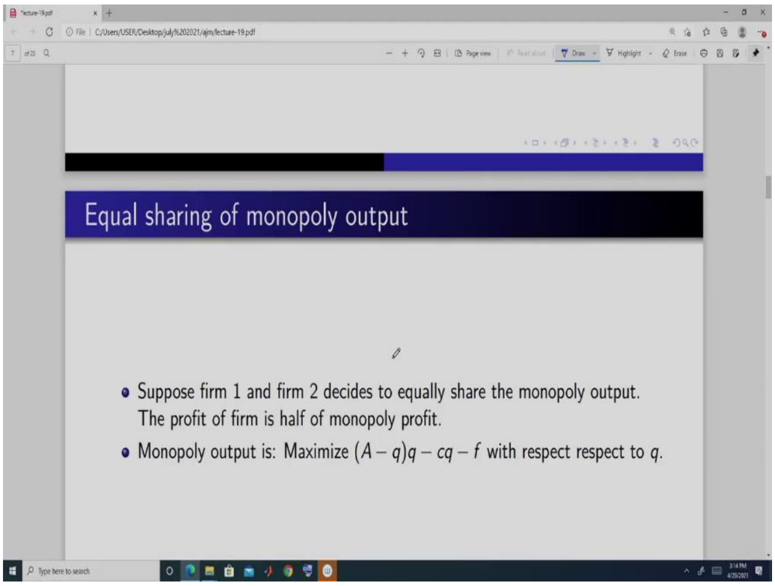

Now we ask the question can we improve upon this? So suppose instead of playing the Cournot thing what happens firm 1 and firm 2 they have a discussion or a negotiation and they say that let us produce half of monopoly output each and then we will sell that half of monopoly output, okay. Now the question is whether that is a Nash equilibrium outcome or not. To show that we will now do the following steps.

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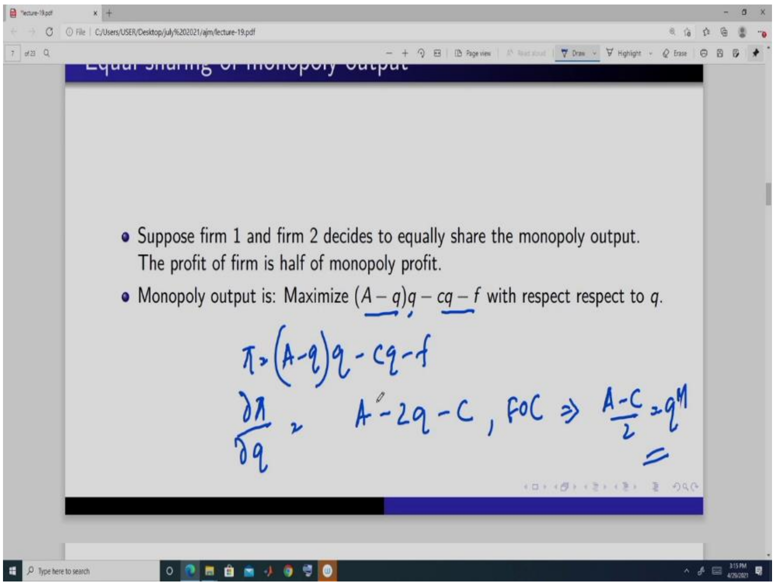

So first suppose each firm decides that we are going to produce half of monopoly. So how to find the monopoly output in this case. So this is market demand and this is the cost function of each firm- $(A-q)q-cq-f$ and this is the output of each firm. Since if there is a monopoly then that means only one firm so this output of one firm can be firm 1 or firm 2 that is giving you the market price A minus q is giving you the market price into q output of that single firm this. So profit is of a monopolist is simply this- $\pi = (A - q)q - cq - f$ and we maximize this with respect to so we get, so then first order condition gives us so this is the monopoly output- $\frac{A-c}{2} = q$

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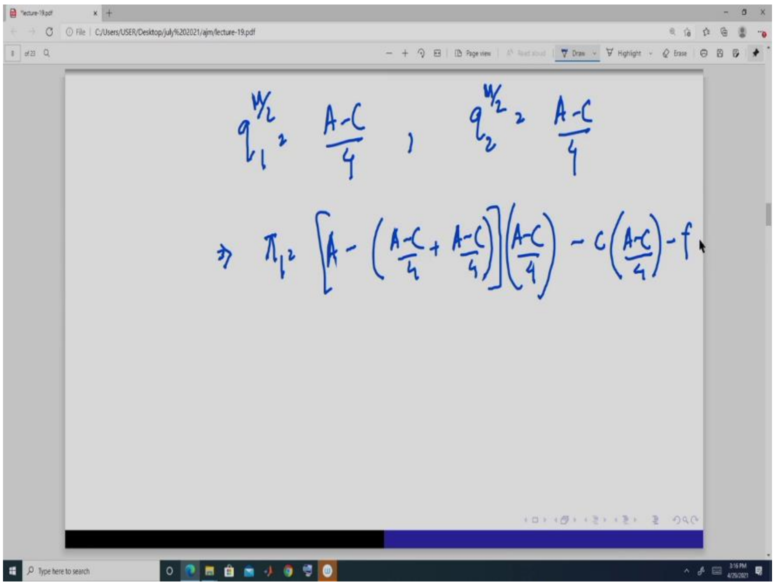

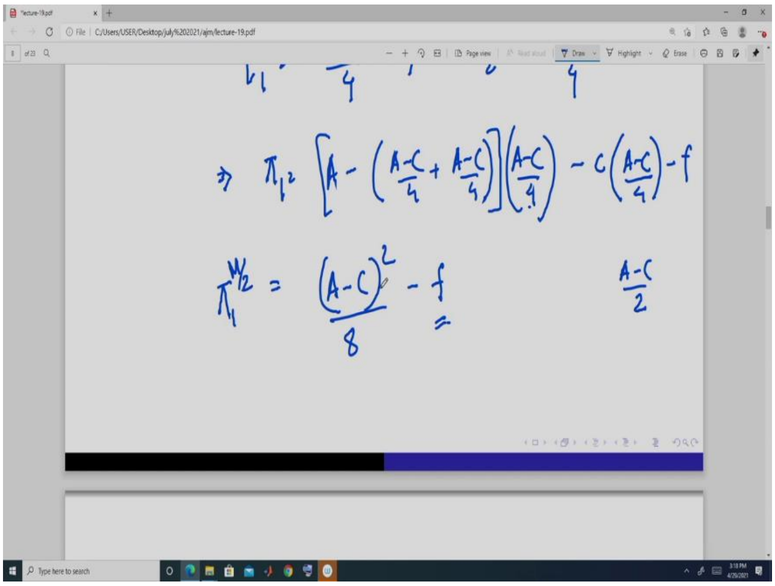

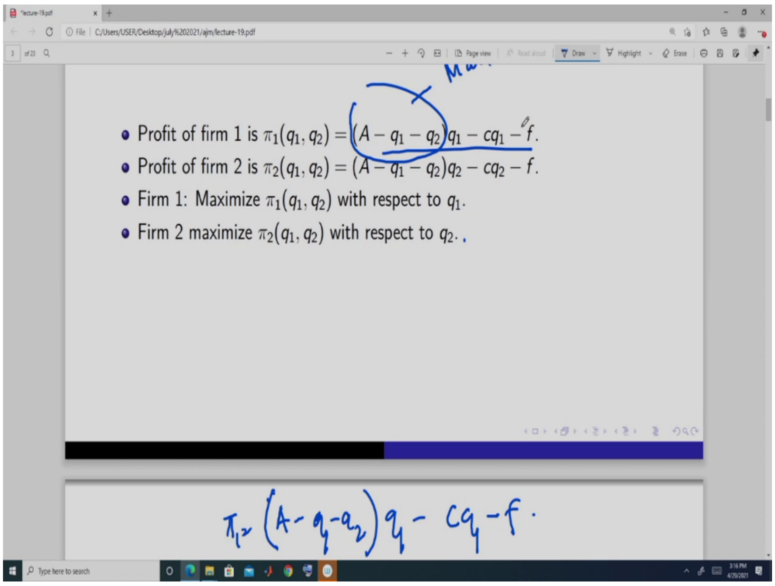

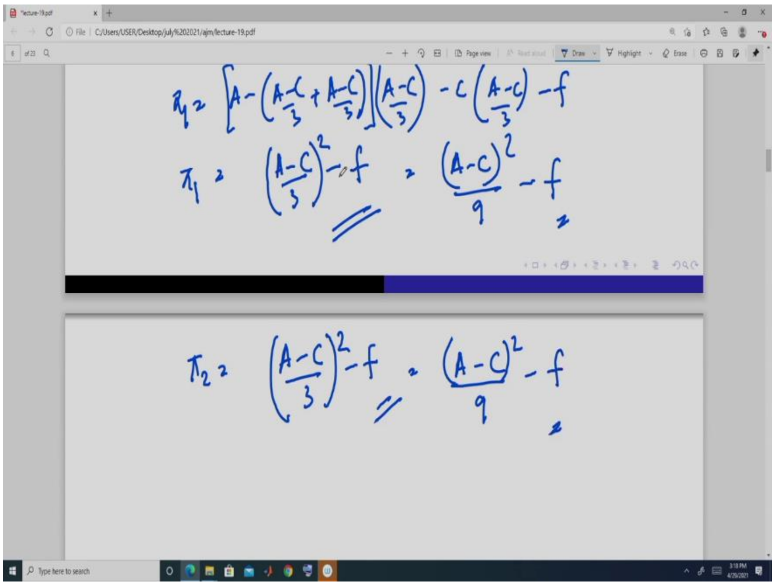

And so if each firm decides to produce half of monopoly output so firm 1 is going to produce this-$\frac{A-c}{4}$, firm 2 is going to produce this-$\frac{A-c}{4}$, right? Now let us look at the profit. So profit of firm 1 so this is f the fixed cost. This is the profit-$\pi_1 = \left[A - \left(\frac{A-c}{4} + \frac{A-c}{4}\right)\right].\left(\frac{A-c}{4}\right) - c\left(\frac{A-c}{4}\right) - f$. We know the profit function in the duopoly thing is given by this so we got similar kind of profit thing.

Now if we simply it, we will get, it will be this-$\left(\frac{A-c}{4}\right)^2 - f$. So each firm producing this, so the total output is going to be a monopoly output, this much- $\frac{A-c}{2}$. Now if you plug in here you will get, so this portion it will be this-$\frac{(A-c)^2}{8} - f$, okay. So this profit of firm 2 have this, it is easy to see that this is greater than this thing-$\left(\frac{A-c}{3}\right)^2 - f.$, because this is, right? so now the question is whether this- $\frac{(A-c)^2}{8} - f$ can be a possible outcome of some game and we will define the game slightly later on, okay. But we know that this is giving a higher profit, okay.

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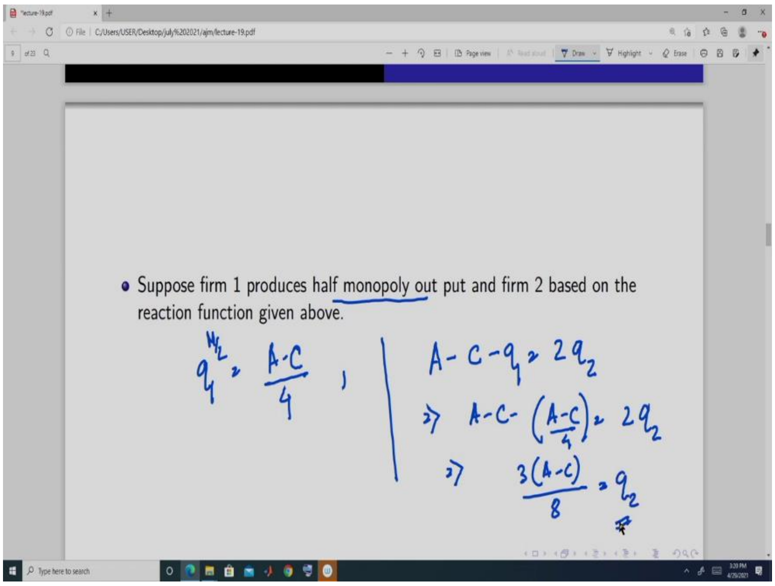

Now suppose we take another one. Firm 1 produces half monopoly output that is q1 is this-$\frac{A-c}{4}$ and firm 2 produces based on the reaction function of that we have got in the Cournot thing. So the reaction function of firm 2 is this- $A - q_1 - c = 2q_2$, so you plug in the output of firm 1 you will get the optimal output of firm 2 so here firm 1 produces this so the optimal output for firm 2 is this, this is the optimal output of firm 2- $\frac{3(A-c)}{8} = q_2$.

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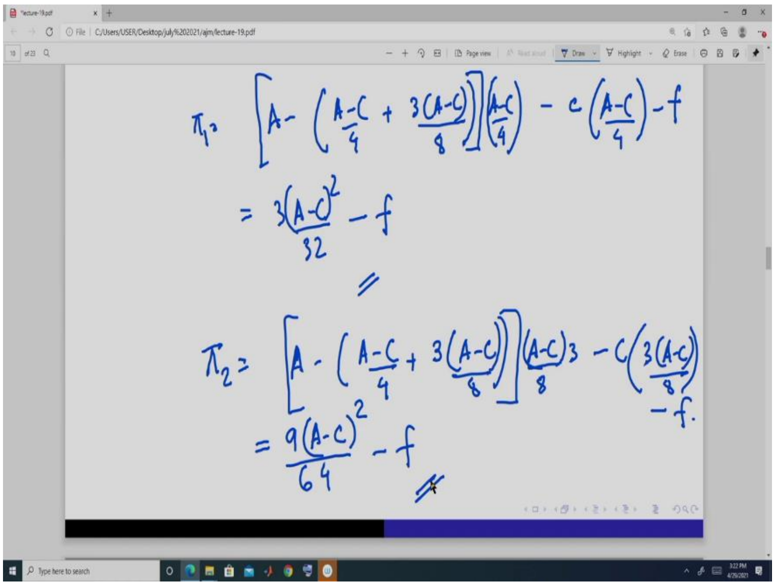

Now let us find out the profit in this case. So profit of firm 1 is again this, so this is going to be 5 by 8 and so this is so again this is going to be 32 minus f. This is the profit of firm 1-$\frac{3(\mathrm{A}-\mathrm{c})^2}{32} -$ f. And profit of firm 2, it is this, so this is equal to this-$\pi_2 = \frac{9(\mathrm{A}-\mathrm{c})^2}{64} -$ f. So this is the profit of firm 2, when firm 2 decides its output based on the Cournot reaction function. This is the Cournot reaction function and firm 1 decides that it is going to produce half of monopoly output.

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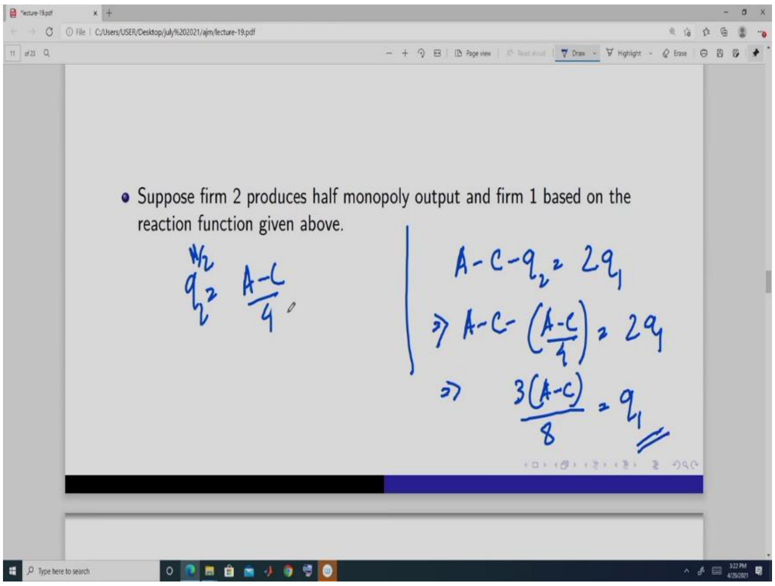

Now suppose firm 2 decides that it is going to produce half of monopoly output. So it is this-$\frac{\mathrm{A}-\mathrm{c}}{4}$ and firm 1 produces based on the Cournot reaction function. So firm 1's Cournot reaction function is this- $A - c - q_2 = 2q_1$, so you will get this as this- $A - c - \left(\frac{A-c}{4}\right) = 2q_1 \Rightarrow \frac{3(A-c)}{8} - q_1$.

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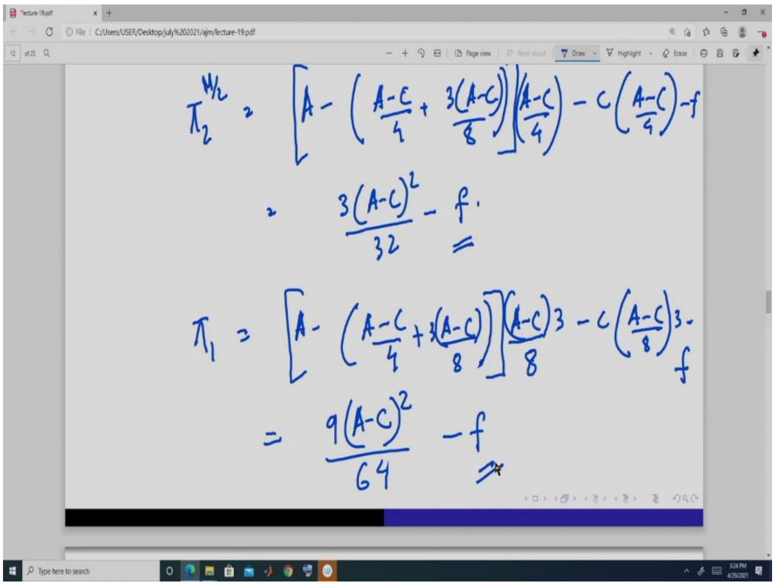

Now let us find out the profits. Profit of firm 2 when it produces monopoly output. Half of monopoly output is so market price is determined by the aggregate output so it is this. You get this- $\frac{3(A-c)^2}{32} - f$ and the profit of firm 1 when it produces based on the Cournot reaction function is, and this will give profit to be this- $\frac{9(A-c)^2}{64} - f$, right?

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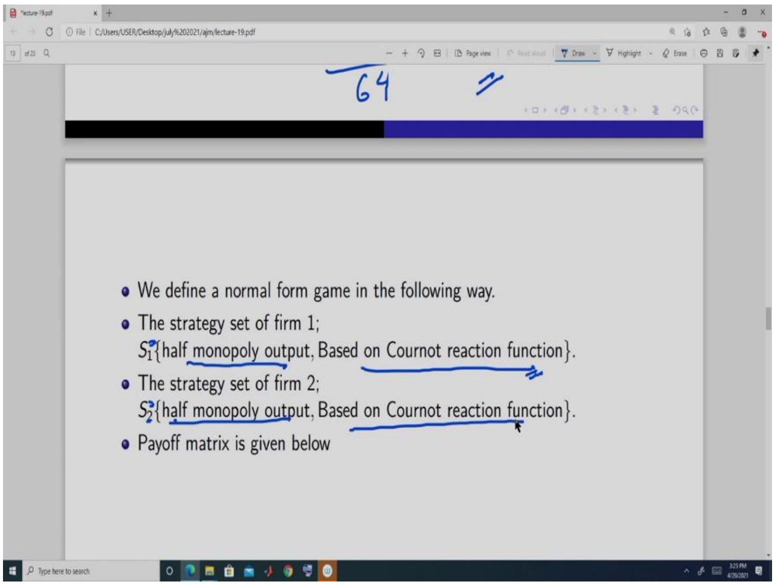

Now let us define a game. Game is a normal form game where it is played only once, and, okay. So this should be S21, S2 played once and it is played simultaneously between firm 1 and firm 2. Strategy of firm 1 it is either choose half monopoly output or it can choose based on Cournot reaction function. If both the firms chooses Cournot reaction function, we get the Cournot outcome. If both the firm chooses half monopoly then we get the half monopoly outcome. And reaction strategy set of firm 2 is half monopoly or based on reaction Cournot reaction function, okay.

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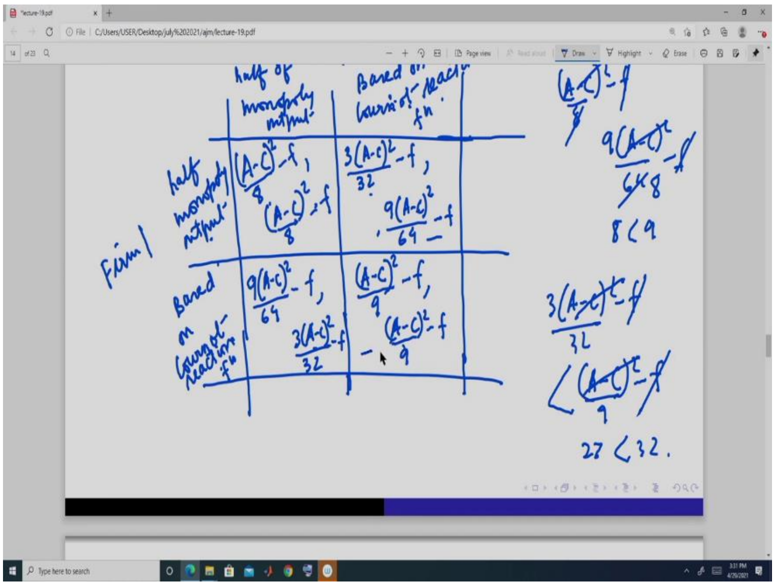

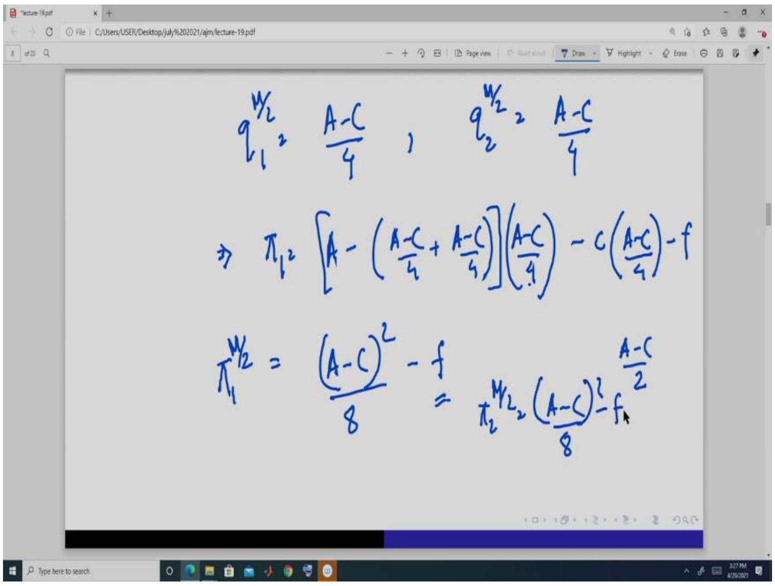

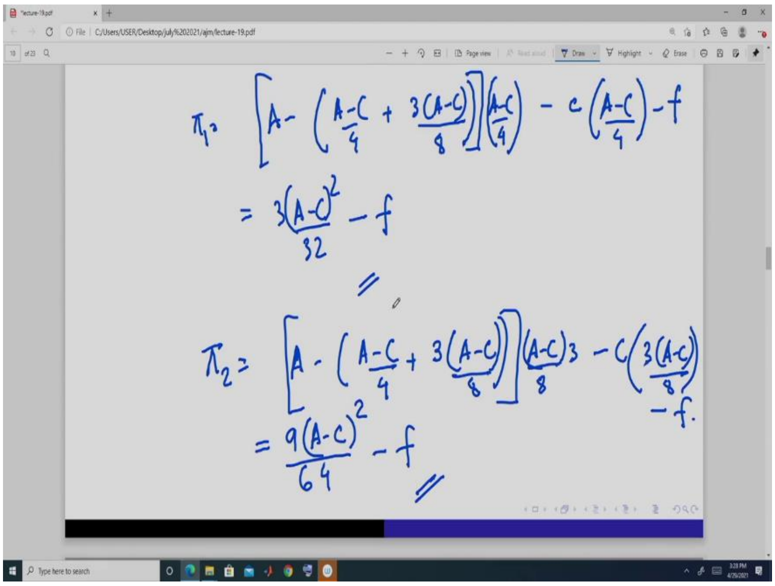

Now how do we define the payoff matrix, payoff matrix is given in this way. So let us make this block a slightly bigger one so this is for firm 2. Firm 2 is a column player; firm 1 is a row player and this is half monopoly output and this is based on Cournot reaction function. Similarly, this is half of monopoly output, so these are the strategies, okay. And this is based on Cournot reaction function. If no half monopoly half monopoly this is both this payoff in this block is both the firms are playing half monopoly output.

So if both are playing half monopoly we know the profits are this, this-$\frac{(A-c)^2}{8} - f$ for both the firms right. So since it is profit of A is this so profit of firm 2 is also going to be same because the cost function is same, so payoff are this. So in this normal firm game we write the payoffs. This is the payoff of firm 1-$\frac{(A-c)^2}{8} - f$, okay, this is the payoff of firm 2-$\frac{(A-c)^2}{8} - f$. Then firm 1 produces half monopoly output and firm 2 produces based on the Cournot reaction function. This case, payoff of firm 1 is this, and payoff of firm 2 is this.

And payoff of this similarly this is firm 2 produces, firm 1 produces based on Cournot reaction function and firm 2 produces half monopoly profit so this is this-$\frac{9(A-c)^2}{64} - f$ and this is the Cournot outcome and the Cournot outcome is we know firm 1, firm 2-$\frac{(A-c)^2}{9} - f$. Now we have to find the pure-strategy Nash equilibrium of this game.

Suppose this firm 1 plays half monopoly output, then firm 2 will decide this and this. So from here we know if we compare this, this cancels 8, this cancels, so 8 is less than 9 so we get that for firm 2 this is greater than this. So if firm 1 produces half monopoly output, firm 2 is going to produce based on the Cournot reaction function, not this-$\frac{(A-c)^2}{8} - f$, okay**.**

Now suppose firm 2 produces based on Cournot reaction function. So whether firm one is going to choose this, half monopoly output or based on Cournot reaction function. So we have to compare this payoff- $\frac{3(A-c)^2}{32} - f$ with this payoff-$\frac{(A-c)^2}{9} - f$, right? Now here it is easy to see that this is greater than this because this cancels out this, cancels out and 27 is less than 32.

So when firm 2 plays best, chooses output based on Cournot reaction function, this. Firm 1 also chooses output based on Cournot reaction function. Now we have to choose whether when firm 2, firm 1 chooses based on Cournot reaction function, whether firm 2 chooses half monopoly or this. Now if we compare this and this, we know this is so it will choose this.

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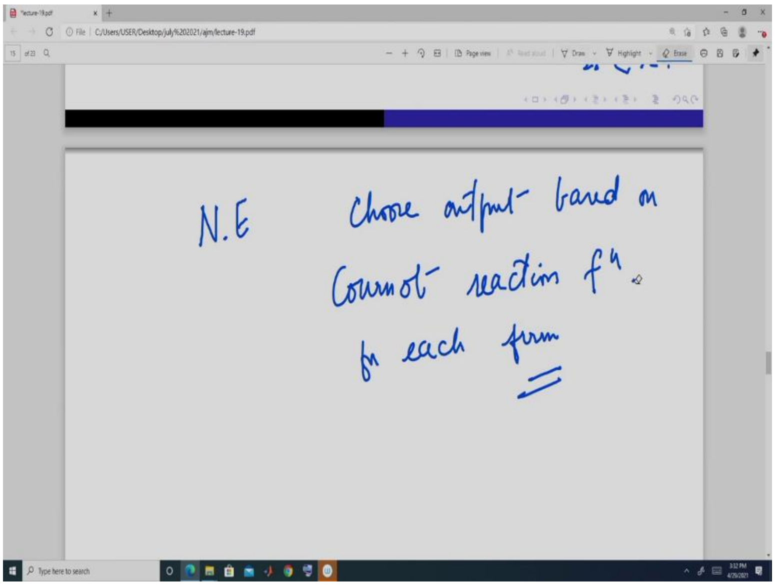

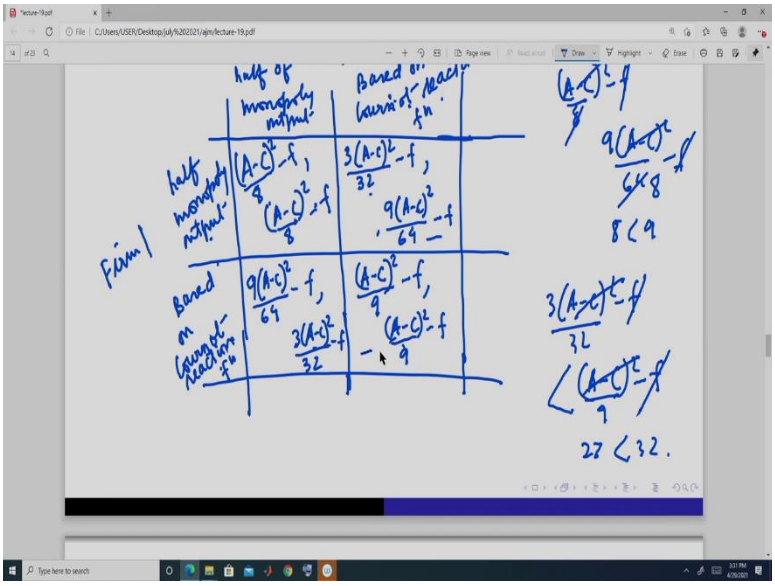

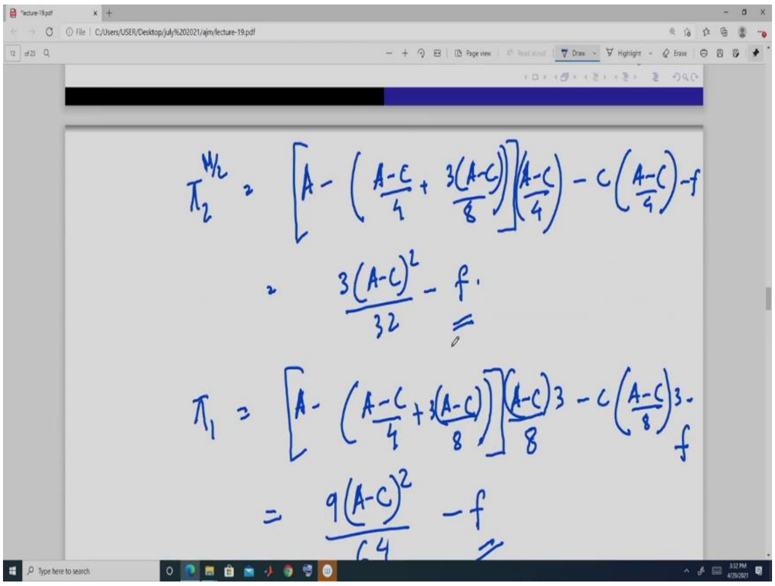

So the outcome is, so the Nash equilibrium, Nash equilibrium is to choose output based on Cournot reaction function for each firm. So this if you compare here this, this- $\frac{(A-c)^2}{8} - f$ is a better outcome for each firm than this outcome-$\frac{(A-c)^2}{9} - f$. But still firms choose this. Why? Because choosing output based on Cournot reaction here it is a Nash equilibrium and also it is a dominant strategy if you look at it, okay. So because of this reason we do not get the Cournot outcome. We do not get the sharing of monopoly market between the two firms. Instead they are going to play the Cournot game, okay.

So this is an important result. In a Cournot competition we will see that the firms if they collude or if they form any cartel then what they are going to do, the maximum profit is the monopoly profit in that market. So they are going to share the monopoly profit but that is not possible in this case if the game is played only once between 2 firms, okay.

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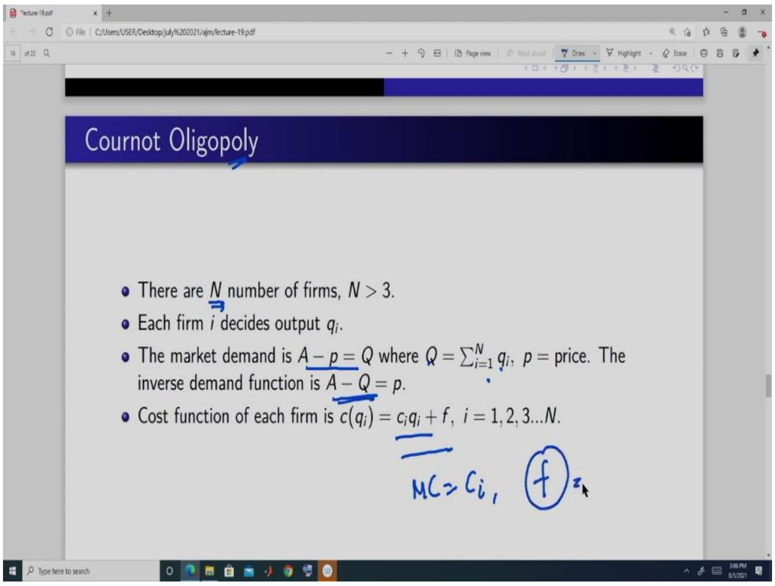

Now we will extend this model into n firms and that is why it is oligopoly. Oligopoly means that there are many firms. So there are n firms. So n is some positive integer, which is greater than 3, okay. Market demand we have kept same it is this-A-p=Q and so this capital Q is sum of all the outputs of each n firm each firm and there are n firms this p is the market price and the inverse demand function is this- A-Q=p, so cost function is of this firm is of this form- $c(q_i) = c_i q_i + f, i = 1,2,3..N$.

So it means that the marginal cost is Ci, i denote for each firm and it is constant for each firm and we have a fixed cost and fixed cost is given by this f. So we can see this kind of cost function when one of the factor is fixed and other factor is variable.

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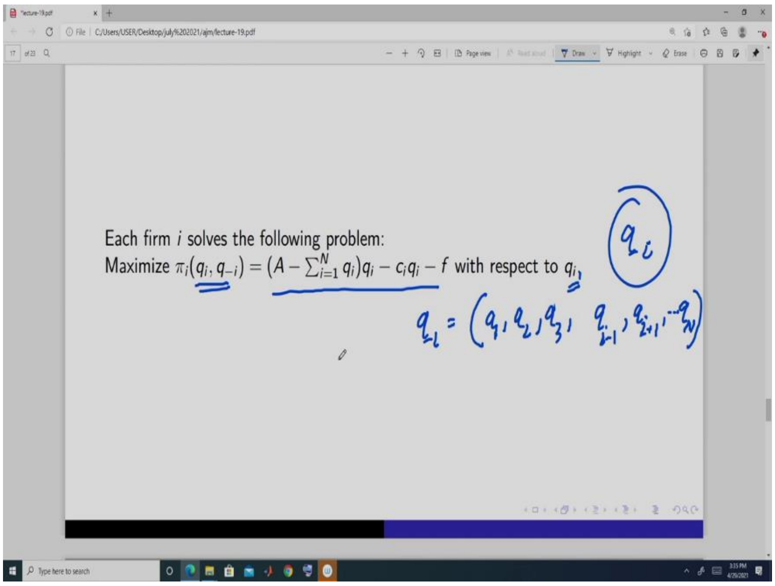

So now what each firm is going to do. The profit here when we write the profit function as a function of firm i as output of firm i and this. This q minus i this means actually this-$q_{-i} = (q_1, q_2, q_3, q_{i-1}, q_{i+1} \ldots)$, i plus 1 this whole vector is denoted by this q. So this is a short firm notation that is used in game theory, okay. So the profit of firm A is this- $\pi_i(q_i, q_{-i}) = (A - \Sigma_{i=1}^{N} q_i)q_i - c_i q_i - f$ and firm i is going to maximize this profit with respect to its output taking output of all the firms as given, okay, taking q this as given. So since it is a differentiable profit function, so we can solve it in this way.

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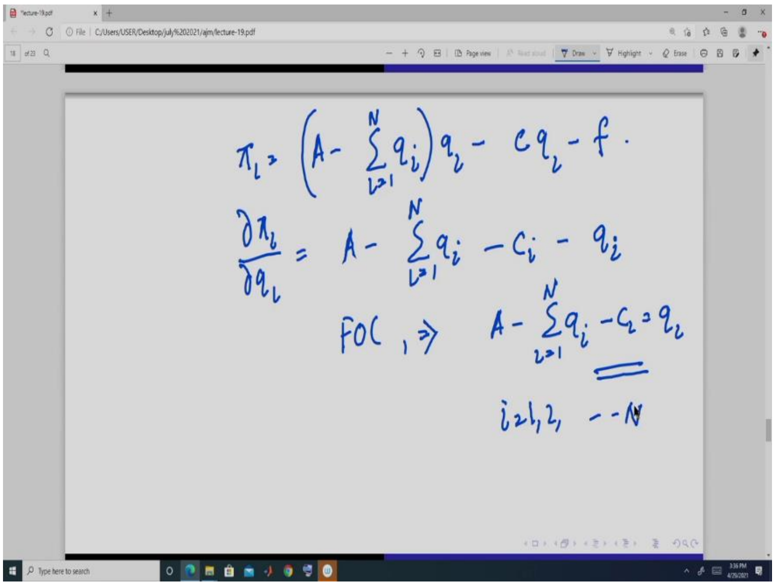

Then here if we differentiate this with respect to qI, we will this is so we will get qi so this is qi then the first order condition this will imply- $\mathrm{A} - \Sigma_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{q}_{\mathrm{i}} - \mathrm{c}_2 = \mathrm{q}_2$. Now we will have such for each i, i is equal to 1 to 2 and so we will get this.

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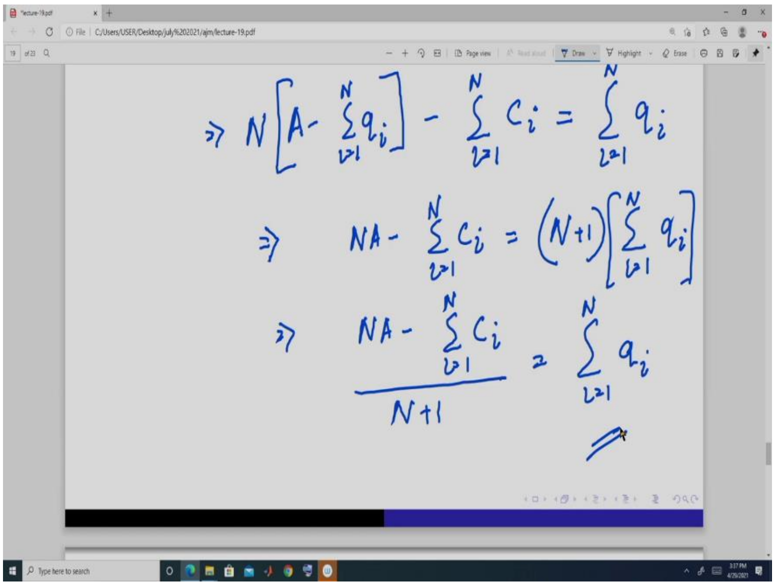

So if we sum this, so the summation, we can write so since there are n equations so this part is going to be same for each equation. So this marginal cost is going to be different for each firm and we will take the sum of this and here RHS is also going to be the sum of all the outputs like this. So from this we get-$N\left[A - \Sigma_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{q}_{\mathrm{i}}\right] - \Sigma_{\mathrm{i}=1}^{\mathrm{N}} c_i = \Sigma_{\mathrm{i}=1}^{\mathrm{N}} q_i => NA - \Sigma_{\mathrm{i}=1}^{\mathrm{N}} c_i = (N + 1)[\Sigma_{\mathrm{i}=1}^{\mathrm{N}} q_i]$, so the aggregate output in the market is going to be, aggregate output is this in the market- $\frac{\mathrm{NA} - \Sigma_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{c}_{\mathrm{i}}}{\mathrm{N}+1} = \Sigma_{\mathrm{i}=1}^{\mathrm{N}} q_i$.

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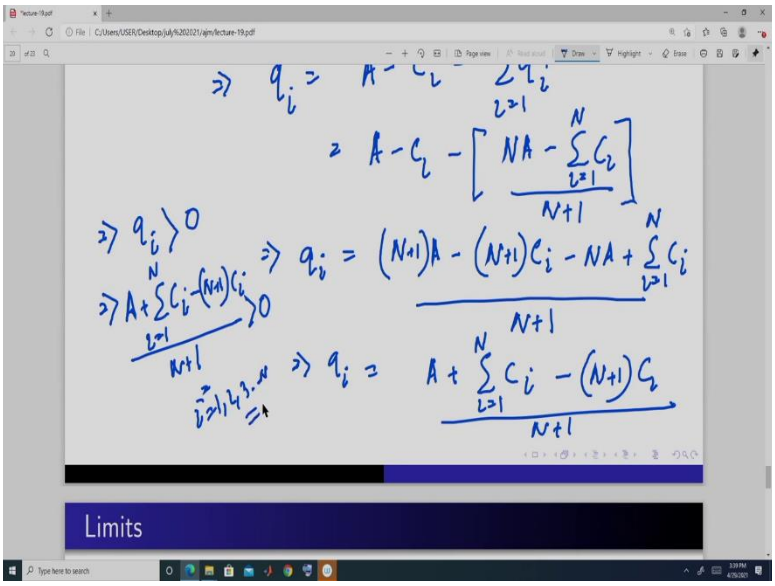

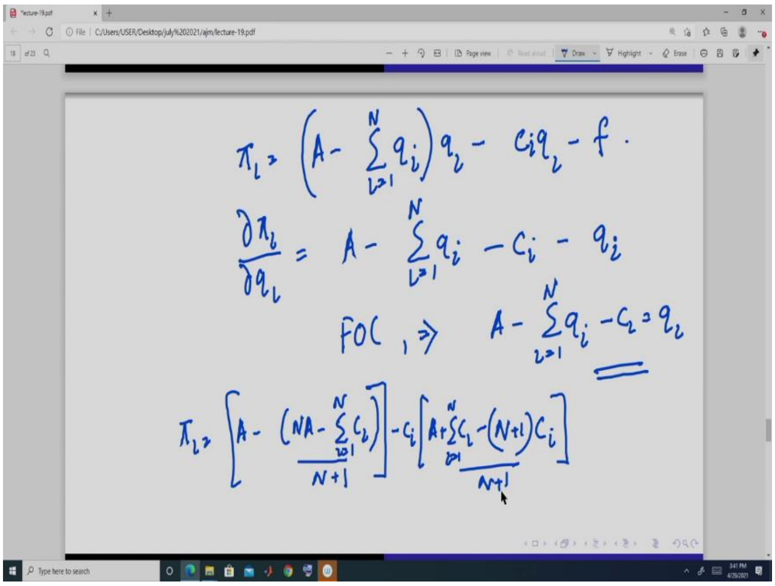

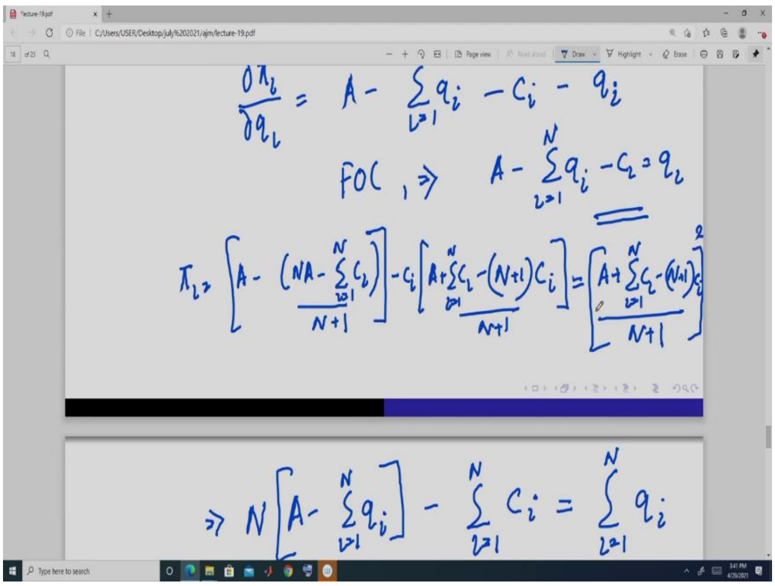

Now so the output of each firm qi you can simply get it, it is this- $q_i = A - c_i - \Sigma_{i=1}^{N} q_i$ from the reaction function of each firm and we get. It is this- $A - c_i - \left[\frac{NA - \Sigma_{i=1}^{N} c_i}{N+1}\right]$ so this is going to be, it is going to be this one-$q_i = \frac{(N+1)A - (N+1)c_i - NA + \Sigma_{i=1}^{N} c_i}{N+1}$. So q1 is actually A plus sum of all the marginal cost of all the firms and plus 1 into Ci marginal cost of firm i divided by, so this is the output of each firm- $q_i = \frac{A + \Sigma_{i=1}^{N} c_i - (N+1)c_i}{N+1}$, right? We get this as the output. For this to be a solution, we put the condition that this should be greater than A, so this means that should be greater than 0, okay. So for i equal to 1 for each firm i.

Now we plug in this in this profit function. This profit, this profit function- $\pi_1 = \left(A - \Sigma_{i=1}^{N} q_i\right) q_2 - cq_2 - f$ and then we will give the Cournot profit and if we do it here you will see this is and if you do the simple calculation, this is going to be. it is going to be this one $\left[\frac{A + \Sigma_{i=1}^{N} c_i - (N+1)c_i}{N+1}\right]^2$, okay.

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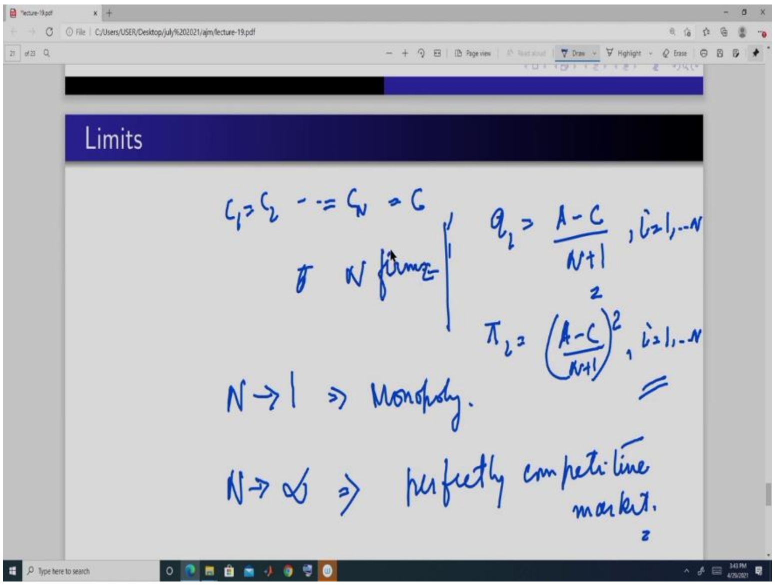

So and now here if you take the C to be common, same for each firm marginal cost is same that is C n is equal to this then qi is this- $\frac{A-c}{N+1}$, okay. And profit of so i equal to 1 to N, profit is this- $\pi_2 = \left(\frac{A-c}{N+1}\right)^2$, right? Now here we look at the limit now. What do we mean by limit? So when this n tends to 1 then it is implies monopoly and when n tends to infinity then it implies that we have perfectly competitive market. You can see this. So this is the, so we have assumed that the cost is same and our output of each firm is this when there are n firms. This is when we have n firms, right?

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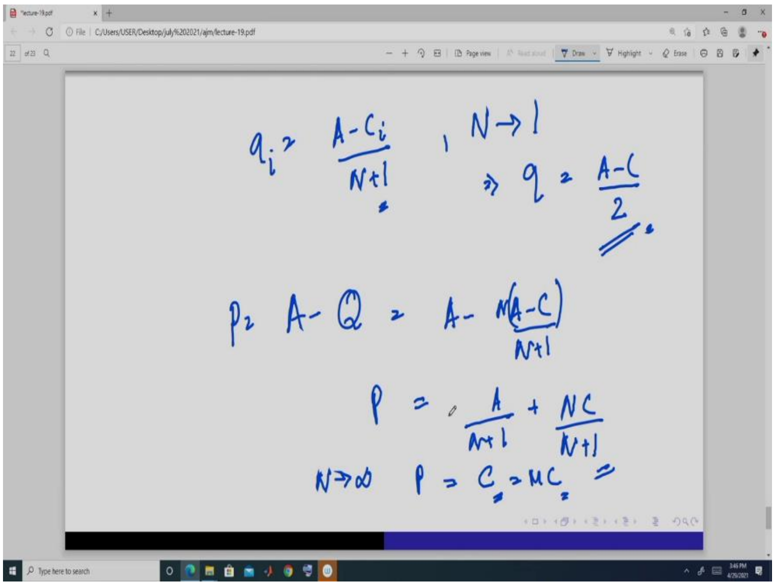

Equal sharing of monopoly output

- Suppose firm 1 and firm 2 decides to equally share the monopoly output. The profit of firm is half of monopoly profit.
- Monopoly output is: Maximize $(A-q)q - cq - f$ with respect respect to q.

$$\pi = (A-q)q - cq - f$$

$$\frac{\partial \pi}{\partial q} = A - 2q - c \,, \quad FOC \Rightarrow \frac{A-c}{2} = q^M$$

$$\Rightarrow q_i = A - c_i - \sum_{i=1} q_i$$

$$= A - c_i - \left[\frac{NA - \sum_{i=1}^{N} c_i}{N+1} \right]$$

$$\Rightarrow q_i = \frac{(N+1)A - (N+1)c_i - NA + \sum_{i=1}^{N} c_i}{N+1}$$

$$\Rightarrow q_i = \frac{A + \sum_{i=1}^{N} c_i - (N+1)c_i}{N+1}$$

$$\Rightarrow q_i > 0$$

$$\Rightarrow \frac{A + \sum_{i=1}^{N} c_i - (N+1)c_i}{N+1} > 0 \,, \quad i = 1, 2, 3 \ldots N$$

Limits

Limits

$$c_1 = c_2 = \cdots = c_N = c$$

$\#$ N firms

$$q_i = \frac{A-c}{N+1} \,, \quad i = 1, \ldots N$$

$$\pi_i = \left(\frac{A-c}{N+1}\right)^2 , \quad i = 1, \ldots N$$

$N \to 1 \Rightarrow$ Monopoly.

$N \to \infty \Rightarrow$ perfectly competitive market.

So it is what? this N tends to 1, so this q is and this we know we have found the monopoly. We have already calculated the monopoly profit and the monopoly profit is given like this and it is this monopoly output is this. So here again we have got the monopoly profit and that is this- $q = \frac{A-c}{2}$.

So we know the Cournot, from Cournot we can derive the monopoly output so in a Cournot market there are n firms, Cournot oligopoly market and there we have found the Cournot outcome. Cournot outcome is this. Each output of each firm is this. Profit we have calculated it and then when we take the limit on n and the limit is that suppose from n firm we reduce it to one so they are only one firm then the outcome is same as the monopoly output and the profit is also going to be the same of monopoly profit.

Next what is the price in this Cournot thing, this, it is going to be this one- $P = A - \frac{N(A-c)}{N+1}$ because output of each firm is this- $\frac{A-c}{N+1}$ and there are n firms so it is n into this. Cost is same. So it is going to be this. So this is what, this is, this price is this in the Cournot oligopoly when the cost is same we get this as the market price- $P = \frac{A}{N+1} + \frac{NC}{N+1}$.

Now here when you take N tends to infinity this price you see is going to be equal to this which is equal to marginal cost and we know the when moment of price is equal to marginal cost then the pricing is same as the perfectly competitive pricing.

So in this case when the firms decide its output but there are infinite number of firms then it is same as perfectly competitive. So you can say the Cournot thing is in between monopoly and a perfectly competitive and the main strength of this result is this that at the limit we get monopoly that is when there is one firm and when there are many firms, infinitely many then the price is same as the marginal cost. So it is outcome is same as competitive market.

So when each firm is deciding its output taking the output of other firm as given we know in that case it is the Cournot market. In that case when the number of firms are very large then this same outcome tends to competitive pricing or the outcome is same as the competitive market. So this is the importance of Cournot market. So Cournot market is you can say from there it is lying in between monopoly and the competitive market.

And we know that in reality competitive market hardly we see. In the competitive market mainly the firms are price taker so and there is a homogenous output and there is free entry and exit of firms. So these characteristics are perfectly competitive market, it is very difficult to see in the real life.

Monopoly and 100 percent monopoly is also very difficult to see but what do we see in between these two and that is Cournot kind of thing where each firm if they are producing homogenous output they are choosing their own output given taking the output of other firm as given, okay.

So this is the end of Cournot competition, Cournot model, so we have done Cournot duopoly where we have taken 2 firms and then we have form the pure-strategy Nash equilibrium. Then we have shown that there is a possibility of a better outcome where they can share the equally share the monopoly market but monopoly output but instead of that firm chooses to play the Cournot kind of competition, okay. That we have shown.

We have shown in the last class also and today also we have shown it in detail and then we have derived the Cournot oligopoly result taking CRS production function so that the cost function has a variable component and a fixed component.

And we have assumed a linear demand curve and based on that we have got the market outcome and market outcome is given by this and this is the pure-strategy Nash equilibrium of this game. Here we have to assume this condition so that the output is positive for each firm, okay. And from this Cournot oligopoly outcome we have derived the monopoly thing taking the limit when n tends to 1 and the perfectly competitive outcome when n tends to infinity, okay. So this is what we require in a oligopoly, it is not oligopoly in a Cournot oligopoly, okay.

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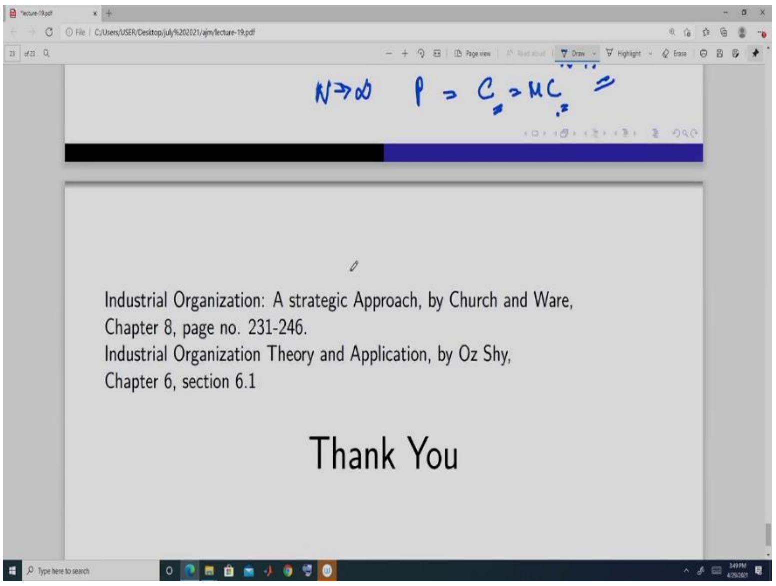

So you can read this portion from chapter 8 of Church and Ware Industrial Organization: A Strategic Approach. These are the page numbers. Also you can read from another book Industrial Organization Theory and Application by Oz Shy and this is chapter 6, section 6.1, okay. Thank you very much.